Introduction to Black Hole Astrophysics

Ervin Kafexhiu

Data Science, Particle and Astroparticle Physics School
Tirana, June 14–18, 2022

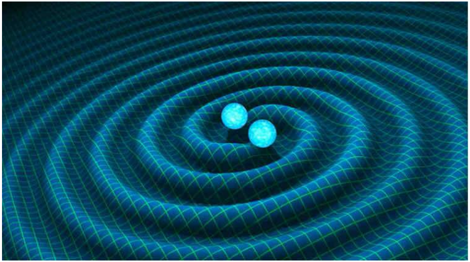
Content

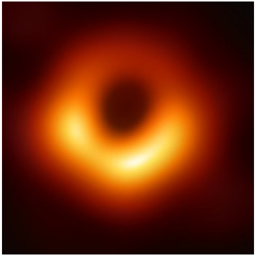
Motivation

Motivation for this lecture:

- First Detection of **Gravitational Waves from Black Hole Mergers** from LIGO & VIRGO Collaborations in 2016.

- **Direct Imaging of the Super Massive Black Hole** (Sagittarius A^*) in the Center of our Milky Way Galaxy by the Event Horizon Telescope Collaboration in 2022.





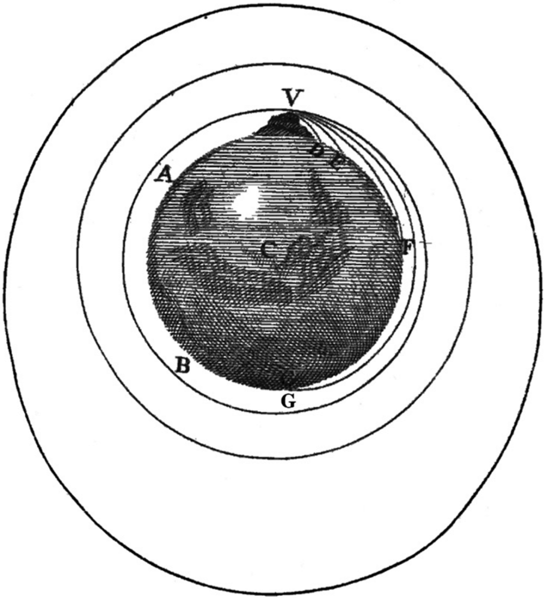
What is a Black Hole (BH)

Definition

A Black Hole (BH) is a region of spacetime where gravity is so strong that nothing can escape from it – not even light. [Ward 1984]

Dark Stars in Newtonian Gravity

Newtonian Gravity:Escape Velocity



Escape Velocity v_{esc}

Mechanical energy of a particle when $r \to \infty \Rightarrow v \to 0 \Rightarrow E_m \to 0$.

$$E_m = \frac{m\,v^2}{2} - \frac{G\,M\,m}{R} = 0$$

$$\boxed{v_{\text{esc}} = \sqrt{\frac{2\,G\,M}{R}}}$$

Earth: $v_{\text{esc}} \approx 11.2\,\text{km/s}$.
Sun: $v_{\text{esc}} \approx 618\ \text{km/s}$.

Newtonian Gravity: Dark Stars

- Based on the corpuscular theory of light, **Michell (1784) & Laplace (1796)** postulated the existence of **Dark Stars**.
- They assumed a star of density equal to Earth's average density $\rho \approx 5.5\,\mathrm{g/cm^3}$ and estimated its M & R so that the $v_{\rm esc} \geq c$.

