MODEL PREDICTIVE CONTROL FOR VELOCITY CONTROL OF PERMANENT MAGNET SYNCHRONOUS MOTOR (PMSM) IN AUTOMOTIVE ELECTRICAL-TRACTION DRIVE

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ABSTRACT

It is necessary to develop high-performance motor technologies in order to popularize the use of energy-efficient PMSMs. As known, conventional control methods such as field oriented control (FOC) method are widely used in industry. However, MPC for PMSMs has been found to enhance the speed-tracking performance compared to conventional controllers. In this paper, an MPC method established by the linearized continuous-time and discrete-time model of the PMSM is developed and implemented. Furthermore, the dynamic performance and steady-state behavior of the MPC controller are evaluated and discussed by means of motor speed tracking. From the simulation results, the MPC method performs incredibly well with fast response time and reduced oscillations.

1. INTRODUCTION

The permanent magnet synchronous motor (PMSM) has been utilized extensively in low- and medium-power systems, such as robots, agricultural applications, and electric vehicles. A PMSM drive system is an electric motor that uses permanent magnets in the rotor to produce a magnetic field that interacts with the stator's magnetic field to generate torque. It is made up of a PMSM motor, an inverter, a controller, and a power supply. The inverter converts the DC voltage from the power supply into an AC voltage that drives the motor, and the controller monitors the motor's speed and stator winding currents to adjust the inverter's output voltage and frequency to maintain the desired speed and torque.

The main control techniques currently deployed for PMSM systems include field-oriented control (FOC) and direct torque control (DTC). However, the PI controller used for FOC has shortcomings in drive applications requiring overshoot and high dynamic performance, whilst the DTC is impacted by large torque fluctuations [1]. Model predictive control (MPC) has garnered considerable interest in the field of power drives as an alternative algorithm [2], [3]. Unlike the traditional PI control method, MPC incorporates future output, whereas the traditional PI algorithm only considers past and present tracking errors and not future errors [4]. Therefore, the MPC has theoretically superior performance.

In this paper, MPC is applied to a PMSM operating at rated conditions. In Chapter 2, developments in the implementation of MPC in motor drive systems are discussed. In Chapter III, the linearized PMSM model and open loop system analysis are presented. Based on theory, the model predictive controller is designed in Chapter IV. The simulation results are discussed in chapter V. Finally, chapter VI concludes this paper.

2. LITERATURE REVIEW

MPC used in PMSM control can be separated into finite control set MPC (FCS-MPC) and continuous control set MPC (CCS-MPC) [5]. During the process of establishing a predictive model, the converter model is considered for FCS-MPC, so the output of this type of controller is the control signal for the switch device [6]. CCS-MPC establishes the predictive model of PMSM and then implements motor control via modulation, such as space vector pulse width modulation. CCS-MPC is more appropriate for PMSM drives because it is simpler to guarantee high dynamics and low current and speed ripple. e.g. [7], [8].

The speed control system of PMSM based on MPC also can be divided into cascade structure and non-cascading structure. The speed control system based on cascade MPC structure can reduce the motor speed ripple and improve the system robustness [9], [10]. Due to the absence of a cascade bandwidth limitation, the non-cascading structure can further enhance the dynamic performance of the system [11]. The various non-cascading structures have been proposed in [11].

[12] proposes and implements model predictive direct speed control (MP-DSC), a technique that overcomes the limitations of cascaded linear controllers. The MP-DSC employs a finite control set approach in which possible plant inputs are applied to an online plant model and their effects are predicted until the prediction horizon is attained. The results are supplied to a decision or cost function, which determines the plant input, i.e. the switching state of the converter for the subsequent sampling interval. Included are secondary control objectives such as tracking maximum torque per ampere.

In [13], an MPC based on a full-speed domain control

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strategy is proposed for electric vehicles with high dynamic performance. The diverse control strategies for PMSM at various speeds necessitate the calculation of the current control principle. Then, the MPC controller is designed to supplant the PI controller, and the dynamic performance of the PMSM system based on non-cascading MPC is significantly enhanced compared to traditional PI.

