# L4
# Relational Algebra

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## Reading

Ramakrishnan
  Sections 4.1 and 4.2

Helpful References
  https://en.wikipedia.org/wiki/Relational_algebra

## Overview

Last time, learned about
- pre-relational models
- an informal introduction to relational model
- an introduction to the SQL query language.

Learn about formal relational query languages
- Relational Algebra (algebra: perform operations)
- Relational Calculus (logic: are statements true?)

Keys to understanding SQL and query processing

## Who Cares?

Clean query semantics & rich program analysis
Helps/enables optimization
Opens up rich set of topics
- Materialized views
- Data lineage/provenance
- Query by example
- Distributed query execution
- …

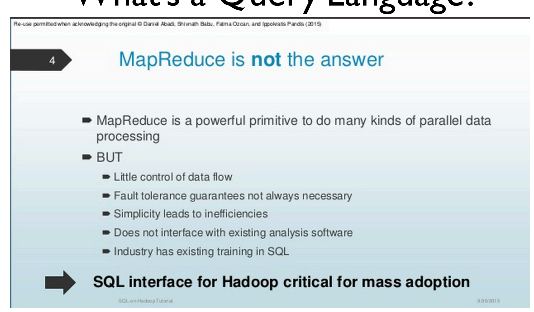
## What's a Query Language?

Allows manipulation and retrieval of data from a database.

Traditionally: QL != programming language
- Doesn't need to be turing complete
- Not designed for computation
- Supports easy, efficient access to large databases

Recent Years
- Scaling to large datasets is a reality
- Query languages are a powerful way to
  - think about data algorithms scale
  - think about asynchronous/parallel programming

## What's a Query Language?




Tutorial at VLDB 2015 Abadi et al.
http://www.slideshare.net/abadid/sqlonhadoop-tutorial

## Formal Relational Query Languages

Formal basis for real languages e.g., SQL

Relational Algebra
  Operational, used to represent execution plans

Relational Calculus
  Logical, describes what data users want
  (not operational, fully declarative)

## Prelims

Query is a function over relation instances

$Q(R_1,\ldots,R_n) = R_{result}$

Schemas of input and output relations are *fixed* and well defined by the query Q.

Positional vs Named field notation
  Position easier for formal defs
    one-indexed (not 0-indexed!!!)
  Named more readable
  Both used in SQL

## Prelims

Relation (for this lecture)
  Instance is a set of tuples
  Schema defines field names and types (domains)
  Students(<u>sid</u> int, name text, major text, gpa int)

How are relations different than generic sets ($\mathbb{R}$)?
  Can assume item structure due to schema
  Some algebra operations (x) need to be modified
  Will use this later

## Relational Algebra Overview

Core 5 operations
  PROJECT ($\pi$)
  SELECT ($\sigma$)
  UNION (U)
  SET DIFFERENCE (-)
  CROSSPRODUCT (x)

Additional operations
  RENAME (p)
  INTERSECT ($\cap$)
  JOIN ($\bowtie$)
  DIVIDE (/)

## Instances Used Today: Library

Students, Reservations

Use positional or named field notation

Fields in query results are inherited from input relations (unless specified)

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

## Project

$\pi_{<attr1,\ldots>}(A) = R_{result}$

Pick out desired attributes (subset of columns)
Schema is subset of input schema in the projection list
  $\pi_{<a,b,c>}(A)$ has output schema (a,b,c) w/ types carried over

## Project

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$\pi_{name,age}(S2) =$

| name | age |
|---|---|
| aziz | 21 |
| barak | 21 |
| trump | 88 |
| rusty | 21 |

## Project

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$\pi_{age}(S2) =$

| age |
|---|
| 21 |
| 88 |

Where did all the rows go?
Real systems typically don't remove duplicates. Why?

