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COURSE: CSC 333
LEVEL: 300L
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LINEAR PROGRAMMING PROBLEMS

1 Maximizing Profit for a Factory

Objective function (maximize): $3x + 4y$

Constraints

Machine Time = $2x + 3y \leq 12$
Raw material = $x + 2y \leq 8$
non-Negativity = $x \geq 0, \ y \geq 0$

Decision variables:
x = Number of units of product A produced
y = Number of units of product B produced

	Product A	Product B	Total
machine time	2	3	12
Raw material	1	2	8
Profit	3	4	

Corner point coordinates

$x = 0$ and $y = 0$

let $y = 0$ in the machine Time Constraint

$2x + 3(0) = 12$

$\frac{2x}{2} = \frac{12}{2} = 6 \therefore x = 6 \Rightarrow (6, 0)$

let $x = 0$ in the machine time constraint

$2(0) + 3y = 12$

$\frac{3y}{3} = \frac{12}{3} = 4 \quad \therefore \; y = 4 \quad \rightarrow (0,4)$

machine time $\rightarrow 2x + 3y = 12 \; \cdots$ ①

Raw material $\rightarrow x + 2y = 8 \; \cdots$ ②

$x = 8 - 2y \; \cdots\cdots$

Put $x = 8 - 2y$ into equation 1

$2(8-2y) + 3y = 12$

$16 - 4y + 3y = 12$

$\frac{-1y}{-1} = \frac{-4}{-1}$

$y = 4$

Put $y = 4$ into equation ②

$x = 8 - 2y$

$x = 8 - 2(4)$

$x = 0. \quad \Rightarrow (0,4)$

Corner point	(x,y)	objective function $(3x+4y)$
A	$(0,0)$	$3(0) + 4(0) = 0$
B	$(6,0)$	$Z = 18$
C	$(0,4)$	$Z = 16$
D	$(0,4)$	$Z = 16$

Optimal Solution:

The maximum profit is $Z = 18$ achieved at the corner point $(6,0)$. Factory should produce 6 unit of product A and 0 units of product B to maximize profit

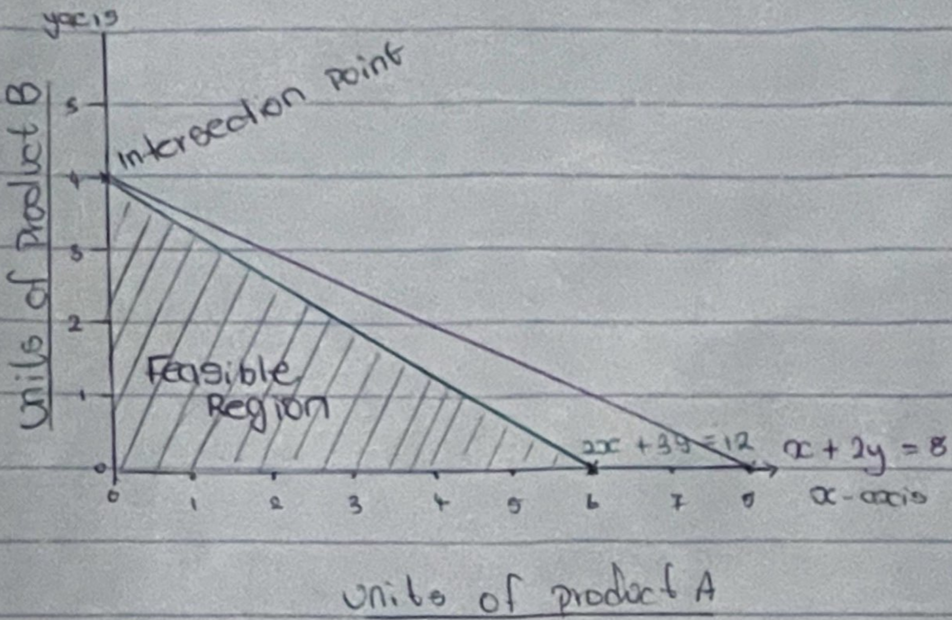
Maximize Machine time $= 2x + 3y = 12$

Boundary line : $(6, 0)$ and $(0, 4)$
$\quad x \; y \qquad x \; y$

Raw material $= x + 2y = 8$

Boundary line : $(8, 0)$ and $(0, 4)$
$\quad x \; y \qquad x \; y$

Graphical solution for LP



The Intersection point is $(0, 4)$ for $2x + 3y = 12$ and $x + 2y = 8$

2. Minimizing Cost for a Manufacturer

Objective function (minimize) $2x + 5y$

Constraints

Labour : $x + 2y \leq 6$

Material : $2x + y \leq 5$

non-negativity : $x \geq 0$, $y \geq 0$

Decision variables :

x = Numbers of hours of Product A produced

y = Number of units of

	Labour	material	Total ~~Cost~~
Product x	1	2	6
Product y	2	1	5
Cost ~~Total~~	2	5	

Corner Point Coordinates

Let $x = 0$ and $y = 0$

$(0,0)$

$x + 2y = 6$

$(0) + \frac{2y}{2} = \frac{6}{2} = 3$

$y = 3 \quad (0,3)$

$x + 2(0) = 6$

$x = 6 \quad (6,0)$

$x + 2y = 6 \text{ ---- } ①$

$2x + y = 5 \text{ ---- } ②$

$x = 6 - 2y$

Put $x = 6 - 2y$ into equation ②

$2(6 - 2y) + y = 5$

$12 - 4y + y = 5$

$12 - 3y = 5$

$-3y = 5 - 12$

$\frac{-3y}{-3} = \frac{-7}{-3} \quad \therefore y = \frac{7}{3}$ or 2.33

Put $y = 7/3$ into equation ②

$2x + 7/3 = 5$

$2x = 5 - \frac{7}{3}, \frac{2x}{2} = \frac{2.67}{2} \qquad \therefore x = 0.67 \,/\, \frac{2}{3}$

$\therefore \left(\frac{2}{3}, \frac{7}{3}\right)$

Corner point	(x, y)	Objective function $(2x + 5y)$
A	$(0,0)$	$2(0) + 5(0) = 0$
B	$(0,3)$	$2(0) + 5(3) = 15$
C	$(6,0)$	$2(6) + 5(0) = 12$
D	$(2/3, 7/3)$	$2(2/3) + 5(7/3) = 13$

Optimal Solution :

The minimum production cost occurs at $(0,0)$ with $Z = 0$

Labour : $x + 2y = 6$

Let $x = 0$ $\therefore$ $0 + \frac{2y}{2} = \frac{6}{2}$ 3 $\therefore$ $y = 3$ $(0,3)$

let $y = 0$ $\therefore$ $x + 2(0) = 6$ $\therefore$ $x = 6$ $(6,0)$

Labour boundary line : $\overset{x}{(0}.\overset{y}{3)}$ $\overset{x}{(6},\overset{y}{0)}$