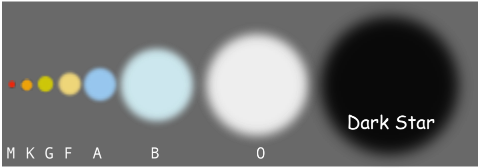
$$\boxed{c^2 \leq \frac{2\,G\,M}{R}, \quad M = \frac{4\,\pi\,\rho\,R^3}{3}} \Rightarrow \boxed{R \geq \sqrt{\frac{3\,c^2}{8\,\pi\,G\,\rho}}}$$

[Montgomery et al.(2009)]

Michell (1784) & Laplace (1796) concluded:

Every star in the Universe with $R \gtrsim 250\,R_\odot$ **or** $M \gtrsim 10^4\,M_\odot$ **will be invisible.**

Stars Classification:



Newtonian Gravity: Dark Stars

The Forgotten Dark Stars

- Despite the initial enthusiasm, the interest on Dark Stars faded away with the discovery of the wave–like nature of light.
- It was not clear at the time what gravity can do to waves!

- Stars do not have constant density, therefore, massive stars are not stable and collapse!
- Instead of fixing density, we fix the mass M of a star and ask what would be its radius R_s so that the $v_{\mathrm{esc}} = c$:

$$\boxed{R_s = \frac{2\,G\,M}{c^2}}$$

Newtonian Gravity: Dark Stars

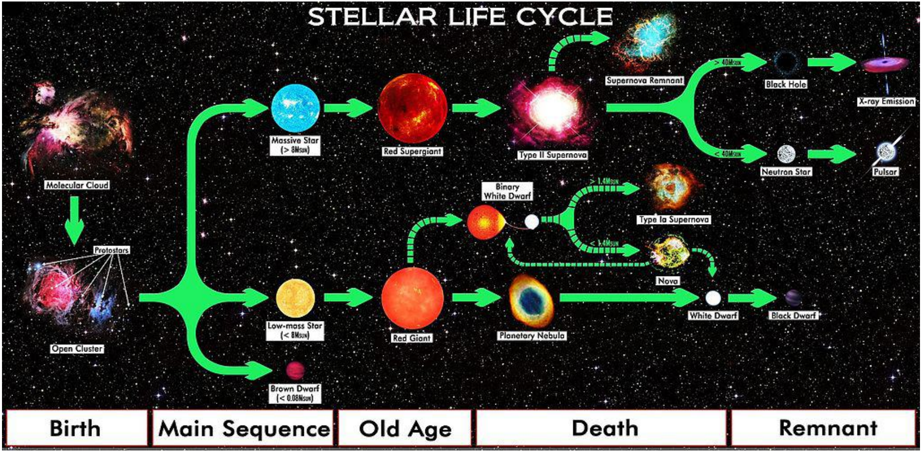
Examples

- The Sun: $R_s \approx 3\,\mathrm{km}$ ($M \approx 2 \times 10^{30}\,\mathrm{kg}$, $g_s \sim 10^{13}\,\mathrm{m/s^2}$).
- The Earth: $R_s \approx 9\,\mathrm{mm}$ ($M \approx 6 \times 10^{24}\,\mathrm{kg}$, $g_s \approx 5 \times 10^{18}\,\mathrm{m/s^2}$).
- The Moon: $R_s \approx 0.1\,\mathrm{mm}$ ($M \approx 7 \times 10^{22}\,\mathrm{kg}$, $g_s \sim 10^{20}\,\mathrm{m/s^2}$).

Extreme Gravity!

Gravitational Collapse

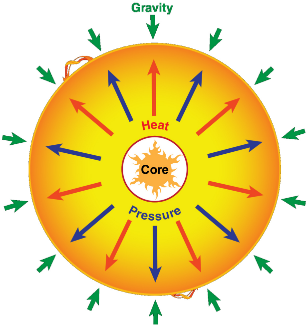
Evolution of Stars



Black Hole Formation

- Giant Stars $(M > 8\,M_\odot) \Longrightarrow$ Type II Supernova.
- If the supernova core has $M > 40\,M_\odot \Longrightarrow$ a Black Hole forms.

How can a Massive Star turn into a Black Hole?



Stability of a Star

A star remains stable if its internal pressure balances out the gravitational collapse.

- Energy generated in the core is transported outward to the cooler surface.
- Inside the star, the inward force of gravity is balanced by the outward force of pressure.
- The star is stabilized by pressure–temperature thermostat.

