Tree cover variability increases from 2005 to 2100 in Sub-Saharan Africa

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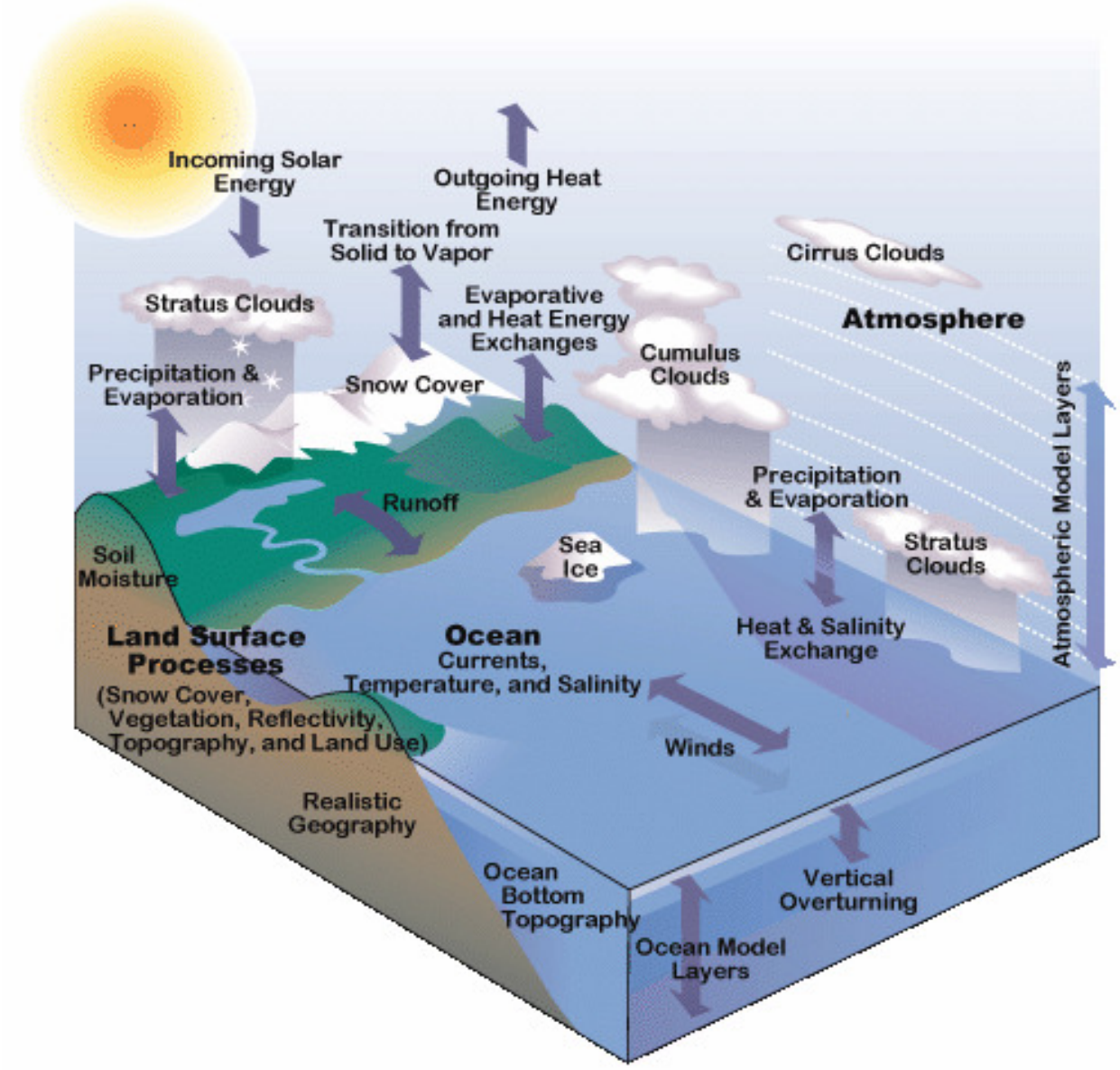
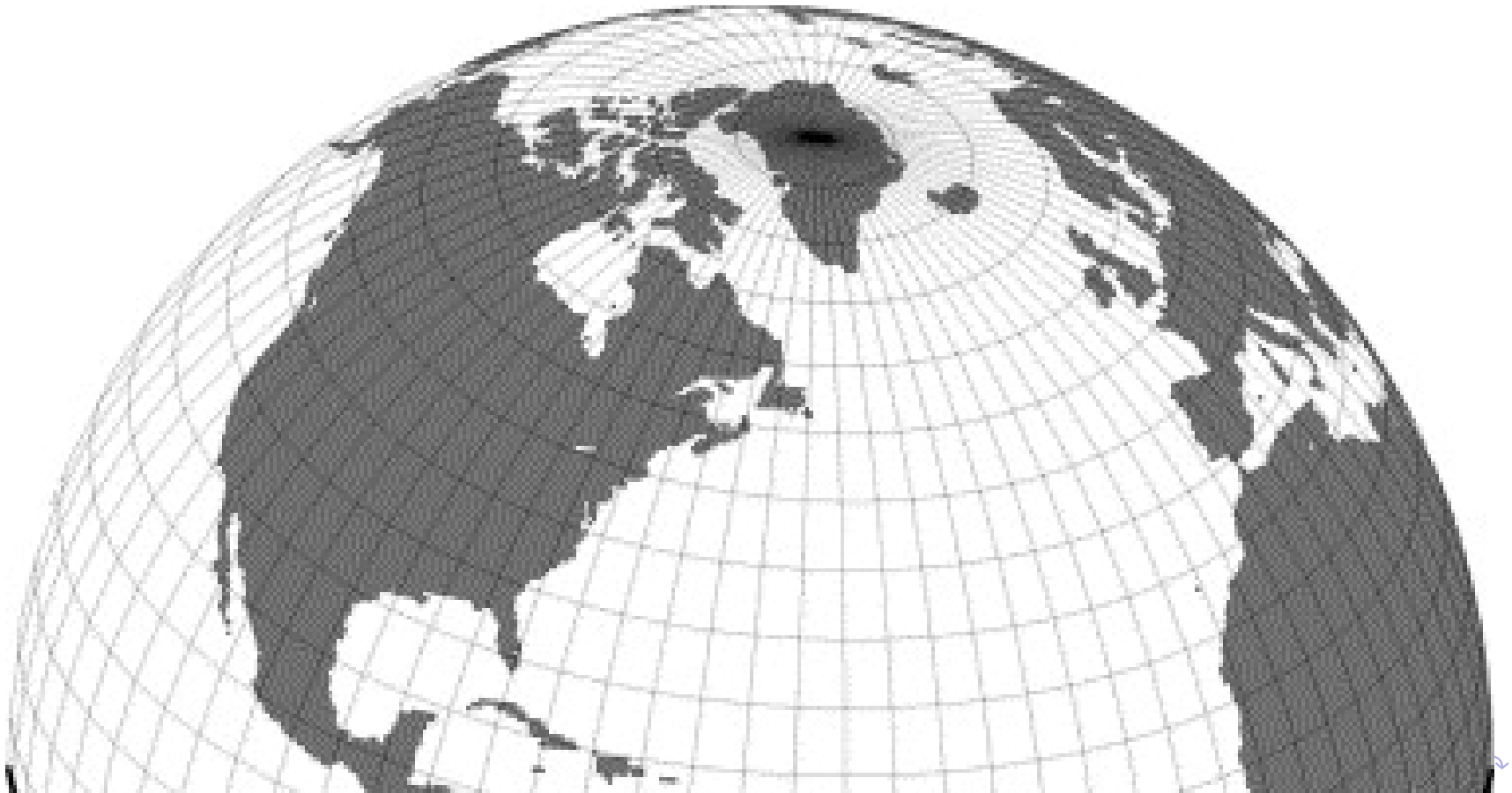
Introduction: climate system



https://www.ucar.edu/communications/CCSM/overview.html

Introduction: climate system modeling

- Climate change is driven by anthropogenic carbon forcing
- IPCC has developed representative carbon forcing trajectories
- Computer simulations are run predicated on particular forcing pathways
- These simulations are realizations of gridded meteorological PDEs