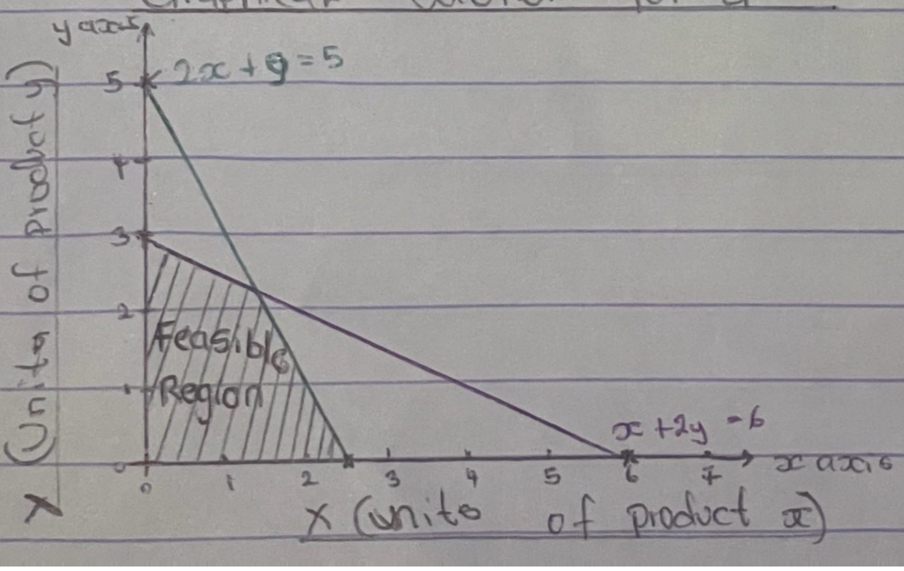
Material : $2x + y = 5$

let $x = 0$ $\therefore$ $2(0) + y = 5$ $\therefore$ $y = 5$ $(0,5)$

let $y = 0$ $\therefore$ $\frac{2x}{2} + 0 = \frac{5}{2}$ $\therefore$ $x = 2.5$ $(2.5,0)$

material boundary line : $(0,5)$ $(2.5,0)$

Graphical Solution for LP



3. Maximizing production with multiple Resources

Objective Function (maximize) : $5A + 4B = Z$

Constraints

Labour : $2A + B \le 20$

Material : $3A + 2B \le 30$

Machine Time : $A + 2B \le 18$

Non-Negativity : $A \ge 0$, $B \ge 0$

Decision Variables :

x = number of unit product A produced

y = number of unit product B produced

Graphical solution

let $A = 0$ and $B = 0$ $(\overset{x}{0}, \overset{y}{0})$

Labour : $2(0) + B = 20$ $\therefore B = 20$ $(0, 20)$ A

$\frac{2A}{2} + 0 = \frac{20}{2}^{10}$ $\therefore A = 10$ $(10, 0)$ B

material : $3(0) + \frac{2B}{2} = \frac{30}{2}^{15}$ $\therefore B = 15$ $(0, 15)$ C

$\frac{3A}{3} + 2(0) = \frac{30}{3}^{10}$ $\therefore A = 10$ $(10, 0)$ D

machine Time : $(0) + \frac{2B}{2} = \frac{18}{2}^{9}$ $\therefore B = 9$ $(0, 9)$ E

$A + 2(0) = 18$ $\therefore A = 18$ $(18, 0)$ F

	Resources			
	Labour	material	machine Time	Profit
Product A	2	3	1	5
product B	1	2	2	4
Total	20	30	18	

The Intersection point

Labour : $2A + B = 20 \quad \ldots\ldots (1)$

material : $3A + 2B = 30 \quad \ldots (2)$

Machine Time : $A + 2B = 18 \quad \ldots\ldots (3)$

$A = 18 - 2B$

Put A into equ (1)

$2(18 - 2B) + B = 20$

$36 - 4B + B = 20 \quad \therefore -3B = 20 - 36 \quad B = \frac{16}{3}$ or 5.33

$\frac{-3B}{-3} = \frac{-16}{-3}$

Put B into equation (3)

$A = 18 - 2B \quad A = 18 - 2\left(\frac{16}{3}\right) = 18 - \frac{32}{3} = 18 - 10.67 = 7.33 = \frac{22}{3}$

$\left(\frac{22}{3}, \frac{16}{3}\right)$

$3A + 2B = 30$

$-\ A + 2B = 18$

$2A + 0 = 12 \quad \therefore \frac{2A}{2} = \frac{12}{2}\,6 \quad \therefore A = 6$

Solve for B (Put A = 6 in equation 3)

$A + 2B = 18$

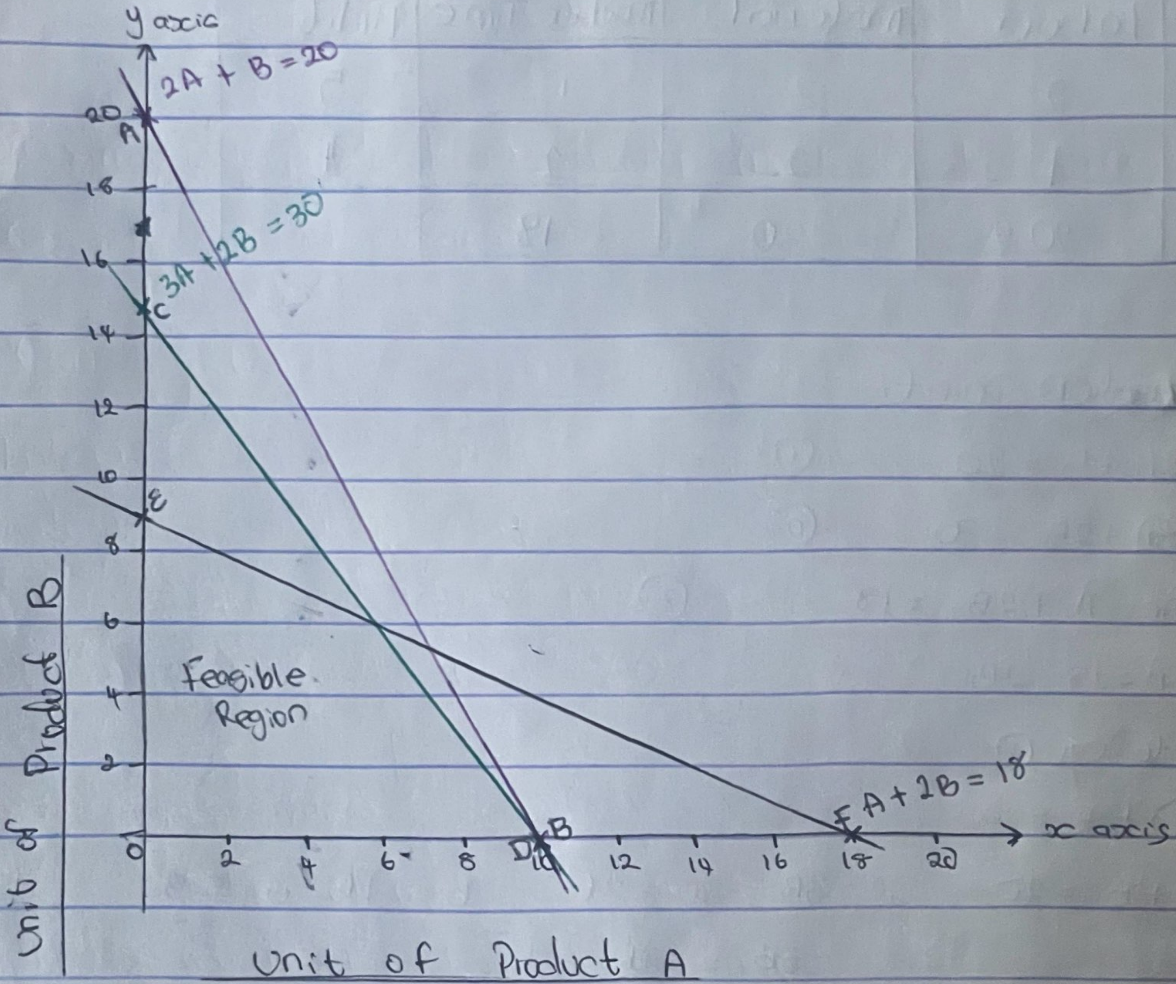
$6 + 2B = 18$

$2B = 18 - 6$

$\frac{2B}{2} = \frac{12}{2}\,6 \quad \therefore B = 6$

$(6, 6)$

Graphical solution for LP



The Interception points are (6, 6) for 3A + 2B = 30 and A + 2B = 18 while $\left(\frac{22}{3}, \frac{16}{3}\right)$ for 2A + B = 20 and A + 2B = 18