Self-Regulation of Stars' Stability

Suppose the fusion rate $\dot{\epsilon}\uparrow$ of a star increases then:

- $\Rightarrow$ Temperature increases $T\uparrow$,
- $\Rightarrow$ Pressure increases $p\uparrow$ (thermal & radiative pressure),
- $\Rightarrow$ Core expansion $V_{\text{core}}\uparrow$,
- $\Rightarrow$ Density & Temperature decrease $\rho\downarrow$ & $T\downarrow$,
- $\Rightarrow$ Fusion rate decrease $\dot{\epsilon}\downarrow$!

This cycle works backwards, too.

Self-Regulating Cycle

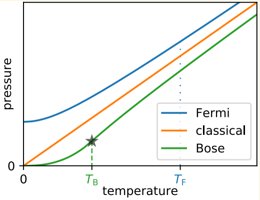
$$\dot{\epsilon}\uparrow\Leftrightarrow\left\{\begin{array}{c}\Rightarrow\ T\uparrow\Rightarrow\ p\uparrow\Rightarrow\ V_{\text{core}}\uparrow\Rightarrow\\ \Leftarrow\ \rho\ \&\ T\uparrow\Leftarrow\ V_{\text{core}}\downarrow\Leftarrow\end{array}\right\}\Leftrightarrow\rho\ \&\ T\downarrow\Leftrightarrow\ \dot{\epsilon}\downarrow$$

If $\dot{\epsilon} \to 0$

If the star stops generating energy (no more fuel)

- Pressure is still high and the core is hotter than the star envelope.
- The star would cool down via convection and radiation and lower its pressure.
- The stars' core will shrink from gravity and become hotter.
- It will continue to cool via convection and radiation and shrink continuously
- For stars with $\dot{\epsilon} \to 0$, three outcomes are believed:
  - $M < 1.4 \times M_\odot$ shrinks & stabilize into a White Dwarf.
  - $1.4 \times M_\odot < M < 3 \times M_\odot$ collapse into a Neutron Star.
  - $M > 3 \times M_\odot$ **collapse into a Black Hole**.
- $M = 1.4 \times M_\odot$ is called the Chandrasekhar limit.
- $M = 3 \times M_\odot$ is called Tolman–Oppenheimer–Volkoff limit.

Gas Pressure as a function of Temperature and Density



- Ideal gas: $\boxed{p = n^1 \times kT}$
- Degenerate Fermi gas:
$$\boxed{p = \frac{Const}{m} \times n^{\frac{5}{3}}}$$

The Degenerate State of Matter can stop the collapse

The collapse is stopped by these states:

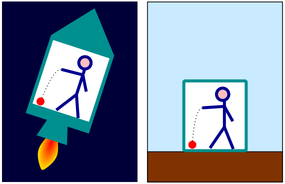
- Electron degenerate gas, e.g. White Dwarf.
- Neutron degenerate gas, e.g. Neutron Stars.
- Speculative quark degenerate gas, Quark Stars!!!

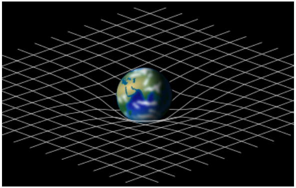
If no state of the matter can stop the gravitational collapse, the star turns into a Black Hole.

General Relativity Theory

Einstein's Theory of General Relativity (GR)

- Black Holes are objects of strong gravitational fields.
- Matter moves relativistically close to them.
- Therefore, Black Holes are described by GR.





Foundations of GR

- Einstein's Equivalence Principle ($m_i \equiv m_g$, gravity can be eliminated locally, local flatness of spacetime).
- Spacetime Geometry $\equiv$ Gravity.
- Energy/matter shape spacetime & spacetime "tells matter" how to move.

Einstein's Field Equations

How matter shapes spacetime? Answer: Find the Metric Tensor!

$$\boxed{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}} \qquad (1)$$

$g_{\mu\nu}$–metric tensor, $R_{\mu\nu}$–Ricci tensor, R–Ricci scalar, $T_{\mu\nu}$–Stress–Energy tensor.