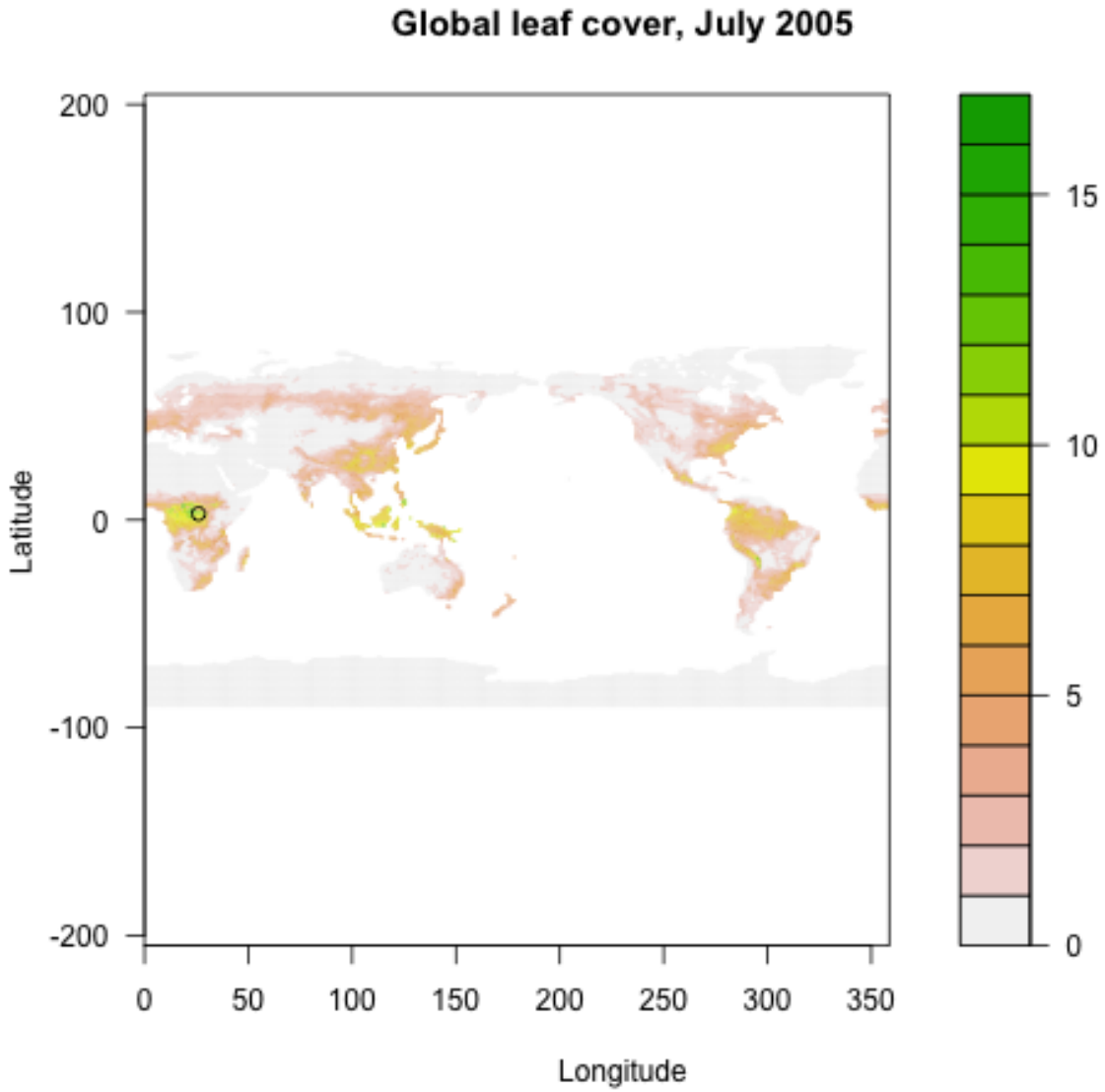


Introduction: climate system modeling

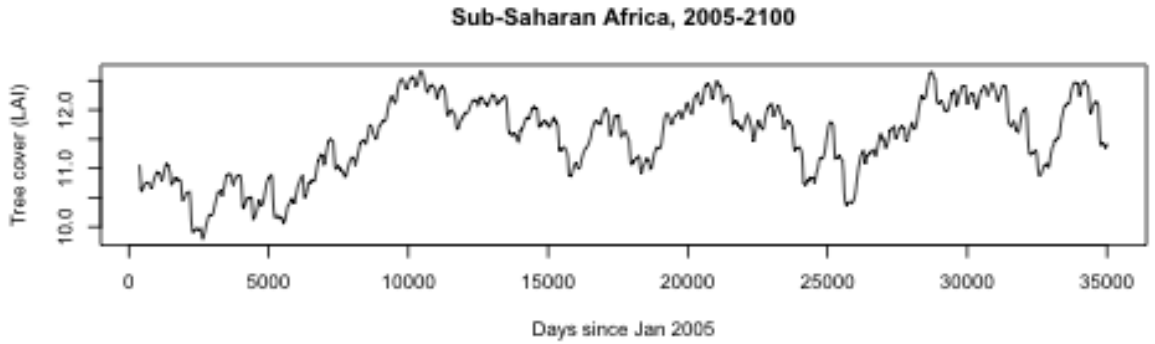
- The Community Climate Model System has 5 components and a coupler: atmosphere, sea, land, sea ice, and land ice.
- We focus on the land system output from a single ensemble under the RCP4.5 experiment.
- $\theta \in \Theta, \{X_t | t \in \mathbb{Z}\}$ where $X_t \sim \mathrm{ARMA}_\theta$

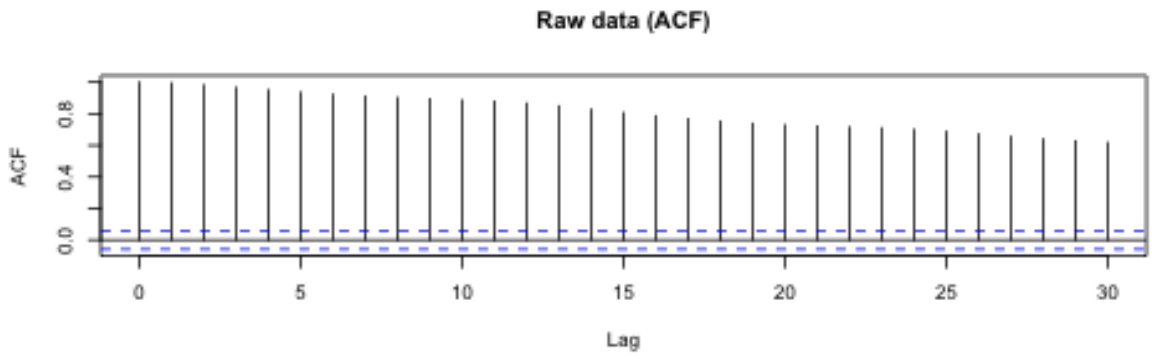
Introduction: leaf area index (LAI)

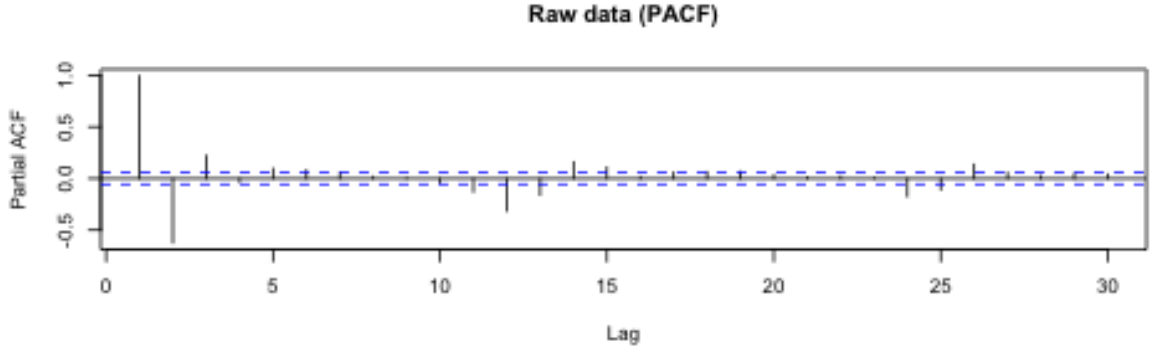
- LAI is unitless quantity (leaf area (m^2))/(ground area (m^2))
- "Comparative Physiological Studies on the Growth of Field Crops" Watson 1947 Annals of Botany
- Bounded below by 0, above by physiological limits
- We interrogate a location in a tropical rainforest



