L4
Relational Algebra

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Overview

Last time, learned about
- pre-relational models
- an informal introduction to relational model
- an introduction to the SQL query language.

Learn about formal relational query languages
- Relational Algebra (algebra: perform operations)
- Relational Calculus (logic: are statements true?)

Keys to understanding SQL and query processing

Who Cares?

Query Semantics
- Concise *domain specific language (DSL)*
- Rich program analysis

Helps/enables optimization

Opens up rich set of topics
- Materialized views
- Data lineage/provenance
- Query by example
- …

Reading

Ramakrishnan

Optional

Helpful References
- https://en.wikipedia.org/wiki/Relational_algebra

What's a Query Language?

Allows manipulation and retrieval of data from a database.

Traditionally: QL != programming language
- Doesn't need to be turing complete
- Not designed for computation
- Supports easy, efficient access to large databases

Recent Years
- Scaling to large datasets is a reality
- Query languages are a powerful way to
  - think about data algorithms scale
  - think about asynchronous/parallel programming

Formal Relational Query Languages

Formal basis for real languages e.g., SQL

Relational Algebra
- Operational, used to represent execution plans

Relational Calculus
- Logical, describes what data users want
- (not operational, fully declarative)

Prelims

Query is a function over relation instances

$Q(R_1,\ldots,R_n) = R_{result}$

Schemas of input and output relations are *fixed* and well defined by the query Q.

Positional vs Named field notation
- Position easier for formal defs
  - one-indexed (not 0-indexed!!!)
- Named more readable
- Both used in SQL

Prelims

Relation (for this lecture)
- Instance is a set of tuples
- Schema defines field names and types (domains)
- Students(sid, name, bday, major, gpa)

How are relations different than generic sets ($\mathbb{R}$)?
- Can assume item structure due to schema
- Some algebra operations (x) need to be modified
- Will use this later

Relational Algebra Overview

Core 5 operations
- PROJECT (π)
- SELECT (σ)
- UNION (U)
- SET DIFFERENCE (-)
- CROSSPRODUCT (x)

Additional operations
- RENAME (ρ)
- INTERSECT ($\cap$)
- JOIN ($\bowtie$)
- DIVIDE (/)

Instances Used Today: Library

Students, Reservations, Books

Use positional or named field notation

Fields in query results are inherited from input relations (unless specified)

R1

sid	rid	day
1	101	10/10
2	102	11/11

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

Project

$\pi_{<attr1,\ldots>}(R_{in}) = R_{result}$

Pick out desired attributes (subset of columns)

Schema is subset of input schema in the projection list
- $\pi_{<a,b,c>}(R_{in})$ has output schema (a,b,c) with types carried over

Project

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$\pi_{name,age}(S2) =$

name	age
aziz	21
barak	21
trump	88
rusty	21

Project

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$\pi_{age}(S2) =$

age
21
88

Where did all the rows go?
Real systems typically don't remove duplicates. Why?

Select

$\sigma_{<p>}(R_{in}) = R_{result}$

Select subset of rows that satisfy condition *p*
Won't have duplicates in result. Why?
Result schema same as input

Select

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

$\sigma_{age<30}\ (S1) =$

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21

$\pi_{name}(\sigma_{age<30}\ (S1)) =$

name
eugene
barak

Union, Set-Difference

R_1 op $R_2 = R_{result}$

R_1, R_2 must be *union-compatible*
- Same number of fields
- Field i in each schema have same type

Result Schema borrowed from first arg (R_1)
- R1 (big int, poppa int) U R2(thug int, life int) = ?