## Select

$\sigma_{<p>}(A) = R_{result}$

Select subset of rows that satisfy condition *p*
Won't have duplicates in result. Why?
Result schema same as input

## Select

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

$\sigma_{age<30}\ (S1) =$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |

$\pi_{name}(\sigma_{age<30}\ (S1)) =$

| name |
|---|
| eugene |
| barak |

## Commutatively

$$A + B = B + A$$
$$A * B = B * A$$
$$A + (B * C) = (B * C) + A$$

$$A + (B + C) = (A + B) + C$$
$$A + (B * C) = (A + B) * C$$

## Commutatively

$$A + B = B + A$$
$$A * B = B * A$$
$$A + (B * C) = (B * C) + A$$

$$A + (B + C) = (A + B) + C$$
$$\cancel{A + (B * C) = (A + B) * C}$$

## Commutatively

$\pi_{age}(\sigma_{age<30}(S1))$

$\sigma_{age<30}$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |

## Commutatively

$\pi_{age}(\sigma_{age<30}(S1))$

$\sigma_{age<30}$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

$\pi_{age}$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |

=

| age |
|---|
| 20 |
| 21 |

## Commutatively

$\sigma_{age<30}(\pi_{age}(S1))$

$\pi_{age}$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

| age |
|---|
| 20 |
| 21 |
| 88 |

## Commutatively

$\sigma_{age<30}(\pi_{age}(S1))$

$\pi_{age}$

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

$\sigma_{age<30}$

| age |
|---|
| 20 |
| 21 |
| 88 |

=

| age |
|---|
| 20 |
| 21 |

## Commutatively

Does Project and Select commute?

$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$

What about

$\pi_{name}(\sigma_{age<30}(S1))$?

## Commutatively

Does Project and Select commute?

$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$

What about

$\pi_{name}(\sigma_{age<30}(S1)) \mathrel{!=} \sigma_{age<30}(\pi_{name}(S1))$

## Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) \mathrel{!=} \sigma_{age<30}(\pi_{name,\ age}(S1))$$

## Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30}(S1)) = \sigma_{age<30}(\pi_{age}(S1))$$

What about

$$\pi_{name}(\sigma_{age<30}(S1)) = \pi_{name}(\sigma_{age<30}(\pi_{name,\ age}(S1)))$$

OK!

## Union, Set-Difference

A op B = $R_{result}$

A, B must be *union-compatible*

- Same number of fields
- Field i in each schema have same type

Result Schema borrowed from first arg (A)

A(big int, poppa int) U B(thug int, life int) = ?

## Union, Set-Difference

A op B = $R_{result}$

A, B must be *union-compatible*

- Same number of fields
- Field i in each schema have same type

Result Schema borrowed from first arg (A)

A(big int, poppa int) U B(thug int, life int) = $R_{result}$(big int, poppa int)

## Union, Intersect, Set-Difference

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

S1US2 =

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 4 | aziz | 3.2 | 21 |
| 5 | rusty | 3.5 | 21 |
| 3 | trump | 2 | 88 |
| 2 | barak | 3 | 21 |

## Union, Intersect, Set-Difference

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

S1 - S2 =

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |

## Note on Set Difference & Performance

Notice that most operators are monotonic

- increasing size of inputs → outputs grow
- if $A \supseteq B \rightarrow Q(A,T) \supseteq Q(B,T)$
- can compute *incrementally*

Set Difference is *not monotonic*

- if $A \supseteq B \rightarrow T - A \subseteq T - B$
- e.g., $5 > 1 \rightarrow 9 - 5 \subseteq 9 - 1$

Set difference is *blocking*:

- For T – S, must wait for all S tuples before any results

## Cross-Product

$$A(a_1,\ldots,a_n) \times B(a_{n+1},\ldots,a_m) = R_{result}(a_1,\ldots,a_m)$$

Each row of A paired with each row of B

- Result schema concats A and B's fields, inherit if possible
- Conflict: students and reservations have *sid* field

Different than mathematical "X" by flattening results:

- math $A \times B = \{ (a, b) \mid a \in A \wedge b \in B \}$
- e.g., $\{1, 2\} \times \{3, 4\} = \{ (1, 3), (1, 4), (2, 3), (2, 4) \}$
- what is $\{1, 2\} \times \{3, 4\} \times \{5, 6\}$?