Corner Points	Coordinates (x, y)	Objective function (Z) 5A + 4B
O	(0, 0)	Z = 5(0) + 4(0) = 0
A	(0, 20)	Z = 5(0) + 4(20) = 80
B	(10, 0)	Z = 5(10) + 4(0) = 50
C	(0, 15)	Z = 5(0) + 4(15) = 60
D	(10, 0)	Z = 5(10) + 4(0) = 50
E	(0, 9)	Z = 5(0) + 4(9) = 36
F	(18, 0)	Z = 5(18) + 4(0) = 90

Optimal Solution : The maximum profit is Z = 90 achieved at the Corner point (18, 0).

4 Maximizing Revenue from sales

Objective Function (maximum revenue): $Z = 4x + 5y$

$Z = 4A + 5B$

Constraints

Advertising budget : $A + 2B \leq 20$

production Capacity : $A + 2B \leq 15$

non-negativity : $A \geq 0$, $B \geq 0$

Decision variables:

x : Number of product A produced and sold

y : Number of product B produced and sold

	Advertising budget	Production Capacity	Profit
Product A	1	1	4
product B	2	20	5
Total	20	15	

Graphical Solution

non-negativity Let $A=0$ and $B=0$

$(0,0)$

Advertising budget: $A + 2B = 20$ ∴ $0 + \frac{2B}{2} = \frac{20}{2}^{10}$ $B = 10$ $(0, 10)$A

$A + 2(0) = 20$ ∴ $A = 20$ $(20, 0)$B

Production capacity: $A + 2B \leq 15$ ∴ $0 + \frac{2B}{2} = \frac{15}{2}$ ∴ $B = \frac{15}{2} / 7.5$ C

$(0, \frac{15}{2})$

$A + 2(0) = 15$ ∴ $A + 0 = 15$ ∴ $A = 15$

$(15, 0)$ D

The Intersection point

advertising bud. : A + 2B = 20 .... (1)

Production cap. : A + 2B = 15 .... (2)

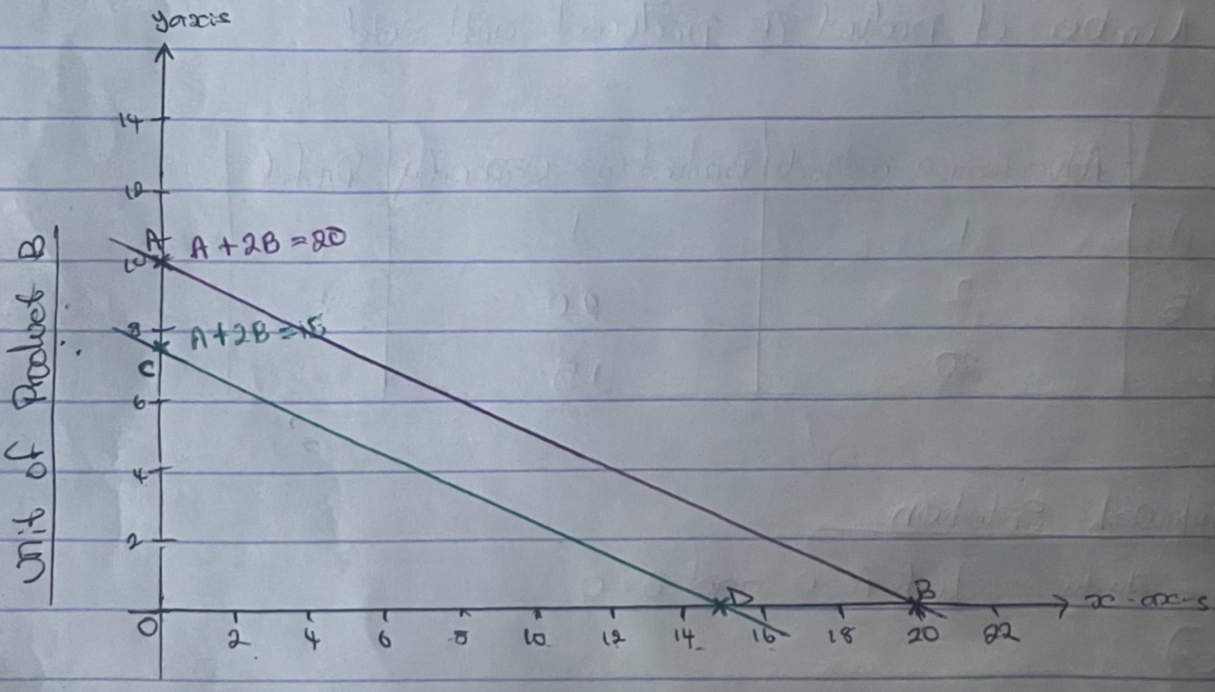
$A = 15 - 2B$

Put A into equation (1)

$15 - 2B + 2B = 20$

Both lines are parallel, There is no Intersection.

Graphical Solution For LP



unit of Product A

Corner points	Coordinates (x,y)	objective function (Z = 4x + 5y)
O	(0,0)	Z = 4(0) + 5(0) = 0
A	(0,10)	Z = 4(0) + 5(10) = 50
B	(20,0)	Z = 4(20) + 5(0) = 80
C	(0, 15/2)	Z = 4(0) + 5(15/2) = 75/2
D	(15,0)	Z = 4(15) + 5(0) = 60

The optimal solution : The maximum profit for revenue sales

5. Resource Allocation for Two projects.

Objective function (maximize) : $8x + 7y$

Decision variables:

x : numbers of units of project P1

y : numbers of units of project P2

	Labour	Capital Investment	Profit
P1	3	2	8
P2	4	1	7
Total	12	6	

Constraints

Labour hour : $3x + 4y \leq 12$

Capital Investment : $2x + y \leq 6$

Non-negativity : $x \geq 0,\ y \geq 0$

Graphical solution

$(0,0)$

Let $x = 0$ and $y = 0$

Labour hour : $3(0) + 4y = 12 \therefore \frac{\cancel{4}y}{\cancel{4}} = \frac{\cancel{12}^{3}}{\cancel{4}} \therefore y = 3$ $(0,3)$ A

$3x + 4(0) = 12 \therefore \frac{\cancel{3}x}{\cancel{3}} = \frac{\cancel{12}^{4}}{\cancel{3}} = x = 4$ $(4,0)$ B

Capital Investment : $2(0) + y = 6 \therefore y = 6 \therefore (0,6)$ C

$2x + 0 = 6 \therefore \frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2} \therefore x = 3 \therefore (3,0)$