Geodesic Equation

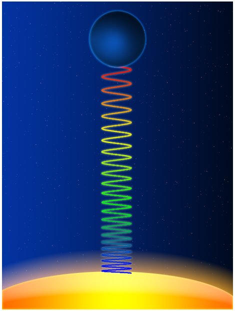
How matter moves in spacetime? Answer: Use the Metric Tensor!

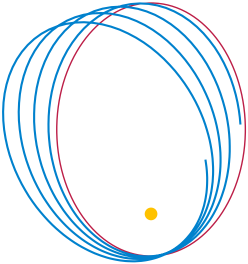
$$\boxed{\ddot{x}^{\mu} + \Gamma^{\mu}_{\nu\lambda}\, \dot{x}^{\nu}\, \dot{x}^{\lambda} = 0} \qquad (2)$$

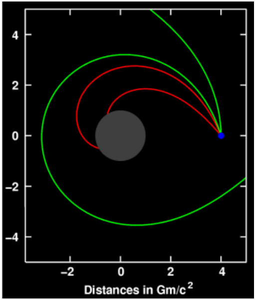
x^{μ}–coordinate, $\dot{x}^{\mu} = dx^{\mu}/d\tau$–derivative with respect to the affine parameter τ, $\Gamma^{\mu}_{\nu\lambda}$– metric connection, called the Christoffel's symbols).

Predictions of GR

- Time Delay and Gravitational Redshift
- Precession of Apsides (closest and farthest points in orbit)
- Bending of Light Rays
- Gravitational Waves
- **Black Holes**
- Big Bang ...







Black Holes of General Relativity

Exact Solutions of Einstein's Equations

- Schwarzschild solution (spherically symmetric, asymptotically flat spacetime)
- Kerr solution (rotating, spherically symmetric, asymptotically flat spacetime)
- Reissner–Nordström solutions (charged, spherically symmetric, static solution of Einstein–Maxwell equations)

No-hair theorem

After formation, black holes in GR "loose their hairs (complexity of collapse)", and are characterized only by three parameters, namely:

- their mass M,
- angular momentum J,
- electric charge Q.

Black Hole Properties

Mechanics of a BH is very simple and is described by:

- the spacetime position X and momentum P (translation), and
- its M, J and Q and nothing else.

Classification

All black holes have mass M.

- if $J = 0$ & $Q = 0 \Rightarrow$ Schwarzschild type.
- if $J \neq 0$ & $Q = 0 \Rightarrow$ Kerr type.
- if $J = 0$ & $Q \neq 0 \Rightarrow$ Reissner–Nordström type.
- if $J \neq$ & $Q \neq 0 \Rightarrow$ Kerr–Newman type.

Only the Schwarzschild & Kerr Black Holes have Astrophysical relevance, i.e. $Q = 0$ solutions.

Schwarzschild solution

Interval & Metric Tensor

$$\boxed{ds^2 = -\left(1 - \frac{r_\mathrm{s}}{r}\right) c^2\, dt^2 + \frac{dr^2}{\left(1 - \frac{r_\mathrm{s}}{r}\right)} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}$$

The $r_s = 2GM/c^2$ is called the Schwarzschild radius.

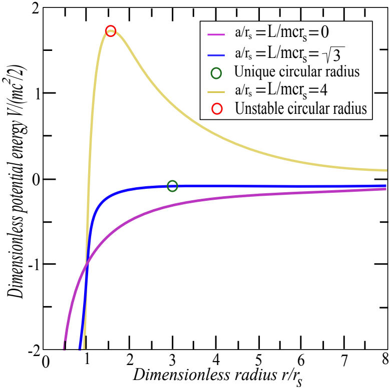
Characteristics

- Relevant surface is $r = r_s$ called the **Event Horizon**.
- Event horizon is a **coordinate singularity**, i.e. is a result of coordinate choice.
- $r = 0$ is a **true singularity** because the curvature goes to ∞.
- Event horizon is a point of no return. Time stops and redshift becomes ∞.

