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CS 301 HW-0

1- Stable Marriage Problem

Stable marriage problem (SMP) is the problem of finding a stable matching between two equally sized set of elements given an order of preferences for each element in these sets.

Matching is stable when there does not exist any match by which both A and B would be individually better off than they are with the element to which they are currently matched.

i)

If we consider it as a computational problem, given the elements and their preferences this problem is expected to find the most accurate and stable pair for every element. Therefore, input and output should be:

Input

- A set of n men M = {m0, m1, m2, m3}
- A set of n women W = {w0, w1,w2, w3}
- A preference list for each man and woman: ex. Pm-i or Pw-i = (P0, P1, P2, P3)

where i is the index of the element in the list

Output

- A stable matching of men and women: $S \subseteq M \times W$
- S is a stable matching if all of the following are true:

$|S| = n$
$\forall m \in M \exists w \in W ((m, w) \in S)$
$\forall w \in W\ \exists m \in M((m, w) \in S)$
$\neg\exists(m,w)\notin S((m, w0) \in S \wedge (m0 , w) \in S \wedge (Pm(w) < Pm(w 0)) \wedge (Pw(m) < Pw(m0)))$

ii)

Detailed main explanation of the algorithm:

1. Begin with every man and woman being free

2. While there exists a free woman w, she proposes to her most preferred man m whom she hasn't proposed to yet as follows:

   (a) w proposes to m
   (b) If m is free: m and w get engaged and (m, w) is added to S
   (c) If m is currently engaged to another woman, w 0 and he prefers w 0 over w, Pm(w) > Pm(w 0): w remains free
   (d) If m is currently engaged to another woman, w 0 and he prefers w over w 0 , Pm(w) < Pm(w 0): m breaks off the engagement with w 0 and gets engaged to w. (m, w0) is removed from S and (m, w) is added to S. w 0 becomes free.

3. return S as a stable matching.

One example to this problem could be between students and universities:

Let's consider that we have 4 students graduated from high school and also 4 colleges for them to study. We know that each student has to create a priority list between these 4 college. Colleges also have a priority list regarding the students they want to pick. We also know that each college can only pick one student. Our main motivation is to create 4 different stable pairs so that it will make both college and student satisfied.

2- Gale-Shapley Algorithm

i) Algorithm Pseudocode

```
function stableMatching {
    Initialize all m ∈ M and w ∈ W to free
    while ∃ free man m who still has a woman w to propose to {
       w = first woman on m’s list to whom m has not yet proposed
       if w is free
         (m, w) become engaged
       else some pair (m', w) already exists
         if w prefers m to m'
            m' becomes free
           (m, w) become engaged
         else
           (m', w) remain engaged
    }
}
```

ii) Runtime Analysis

Algorithm begins with Θ(n^2) time to setup the needed data structures, then runs the main proposal loop until there are no more free women or men.

So, inside of the first while loop, algorithm needs to run for N times since we have n many men and all men should have 1 pair.

Then for all men, algorithm needs to find the first woman on m’s list that has not any pair at the moment. Since there is again N many women in the prority list of a man, in the worst case algorithm needs to run for O(n^2) times.

There are no more than N^2 proposals due to the fact that no woman proposes to a man more than once. If the all N women propose to all N men, there would be N^2 proposals. Since no woman proposes to a man more than once, there can’t be any additional proposals.

In conclusion, setup takes Θ(n^2) and there are at most N^2 proposals each taking O(1) time, the Gale-Shapley algorithm has a runtime of O(n^2).