Problem Set 1 – Relativistic Kinematics from Historical Photographs

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Objectives

- Apply relativistic energy–momentum conservation to cosmic–ray events.
- Extract kinematic information from experimental observables (track curvature, decay geometry).
- Connect historical discoveries with quantitative particle identification.
- Develop scientific literacy by locating and reading original discovery papers.

Before the accelerator era, new particles were discovered in cosmic rays through photographic emulsions and cloud or bubble chambers. This problem set guides students to reproduce some of these analyses using real data and historical photographs.

Problem 1 – Energy–Momentum Invariant

Show that the Mandelstam variable

$$s = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2)$$

is Lorentz invariant. Interpret $\sqrt{s}$ physically in fixed–target and collider frames. Recognize $\sqrt{s}$ as the total available energy for new particle creation.

Problem 2 – Threshold for Pion Production

Consider the reaction

$$p + p \to p + p + \pi^0.$$

1. Derive the minimum beam kinetic energy required in a fixed–target experiment.
2. Compare with the case of two colliding proton beams of equal energy.
3. Evaluate numerically for $m_p = 938.27\,\text{MeV}$ and $m_{\pi^0} = 134.98\,\text{MeV}$.

Relate recoil energy loss to the motivation for collider experiments.

Problem 3 – The Young Researcher's Investigation: Learning from Bubble Chamber Data

Scenario: Imagine you are a young researcher in the 1950s, just introduced to a revolutionary new device — the *bubble chamber*. Your task is to understand how this detector works and to analyze real photographic data that led to the discovery of subatomic particles.

Part A – Understanding the Device Read carefully the educational document published by the Fermilab Education Office:

> *Particle Physics with Bubble Chamber Photographs*, Fermilab Education Office (1993).
> Available at: `https://lss.fnal.gov/archive/other/print-93-0553.pdf`

This short monograph explains:

- How the bubble chamber works (superheated liquid, particle ionization, expansion, photography).
- How tracks reveal particle charge, momentum, and interaction type.
- How to interpret curvature, vertex formation, and decay topologies.

Task 1. Write a concise summary (half a page) explaining:

- What physical principles make the bubble chamber work.
- What experimental information can be extracted from a track photograph.
- How the device contributed to discoveries in the 1950s–1970s.

Part B – Reconstructing a Discovery Select one famous bubble-chamber or related photograph from the literature or archives. A few examples (you may choose another of comparable significance):

1. **Discovery of the positron** (C. D. Anderson, 1932) – cosmic-ray track in a cloud chamber.
2. **Charged pion decay** ($\pi^+ \rightarrow \mu^+ + \nu_\mu$) – Powell group, 1947 (emulsion / bubble chamber).
3. **Strange particle (V-shaped decay)** – Rochester & Butler, 1947 (cloud chamber).
4. **Discovery of the Ω^- baryon** – Brookhaven, 1964 (bubble chamber).

Attach the chosen photograph (or cite its exact reference and figure number).
Useful archives for such photographs:

- CERN Bubble Chamber Archive: `https://cds.cern.ch/record/2809280`
- Bubble Chamber pictures for education: `https://cds.cern.ch/record/2307419`
- Science Photo Library: `https://www.sciencephoto.com/search?q=bubble+chamber`

Task 2. Using the information available in the photograph and article:

- Identify the visible charged particles and the decay topology.

- Measure approximate curvature radii r_i for each visible track and estimate their momenta from $p_i = 0.3\,B\,r_i \quad (\mathrm{GeV}/c)$ using the stated magnetic field B of the experiment.
- Determine the charge sign from curvature direction.
- Compute or estimate the mass of the parent particle using relativistic two-body (or multi-body) kinematics.
- State a reasonable uncertainty range, and compare with modern accepted values (from PDG).

Task 3. Write a short reflective note (half a page):

- What reasoning steps allowed you to infer the particle's mass or identity?
- What are the main experimental limitations (projection, unseen neutrals, energy loss, magnetic field precision)?
- If you were the experimentalist at that time, what would you have concluded from your observation?

Optional Extension – Historical Context Include bibliographic references (with links if available) to the original discovery paper corresponding to your chosen event, e.g.:

- Anderson, "The Positive Electron," *Phys. Rev.* **43**, 491 (1933).
- Lattes *et al.*, "Processes Involving Charged Mesons," *Nature* **159**, 694 (1947).
- Barnes *et al.*, "Observation of a Hyperon with Strangeness –3," *Phys. Rev. Lett.* **12**, 204 (1964).

Deliverable: A short report (2-3 pages) combining your measurement results, calculations, and reflections. You are encouraged to attach hand-measured sketches, annotations on the photo, or digital overlays showing your analysis.

Problem 4 – Pion Decay in a Bubble Chamber

Using a historical photograph (see Fig. 3), analyse the decay

$$\pi^+ \to \mu^+ + \nu_\mu$$

in a magnetic field $B \simeq 1.7$ T.