## Cross-Product

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

S1x R1 =

| (sid) | name | gpa | age | (sid) | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 2 | barak | 3 | 21 | 1 | 101 | 10/10 |
| 3 | trump | 2 | 88 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barak | 3 | 21 | 2 | 102 | 11/11 |
| 3 | trump | 2 | 88 | 2 | 102 | 11/11 |

## Rename

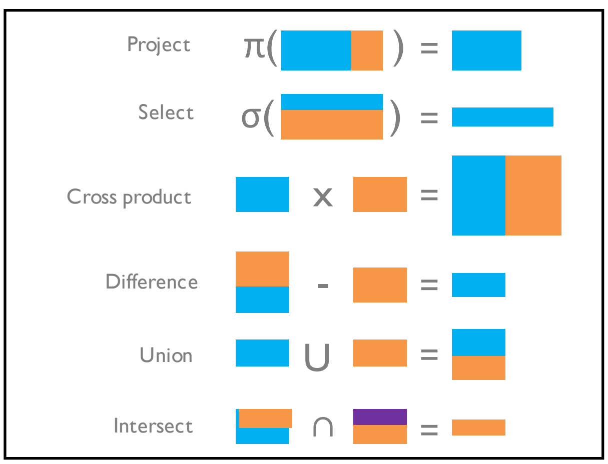
p(<new_name>(<mappings>), Q)

Explicitly defines/changes field names of schema

p(C(1 → sid1, 5 → sid2), S1 x R1)

C =

| sid1 | name | gpa | age | sid2 | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 2 | barak | 3 | 21 | 1 | 101 | 10/10 |
| 3 | trump | 2 | 88 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barak | 3 | 21 | 2 | 102 | 11/11 |
| 3 | trump | 2 | 88 | 2 | 102 | 11/11 |




## Compound/Convenience Operators

INTERSECT (∩)

JOIN (⋈)

DIVIDE (/)

## Intersect

$A \cap B = R_{result}$

A, B must be *union-compatible*

## Intersect

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

S2

| sid | name | gpa | age |
|---|---|---|---|
| 4 | aziz | 3.2 | 21 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

$S1 \cap S2 =$

| sid | name | gpa | age |
|---|---|---|---|
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

## Intersect

$A \cap B = R_{result}$

A, B must be *union-compatible*

Can we express using core operators?

$A \cap B = ?$

## Intersect

$A \cap B = R_{result}$

A, B must be *union-compatible*

Can we express using core operators?

$A \cap B = A - ?$ (think venn diagram)

## Intersect

$A \cap B = R_{result}$

A, B must be *union-compatible*

Can we express using core operators?

$A \cap B = A - (A - B)$

## theta (θ) Join

$A \bowtie_c B = \sigma_c(A \times B)$

Most general form

Result schema same as cross product

Often *far* more efficient to compute than cross product

Commutative

$(A \bowtie_c B) \bowtie_c C = A \bowtie_c (B \bowtie_c C)$

# theta (θ) Join

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$S1 \bowtie_{S1.sid \leq R1.sid} R1 =$

| (sid) | name | gpa | age | (sid) | rid | day |
|---|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 1 | 101 | 10/10 |
| 1 | eugene | 4 | 20 | 2 | 102 | 11/11 |
| 2 | barak | 3 | 21 | 2 | 102 | 11/11 |

# Equi-Join

$A \bowtie_{attr} B = A \bowtie_{A.attr = B.attr} B$

Special case where the condition is attribute equality

Result schema only keeps one copy of equality fields

Natural Join ($A \bowtie B$):