Union, Intersect, Set-Difference

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

S1 U S2 =

sid	name	gpa	age
1	eugene	4	20
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

Union, Intersect, Set-Difference

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

S1 - S2 =

sid	name	gpa	age
1	eugene	4	20

Note on Set Difference & Performance

Notice that most operators are monotonic
- increasing size of inputs → outputs grow
- if $S1 \supseteq S2 \rightarrow Q(S1,T) \supseteq Q(S2,T)$
- can compute *incrementally*

Set Difference is *not monotonic*
- e.g., $T - S1$
- if $S1 \supseteq S2 \rightarrow T - S1 \subseteq T - S2$

Set difference is *blocking*:
- For $T - S$, must wait for all S tuples before any results

Cross-Product

$$R_1(a_1,\dots,a_n) \times R_2(a_{n+1},\dots,a_m) = R_{result}(a_1,\dots,a_m)$$

Each row of S1 paired with each row of R1
- Result schema concats S1 and R1's fields, inherit if possible
- Conflict: S1 and R1 have *sid* field

Different than mathematical "X" by flattening results:
- math $A \times B = \{ (a, b) \mid a \in A \wedge b \in B \}$
- e.g., $\{1, 2\} \times \{3, 4\} = \{ (1, 3), (1, 4), (2, 3), (2, 4) \}$
- what is $\{1, 2\} \times \{3, 4\} \times \{5, 6\}$?

Cross-Product

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

S1 x R1 =

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
2	barak	3	21	1	101	10/10
3	trump	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11
3	trump	2	88	2	102	11/11

Rename

p(<new_name>(<mappings>), Q)

Explicitly defines/changes field names of schema

p(C(1 → sid1, 5 → sid2), S1 x R1)

C =

sid1	name	gpa	age	**sid2**	rid	day
1	eugene	4	20	1	101	10/10
2	barak	3	21	1	101	10/10
3	trump	2	88	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11
3	trump	2	88	2	102	11/11

Compound/Convenience Operators

Convenience operations
- INTERSECT (∩)
- JOIN (⋈)
- DIVIDE (/)

Intersect

$$R_1 \cap R_2 = R_{result}$$

R_1, R_2 must be *union-compatible*

Intersect

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

S2

sid	name	gpa	age
4	aziz	3.2	21
2	barak	3	21
3	trump	2	88
5	rusty	3.5	21

$S1 \cap S2 =$

sid	name	gpa	age
2	barak	3	21
3	trump	2	88

Intersect

$R_1 \cap R_2 = R_{result}$

R_1, R_2 must be *union-compatible*

Can we express using core operators?

$S \cap T = ?$

Intersect

$R_1 \cap R_2 = R_{result}$

R_1, R_2 must be *union-compatible*

Can we express using core operators?

$S \cap T = S - ?$ (think venn diagram)

Intersect

$R_1 \cap R_2 = R_{result}$

R_1, R_2 must be *union-compatible*

Can we express using core operators?

$S \cap T = S - (S - T)$

theta (θ) Join

$R \bowtie_c S = \sigma_c(R \times S)$

Most general form

Result schema same as cross product

Often *far* more efficient to compute than cross product

Commutative

$(A \bowtie_c B) \bowtie_c C = A \bowtie_c (B \bowtie_c C)$

theta (θ) Join

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

$S1 \bowtie_{S1.sid \leq R1.sid} R1 =$

(sid)	name	gpa	age	(sid)	rid	day
1	eugene	4	20	1	101	10/10
1	eugene	4	20	2	102	11/11
2	barak	3	21	2	102	11/11

Equi-Join

$R \bowtie_{attr} S = R \bowtie_{R.attr\ =\ S.attr} S$

Special case where the condition is attribute equality
Result schema only keeps one copy of equality fields
Natural Join ($R \bowtie S$):
Equijoin on *all* shared fields

Equi-Join

S1

sid	name	gpa	age
1	eugene	4	20
2	barak	3	21
3	trump	2	88

R1

sid	rid	day
1	101	10/10
2	102	11/11

$S1 \bowtie_{sid} R1 =$

sid	name	gpa	age	rid	day
1	eugene	4	20	101	10/10
2	barak	3	21	102	11/11

Division

Let us have relations A(x, y), B(y)

$A/B = \{ \langle x \rangle \mid \exists \langle x,y \rangle \in A\ \forall A\ \langle y \rangle \in B \}$

Good to ponder, not supported in most systems (why?)
Find all students that have reserved all books
A/B = all x (students) s.t. for every y (reservation), $\langle x,y \rangle \in A$
Generalization
y can be a list of fields in B
x U y is fields in A

Examples

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

R1

rid
2

sid
1
2
3
4

A/R1

R2

rid
2
4

sid
1
4

A/R2

R3

rid
1
2
4

sid
1

A/R3

Is A/B a Fundamental Operation?

