L4 Relational Algebra

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Reading

Ramakrishnan

Sections 4.1 and 4.2

Helpful References

https://en.wikipedia.org/wiki/Relational_algebra

Overview

Last time, learned about pre-relational models an informal introduction to relational model an introduction to the SQL query language.

Learn about formal relational query languages
Relational Algebra (algebra: perform operations)
Relational Calculus (logic:are statements true?)

Keys to understanding $\,\mathrm{SQL}\,$ and $\,\mathrm{query}\,$ processing

Who Cares?

Clean query semantics & rich program analysis Helps/enables optimization

Opens up rich set of topics

Materialized views

Data lineage/provenance

Query by example

Distributed query execution

. . .

What's a Query Language?

Allows manipulation and retrieval of data from a database.

Traditionally: QL != programming language

Doesn't need to be turing complete

Not designed for computation

Supports easy, efficient access to large databases

Recent Years

Scaling to large datasets is a reality

Query languages are a powerful way to
think about data algorithms scale
think about asynchronous/parallel programming

What's a Query Language? All the particular is a conveniently the activate that the count and upward Particular (in the answer) MapReduce is a powerful primitive to do many kinds of parallel data processing BUT Little control of data flow Fault tolerance guarantees not always necessary Simplicity leads to inefficiencies Does not interface with existing analysis software Industry has existing training in SOL SQL interface for Hadoop critical for mass adoption Tutorial at VLDB 2015 Abadii et al. http://www.slideshare.net/abadiid/sqlonhadoop-tutorial

Formal Relational Query Languages

Formal basis for real languages e.g., SQL

Relational Algebra

Operational, used to represent execution plans

Relational Calculus

Logical, describes what data users want (not operational, fully declarative)

Prelims

Query is a function over relation instances

$$Q(R_1,...,R_n) = R_{result}$$

Schemas of input and output relations are fixed and well defined by the query Q.

Positional vs Named field notation Position easier for formal defs one-indexed (not 0-indexed!!!)

Named more readable Both used in SQL

Prelims

Relation (for this lecture) Instance is a set of tuples Schema defines field names and types (domains) Students(sid int, name text, major text, gpa int)

How are relations different than generic sets (\mathbb{R})? Can assume item structure due to schema Some algebra operations (x) need to be modified Will use this later

Relational Algebra Overview

Core 5 operations Additional operations PROJECT (π) SELECT (σ) UNION (U) SET DIFFERENCE (-) CROSSPRODUCT (x)

RENAME (p) INTERSECT (∩) JOIN (⋈) DIVIDE (/)

Instances Used Today: Library

Students, Reservations

Use positional or named field notation

Fields in query results are inherited from input relations (unless specified)

| | • | | • |
|----|-----|-----|-------|
| RΙ | sid | rid | day |
| | 1 | 101 | 10/10 |
| | 2 | 102 | 11/11 |

| I | sid | name | gpa | age |
|---|-----|--------|-----|-----|
| | 1 | eugene | 4 | 20 |
| | 2 | barak | 3 | 21 |
| | 3 | trump | 2 | 88 |
| | | | | |
| _ | | | | |

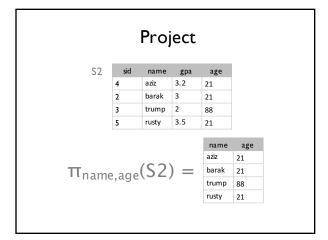
| 2 | sid | name | gpa | age |
|---|-----|-------|-----|-----|
| | 4 | aziz | 3.2 | 21 |
| | 2 | barak | 3 | 21 |
| | 3 | trump | 2 | 88 |
| | 5 | rustv | 3.5 | 21 |

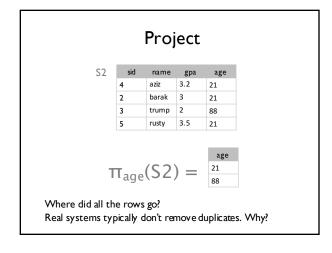
Project

 $\pi_{<attr1,...>}(A) = R_{result}$

Pick out desired attributes (subset of columns) Schema is subset of input schema in the projection list