1. Measure the radii of curvature of the incoming pion (r_π) and outgoing muon (r_μ) tracks.
2. Compute the corresponding momenta using $p = 0.3\,B\,r(\mathrm{GeV/c})$).
3. Using two–body decay kinematics, estimate the pion mass and compare with the PDG value.
4. Discuss sources of uncertainty (projection, scattering, unseen neutrino, etc.).

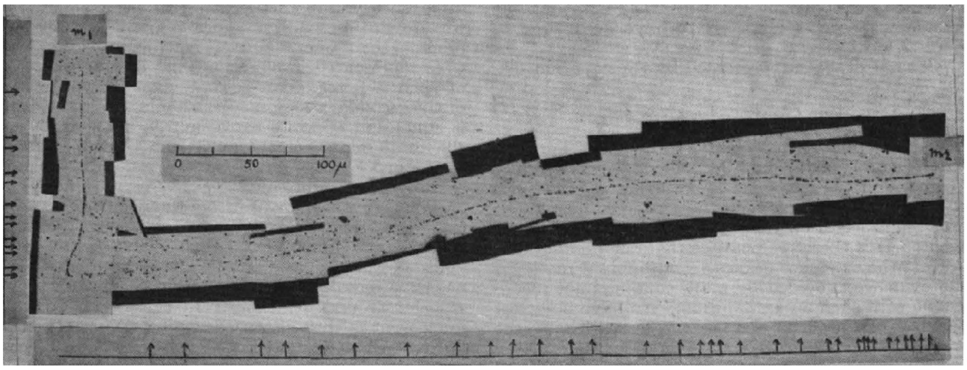

Figure 1: Example of $\pi^+ \to \mu^+ + \nu_\mu$ track in a bubble chamber (Griffiths Fig. 1.2 or CERN archive).

Problem 5 – High Energy Electron

A high–energy electron crosses a thin Pb plate inside a bubble chamber, see Fig. 2. Its curvature radius changes from r_1 to r_2 in a field $B = 2.0\,\mathrm{T}$ see figure.

1. Determine the fractional momentum loss $\Delta p/p = (r_1 - r_2)/r_1$.
2. Assuming the loss is dominated by bremsstrahlung, estimate the radiation length X_0 in lead.
3. What would change if the particle were a muon instead of an electron?

Hints: For the energy loss (ionization, radiation, bremsstrahlung etc) and the radiation length see PDG review on Passage of Particles Through Matter `https://pdg.lbl.gov/2025/reviews/contents_sports.html`.

Problem 6 – $K^+ \to \pi^+ + \pi^+ + \pi^-$ Decay

1. From the photograph Fig. 3, determine the direction of the magnetic field B (perpendicular te the photographic plate).
2. Identify the pions path (sign of the charge).
3. Identify which of the pion tracks has higher energy.

Problem 7 – Literature Exercise: Classic Cosmic–Ray Discoveries

Each student chooses one of the following papers (freely accessible):

1. **Discovery of the pion:**
   C. M. G. Lattes, H. Muirhead, G. P. S. Occhialini, and C. F. Powell, "Processes Involving Charged Mesons," *Nature* **159**, 694–697 (1947).
   `https://www.ghtc.usp.br/clattes.htm`

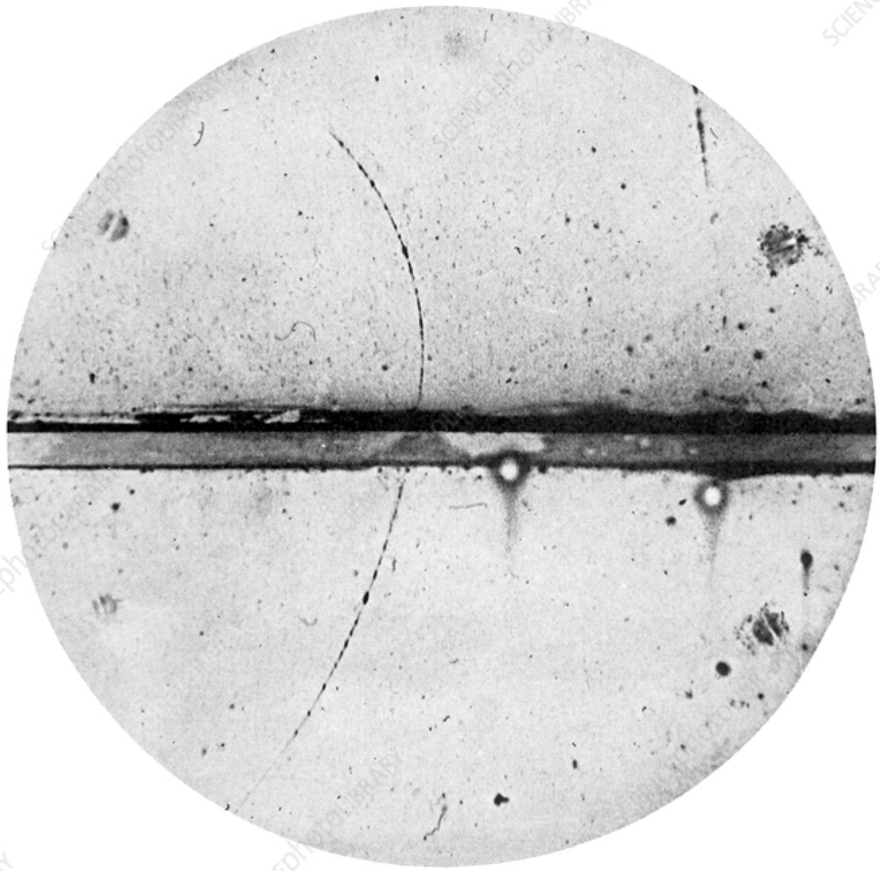

Figure 2: Ref: Science Photo Library `https://www.sciencephoto.com/media/1289910/view`