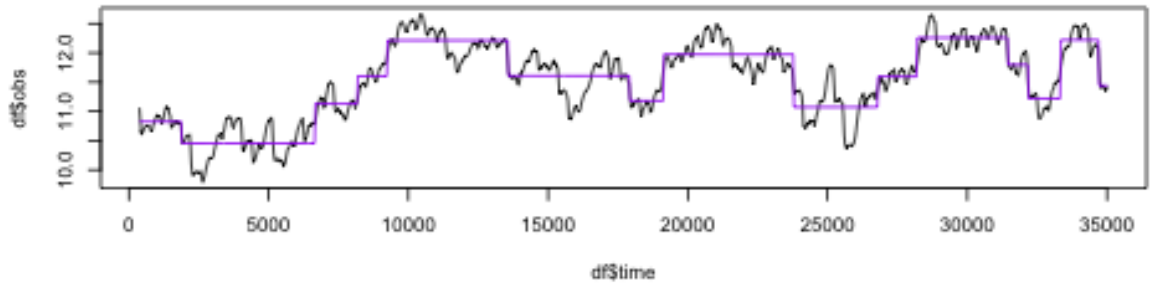
Raw data

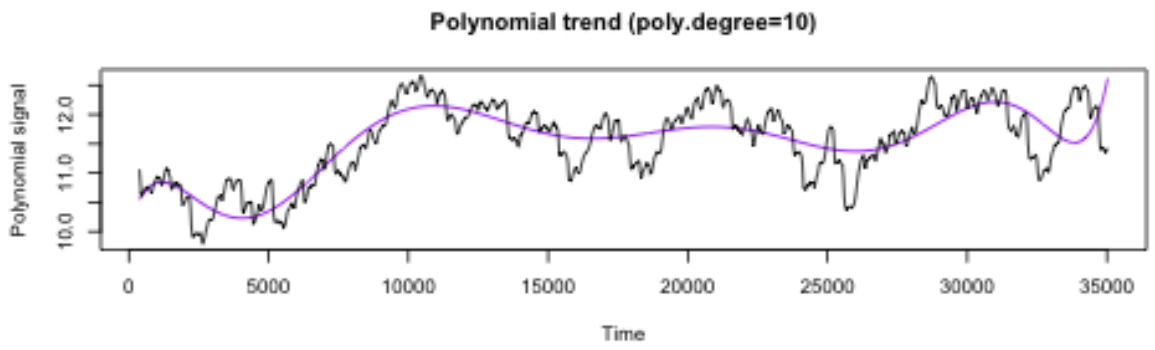

Sub-Saharan Africa, 2005-2100
Tree cover (LAI)
10.0
11.0
12.0
0
5000
10000
15000
20000
25000
30000
35000
Days since Jan 2005