 $\pi_{\langle a,b,c\rangle}(A)$ has output schema (a,b,c) w/ types carried over

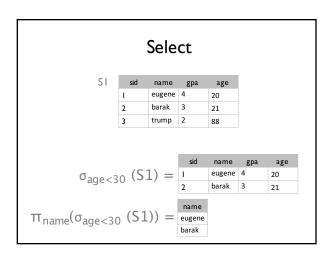




Select

 $\sigma_{\leq p^>}(A) = R_{result}$

Select subset of rows that satisfy condition *p* Won't have duplicates in result. Why?
Result schema same as input



Commutatively

$$A + B = B + A$$

$$A * B = B * A$$

 $A + (B * C) = (B * C) + A$

$$A + (B + C) = (A + B) + C$$

$$A + (B * C) = (A + B) * C$$

Commutatively

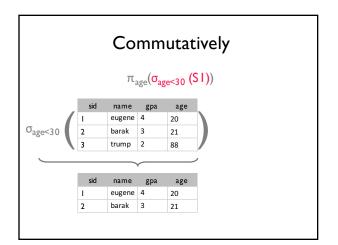
$$A + B = B + A$$

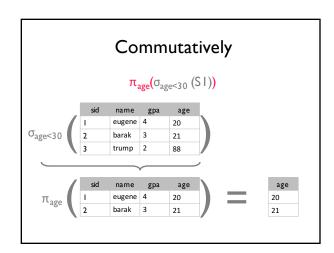
$$A * B = B * A$$

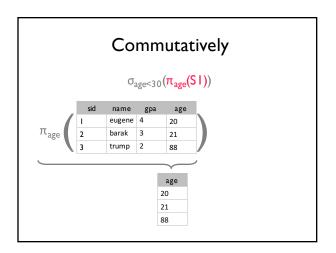
$$A + (B * C) = (B * C) + A$$

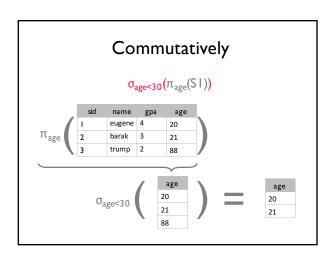
$$A + (B + C) = (A + B) + C$$

$$A + (B * C) = (A + B) * C$$









Commutatively Does Project and Select commute? $\pi_{age}(\sigma_{age<30} \ (SI)) = \sigma_{age<30}(\pi_{age}(SI))$ What about $\pi_{name}(\sigma_{age<30} \ (SI))?$

$$\label{eq:commutatively} \begin{split} & \textbf{Commutatively} \\ & \textbf{Does Project and Select commute?} \\ & \pi_{age}(\sigma_{age<30} \text{ (SI)}) = \sigma_{age<30}(\pi_{age}(\text{SI})) \end{split}$$

$$\label{eq:mage} \\ & \text{What about} \\ & \pi_{name}(\sigma_{age<30} \text{ (SI)}) \ != \sigma_{age<30}(\pi_{name}(\text{SI})) \end{split}$$

Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age < 30} (SI)) = \sigma_{age < 30}(\pi_{age}(SI))$$

What about

$$\pi_{\mathsf{name}}(\sigma_{\mathsf{age} < 30} \; (\mathsf{SI})) \; != \sigma_{\mathsf{age} < 30}(\pi_{\mathsf{name}, \, \mathsf{age}}(\mathsf{SI}))$$

Commutatively

Does Project and Select commute?

$$\pi_{age}(\sigma_{age<30} (SI)) = \sigma_{age<30}(\pi_{age}(SI))$$

What about

$$\pi_{\text{name}}(\sigma_{\text{age} \le 30} \text{ (S I)}) = \frac{\pi_{\text{name}}(\sigma_{\text{age} \le 30}(\pi_{\text{name, age}}(\text{S I})))$$

OK!

Union, Set-Difference

 $A op B = R_{result}$

A, B must be union-compatible

Same number of fields

Field i in each schema have same type

Result Schema borrowed from first arg (A)

A(big int, poppa int) U B(thug int, life int) = ?