3. MODELING AND ANALYSIS

3.1. Continuous-Time and Discrete-Time Model of the PMSM

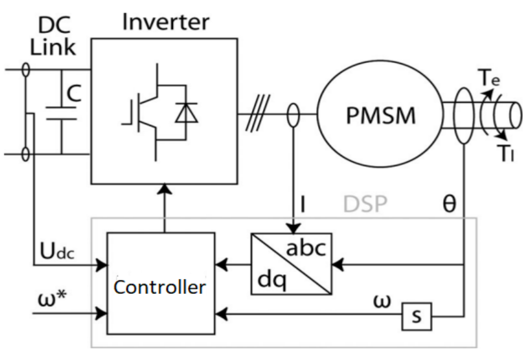

Fig. 1. PMSM - Voltage Source Inverter Drive System

The mathematical model of PMSM in the dq-axis synchronous rotating reference frame is described as (1),

$$\begin{aligned} \frac{di_d}{dt} &= \frac{1}{L_d}\left(u_d - Ri_d + \omega_e L_q i_q\right) \\ \frac{di_q}{dt} &= \frac{1}{L_q}\left(u_q - Ri_q - \omega_e L_d i_d - \omega_e \psi_f\right) \\ \frac{d\omega_e}{dt} &= \frac{3p^2}{2J}\left[\psi_f i_q + \left(L_d - L_q\right) i_d i_q\right] - \frac{B}{J}\omega_e - \frac{p}{J}T_L \end{aligned} \tag{1}$$

Where i_d, i_q are stator dq-axis current, u_d, u_q are the stator dq axis voltage and L_d, L_q are stator dq-axis inductor. R, p, ω_e and ψ_f are stator resistance, number of pole pairs, rotor electrical angular speed and permanent magnet flux linkage, respectively. B, T_L, T_e and J is the viscous coefficient, load torque, electromagnetic torque and the moment of inertia respectively.

It is easy to note that (15) contains nonlinear terms $\omega_e i_d$, $\omega_e i_q, i_d i_q$. Using the first order Taylor series at the equilibrium point ω_e^0, i_d^0 and i_q^0 for linearization. The linear expression is given as follows:

$$\left\{ \begin{array}{l} \frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{D} \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{array} \right.$$

A=

$$\begin{bmatrix} -\frac{R}{L_d} & \frac{L_q}{L_d}\omega_e^0 & \frac{L_q}{L_d} i_q^0 \\ -\frac{L_d}{L_q}\omega_e^0 & -\frac{R}{L_q} & -\frac{L_d}{L_q} i_d^0 - \frac{\psi_f}{L_q} \\ \frac{3p^2}{2J}\left(L_d - L_q\right) i_q^0 & \frac{3p^2}{2J}\left[\left(L_d - L_q\right) i_d^0 + \psi_f\right] & -\frac{B}{J} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} \frac{T_s}{L_d} & 0 \\ 0 & \frac{T_s}{L_q} \\ 0 & 0 \end{pmatrix}, \boldsymbol{D} = \begin{pmatrix} -\frac{L_q}{L_d}\omega_e^0 i_q^0 \\ -\frac{L_d}{L_q}\omega_e^0 i_d^0 \\ \frac{3p^2}{2J}\left(L_d - L_q\right) i_d^0 i_q^0 - \frac{p}{J}T_L \end{pmatrix}$$

$$\boldsymbol{y} = \boldsymbol{x} = \begin{bmatrix} i_d \\ i_q \\ \omega_e \end{bmatrix} \boldsymbol{u} = \begin{bmatrix} u_d \\ u_q \end{bmatrix} \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.2. Analysis of System

The drive system (plant) is found to be internally stable i.e. matrix A is Hurwitz. This can also be seen in Figure 2 where the states approach zero with time. Internal stability of the system sets the foundation for designing effective control algorithms to achieve desired performance objectives.

It is possible to find an input sequence that can drive the system from any initial state to any desired state in a finite time. In other words, the drive system is controllable i.e. the controllability matrix is denoted as

$$C_{ab} = [B \;\; AB \;\; A^{n-1}B]$$

is full rank.

The states of the system can be accurately estimated from the available output measurements. Thus, the drive system is observable i.e. the observability matrix is denoted as

$$O = [C \;\; CA \;\; CA^{n-1}]^T$$

is full rank.

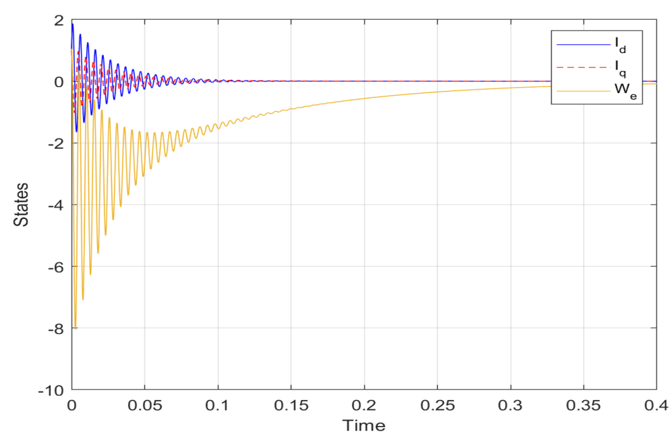

Fig. 2. Simulation of open loop system

Figure 2 plots the states of the open loop system from some initial conditions with time. Though the open loop system is internally stable, the oscillations in the states, as well as their slow response, necessitate the need for a controller.