Schwarzschild solution

Characteristics

- The last circular orbit for matter is $r = 3\,r_s$ (stable).
- The last circular orbit for light is $r = 1.5\,r_s$ (unstable).



Kerr Solution

Interval & Metric Tensor

Boyer–Lindquist coordinates:

$$ds^2 = -\left(1 - \frac{r_s r}{\Sigma}\right) c^2 dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{r_s r a^2}{\Sigma}\sin^2\theta\right)\sin^2\theta\, d\phi^2 - \frac{2 r_s r a \sin^2\theta}{\Sigma} c dt\, d\phi$$

- $r_s = 2GM/c^2$ is the Schwarzschild radius.
- $a = J/Mc$,
- $\Sigma = r^2 + a^2\cos^2\theta$,
- $\Delta = r^2 - r_s r + a^2$

Kerr Solution

Characteristics

Relevant Surface:

- **Event Horizon** surface at $g_{rr} \to \infty$ for

$$r_H^{\pm} = \frac{r_s \pm \sqrt{r_s^2 - 4\,a^2}}{2}$$

- $r < r_H^{\pm}$ are the points of no return.
- **Ergosphere** for which $g_{tt} \to 0$

$$r_E^{\pm} = \frac{r_s \pm \sqrt{r_s^2 - 4\,a^2\,\cos^2\theta}}{2}$$

- Ergosphere touches event horizon at the poles & is oblate at the equator.

Kerr Solution

- Particles in the Ergosphere co-rate with inner mass due to frame dragging effect of spacetime – dragging has the speed of light and therefore, forces particles to co-rotate.
- Particles in the Ergosphere can escape from black hole but not if they cross the event horizon.
- Particle trajectories are more complicated than in Schwarzschild BH.

Penrose process

- Particles falling in the ergosphere are forced to rotate fast and gain energy.
- Being outside the event horizon, they can escape from the black hole.
- Thus escaping with more energy than entered in.
- **Rotating black holes can accelerate particles at very high energies [Penrose (1969)]**
- They explain e.g. gamma-ray burst events – mysterious flashes of energetic gamma-rays from the universe.

Other Properties of Black Holes

Surface Gravity κ

Newtonian limit of GR give $g_{tt} = c^2 \left(1 + \frac{2\Phi}{c^2}\right)$ therefore, the surface gravitational intensity $(a_g = -\partial_r \Phi)$ is generalized as

$$\boxed{\kappa = -\frac{1}{2}\partial_r \, g_{tt}}$$

Surface Gravity at the Event Horizon

- $\boxed{\kappa = \frac{c^2}{2\, r_s}}$ for Schwarzschild BH.
- $\boxed{\kappa = \frac{c^2}{2\, r_s} - M\, \Omega_+}$ for Kerr BH. (Ω_+ is the angular velocity at the event horizon.

Gravitational singularity

Definition:

Spacetime singularity (or singularity for short) is a region of extreme gravity where spacetime itself breaks down, catastrophically. Therefore, in a singularity, one cannot determine "where" or "when" an event occurred. In GR, singularities are defined as regions where the curvature tensor goes to ∞.

Cosmic Censorship Hypothesis

Weak version states: Infinities/singularities are hidden behind Event Horizons [Penrose (1969)].

Gravitational singularity

Naked Singularities

Naked singularities in GR are possible under specific conditions as has been shown by e.g. computer models of gravitational collapse [Shapiro& Teukolsky(1991)].

Singularities "Topology"

- Schwarzschild black holes have singularity at a single point $r = 0$.
- Kerr black holes have singularity at $r = 0$ & $\theta = \pi/2$ which is not a point but a **ring**.
- Kerr black holes ring singularity form a time loop!!!

Black Hole Laws

For stationary black holes:

Zeroth Law

The horizon has constant surface gravity $\boxed{\kappa = const}$.

