A Gradient Vector based Algorithm for *Reduction of N - Dimensional Piecewise Affine Models for Real-Time Motor Control*

ABSTRACT

The use of Piecewise Affine (PWA) models has gained popularity in various control applications including motor control due to their ability to approximate non-linear functions for which only sample points are available. In motor control, PWA functions can be used to build locally linear flux-linkage magnetic models of motors enabling robust controls and efficient operation. High-performance control of electric machines utilizing higher dimension PWA models can capture other non-linearities such as temperature, position-dependent parameters, and inverter nonlinearity. However, as the dimensionality of the model increases, the memory size and processing time of the PWA model drastically increases. Data reduction schemes are essential for efficient data storage. This digest presents a gradient vector-based algorithm to reduce n-dimensional PWA models by selecting data points for removal.

1. INTRODUCTION

Piecewise Affine (PWA) also known as Piecewise Linear (PWL) functions are used to model the control plant in many control problems because they possess desirable properties [1,2]. PWA functions are linear, bidirectional, and continuous. Control algorithms, such as MPC, exploit piecewise linearity of PWA models to design control actions and constraints effectively [3]. Particularly in motor control, the non-linear approximation capability allows PWA models to represent different operating modes of the motor and capture variations in behavior under different conditions. Computationally efficient representation of PWA models is suitable for embedded systems with limited resources. Lastly, PWA models can be easily adapted and identified based on available data to maintain accuracy.

The controller performance for an electric motor depends on the accuracy of the machine parameters [4,5]. The machine parameters are typically accounted for by online parameter estimation, offline parameter look-up tables (LUTs), or a combination of two. The offline method is preferred as it requires less real-time control computation time as compared to the online method. The offline parameter LUTs use the data of the machine from analytical calculations, FEA analysis, and experimentation (or any combination) to approximate the parameters given various operating points of the machine.

The many datapoints are interpolated by a PWA function through Delaunay triangulation. However, overly dense data results in an unnecessarily large amount of piecewise affine functions. This results in a large memory sized offline parameter LUT which must be uploaded to a limited storage microcontroller. The simplification of high-fidelity PWA frees ups space for higher dimensional PWA models, capturing more machine parameters and their interdependencies while still maintaining sufficient accuracy for efficient motor control. A survey of several schemes of data reduction of piecewise linear curves are provided in [6]. Curvature approximation and data reduction for triangulated surfaces are discussed in [7,8]. A method for selecting data points from a given finite set of curve points is presented in [9]. The methods presented in [6-9] fail to generalize their reduction schemes to higher dimensional PWA functions.

A heuristic N-dimensional PWA model reduction method is presented [2] to optimize the magnetic model of a Permanent Magnet Synchronous Machine (PMSM). The input domain is current while the output domain is flux linkage. If the number of datapoints is less than the allotted number of points, the algorithm iteratively adds points to minimize the maximum error until the desired number is reached. The digest is structured as follows. The simplification algorithm is described in Section 2, and its effectiveness assessed in Section 3. Section 4 concludes the digest.

2. N DIMENSIONAL PWA REDUCTION ALGORITHM

Consider the most general PWA model given by $f: R^N \rightarrow R^M$ where N and M denotes the input and output space dimension respectively. It is defined by dividing the input space into non-overlapping regions, also known as simplices or polytopes, and assigning an affine function to each region. Mathematically, it can be expressed as:

$$f_i(x) = \{A_i x + b_i\} \, \forall \, x \in S_i$$

where $f_i(x)$ represents the affine function for the i-th region, A_i is the M x N matrix specific to the i-th region, b_i is the M-dimensional vector specific to the i-th region and S_i is the i-th region (simplex or polytope) in the input space. It is important to note that each row of A_i is the gradient vector of the i-th region with respect to one output variable. The algorithm begins with as many datapoints as possible, and is implemented as follows:

1. Iterate through each vertex (datapoint) and at each datapoint,

a. Identify the index of all simplices common to that datapoint.

b. Calculate the Euclidean distance (E(·)) between each pair of gradient vectors associated with the simplices.

For A_i, $A_j \in R^{M \times N}$, E (A_i, A_j) = $\|A_i - A_j\|_2$

c. The arithmetic mean of the calculated Euclidean distances is taken as the similarity value for that vertex.

2. Each vertex is ranked based on its similarity value with lower similarity values indicating points more likely to be eliminated. The user may decide to keep certain points in the list to preserve some desirable characteristics just as boundary points.

3. Remove desired points in order of rank until the desired number of points remaining are left.

The Euclidean distance measures the geometric distance between gradient vectors capturing both the direction and magnitude of the vectors.

3. TEST RESULTS AND APPLICATIONS

The N-dimensional PWA model reduction technique has been tested for univariate (1D) and bivariate (2D) functions. RMS error is used as the error metric and is computed as $\sqrt{\frac{\sum_{i=1}^{n}\|Y_o - Y_{pwa}\|_2}{n}}$, where Y_o is the original function output, Y_{pwa} is the PWA function approximation output and n is the initial number of datapoints. The functions used in this test are of parabola, exponential function, and trigonometric function forms as described in Table 1. The initial datapoints for the 1D and 2D tests are 101 and 121 respectively.

Table 1 Properties of the functions sampled and used for testing our reduction algorithm.