Equijoin on *all* shared fields

# Equi-Join

S1

| sid | name | gpa | age |
|---|---|---|---|
| 1 | eugene | 4 | 20 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |

R1

| sid | rid | day |
|---|---|---|
| 1 | 101 | 10/10 |
| 2 | 102 | 11/11 |

$S1 \bowtie_{sid} R1 =$

| sid | name | gpa | age | rid | day |
|---|---|---|---|---|---|
| 1 | eugene | 4 | 20 | 101 | 10/10 |
| 2 | barak | 3 | 21 | 102 | 11/11 |

# Division

Let us have relations A(x, y), B(y)

$$A/B = \{ \langle x \rangle \mid \exists \langle x,y \rangle \in A \ \forall A \ \langle y \rangle \in B \}$$

Good to ponder, not supported in most systems (why?)

*Find all students that have reserved all books*

A/B = all x (students) s.t. for every y (reservation), $\langle x,y \rangle \in A$

Generalization

y can be a list of fields in B

x U y is fields in A

# Examples

A

| sid | rid |
|---|---|
| 1 | 1 |
| 1 | 2 |
| 1 | 3 |
| 1 | 4 |
| 2 | 1 |
| 2 | 2 |
| 3 | 2 |
| 4 | 2 |
| 4 | 4 |

R1

| rid |
|---|
| 2 |

| sid |
|---|
| 1 |
| 2 |
| 3 |
| 4 |

A/R1

R2

| rid |
|---|
| 2 |
| 4 |

| sid |
|---|
| 1 |
| 4 |

A/R2

R3

| rid |
|---|
| 1 |
| 2 |
| 4 |

| sid |
|---|
| 1 |

A/R3

# Is A/B a Fundamental Operation?

No. Shorthand like Joins

joins so common, it's natively supported

Hint: Find all *x*s not 'disqualified' by some *y* in *B*.

*x* value is *disqualified* if by attaching *y* value from *B*, we obtain an *x,y* tuple that is not in *A*.

A

| sid | rid |
|---|---|
| 1 | 1 |
| 1 | 2 |
| 1 | 3 |
| 1 | 4 |
| 2 | 1 |
| 2 | 2 |
| 3 | 2 |
| 4 | 2 |
| 4 | 4 |

B

| rid |
|---|
| 2 |
| 4 |

Disqualified =
A/B =

---

A

| sid | rid |
|---|---|
| 1 | 1 |
| 1 | 2 |
| 1 | 3 |
| 1 | 4 |
| 2 | 1 |
| 2 | 2 |
| 3 | 2 |
| 4 | 2 |
| 4 | 4 |

B

| rid |
|---|
| 2 |
| 4 |

$\pi_x(A) \times B$

| sid | rid |
|---|---|
| 1 | 2 |
| 1 | 4 |
| 2 | 2 |
| 2 | 4 |
| 3 | 2 |
| 3 | 4 |
| 4 | 2 |
| 4 | 4 |

Disqualified = $(\pi_x(A) \times B)$
A/B =

---

A

| sid | rid |
|---|---|
| 1 | 1 |
| 1 | 2 |
| 1 | 3 |
| 1 | 4 |
| 2 | 1 |
| 2 | 2 |
| 3 | 2 |
| 4 | 2 |
| 4 | 4 |

B

| rid |
|---|
| 2 |
| 4 |

$\pi_x(A) \times B$

| sid | rid |
|---|---|
| 1 | 2 |
| 1 | 4 |
| 2 | 2 |
| 2 | 4 |
| 3 | 2 |
| 3 | 4 |
| 4 | 2 |
| 4 | 4 |

$\pi_x(A) \times B) - A$

| sid | rid |
|---|---|
| 2 | 4 |
| 3 | 4 |

Disqualified = $((\pi_x(A) \times B) - A)$
A/B =

---

A

| sid | rid |
|---|---|
| 1 | 1 |
| 1 | 2 |
| 1 | 3 |
| 1 | 4 |
| 2 | 1 |
| 2 | 2 |
| 3 | 2 |
| 4 | 2 |
| 4 | 4 |