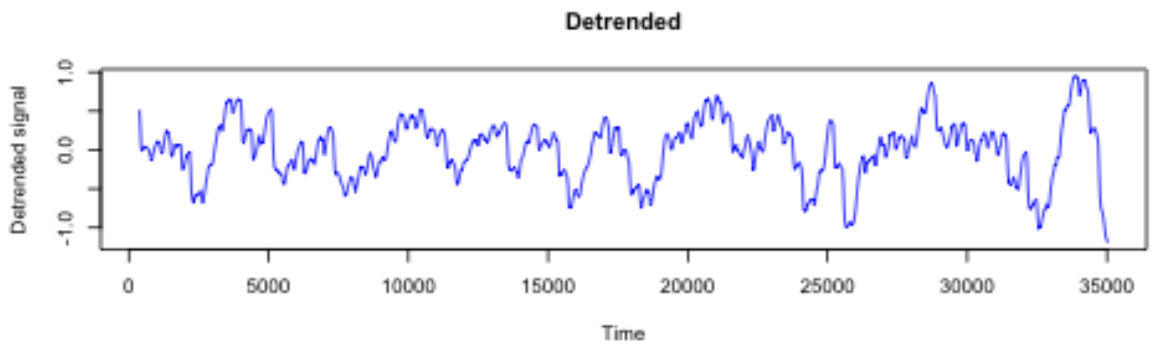

Raw data (ACF)
ACF
0.0
0.4
0.8
0
5
10
15
20
25
30
Lag


Raw data (PACF)
Partial ACF
-0.5
0.0
0.5
1.0
0
5
10
15
20
25
30
Lag

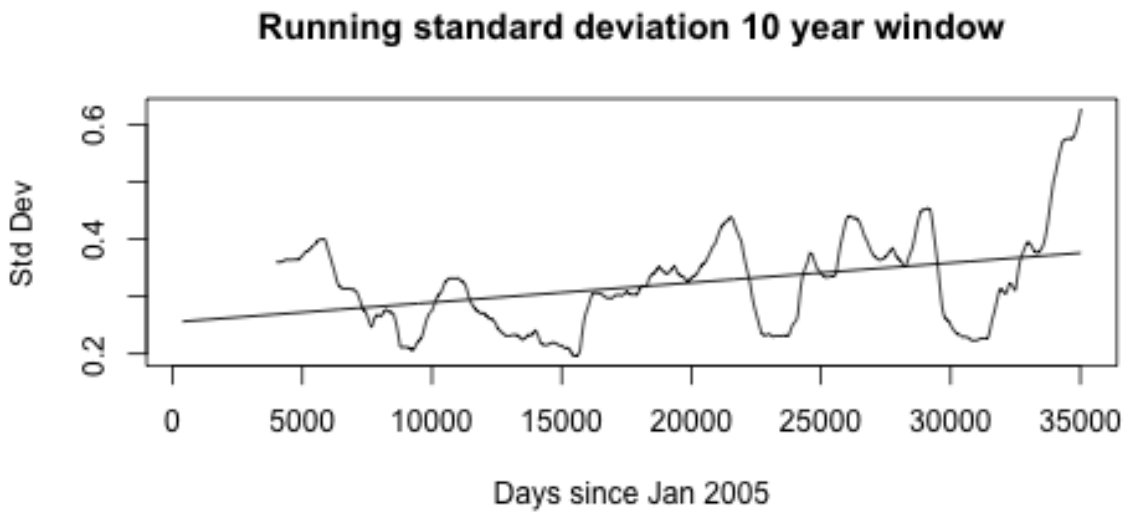
Detrended data

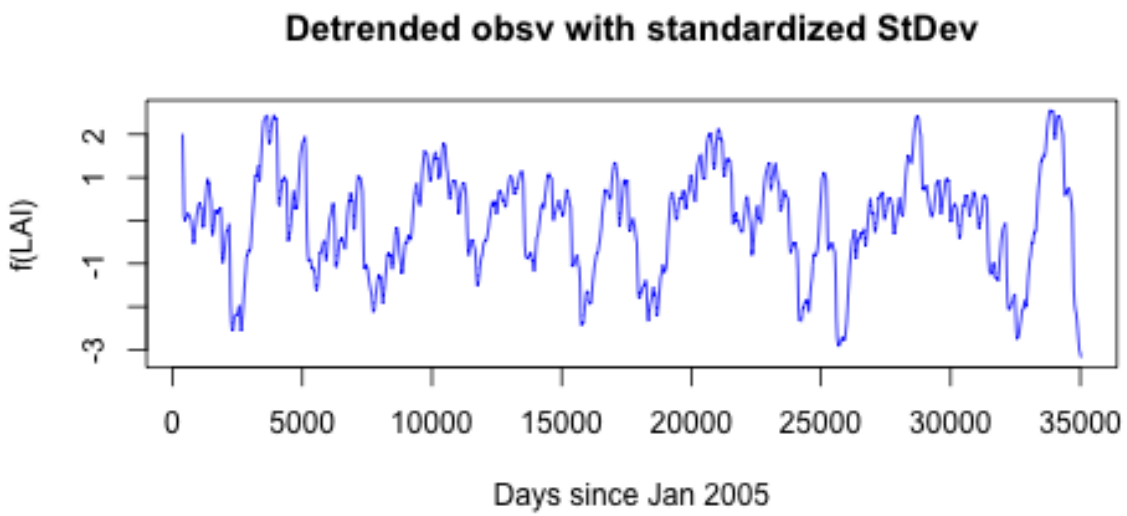




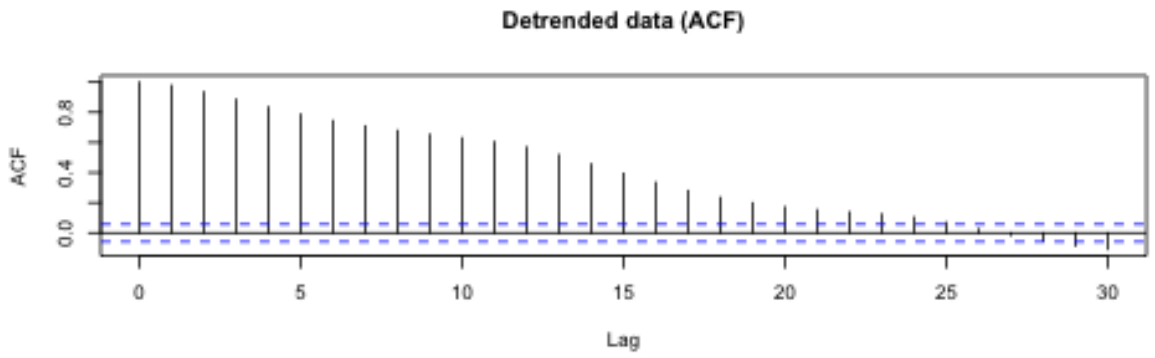


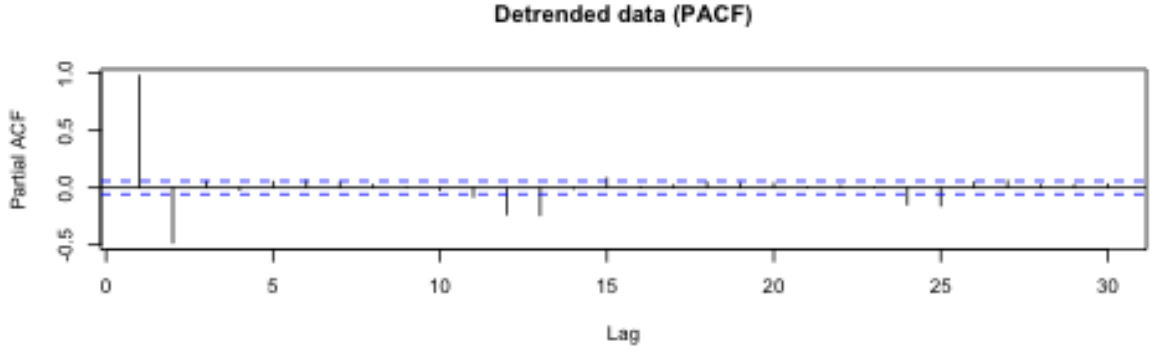
Standardize variance