Union, Set-Difference

 $A op B = R_{result}$

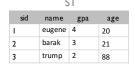
A, B must be union-compatible

Same number of fields
Field i in each schema have same type

Result Schema borrowed from first arg (A)

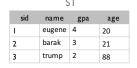
A(big int, poppa int) U B(thug int, life int) = R_{result} (big int, poppa int)

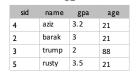
Union, Intersect, Set-Difference



| | 3 | 2 | |
|-----|-------|-----|-----|
| sid | name | gpa | age |
| 4 | aziz | 3.2 | 21 |
| 2 | barak | 3 | 21 |
| 3 | trump | 2 | 88 |
| 5 | rusty | 3.5 | 21 |

Union, Intersect, Set-Difference





Note on Set Difference & Performance

Notice that most operators are monotonic increasing size of inputs \Rightarrow outputs grow if $A \supseteq B \Rightarrow Q(A,T) \supseteq Q(B,T)$ can compute incrementally

Set Difference is not monotonic

if
$$A \supseteq B$$
 \rightarrow $T - A \subseteq T - B$
e.g., $5 > I$ \rightarrow $9 - 5 < 9 - I$

Set difference is blocking:

For T - S, must wait for all S tuples before any results

Cross-Product

$$A(a_1,...,a_n) \times B(a_{n+1},...,a_m) = R_{result}(a_1,...,a_m)$$

Each row of A paired with each row of B

Result schema concats A and B's fields, inherit if possible

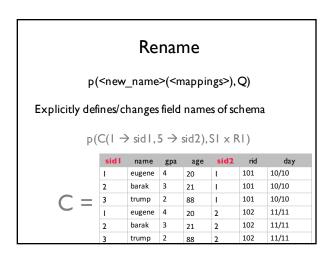
Conflict: students and reservations have sid field

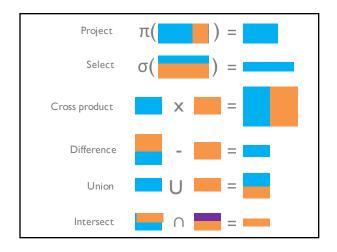
Different than mathematical "X" by flattening results: $\operatorname{math} A \times B = \{ (a, b) \mid a \subseteq A \land b \subseteq B \}$

e.g.,
$$\{1, 2\} \times \{3, 4\} = \{ (1, 3), (1, 4), (2, 3), (2, 4) \}$$

what is $\{1, 2\} \times \{3, 4\} \times \{5, 6\}$?

| | | | С | ross | -Pr | 00 | du | ct | | | |
|-----|------|--------|-------|--------|-----|---------------------------------------|----|-------|-------|-------|---|
| | | 9 | S I | | | | | | RI | | |
| | sid | name | gpa | age | | | : | sid | rid | day | |
| | 1 | eugene | 4 | 20 | | | 1 | | 101 | 10/10 | |
| | 2 | barak | 3 | 21 | | | 2 | | 102 | 11/11 | |
| | 3 | trump | 2 | 88 | | | | | | | |
| | | | (sid) | name | gpa | a | ge | (sid) | rid | day | , |
| | | | I | eugene | 4 | 20 | | ı | 101 | 10/10 | |
| | | | 2 | barak | 3 | 2 age (sic 20 I 21 I 88 I 20 2 21 2 | | ı | 101 | 10/10 | |
| SI | x R | 1 = | 3 | trump | 2 | age (sid) 20 1 21 1 88 1 20 2 | | 101 | 10/10 | | |
| 0 1 | / 11 | _ | I | eugene | 4 | 20 | | 2 | 102 | 11/11 | |
| | | | 2 | barak | 3 | 21 | | 2 | 102 | 11/11 | |
| | | | 3 | trump | 2 | 88 | | 2 | 102 | 11/11 | |





INTERSECT (∩)

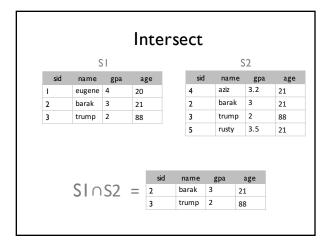
JOIN (⋈)

DIVIDE (/)

Intersect

 $A \cap B = R_{result}$

A, B must be union-compatible



Intersect

 $A \cap B = R_{result}$

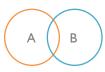
A, B must be union-compatible

Can we express using core operators?