The Interception Point

Labour hour : $3x + 4y = 12$ ...... ①

Capital Invest. : $2x + y = 6$ .... ②

$y = 6 - 2x$

Put $y = 6 - 2x$ into equation ①

$3x + 4y = 12$

$3x + 4(6 - 2x) = 12$

$3x + 24 - 8x = 12$

$-5x = 12 - 24$

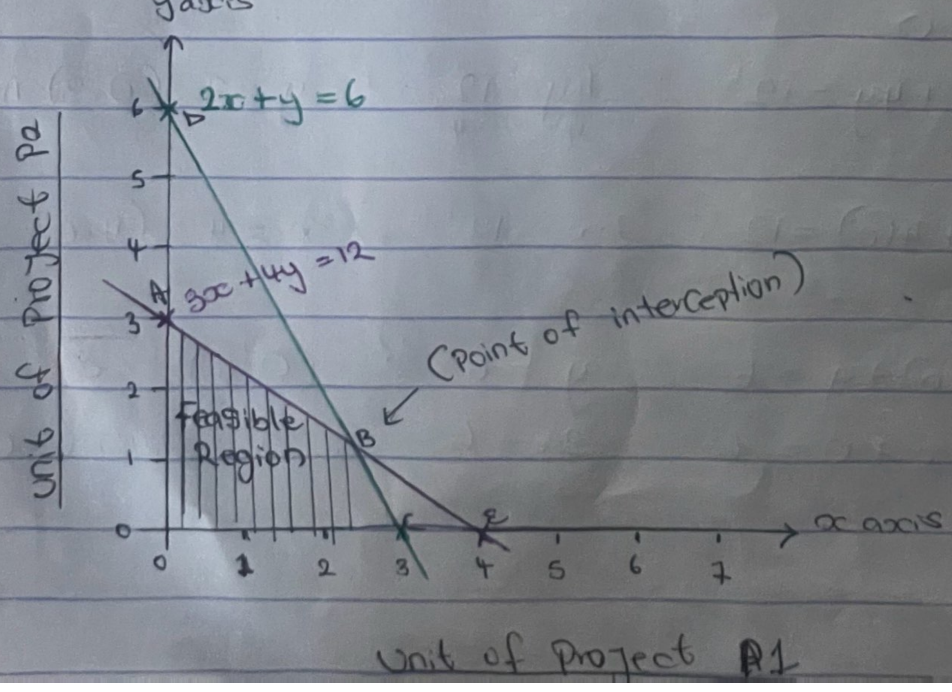
$\frac{\cancel{-5}x}{\cancel{-5}} = \frac{\cancel{-}12}{\cancel{-}5}$ $\therefore$ $x = \frac{12}{5}$ or 2.4

Put $x = \frac{12}{5}$ in equation $y = 6 - 2x$

$y = 6 - 2\left(\frac{12}{5}\right)$

$= 6 - \frac{24}{5}$ $\therefore$ $\frac{6}{5}$ $\therefore$ $y\ \frac{6}{5}$ $\Rightarrow \left(\frac{12}{5}, \frac{6}{5}\right)$

Graphical Solution for LP



Corner points	Coordinates (x,y)	Objective function $(Z = 8x+7y)$
A	$(0,0)$	$Z = 8(0)+7(0) = 0$
B	$(0,3)$	$Z = 8(0)+7(3) = 21$
C	$(4,0)$	$Z = 8(4)+7(0) = 32$
D	$(0,6)$	$Z = 8(0)+7(6) = 42$
E	$(3,0)$	$Z = 8(3)+7(0) = 24$
F	$(\frac{12}{5}, \frac{6}{5})$	$Z = 8(\frac{12}{5})+7(\frac{6}{5}) = \frac{138}{5} / 27.6$

The optimal solution : is the corner point with the maximum value of objective function and that $(0,6) = 42$.

6. Production planning for a Bakery

Objective function (maximize) : $5x + 3y$ (Total Profit)

Decision variables

x : Number of chocolate cakes produced

y : Number of vanilla cakes produced

	Baking time	flour	Profit
Chocolate Cakes	1	3	5
Vanilla cakes	2	2	3
Total	8	12	

Constraints

~~Chocolate cakes~~ : Baking Time : $x + 2y \leq 8$

Flour : $3x + 2y \leq 12$

Non-negativity : $x \geq 0$, $y \geq 0$

Graphical Solution (Let $x=0$ and $y=0$)

Baking time: $x + 2y = 8$

$0 + \frac{2y}{2} = \frac{8}{2}$ $\therefore$ $y = 4$ $\therefore$ $(0,4)$

$x + 2(0) = 8$ $\therefore$ $x = 8$ $\therefore$ $(8,0)$

Flour: $3x + 2y = 12$

$3(0) + \frac{2y}{2} = \frac{12}{2}$ $\therefore$ $y = 6$ $\therefore$ $(0,6)$

$\frac{3x}{3} + 2(0) = \frac{12}{3}$ $\therefore$ $x = 4$ $\therefore$ $(4,0)$

Non-negativity: $(0,0)$ $x=0$ $y=0$

The Interception Point

Baking time : $x + 2y = 8$ ....(1)

flour : $3x + 2y = 12$ ...(2)

$x = 8 - 2y$

Put $x = 8 - 2y$ into equation (2)

$3(8-2y) + 2y = 12$

$24 - 6y + 2y = 12$

$-6y + 2y = 12 - 24$

$\frac{-4y}{-4} = \frac{-12}{-4}$

$y = 3$

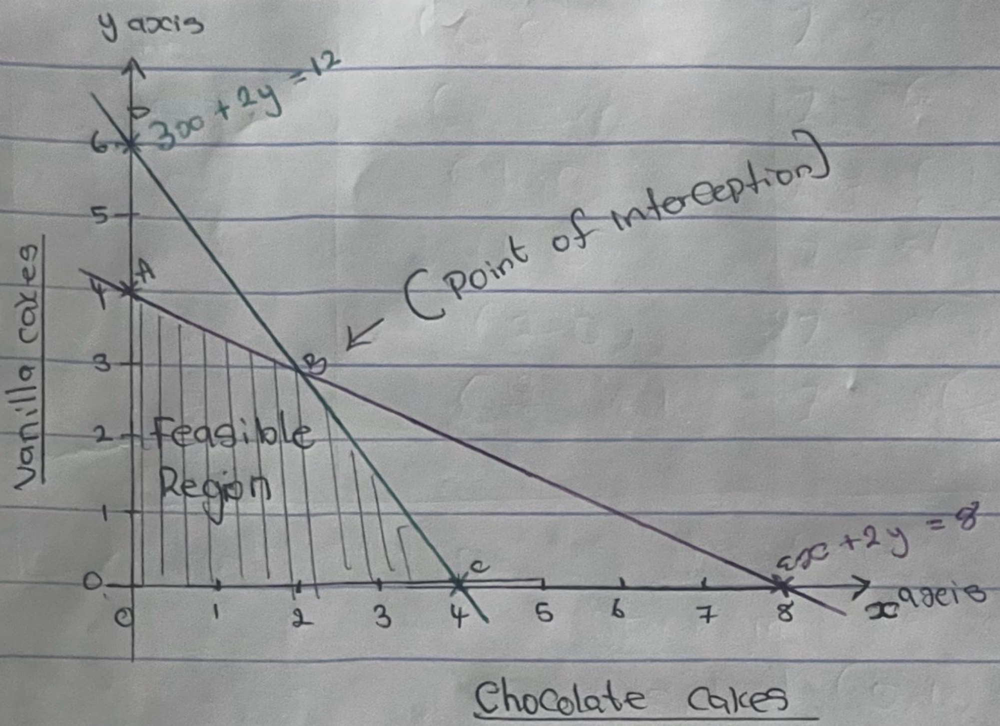
Put $y = 3$ into $x = 8 - 2y$

$x = 8 - 2(3)$

$x = 8 - 6$

$x = 2$ $\Rightarrow$ $(2,3)$

Graphical Solution For LP



Corner Point	Coordinates (x, y)	Objective Function (Z = 5x + 3y)
O	(0, 0)	Z = 5(0) + 3(0) = 0
A	(0, 4)	Z = 5(0) + 3(4) = 12
B	(2, 3)	Z = 5(2) + 3(3) = 19
C	(4, 0)	Z = 5(4) + 3(0) = 20
D	(0, 6)	Z = 5(0) + 3(6) = 18
E	(8, 0)	Z = 5(8) + 3(0) = 40