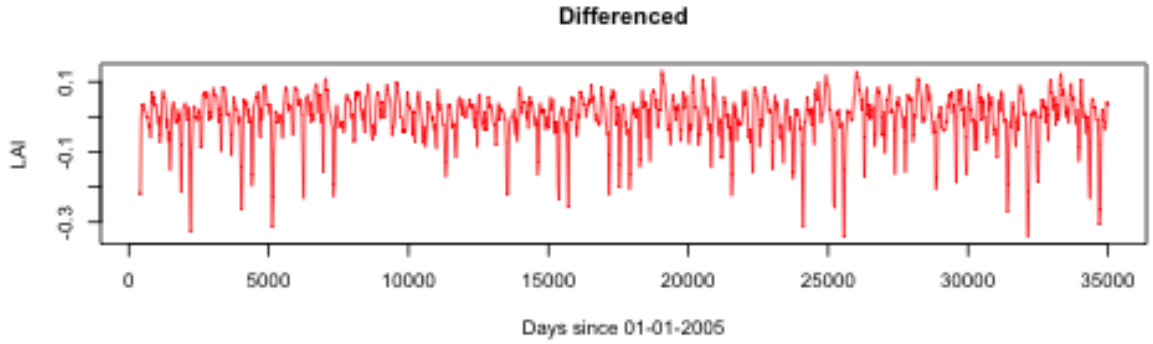


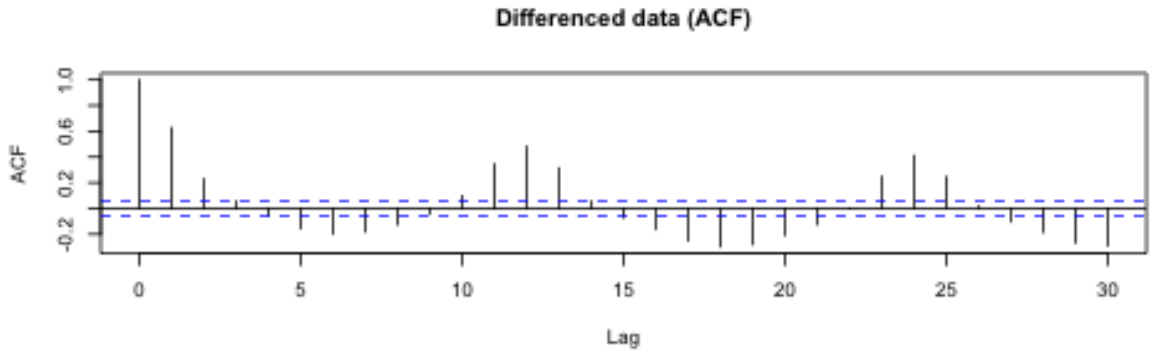
Standardized variance

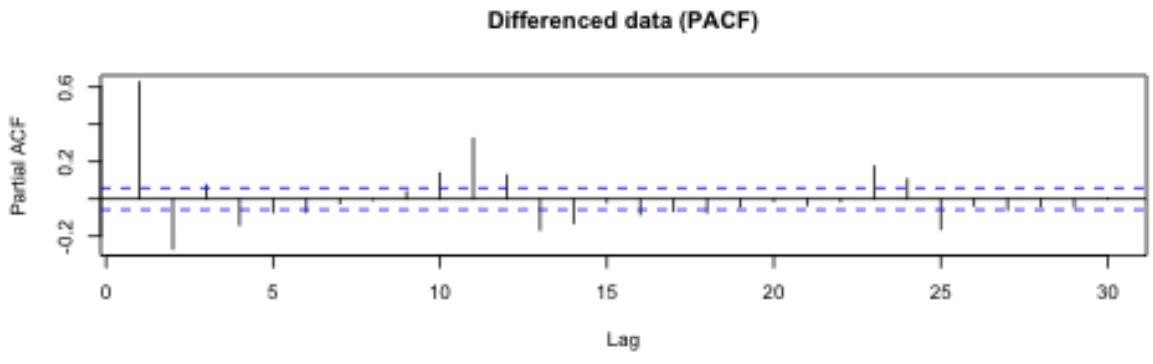




Differenced data

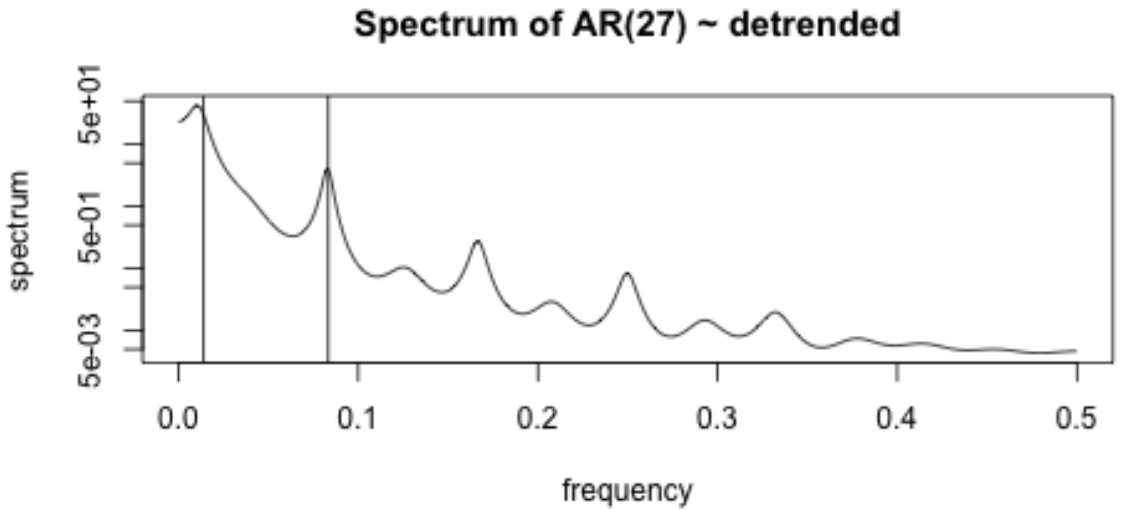


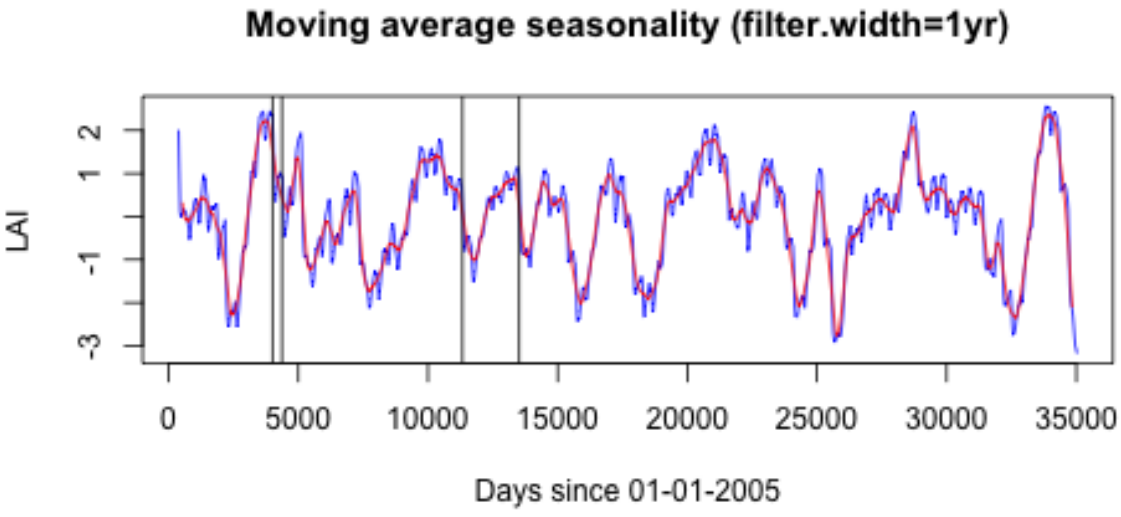




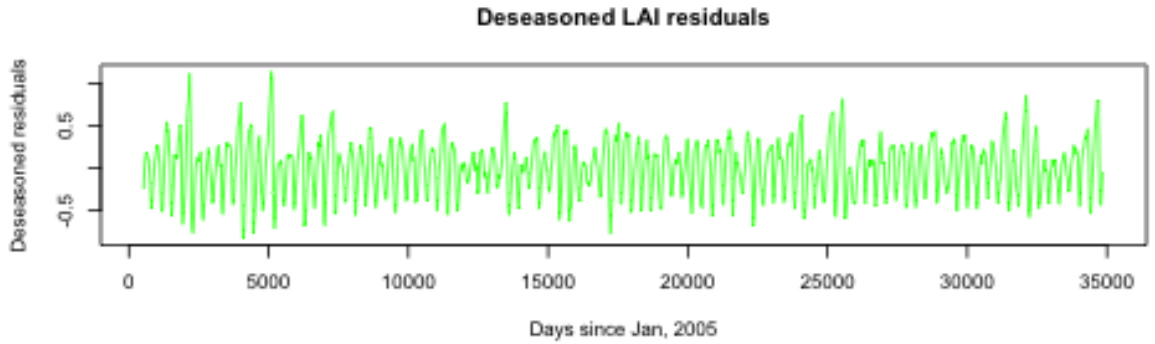
Perhaps we need to deseason the data before differencing to find stationarity.