Curve	1D function	2 D function	Input interval (1D)	Input interval (2D)
Parabola	$y_1 = {x_1}^2$	$y_1 = {x_1}^2 + {x_2}^2$	1:0.02:1	-1:0.2:1, -1:0.2:1
Exponential	$y_1 = e^{x_1}$	$y_1 = e^{x_1} + e^{x_2}$	0:0.02:2	0:0.2:2, 0:0.2:2
Trigonometric	$y_1 = \sin(\pi x_1)$	$y_1 = \sin(\pi x_1) + \sin(\pi x_2)$	0:0.02:2	0:0.2:2, 0:0.2:2

The algorithm in its current state has shown promising results as seen in Figure 1, Figure 2, and Figure 3. It is observed that the algorithm achieves an acceptable margin of error when about 50% of the initial datapoints are removed. It is our aim to test the algorithm against the method given in [2,7]. Additionally, it would be useful to apply the algorithm to PMSM and Wound Rotor Synchronous Machine (WRSM) data. This would help assess its efficiency and applicability.

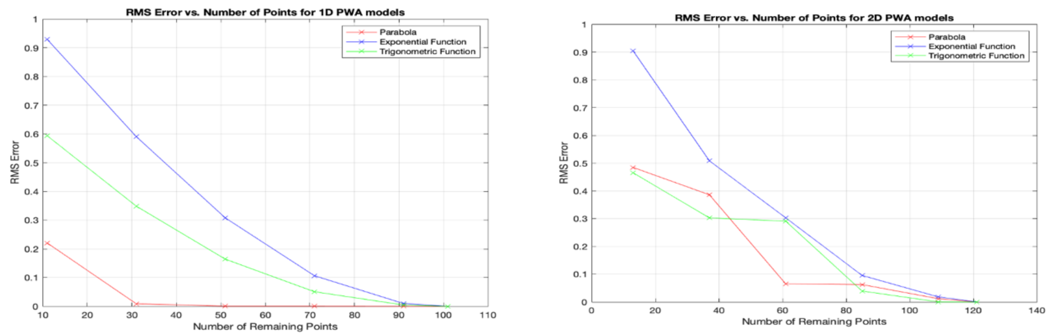

Figure 1: RMS error for the various 1D and 2D PWA function approximations variation with datapoints in the PWA model

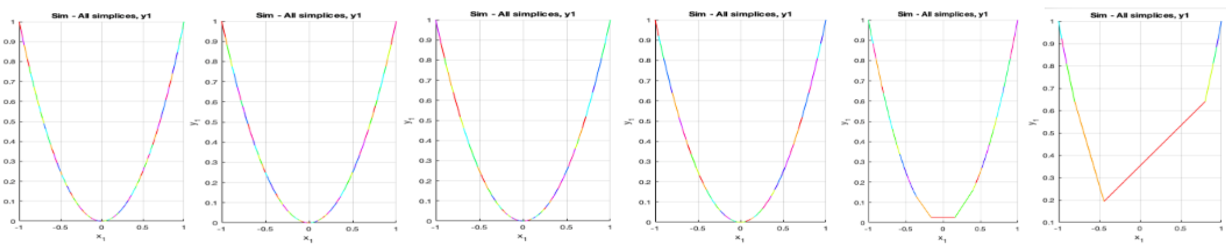

Figure 2: PWA approximation of 1D parabola function with n = 101, 91, 71, 51, 31 and 11 datapoints (from left to right)

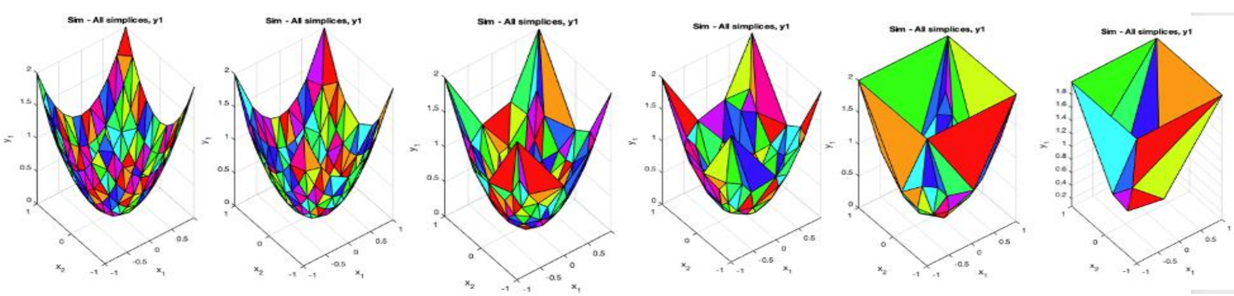

Figure 3: PWA approximation of 2D parabola function with n = 121, 109, 85, 61, 37 and 13 datapoints (from left to right)

4. CONCLUSIONS AND FUTURE WORK

A novel N-dimensional PWA reduction algorithm is presented in this digest. The algorithm can be improved by updating the list of best datapoints to be removed after a datapoint is removed. This would tend to be computationally expensive but would yield better levels of accuracy. The algorithm contributes to the advancement of motor control technologies by offering significant benefits in terms of memory usage, processing time, and high-fidelity model approximation. High fidelity in higher PWA dimensions would enable efficient and robust motor control by capturing more variations in motor dynamics.

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