First Law

Change of energy is related to change of area, angular momentum, and electric charge by:

$$\boxed{dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ}$$

where E is the energy, κ is the surface gravity, A is the horizon area, Ω is the angular velocity, J is the angular momentum, Φ is the electrostatic potential and Q is the electric charge.

Black Hole Laws

For stationary black holes:

Second Law

The horizon area is a non-decreasing function of time,

$$\boxed{\frac{dA}{dt} \geq 0}.$$

Third Law

It is not possible to form a black hole with vanishing surface gravity, i.e. $\kappa \neq 0$.

Black Holes & Thermodynamics

Application of Statistical Mechanics on the Black-Body Radiation led to the development of Quantum Mechanics.
Application of Statistical Mechanics on Black Holes has led to a deeper understanding of Quantum Gravity and the so-called Holographic Principle.

in units $G = k_B = c = 1$ (Planck units)

- **Hawking Temperature**: Surface gravity is related to BH temperature $\boxed{T_H = \kappa/8\pi}$ ($T_H = 6 \times 10^{-8} (M/M_\odot)^{-1}$ K).
- **Bekenstein–Hawking entropy**: Surface area is related to BH entropy $\boxed{S_{BH} = A/4}$.
- **Hawking Radiation**: Black Holes radiate like a black body of temperature T_H. Pair production near the event horizon – one particle falls in and the other one is radiated away.

Black Holes & Thermodynamics

- **Hawking Radiation is relevant only for micro BH.**
  - Micro BH with $M = 1$ kg has $T_H \sim 10^{23}$ K, luminosity $L \sim 10^6 \times L_\odot$ and lifetime $\tau_{BH} \sim 10^{-17}$ s.
  - A solar mass BH has has $T_H \sim 10^{-8}$ K, luminosity $L \sim 10^{-55} \times L_\odot$ and lifetime $\tau_{BH} \sim 10^{67}$ years.
- Micro BH will evaporate quickly leading to the information loss, the so-called **Information paradox**. [Hawking (1975,1976)]

Black Holes & Quantum Mechanics

- The pair of particles that produces Hawking radiation are **Entangled (quantum entanglement)**.
- Suppose Alice is an observer outside the BH and, Bob an observer inside the BH event horizon.
- The quantum entanglement allows Alice and Bob to communicate via measurement entangled Hawking radiation particles inside and outside the BH, respectively!!!
- Resolution of the so-called **Information paradox**, has led to many breakthroughs in theoretical physics and advances in **Quantum Information** and **Quantum gravity** studies.

Black Holes & Quantum Mechanics

Resolution of Information paradox

- **ADS/CFT correspondence, Holographic principle, ER=EPR** (wormholes & entangled BH). [Maldacena (1998,2003), Maldacena & Susskind (2013)]
- Most striking result: **Geometry–Entanglement relation** where quantum entanglement act as a "geometric glue" for spacetime. Spacetime bulk & gravity emerges from entanglement of fields in the surface boundary of the universe (the hologram). [Van Raamsdonk (2010)]

for more details see: [Raju(2020) arXiv:2012.05770, Cowen(2015) Nature doi:10.1038/527290a] (open access)

Black Holes in Astrophysics

Black Hole Classes

- Supermassive black holes $M \sim 10^5 - 10^{10}\, M_\odot$
- Intermediate-mass black holes $M \sim 10^3\, M_\odot$
- Stellar black holes $M \sim 10\, M_\odot$
- Micro black holes $M < M_{Moon}$ (primordial black holes)

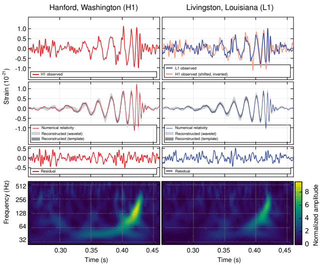
In GR, black holes can have any mass. Quantum mechanics, however, imposes a lower limit to BH mass equal to the Plank's mass $M_{\rm pl} \approx 2 \times 10^{-8}$ kg. The black hole evaporation timescale is [Hawking (1974)]:

$$\tau_{\rm evap} \approx 2 \times 10^{61} \times \left(\frac{M}{M_\odot}\right)^3 \ \text{years.}$$