No. Shorthand like Joins
joins so common, it's natively supported

Hint: Find all xs not 'disqualified' by some *y* in *B*.
x value is *disqualified* if by attaching *y* value from *B*, we obtain an *xy* tuple that is not in *A*.

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

Disqualified =
A/B =

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

$\pi_x(A) \times B$

sid	rid
1	2
1	4
2	2
2	4
3	2
3	4
4	2
4	4

Disqualified = $(\pi_x(A) \times B)$
A/B =

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

$\pi_x(A) \times B$

sid	rid
1	2
1	4
2	2
2	4
3	2
3	4
4	2
4	4

$\pi_x(A) \times B) - A$

sid	rid
2	4
3	4

Disqualified = $((\pi_x(A) \times B) - A)$
A/B =

A

sid	rid
1	1
1	2
1	3
1	4
2	1
2	2
3	2
4	2
4	4

B

rid
2
4

$\pi_x(A) \times B$

sid	rid
1	2
1	4
2	2
2	4
3	2
3	4
4	2
4	4

$\pi_x(A) \times B) - A$

sid	rid
2	4
3	4

sid
1
4

A/B

Disqualified = $\pi_x((\pi_x(A) \times B) - A)$
A/B = $\pi_x(A)$ - Disqualified

Names of students that reserved book 2

$\pi_{name}(\sigma_{rid=2}\ (R1) \bowtie S1)$

Equivalent Queries

$p(Tmp1, \sigma_{rid=2}\ (R1))$
$p(Tmp2, Tmp1 \bowtie S1)$
$\pi_{name}(Tmp2)$

$\pi_{name}(\sigma_{rid=2}(R1 \bowtie S1))$

Names of students that reserved db books

Book(bid, type) Reserve(sid, bid) Student(sid)

Need to join with Instructors to access role

$\pi_{name}(\sigma_{type='db'}\ (Book) \bowtie Reserve \bowtie Student)$

More efficient query

$\pi_{name}(\pi_{sid}((\ \pi_{cid}\sigma_{type='db'}\ (Book)) \bowtie Reserve) \bowtie Student)$

Query optimizer can find the more efficient query!

Students that reserved DB or HCI book

1. Find all DB or HCI books
2. Find students that reserved one of those books

$p(tmp, (\sigma_{type='DB' \vee type='HCI'}\ (Book))$
$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$

Alternatives

define tmp using UNION (how?)
what if we replaced v with ^ in the query?

Students that reserved a DB and HCI book

Does previous approach work?

$\rho(tmp, (\sigma_{type='DB' \wedge type='HCI'} (Book))$
$\pi_{name}(tmp \bowtie Reserve \bowtie Student)$

NO

Students that reserved a DB and HCI book

Does previous approach work?
1. Find students that reserved DB books
2. Find students that reversed HCI books
3. Intersection

$\rho(tmpDB, \pi_{sid}(\sigma_{type='DB'} Book) \bowtie Reserve)$
$\rho(tmpHCI, \pi_{sid}(\sigma_{type='HCI'} Book) \bowtie Reserve)$
$\pi_{name}((tmpDB \cap tmpHCI) \bowtie Student)$

Is the intersection always allowed?
What if it projected book name?

Students that reserved all books

Use division
Be careful with schemas of inputs to / !

$\rho(tmp, (\pi_{sid,bid} Reserves) / (\pi_{bid} Books))$
$\pi_{name}(tmp \bowtie Student)$

What if want students that reserved all horror books?

$\rho(tmp, (\pi_{sid,bid} Reserves) / (\pi_{bid}(\sigma_{type='horror'} Book)))$

Summary

Relational Algebra (RA) operators

Operators are closed (inputs & outputs are relations)

Multiple Relational Algebra queries can have same semantics (always return same results)
Forms basis for optimizations

Next Time

Relational Calculus