 $A \cap B = ?$



 $A \cap B = R_{result}$



Can we express using core operators?

 $A \cap B = A - ?$ (think venn diagram)

Intersect

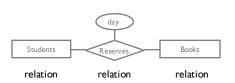
 $A \cap B = R_{result}$



Can we express using core operators?

 $A \cap B = A - (A - B)$

Joins (high level)

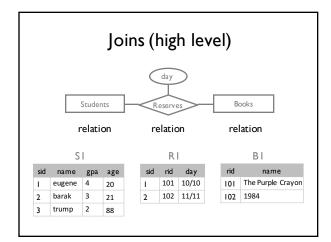


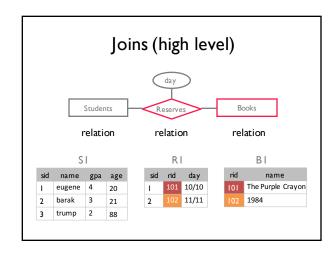
What if you want to query across all three tables?

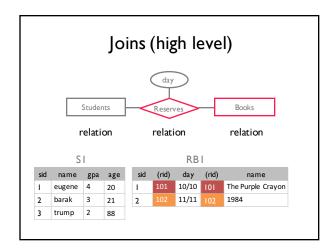
e.g., all names of students that reserved "The Purple Crayon"

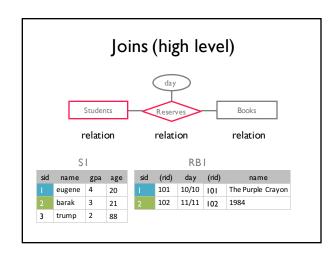
Need to combine these tables

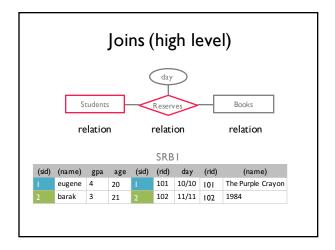
Cross product? But that ignores foreign key references

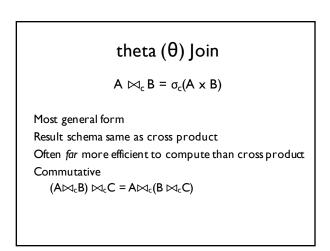


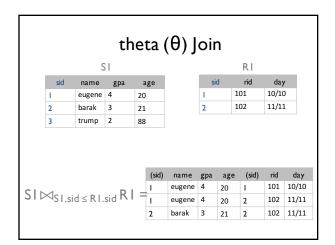










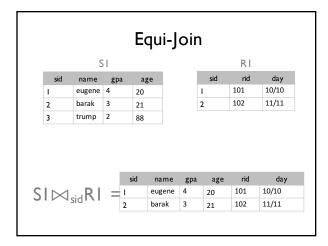


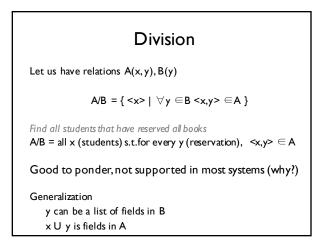


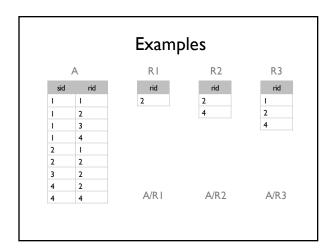
 $A\bowtie_{attr} B = \pi_{all\ attrs\ except\ Battr}(A\bowtie_{Aattr\ =\ Battr} B)$