Detection

Stellar mass black holes are observed via

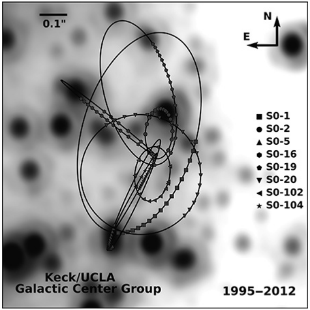
- Accretion disk & relativistic jets: X-ray binaries, microquasar ...
- Examples: Cygnus X-1, SS 433, GRS 1915+105 ...
- Gravitational waves: Merging black holes (LIGO/VIRGO), e.g. GW150914 ...

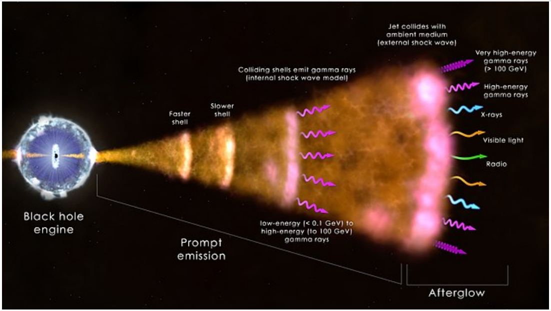


Detection

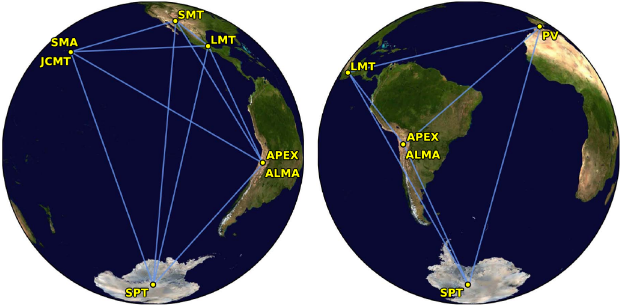
Supermassive black holes

- Active Galactic Nuclei (quasars, different Electromagnetic bands)
- Mapping stars orbits close to supermassive black holes at the center of our galaxy.
- Direct detection of the event horizon (radio waves Event Horizon Telescope)

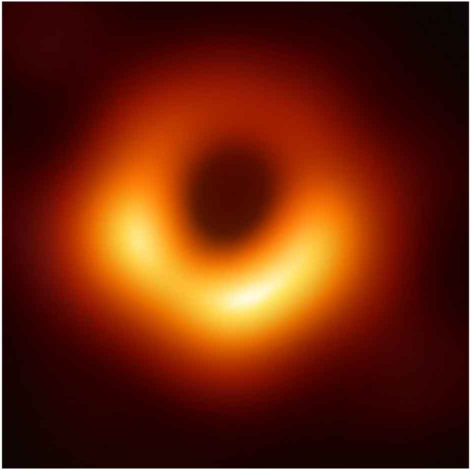




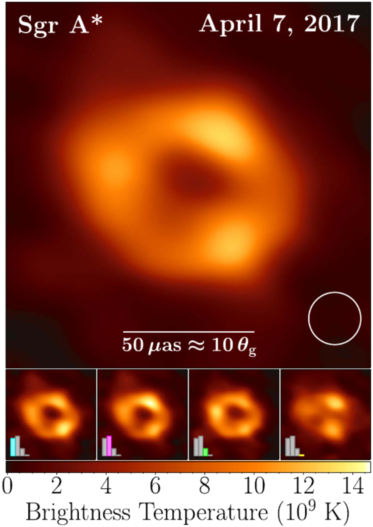
Event Horizon Telescope (EHT)



EHT Direct Observation of M87 Supermassive BH



EHT Direct Observation of Sag A* Supermassive BH in our Galaxy



Thanks