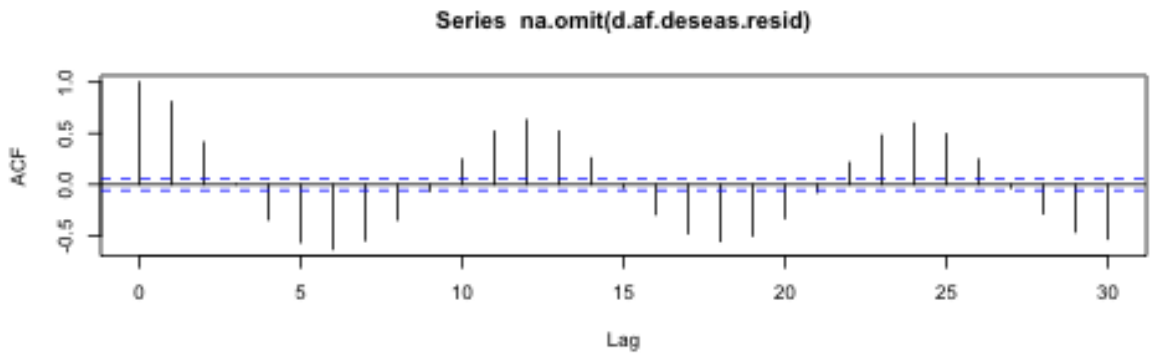
Deseasonized data

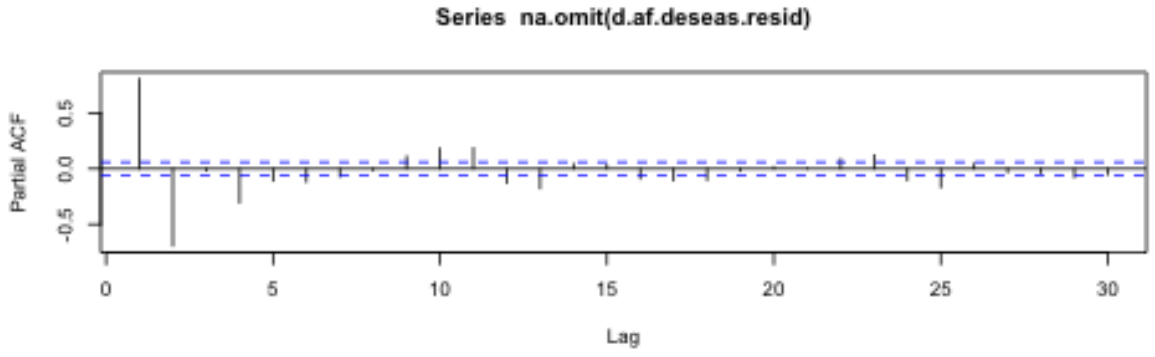




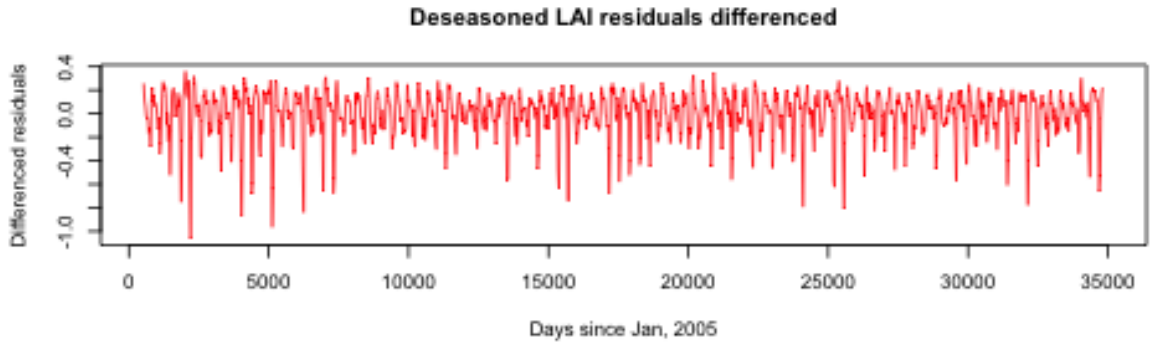
Deseasonized data

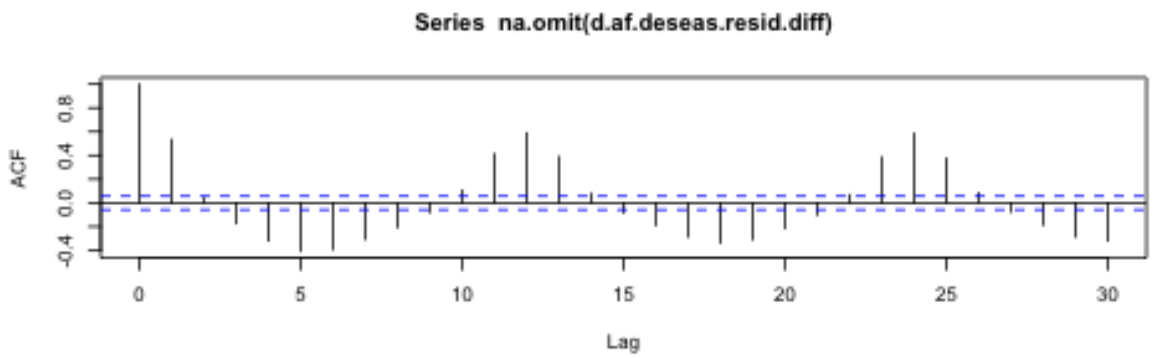


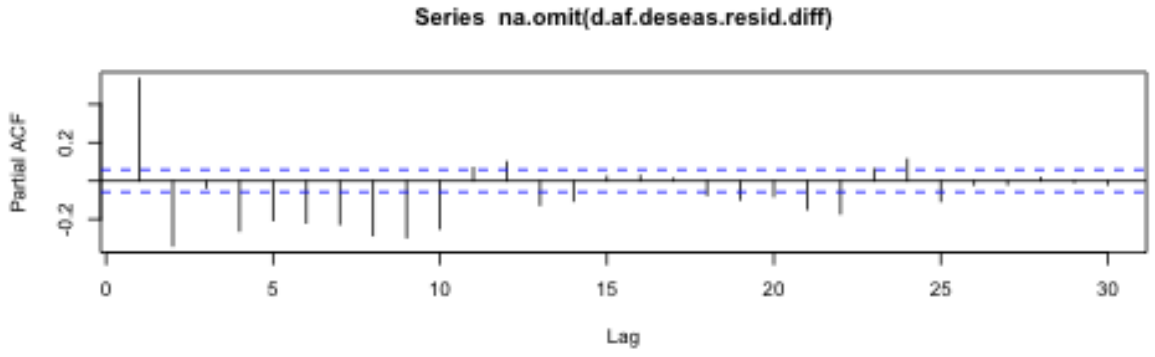




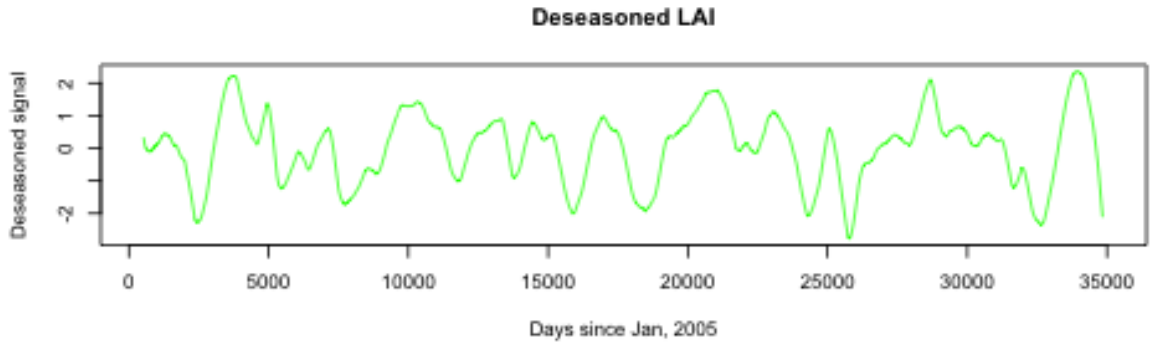
Deseasonized data

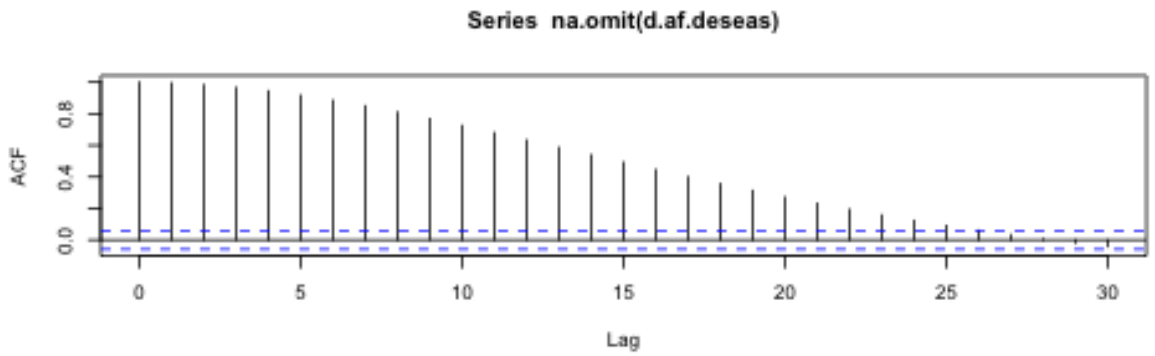


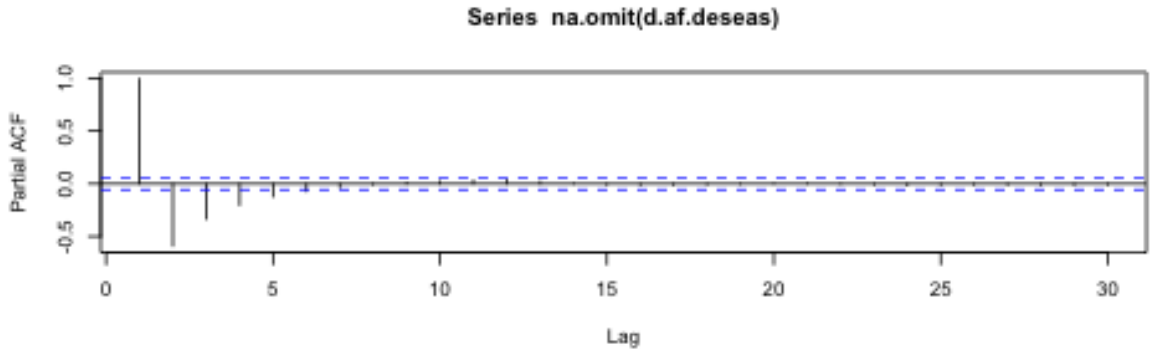




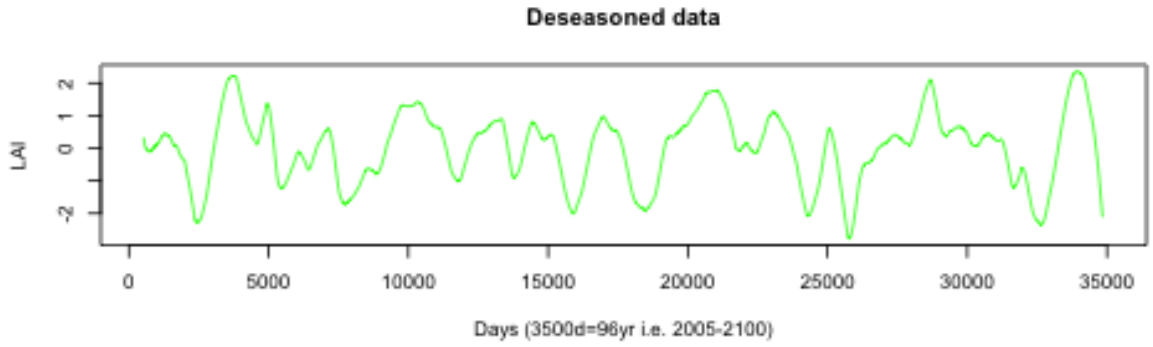
Deseasonized data

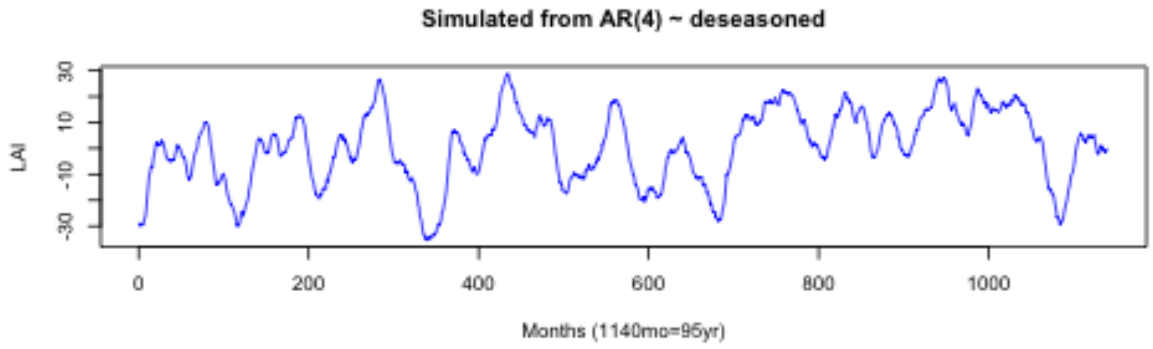


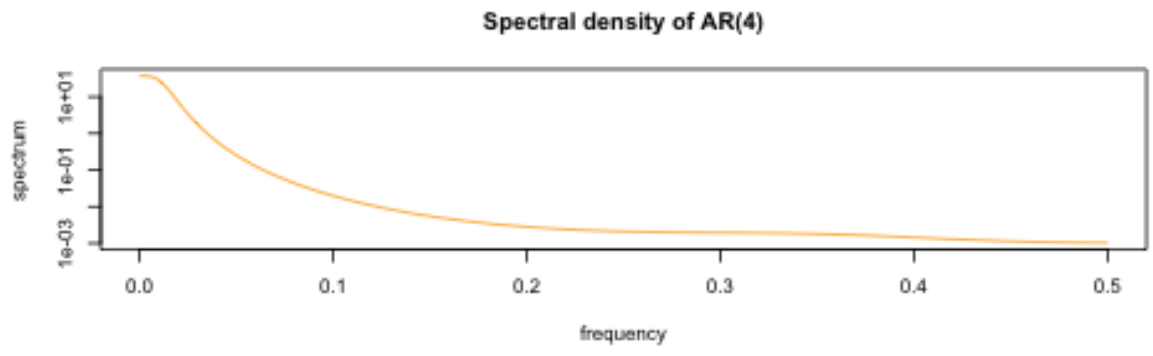




AR(4)


Deseasoned data
LAI
-2
0
1
2
0
5000
10000
15000
20000
25000
30000
35000
Days (3500d=96yr i.e. 2005-2100)


Simulated from AR(4) ~ deseasoned
LAI
-30
-10
10
30
0
200
400
600
800
1000
Months (1140mo=96yr)


Spectral density of AR(4)
spectrum
1e-03
1e-01
1e+01
0.0
0.1
0.2
0.3
0.4