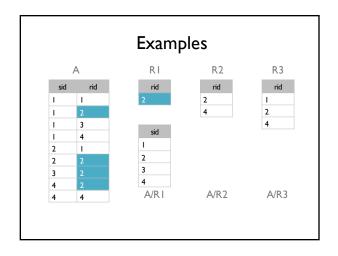
Special case where the condition is attribute equality Result schema only keeps *one copy* of equality fields Natural Join ($A \bowtie B$):

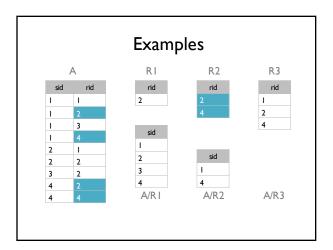
Equijoin on all shared fields (fields w/ same name)

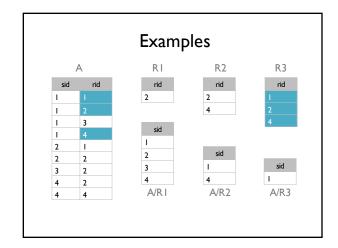












Is A/B a Fundamental Operation?

No. Shorthand like Joins

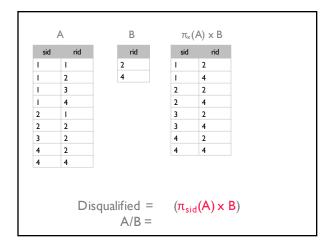
joins so common, it's natively supported

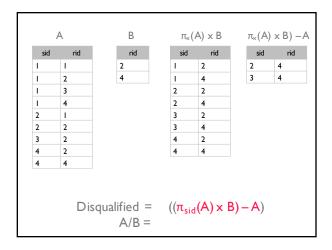
Hint: Find all xs not 'disqualified' by some y in B.

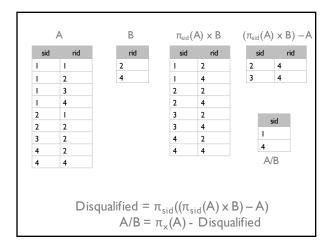
x value is disqualified if

- 1. by attaching y value from B (e.g., create $\langle x, y \rangle$)
- 2. we obtain an $\langle x,y \rangle$ that is not in A.

| sid rid I I I 2 I 3 I 4 2 I 2 2 3 2 4 2 4 4 | | Α | В |
|---|-----|-----|--------------|
| 1 2 1 3 1 4 2 1 2 2 3 2 4 2 | sid | rid | rid |
| 1 3 1 4 2 1 2 2 3 2 4 2 | I | I | |
| 1 4 2 1 2 2 3 2 4 2 | I | 2 | 4 |
| 2 I 2 2 3 2 4 2 | I | 3 | |
| 2 2 3 2 4 2 | I | 4 | |
| 3 2 4 2 | | I | |
| 4 2 | | 2 | |
| | | | |
| 4 4 | | | |
| | 4 | 4 | |
| | | | |
| | | Di | squalified = |
| Disqualified = | | | Λ/R - |
| Disqualified = A/B = | | | A/B - |







Names of students that reserved book 2

 $\pi_{name}(\sigma_{rid=2}(R1) \bowtie SI)$

Equivalent Queries

 $\begin{array}{l} p(tmp1,\,\sigma_{rid=2}\,(R1)) \\ p(tmp2,\,tmp1\,\bowtie SI) \end{array}$

 $\pi_{\text{name}}(\text{tmp2})$

 $\pi_{name}(\sigma_{rid=2}(R1 \bowtie SI))$

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students $\sigma_{\text{type="db'}}(\text{Book})$

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students $\sigma_{tvoe={}^{i}db^{i}} \, (\text{Book}) \bowtie \text{Reserve}$

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students $\sigma_{type={}^{\iota}db^{\iota}}(Book)\bowtie Reserve\bowtie Student$

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students $\pi_{name}(\sigma_{type='db'} (Book) \bowtie Reserve \bowtie Student)$

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students

 $\pi_{name}(\sigma_{type='db'}(Book)\bowtie Reserve\bowtie Student)$

More efficient query

 $\pi_{name}(\pi_{sid}((\pi_{rid} \sigma_{type='db'}(Book)) \bowtie Reserve) \bowtie Student)$

Query optimizer can find the more efficient query!