The optimal solution is to produce 8 chocolate cakes (x = 8) and 0 vanilla cakes (y = 0), which g generates a ~~profit~~ maximum profit of N40

7. Minimizing Cost for a Transport Company

Decision variables

x: number of trips using vehicle x

y: Number of trips using vehicle y

Objective function (minimize the total cost): $6x + 7y$

	Fuel	Driver time	Profit ($)
Vehicle x	3	2	6
Vehicle y	4	1	7
Total	18	10	

Constraints

Fuel: $3x + 4y \leq 18$

Driver time: $2x + y \leq 10$

Non-negativity: $x \geq 0$, $y \geq 0$

Graphical solution (Let $x = 0$ and $y = 0$)

Fuel: $3x + 4y = 18$

$\frac{3(0) + 4y}{4} = \frac{18}{4}$ $\therefore$ $y = \frac{18}{4} = \frac{9}{2}$ or 4.5 $(0, \frac{9}{2})$

$\frac{3x + 4(0)}{3} = \frac{18}{3}$ $\therefore$ $x = \frac{18}{3} = 6$ $\therefore$ $x = 6$ $(6, 0)$

Driver time: $2x + y = 10$

$2(0) + y = 10$ $\therefore$ $y = 10$ $(0, 10)$

$\frac{2x + 0}{2} = \frac{10}{2}$ $\therefore$ $x = \frac{10}{2} = 5$ $\therefore$ $(5, 0)$

Non-negativity: $x = 0$, $y = 0$ $(0, 0)$

The Interception Point

Fuel: $3x + 4y = 18$ ..... ①

Driver time: $2x + y = 10$ ..... ②

$$y = 10 - 2x$$

Put $y = 10 - 2x$ into equation ①

$$3x + 4(10 - 2x) = 18$$

$$3x + 40 - 8x = 18$$

$$3x - 8x = 18 - 40$$

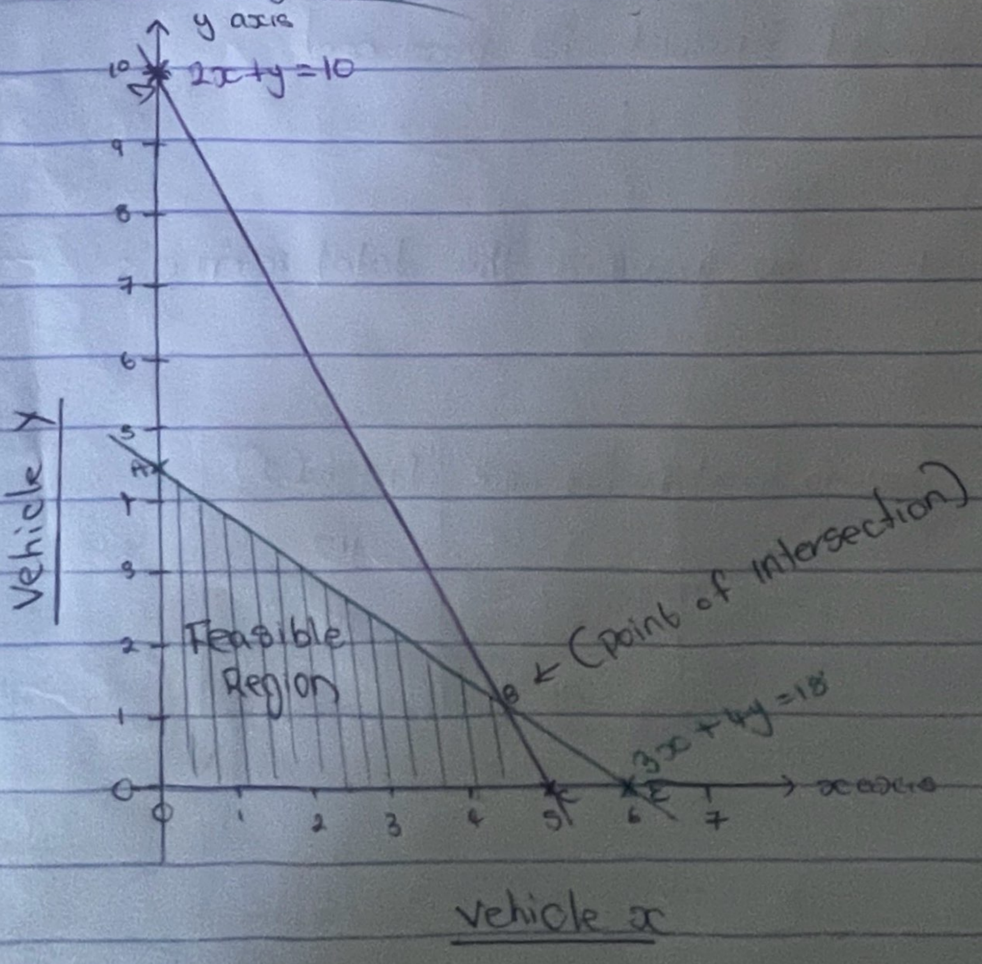
$$\frac{-5x}{-5} = \frac{+22}{+5} \quad \therefore \quad x = \frac{22}{5} \text{ or } 4.4$$

Put x into $y = 10 - 2x$

$$y = 10 - 2\left(\frac{22}{5}\right) \qquad \Rightarrow \left(\frac{22}{5}, \frac{6}{5}\right) / (4.4, 1.2)$$

$$y = \frac{6}{5} \text{ or } 1.2$$

Graphical Solution For LP



Corner points	Coordinates (x, y)	Objective function $(Z = 6x + 7y)$
O	$(0, 0)$	$Z = 6(0) + 7(0) = 0$
A	$(0, \frac{9}{2})$	$Z = 6(0) + 7(\frac{9}{2}) = \frac{63}{2}$ / 31.5
B	$(\frac{22}{5}, \frac{6}{5})$	$Z = 6(\frac{22}{5}) + 7(\frac{6}{5}) = \frac{174}{5}$ / 34.
C	$(5, 0)$	$Z = 6(5) + 7(0) = 30$
D	$(0, 10)$	$Z = 6(0) + 7(10) = 70$
E	$(6, 0)$	$Z = 6(6) + 7(0) = 36$

The optimal solution is to make zero (0) trips using vehicle x $(x = 0)$ and $9/2$ (4.5) trips using vehicle y $(y = 9/2)$, which generates a minimum cost of $\frac{63}{2}$ or \$31.5

8) Maximizing Revenue from Two Products

Decision variables

x : number of units of product P1 produced

y : number of units of Product P2 produced

objective Function

The objective function is to maximize the total revenue :

maximize $10x + 12y$

	Labour	Raw material	machine Time	Profit (\$)
P1	4	1	3	10
P2	3	2	2	12
Total	30	18	24	