0.5
frequency

Principal Component Analysis

- 12818 locations, 1140 time points
- $\mathbf{Z}(s,t) = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T$
- $\mathbf{Z}(s,t) =$

$$\begin{pmatrix} z(s_1,t_1) & z(s_2,t_1) & z(s_3,t_1) & \dots & z(s_{12818},t_1) \\ z(s_1,t_2) & z(s_2,t_2) & z(s_3,t_2) & \dots & z(s_{12818},t_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z(s_1,t_{1140}) & z(s_2,t_{1140}) & z(s_3,t_{1140}) & \dots & z(s_{12818},t_{1140}) \end{pmatrix}$$

Principal Component Analysis

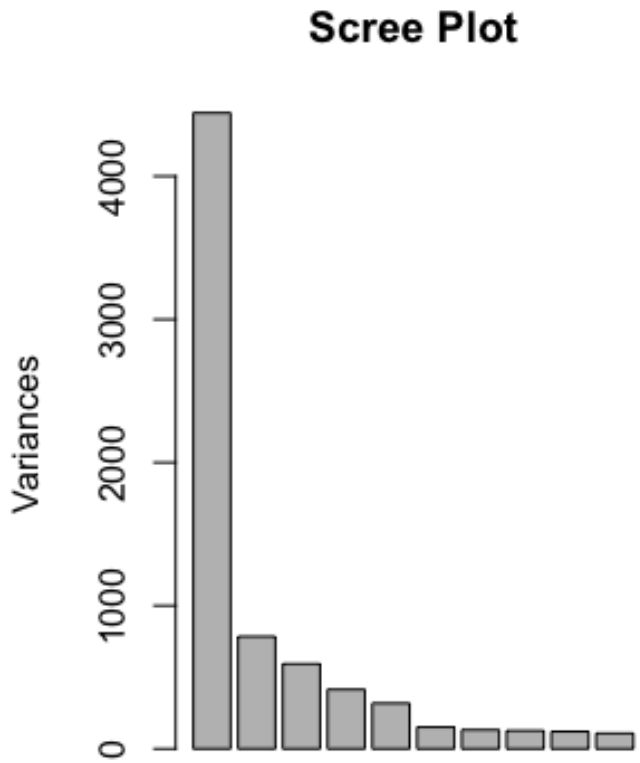


Figure: Raw Data

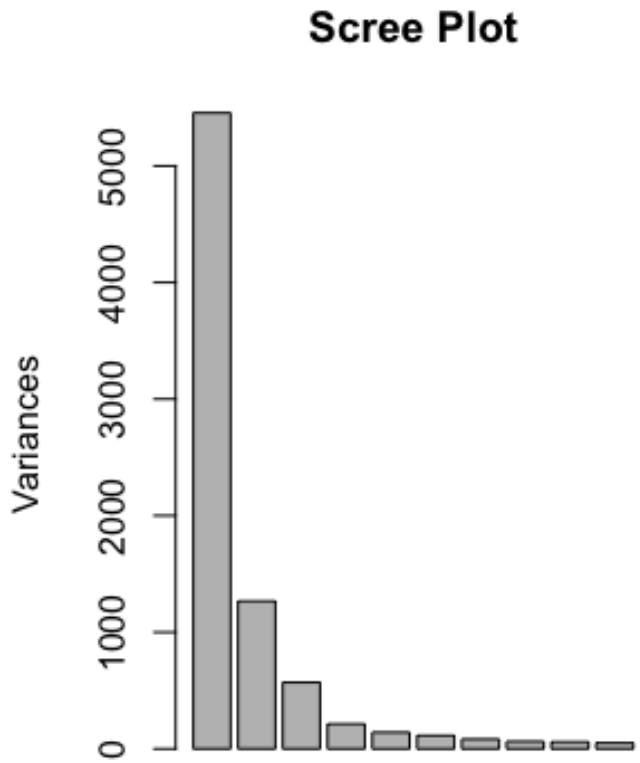


Figure: Detrend Data

Principal Component Analysis

	PC1	PC2	PC3	PC4	PC5	PC6
StDiv	66.650	28.003	24.356	20.348	17.832	12.301
Prop. of Var.	0.347	0.061	0.046	0.032	0.025	0.012
Cum. Prop.	0.347	0.408	0.454	0.486	0.511	0.523