4. WORK DONE - MPC

4.1. FOC theory

The process for FOC has three steps: First, the three-phase currents transform into dq currents. Then, $\alpha\beta$ voltage signals are calculated from the park inverse transformation. Finally, the switch signals of the inverter can be calculated according to the SVPWM modulation.

The three-phase currents i_a, i_b, i_c and motor speed ω are sampled by the sensor, and i_a and i_b are transformed into i_d and i_q by the Clarke and Park transformation. The error of the speed signal is obtained from the outer speed loop and the error of the current signal is obtained from the inner current loop. The error of the speed signal is input to the PI controller to generate the i_q reference current, and the error of the current signal is input to the PI controller to generate the u_d and u_q reference voltage. And voltage signals, u_α and u_β, can be generated through the park inverse transformation from u_d and u_q. Based on u_α and u_β, the PWM duty cycle is calculated, and the SVPWM modulator generates the pulsing signals in order to control the inverter [1].

4.2. MPC theory as applies to the PMSM model

MPC theory consists of three parts: predictive mode establishment, cost function definition, and cycle optimization.

4.2.1. *Predictive Model Establishment*

The motor system is a multiple input and multiple output (MIMO) system used to predict the N_p step ahead states at each time instant k along the prediction horizon. The discrete state-space model of MIMO system can be described as:

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}_m\boldsymbol{x}(k) + \boldsymbol{B}_m\boldsymbol{u}(k) + \boldsymbol{D}\boldsymbol{w(k)} \\ \boldsymbol{y}(k) &= \boldsymbol{C}_m\boldsymbol{x}(k) \end{aligned} \tag{2}$$

Where $\boldsymbol{u}$ is the vector of input variable, $\boldsymbol{y}$ is the vector of output variable, and $\boldsymbol{x}$, is the vector of the state variable. $\boldsymbol{A}_\mathrm{m}, \boldsymbol{B}_\mathrm{m}$ and $\boldsymbol{C}_\mathrm{m}$ are the state matrix, input matrix and output matrix of appropriate dimension, respectively. Besides, $\boldsymbol{D}$ is the disturbance matrix, and the disturbance $(\boldsymbol{w(k)})$ is taken as unity for all k.

4.2.2. *Cost Function Definition*

The cost function is generally expressed as a quadratic function of the difference between the predicted output and the reference trajectory. Assume the present instant is k_i and the state variable can be measurable at this moment. Then the future state variables can be calculated as:

$$\begin{gathered} \boldsymbol{x}(k_i+1 \mid k_i) = \boldsymbol{A}\boldsymbol{x}(k_i) + \boldsymbol{B}\boldsymbol{u}(k_i) \\ \vdots \\ \boldsymbol{x}(k_i+N_\mathrm{p} \mid k_i) = \boldsymbol{A}^{N_\mathrm{p}}\boldsymbol{x}(k_i) + \ldots + \boldsymbol{A}^{N_\mathrm{p}}\boldsymbol{B}\boldsymbol{u}(k_i+1) \end{gathered} \tag{3}$$

The predictive output variables also can be calculated by state variables at k_i :

$$\begin{gathered} \boldsymbol{y}(k_i+1 \mid k_i) = \boldsymbol{C}\boldsymbol{A}\boldsymbol{x}(k_i) + \boldsymbol{B}\boldsymbol{u}(k_i) \\ \boldsymbol{y}(k_i+2 \mid k_i) = \boldsymbol{C}\boldsymbol{A}^2\boldsymbol{x}(k_i) + \boldsymbol{C}\boldsymbol{A}\boldsymbol{B}\boldsymbol{u}(k_i) + \boldsymbol{C}\boldsymbol{B}\boldsymbol{u}(k_i+1) \\ \vdots \\ \boldsymbol{y}(k_i+N_\mathrm{p} \mid k_i) = \boldsymbol{C}\boldsymbol{A}^{N_\mathrm{p}}\boldsymbol{x}(k_i) + \ldots + \boldsymbol{C}\boldsymbol{A}^{N_\mathrm{p}}\boldsymbol{B}\boldsymbol{u}(k_i-1 \end{gathered}$$

(4)