2. **First observation of a strange particle ("V–particle"):**
   G. D. Rochester and C. C. Butler, "Evidence for the Existence of New Unstable Elementary Particles," *Nature* **160**, 855–857 (1947).
   `https://pubmed.ncbi.nlm.nih.gov/18917296/`
   Context summary: `https://timeline.web.cern.ch/clifford-butler-and-george-rochester-c`

3. **Further observations of slow mesons:**
   C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, "Observations on the Tracks of Slow Mesons in Photographic Emulsions," *Nature* **160**, 486–492 (1947).
   `https://link.springer.com/chapter/10.1007/978-3-540-37354-4_5`

For the selected paper:

1. Retrieve and read the article (via the links above or InspireHEP).
2. Identify at least one figure with a particle track or mass measurement.
3. Write a brief note explaining how kinematics was used to deduce the new particle's mass, lifetime, or charge.

Goal: Build familiarity with original literature and understand reasoning from data to discovery.

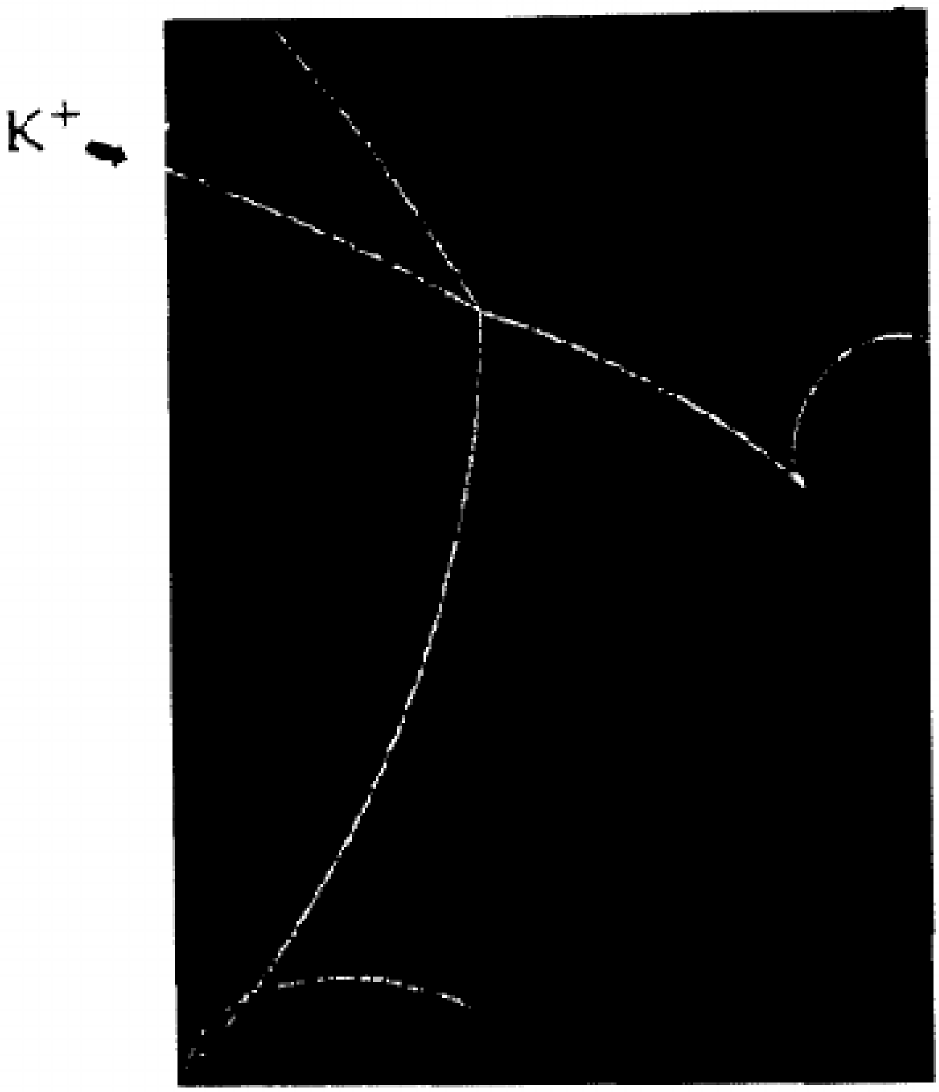

Figure 3: A K^+ meson decays in 3 pions ($K^+ \to \pi^+ + \pi^+ + \pi^-$); the π-mesons then decay (BGRT experiment, by courtesy of CERN, Geneva).