Constraints

Labor : $4x + 3y \leq 30$

Raw material : $x + 2y \leq 18$

Machine Time : $3x + 2y \leq 24$

Non - negativity : $x \geq 0$, $y \geq 0$

Graphical Solution (let $x = 0$, $y = 0$)

Labour : $4x + 3y = 30$

$$4(0) + \frac{3y}{3} = \frac{30}{3} \quad \therefore y = 10 \quad (0, 10)$$

$$\frac{4x}{4} + 3(0) = \frac{30}{4} \quad \therefore x = \frac{30}{4} = \frac{15}{2} \text{ or } 7.5 \quad \left(\frac{15}{2}, 0\right)$$

Raw material : $x + 2y = 18$

$$0 + \frac{2y}{2} = \frac{18}{2} \quad \therefore y = 9 \quad (0, 9)$$

$$x + 2(0) = 18 \quad \therefore x = 18 \quad (18, 0)$$

Machine Time : $3x + 2y = 24$

$$3(0) + \frac{2y}{2} = \frac{\cancel{24}^{12}}{2} \quad \therefore y = 12 \quad (0, 12)$$

$$\frac{3x}{3} + 2(0) = \frac{24}{3} \quad \therefore x = 8 \quad (8, 0)$$

Non - negativity : $x = 0$, $y = 0$ $(0, 0)$

The Interception Points

Labour : $4x + 3y = 30$ ...... ①

Raw material : $x + 2y = 18$ ...... ②

Machine Time : $3x + 2y = 24$ ...... ③

$x = 18 - 2y$

Put $x = 18 - 2y$ into equation ①

$$4x + 3y = 30$$

$$4(18 - 2y) + 3y = 30$$

$$72 - 8y + 3y = 30$$

$$-8y + 3y = 30 - 72$$

$$\frac{-5y}{-5} = \frac{+42}{+5}$$

$$y = \frac{42}{5} \text{ or } 8.4$$

Put $y = \frac{42}{5}$ into ~~eq~~ $x = 18 - 2y$

$$x = 18 - 2\left(\frac{42}{5}\right) \qquad \Rightarrow \left(\frac{6}{5}, \frac{42}{5}\right) \text{ or } (1.2, 8.4)$$

$$x = 18 - \frac{84}{5} \therefore x = \frac{6}{5} \text{ or } 1.2$$

$$\left.\begin{aligned} x + 2y &= 18 \\ 3x + 2y &= 24 \end{aligned}\right\} \quad \begin{aligned} 3x + 2y &= 24 \\ - \; x + 2y &= 18 \\ \hline 2x + 0 &= 6 \end{aligned} \quad \therefore \frac{2x}{2} = \frac{6}{2}\,3 \;\therefore\; x = 3$$

Solve for y

$$x + 2y = 18$$

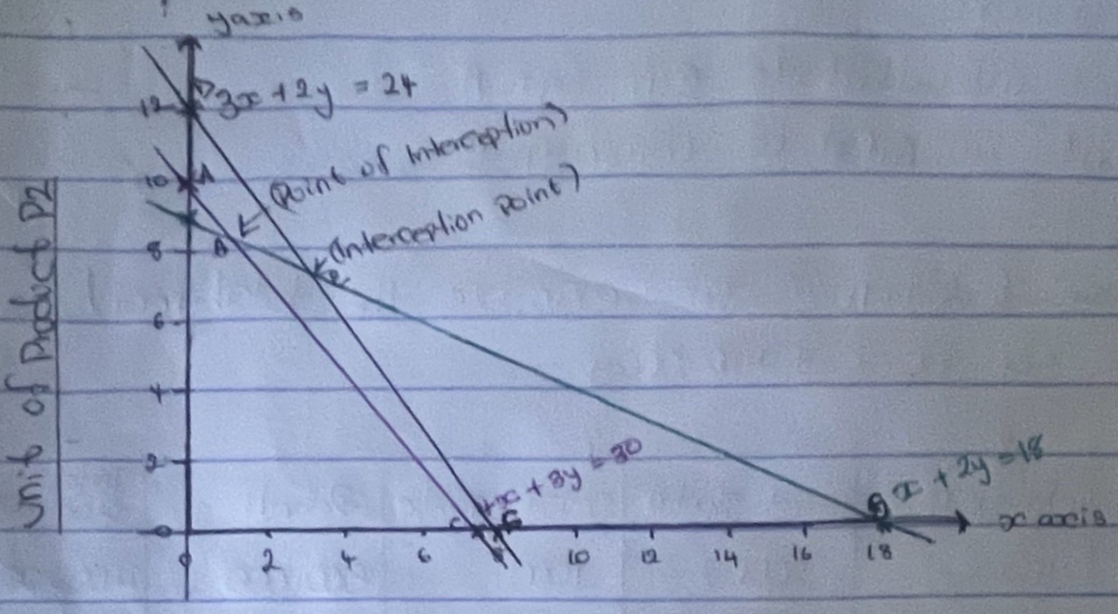
$$3 + 2y = 18$$

$$2y = 18 - 3$$

$$\frac{2y}{2} = \frac{15}{2} \qquad y = \frac{15}{2} \text{ or } 7.5$$

$$\Rightarrow \left(3, \frac{15}{2}\right) \text{ or } (3, 7.5)$$

Graphical Solution For LP



unit of Product P1

The Interception points are $\left(\frac{6}{5}, \frac{42}{5}\right)$ for line $4x+3y=30$ and $x+2y=18$, while $\left(3, \frac{15}{2}\right)$ for line $3x+2y=24$ and $x+2y=18$

Corner points	Coordinates (x,y)	Objective function $(Z = 10x + 12y)$
O	$(0,0)$	$Z = 10(0) + 12(0) = 0$
A	$(0,10)$	$Z = 10(0) + 12(10) = 120$
B	$\left(\frac{6}{5}, \frac{42}{5}\right)$	$Z = 10\left(\frac{6}{5}\right) + 12\left(\frac{42}{5}\right) = 112.8$
C	$\left(\frac{15}{2}, 0\right)$	$Z = 10\left(\frac{15}{2}\right) + 12(0) = 75$
D	$(0,12)$	$Z = 10(0) + 12(12) = 144$
E	$(3, 15/2)$	$Z = 10(3) + 12\left(\frac{15}{2}\right) = 120$
F	$(8,0)$	$Z = 10(8) + 12(0) = 80$
G	$(18,0)$	$Z = 10(18) + 12(0) = 180$

~~The optimal solution to maximize Revenue from Products P1 and P2 is to Increase the unit of~~

Optimal Solution: The maximum profit / revenue is $Z = \$180$ achieved at the Corner Points $(18, 0)$

Q) Advertising Campaign Budget Allocation

Decision variables

x : Amount allocated to Campaign A

y : Amount allocated to Campaign B

The objective function is to maximize the total reach

maximize : $500{,}000x + 400{,}000y$

	Television	Print media	social media	Reach	Total Budget
Campaign A	4000	2000	1000	500,000	4000 $\cancel{x}$
Campaign B	3000	2500	1500	400,000	3000
Total	5000	4500	3000		10 000

Total Budge : 10,000

Constraints

Total Budget : $\cancel{x + y \le 10{,}000}$ $4000\cancel{x} + 3000y \le 10000$

Television : $4000x + 3000y \le 5{,}000$

Print media : $2000x + 2500y \le 4{,}500$

Social media : $1000x + 1500y \le 3000$

Non-Negativity : $x \geq 0$, $y \geq 0$

Graphical solution

let $x = 0$ and let $y = 0$

Television : $4000(0) + 3000y = 5000$

$0 + \frac{\cancel{3000}y}{\cancel{3000}} = \frac{5\cancel{000}}{3\cancel{000}} = \frac{5}{3}$ $\therefore y = \frac{5}{3}$ or 1.67 $(0, \frac{5}{3})$