Where N_p is the prediction horizon. Thus it is obvious that the predicted output is a direct function of the present state variable $\boldsymbol{x}(k_i)$ and future control variables, $\boldsymbol{u}(k_i+j), (j=0,1\ldots,N_\mathrm{c}-1)$ in (4). Then transform (4) into matrix form is described as.

$$\boldsymbol{Y} = \boldsymbol{F}_I(k_i) + \boldsymbol{\Phi}\boldsymbol{U} \tag{5}$$

Where

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{CA} \\ \boldsymbol{CA}^2 \\ \vdots \\ \boldsymbol{CA}^{N_\mathrm{p}} \end{bmatrix}$$

$$\boldsymbol{\Phi}\text{=}\begin{bmatrix} \boldsymbol{CB} & 0 & \ldots & 0 \\ \boldsymbol{CA}B & \boldsymbol{CB} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ \boldsymbol{CA}^{N_p-1}B & \boldsymbol{CA}^{N_\mathrm{p}-2}\boldsymbol{B} & \ldots & \boldsymbol{CA}^{N_p\boldsymbol{B}} \end{bmatrix} \boldsymbol{Y} = $$

$[\boldsymbol{y}(k_i+1), \ldots, \boldsymbol{y}(k_i+N_p)]^T$,

$\boldsymbol{U}$=$[\boldsymbol{u}(k_i), \ldots, \boldsymbol{u}(k_i+N_p-1)]^T$.

The purpose is to find an optimal control variable to minimize the difference between the reference and the predicted output. Therefore, the cost function can be described as a minimization of the function $\mathcal{L}(k, u(k), x(k))$:

$$\min_{\mathbf{y}(1\ldots N_p),\mathbf{u}(1\ldots N_p)} \sum_{i=0}^{N_p-1} \mathcal{L}(k, u(k), x(k)) \tag{6}$$

$\mathcal{L}$ involves a tracking objective term of the objective criteria:

$$(\boldsymbol{r}_\mathrm{s} - \boldsymbol{Y})^T Q (\boldsymbol{r}_\mathrm{s} - \boldsymbol{Y}) \tag{7}$$

Where Q is a symmetric positive semi-definite weighting matrix of appropriate dimension. $\boldsymbol{r}_s$ is the reference. The second criterion is the minimization of the change in the control effort from one time instant to the other (slew rate).

$$\Delta \boldsymbol{U}^T \overline{\boldsymbol{R}} \Delta \boldsymbol{U} \tag{8}$$

$\bar{R}$ is the positive definite weighting coefficient on the slew rate $\Delta \boldsymbol{U}$. The optimization problem is subject to the constraints on the output $y(k)$ and input $u(k)$ and the dynamical system (5). Thus the optimization problem is posed as:

$$\min_{\mathbf{y}(1\ldots N_p),\mathbf{u}(1\ldots N_p)} \sum_{i=0}^{N_p-1} \mathcal{L}(k, u(k), x(k)) \tag{9}$$

$$\text{subject to} \tag{10}$$

$$x(k_i + 1|k) = Ax(k_i|k) + Bu(k_i|k) + Dw \tag{11}$$

$$u_{min} \leq u(k_i|k) \leq u_{max}, \tag{12}$$

$$y_{min} \leq y(k_i + 1|k) \leq y_{max}. \tag{13}$$

4.2.3. Cycle Optimization

After calculating the vector solution from the posed optimization problem

$$\boldsymbol{U} = (u(k_i + 1|k), u(k_i + 2|k), ..., u(k_i + N_p|k))$$

the first entrant in the solution set U, (i.e. $(u(k_i + 1|k))$ is applied to the plant for the output. This output serves as the initial condition for the subsequent time step. At the next sampling time, the calculation of the cost function is repeated online. Through continuous sampling and data updating, the error between the system output and the reference point can be corrected.

4.3. Implementation and Simulation

The MPC algorithm was implemented using yalmip in MATLAB (see appendix for code) as follows:

1. The parameters of the motor and the MPC algorithm are initialized. The system dynamics are defined using the state-space representation of the PMSM. The sampling time for the MPC is also defined.

2. The continuous-time system dynamics are discretized using zero-order hold to obtain the discrete-time system.

3. Next, the optimization problem is defined. The objective function consists of two terms. The first term is the quadratic cost of the control inputs, which penalizes large changes in the control input, called the slew rate. The second term is the quadratic cost of the error between the predicted state and the reference state, which penalizes deviations from the desired state.

4. Constraints may be defined to ensure that the control inputs and the predicted states satisfy physical constraints. These constraints include limits on the control inputs and limits on the predicted states.