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students

 $\pi_{name}(\sigma_{type='db'}(Book) \bowtie Reserve \bowtie Student)$

More efficient query

 $\pi_{\text{name}}(\pi_{\text{sid}}((\pi_{\text{rid}} \sigma_{\text{type='db'}} (\text{Book})) \bowtie \text{Reserve}) \bowtie \text{Student})$

Query optimizer can find the more efficient query!

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students

 $\pi_{name}(\sigma_{type='db'}(Book) \bowtie Reserve \bowtie Student)$

More efficient query

 $\pi_{name}(\pi_{sid}((\pi_{rid} \sigma_{type='db'} (Book))) \bowtie Reserve) \bowtie Student)$

Query optimizer can find the more efficient query!

Names of students that reserved db books

Book(rid, type) Reserve(sid, rid) Student(sid)

Need to join DB books with reserve and students

 $\pi_{name}(\sigma_{type='db'}(Book)\bowtie Reserve\bowtie Student)$

More efficient query

 $\pi_{\text{name}}(\pi_{\text{sid}}((\pi_{\text{rid}} \sigma_{\text{type='db'}} (\text{Book})) \bowtie \text{Reserve}) \bowtie \text{Student})$

Query optimizer can find the more efficient query!

Students that reserved DB or HCl book

- I. Find all DB or HCl books
- 2. Find students that reserved one of those books

 $p(tmp, (\sigma_{type='DB'\ v\ type='HCI'}(Book))$ $\pi_{name}(tmp\bowtie Reserve\bowtie Student)$

Alternatives

define tmp using UNION (how?) what if we replaced v with ^ in the query?

Students that reserved a DB and HCI book

Does previous approach work?

 $p(tmp, (\sigma_{type='DB'^ttype='HCI'}(Book))$ $\pi_{name}(tmp \bowtie Reserve \bowtie Student)$



Students that reserved a DB and HCI book

Does previous approach work?

- 1. Find students that reserved DB books
- 2. Find students that reversed HCl books
- 3. Intersection

$$\begin{split} &p(tmpDB, \pi_{sid}(\sigma_{type="DB"}Book) \bowtie Reserve) \\ &p(tmpHCl, \pi_{sid}(\sigma_{type="HCl"}Book) \bowtie Reserve) \\ &\pi_{name}((tmpDB\cap tmpHCl) \bowtie Student) \end{split}$$

Students that reserved all books

Use division

Be careful with schemas of inputs to / !

p(tmp, ($\pi_{sid,rid}$ Reserves) / (π_{rid} Books)) π_{name} (tmp \bowtie Student)

What if want students that reserved all horror books?

 $p(tmp, (\pi_{sid,rid} Reserves) / (\pi_{rid}(\sigma_{type='horror'} Book)))$

Let's step back

Relational algebra is expressiveness benchmark
A language equal in expressiveness as relational algebra is relationally complete

But has limitations

nulls

aggregation

recursion

duplicates

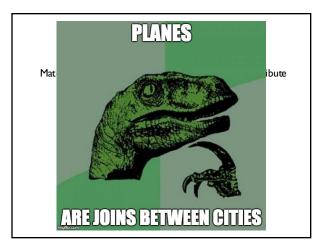
Equi-Joins are a way of life

Matching of two sets based on shared attributes

Yelp: Join between your location and restaurants

Market: Join between consumers and suppliers

High five: Join between two hands on time and space Comm.: Join between minds on ideas/concepts

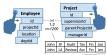


What can we do with RA?

Query by example

Here's my data and examples of the result, generate the query for me

Novel relationally complete interfaces



GestureDB. Nandi et al.

Summary

Relational Algebra (RA) operators

Operators are closed inputs & outputs are relations

Multiple Relational Algebra queries can be equivalent It is operational Same semantics but different performance Forms basis for optimizations

Next Time

Relational Calculus SQL