$4000x + 3000(0) = 5000$

$\frac{\cancel{4000}x}{\cancel{4000}} = \frac{5\cancel{000}}{4\cancel{000}}$ $\therefore x = \frac{5}{4}$ or 1.25 $(\frac{5}{4}, 0)$

Total Budget : $4000(0) + 3000y = 10{,}000$

$0 + \frac{\cancel{3000}y}{\cancel{3000}} = \frac{10\cancel{000}}{3\cancel{000}}$ $\therefore y = \frac{10}{3}$ or 3.33 $(0, \frac{10}{3})$

$4000x + 3000(0) = 10000$

$\frac{4000x}{4000} = \frac{10000}{4000}$

$x = \frac{10}{4} = \frac{5}{2} \therefore x = \frac{5}{2}$ or 2.5 $(\frac{5}{2}, 0)$

Print media: $2000(0) + 2500y = 4500$

$0 + \frac{2500y}{2500} = \frac{4500}{2500} = \frac{9}{5} \therefore y = \frac{9}{5}$ or 1.8

$(0, \frac{9}{5})$

$2000x + 2500(0) = 4500$

$\frac{2000x}{2000} = \frac{4500}{2000} = \frac{9}{4} \therefore x = \frac{9}{4}$ or 2.25

$(\frac{9}{4}, 0)$

Social media: $1000(0) + 1500y = 3000$

$0 + \frac{1500y}{1500} = \frac{3000}{1500} = 2 \therefore y = 2$ $(0, 2)$

$1000x + 1500(0) = 3000$

$\frac{1000x}{1000} = \frac{3000}{1000} = 3 \therefore x = 3$ $(3, 0)$

non-negativity: $x = 0$ and $y = 0$ $(0, 0)$

The Interception points

Television: $4000x + 3000y = 5000$ ...... ①

Print media: $2000x + 2500y = 4500$ ...... ②

Social media: $1000x + 1500y = 3000$ ...... ③

$1x + 1.5y = 3$ | Total Budget: $4000x + 3000y = 10000$ .... ④

$x = 3 - 1.5y$

Put x into equation ①

$4(3 - 1.5) + 3y = 5$

$12 - 6y + 3y = 5$

$-6y + 3y = 5 - 12$

$\frac{-3y}{-3} = \frac{-7}{-3} \therefore y = \frac{7}{3}$ or 2.3

Put y into $x = 3 - 1.5y$

$x = 3 - 1.5(\frac{7}{3}) = 3 - \frac{7}{3} = -\frac{1}{3} \therefore x = \frac{-1}{2}$ (not feasible)

Simultaneously solve equation 4 and 3

$4000x + 3000y = 10000$ ---- 4

$-\ 1000x + 1500y = 3000$ 3

$3000x + 1500y = 7000$ ---- 5

Put $x = 3 - 1.5y$ into equation 5

$3000(3 - 1.5y) + 1500y = 7000$

$9000 - 4500y + 1500y = 7000$

$-4500y + 1500y = 7000 - 9000$

$\frac{-\cancel{3000}y}{-\cancel{3000}} = \frac{+2\cancel{000}}{+3\cancel{000}} \therefore y = \frac{2}{3}$ or 0.67

Put $y = \frac{2}{3}$ into equation 5

$3000x + 1500\left(\frac{2}{3}\right) = 7000$

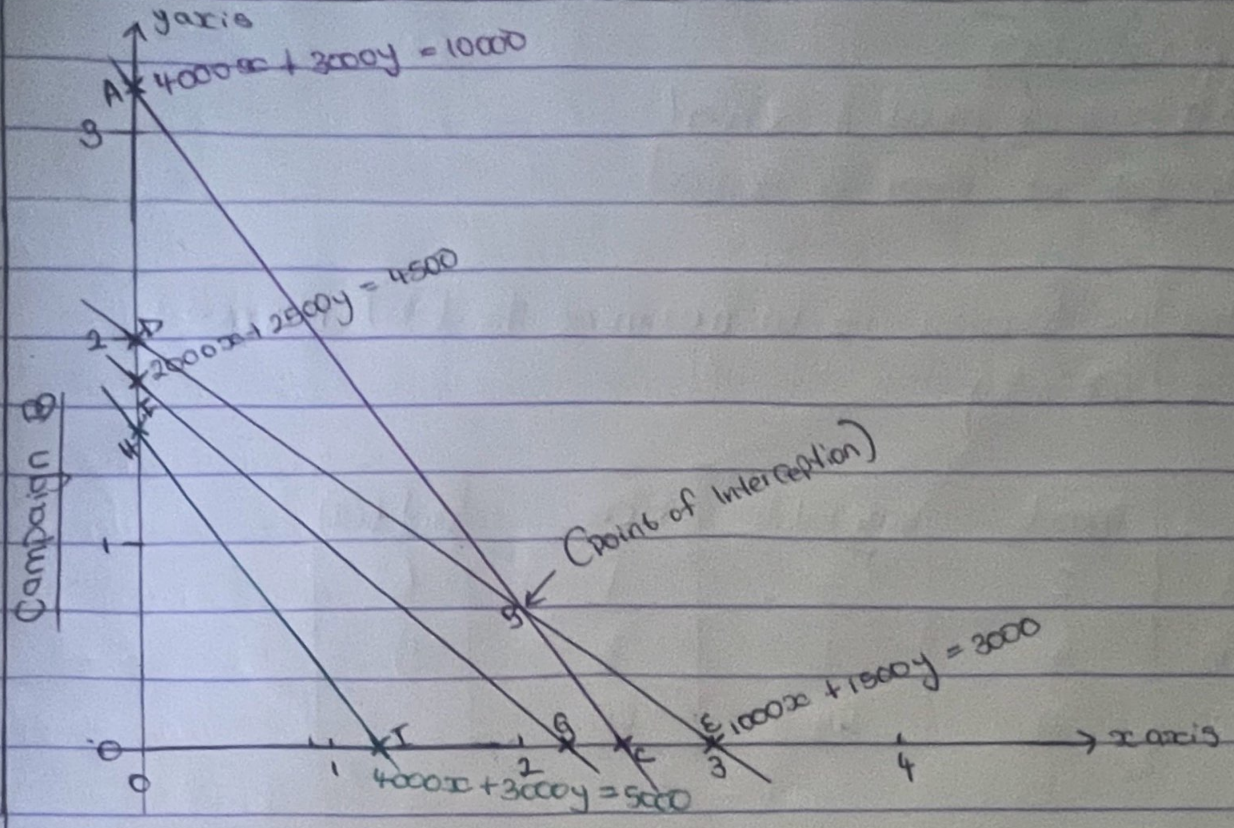
$3000x + 1000 = 7000$

$3000x = 7000 - 1000$

$\frac{\cancel{3000}x}{\cancel{3000}} = \frac{6\cancel{000}}{3\cancel{000}} = \frac{6}{3} = 2 \therefore x = 2$

$\Rightarrow \left(2, \frac{2}{3}\right)$

Graphical Solution For LP



The Interception point is $\left(2, \frac{2}{3}\right)$ for Line Total Budget

$4000x + 3000y = 10000$ and Social Media line $1000x + 1500y = 3000$.