5. The optimization problem is solved at each time step of the MPC using a quadratic programming solver.

6. Finally, the control input for the first time step is applied to the system, and the process is repeated for subsequent time steps. The optimization problem is solved at each time step using the most recent measurements of the system states, and the predicted states and inputs are updated accordingly.

TABLE I. MOTOR PARAMETERS

Parameters	Value
Stator resistance (R)	0.636Ω
d-axis inductance (L_d)	12mH
q-axis inductance (L_q)	20mH
flux linkage (ψ_f)	0.088 Wb
number of pole pairs (p)	5
Rated power (P)	2.3 kW
Rated torque (T_s)	$7.8\ \text{N}\cdot\text{m}$
Rated speed (w_e)	1200r/min
Rated voltage (V)	280 V
Rated current (I)	8 A

5. DISCUSSION OF RESULTS

The goal of our control problem was to achieve reference tracking of motor speed. The reference speed is set to 1200 rpm and MPC is utilized to obtain the otpimal inputs U_d and U_q at each time step. 1200 rpm is the speed we expect the motor to run at rated conditions.

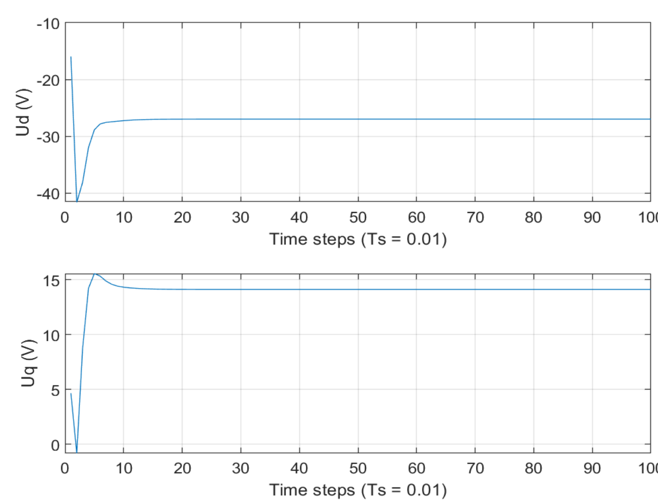

Fig. 3. Optimal inputs U_d and U_q

Figure 3 shows the optimal inputs needed to track the reference speed of 1200 rpm assuming the motor is initially at standstill (speed is 0 rpm).

Figure 4 compares the speed response of the closed loop system with MPC control to the reference speed.

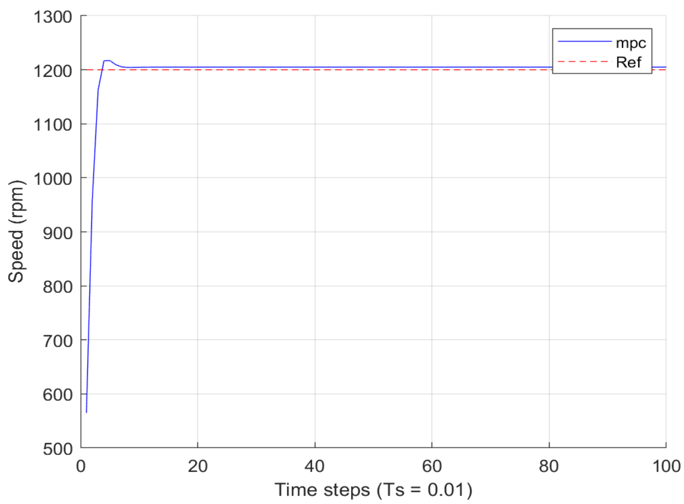

Fig. 4. MPC controller response to reference speed

The MPC controller closely tracks the set speed of 1200 rpm, with the following performance metrics.

TABLE II. MPC PERFORMANCE METRICS

Metric	Value
Root mean square error (RSME)	987.5
Rise time	0.016 s
Settling time	0.035 s
Overshoot	0.996%

The root mean square error was calculated from 10s, so as to calculate the error for steady-state operation.

6. CONCLUSION AND FUTURE WORK

This paper utilizes MPC to successfully track the reference speed of a PMSM drive system. The PMSM continuous time model was linearized and the control problem was formulated as an objective problem. The objective problem was then solved to generate the optimal inputs to control the motor speed to achieve reference tracking. The MPC controller implemented demonstrated a fast tracking response with very little oscillations and damping.

The MPC controller can be compared to other control techniques such as PI and LQR to assess its performance holistically. Additionally, different speed ranges of a PMSM require different control strategies [12]. Hence, the MPC implementation in this paper would have to be modified for full-speed domain control of a PMSM.

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