B

| rid |
|---|
| 2 |
| 4 |

$\pi_x(A) \times B$

| sid | rid |
|---|---|
| 1 | 2 |
| 1 | 4 |
| 2 | 2 |
| 2 | 4 |
| 3 | 2 |
| 3 | 4 |
| 4 | 2 |
| 4 | 4 |

$\pi_x(A) \times B) - A$

| sid | rid |
|---|---|
| 2 | 4 |
| 3 | 4 |

| sid |
|---|
| 1 |
| 4 |

A/B

Disqualified = $\pi_x((\pi_x(A) \times B) - A)$
A/B = $\pi_x(A)$ - Disqualified

---

Names of students that reserved book 2

$\pi_{name}(\sigma_{rid=2}(R1) \bowtie S1)$

**Equivalent Queries**

$\rho(Tmp1, \sigma_{rid=2}(R1))$
$\rho(Tmp2, Tmp1 \bowtie S1)$
$\pi_{name}(Tmp2)$

$\pi_{name}(\sigma_{rid=2}(R1 \bowtie S1))$

---

## Names of students that reserved db books

Book(bid, type) Reserve(sid, bid) Student(sid)

Need to join with Instructors to access role

$\pi_{name}(\sigma_{type='db'}(Book) \bowtie Reserve \bowtie Student)$

More efficient query

$\pi_{name}(\pi_{sid}((\pi_{cid}\sigma_{type='db'}(Book)) \bowtie Reserve) \bowtie Student)$

Query optimizer can find the more efficient query!

## Students that reserved DB or HCI book

1. Find all DB or HCI books
2. Find students that reserved one of those books

$$\rho(tmp, (\sigma_{type='DB' \vee type='HCI'}(Book))$$
$$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$$

Alternatives

define tmp using UNION (how?)

what if we replaced v with ^ in the query?

## Students that reserved a DB and HCI book

Does previous approach work?

$$\rho(tmp, (\sigma_{type='DB' \wedge type='HCI'}(Book))$$
$$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$$

NO

## Students that reserved a DB and HCI book

Does previous approach work?

1. Find students that reserved DB books
2. Find students that reversed HCI books
3. Intersection

$$\rho(tmpDB, \pi_{sid}(\sigma_{type='DB'}\ Book) \bowtie Reserve)$$
$$\rho(tmpHCI, \pi_{sid}(\sigma_{type='HCI'}\ Book) \bowtie Reserve)$$
$$\pi_{name}((tmpDB \cap tmpHCI) \bowtie Student)$$

Is the intersection always allowed?

What if it projected book name?

## Students that reserved all books

Use division

Be careful with schemas of inputs to / !

$$\rho(tmp, (\pi_{sid,bid}\ Reserves) \ / \ (\pi_{bid} Books))$$
$$\pi_{name}(tmp \bowtie Student)$$

What if want students that reserved all horror books?

$$\rho(tmp, (\pi_{sid,bid}\ Reserves) \ / \ (\pi_{bid}(\sigma_{type='horror'}\ Book)\ ))$$

## Let's step back

Relational algebra is expressiveness benchmark

A language equal in expressiveness as relational algebra is relationally complete

But has limitations

nulls

aggregation

recursion

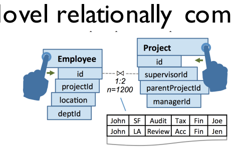
duplicates

## What can we do with RA?

Query by example

Here's my data and examples of the result, *generate the query for me*

Novel relationally complete interfaces




GestureDB. Nandi et al.

## Summary

Relational Algebra (RA) operators

Operators are closed (inputs & outputs are relations)

Multiple Relational Algebra queries can have same semantics (always return same results)

- Forms basis for optimizations

## Next Time

Relational Calculus