Corner points	Coordinates $(x+y)$	Objective function $(Z = 500000x + 400000y)$
O	$(0, 0)$	$Z = 500000(0) + 400000(0) = 0$
A	$(0, \frac{10}{3})$	$Z = 500000(0) + 400000(\frac{10}{3}) = 1,333,333$
B	$(2, \frac{2}{3})$	$Z = 500000(2) + 400000(\frac{2}{3}) = 1.266.666$
C	$(\frac{5}{2}, 0)$	$Z = 500000(\frac{5}{2}) + 400000(0) = 1,250.000$
D	$(0, 2)$	$Z = 500000(0) + 400000(2) = 800000$
E	$(3, 0)$	$Z = 500000(3) + 400000(0) = 1.500.000$
F	$(0, \frac{9}{5})$	$Z = 500000(0) + 400000(\frac{9}{5}) = 750000$
G	$(\frac{9}{4}, 0)$	$Z = 500000(\frac{9}{4}) + 400000(0) = 1,125,000$
H	$(0, \frac{5}{3})$	$Z = 500000(0) + 400000(\frac{5}{3}) = 666.666$
I	$(\frac{5}{4}, 0)$	$Z = 500000(\frac{5}{4}) + 400000(0) = 625,000$

The optimal solution occurs at Corner Point $(3, 0)$ with a maximum ~~Reach~~ Total reach of $Z = 1.500.000$ people

10) Meal planning For a School Cafeteria

Decisio Variables

x : numbers of meal A served

y : number of Meal B served

The objective function is to maximize the total revenue :

maximize : $6x + 5y$

	meat	Vegetable	rice	Profit($)
Meal A	2	3	1	6
Meal B	4	2	2	5
	30	24	20	

Constraints

Meat : $2x + 4y \leq 30$

Vegetable : $3x + 2y \leq 24$

rice : $x + 2y \leq 20$

Non-negativity : $x \geq 0$, $y \geq 0$

Graphical solution (Let $x=0$ and $y=0$)

meat : $2(0) + 4y = 30$

$\frac{\cancel{4}y}{\cancel{4}} = \frac{\cancel{30}^{15}}{\cancel{4}_{2}} = \frac{15}{2}$ or 7.5 $y = \frac{15}{2}$ $(0, \frac{15}{2})$

$2x + 4(0) = 30$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{30}{2} = 15$ $\therefore x = 15$ $(15, 0)$

Vegetable : $3(0) + 2y = 24$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{\cancel{24}^{12}}{2} = 12$ $\therefore y = 12$ $(0, 12)$

$3x + 2(0) = 24$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{\cancel{24}^{8}}{3} = 8$ $\therefore x = 8$ $(8, 0)$

rice : $(0) + 2y = 20$

$\frac{2y}{2} = \frac{20}{2} = 10 \therefore y = 10 \quad (0,10)$

$x + 2(0) = 20$

$x = 20 \quad \therefore (20,0)$

non-negativity : $x = 0$ and $y = 0 \therefore (0,0)$

The Interception point

meat : $2x + 4y = 30$ ...... ①

vegetable : $3x + 2y = 24$ .... ②

rice : $x + 2y = 20$ .... ③

$x = 20 - 2y$

put x into equation ②

$\cancel{2(20-2y) + 4y = 30}$

$\cancel{40 - 4y + 4y = 30}$

$\cancel{-4y + 4y = 30 - 40}$

$3x + 2y = 24$

$3(20-2y) + 2y = 24$

$60 - 6y + 2y = 24$

$-6y + 2y = 24 - 60$

$\frac{-4y}{-4} = \frac{-36}{-4} = 9$

$\therefore y = 9$

Simultaneously solve equation ② and ③

$3x + 2y = 24$

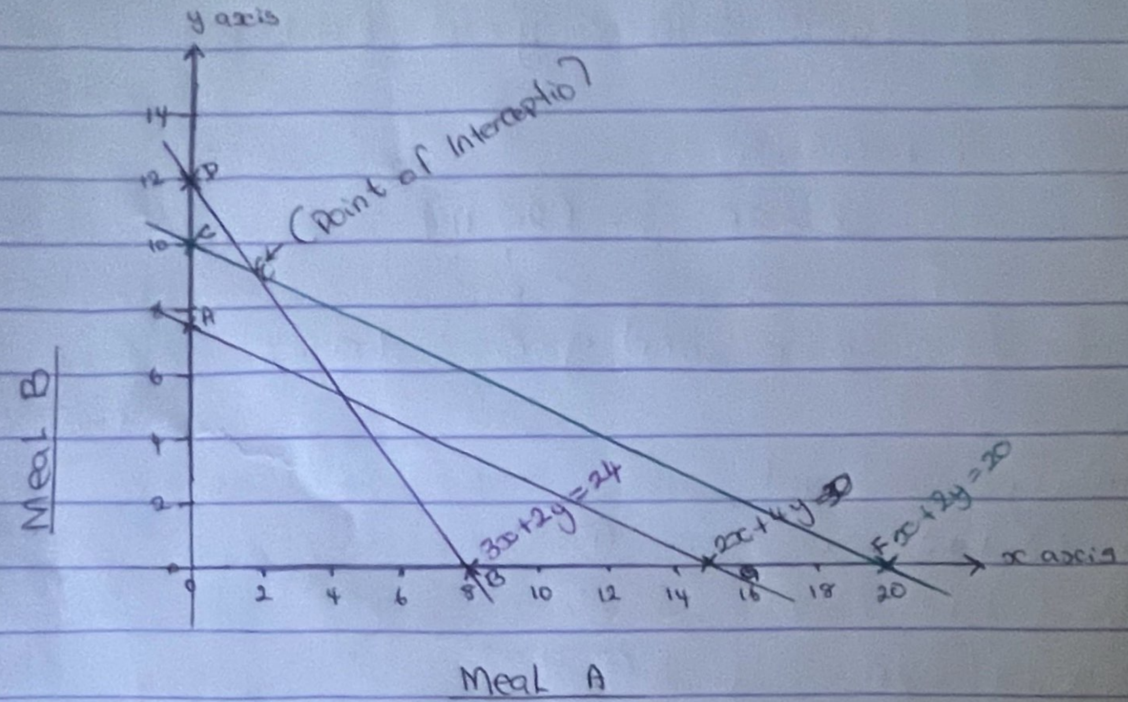
$- \quad x + 2y = 20$

$2x + 0 = 4$

$\therefore \frac{2x}{2} = \frac{4}{2} = 2 \quad \therefore x = 2$

$\Rightarrow (2, 9)$

Graphical Solution FOR LP



The point of Interception is (2,9) Line $3x+2y=24$ and Line $x+2y=20$

Corner points	Coordinates (x,y)	Objective Function $Z = 6x+5y$
O	(0,0)	$Z = 6(0)+5(0) = 0$
A	$(0, \frac{15}{2})$	$Z = 6(0)+5(15/2) = 37.5$
B	(8,0)	$Z = 6(8)+5(0) = 48$
C	(0,10)	$Z = 6(0)+5(10) = 50$
D	(0,12)	$Z = 6(0)+5(12) = 60$
E	(2,9)	$Z = 6(2)+5(9) = 57$
F	(20,0)	$Z = 6(20)+5(0) = 120$
G	(15,0)	$Z = 6(15)+5(0) = 90$

The optimal solution occurs at (20,0), meal A (x): 20 and meal B (y): 0

The solution maximizes the total revenue which

$Z = \$(20) + \$(0) = \$120$