Table: Raw Data

	PC1	PC2	PC3	PC4	PC5	PC6
StDiv	73.857	35.586	23.852	14.581	11.859	10.669
Prop. of Var.	0.426	0.099	0.044	0.017	0.011	0.009
Cum. Prop.	0.426	0.524	0.569	0.585	0.596	0.605

Table: Detrend

Principal Component Analysis

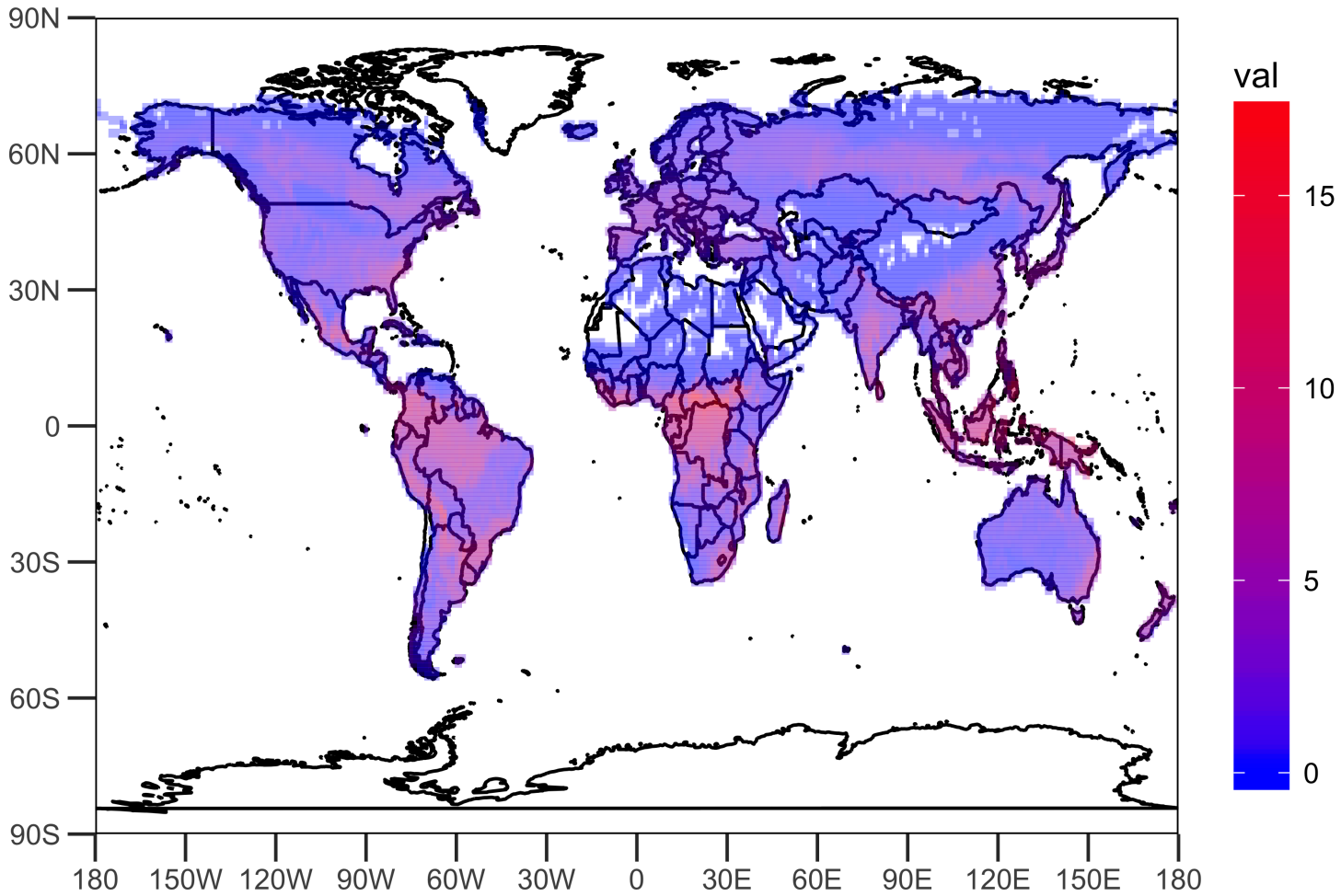


Figure: Global Leaf Index Jan 2005

Principal Component Analysis

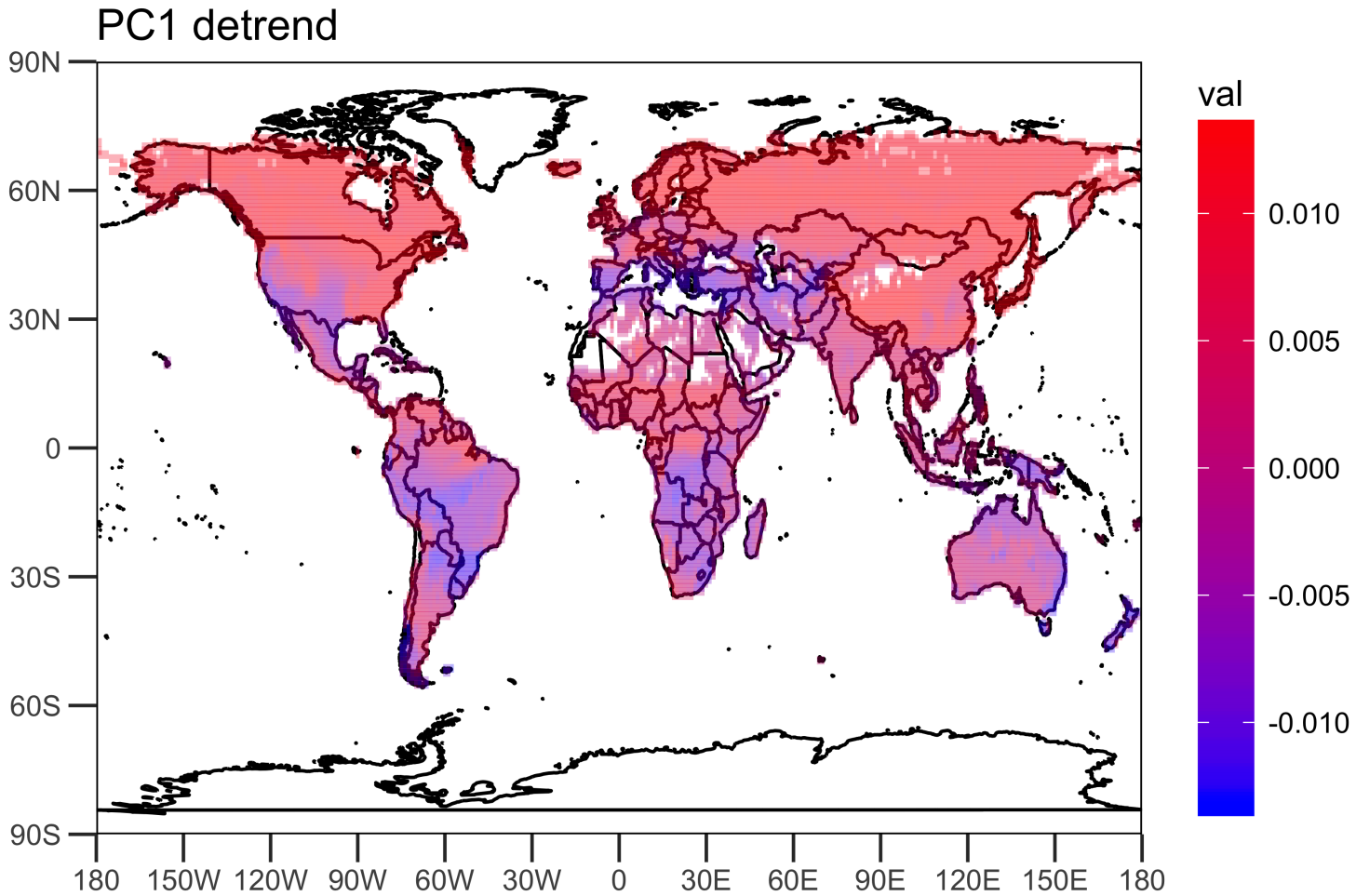


Figure: Loadings of Principal Component 1

Principal Component Analysis

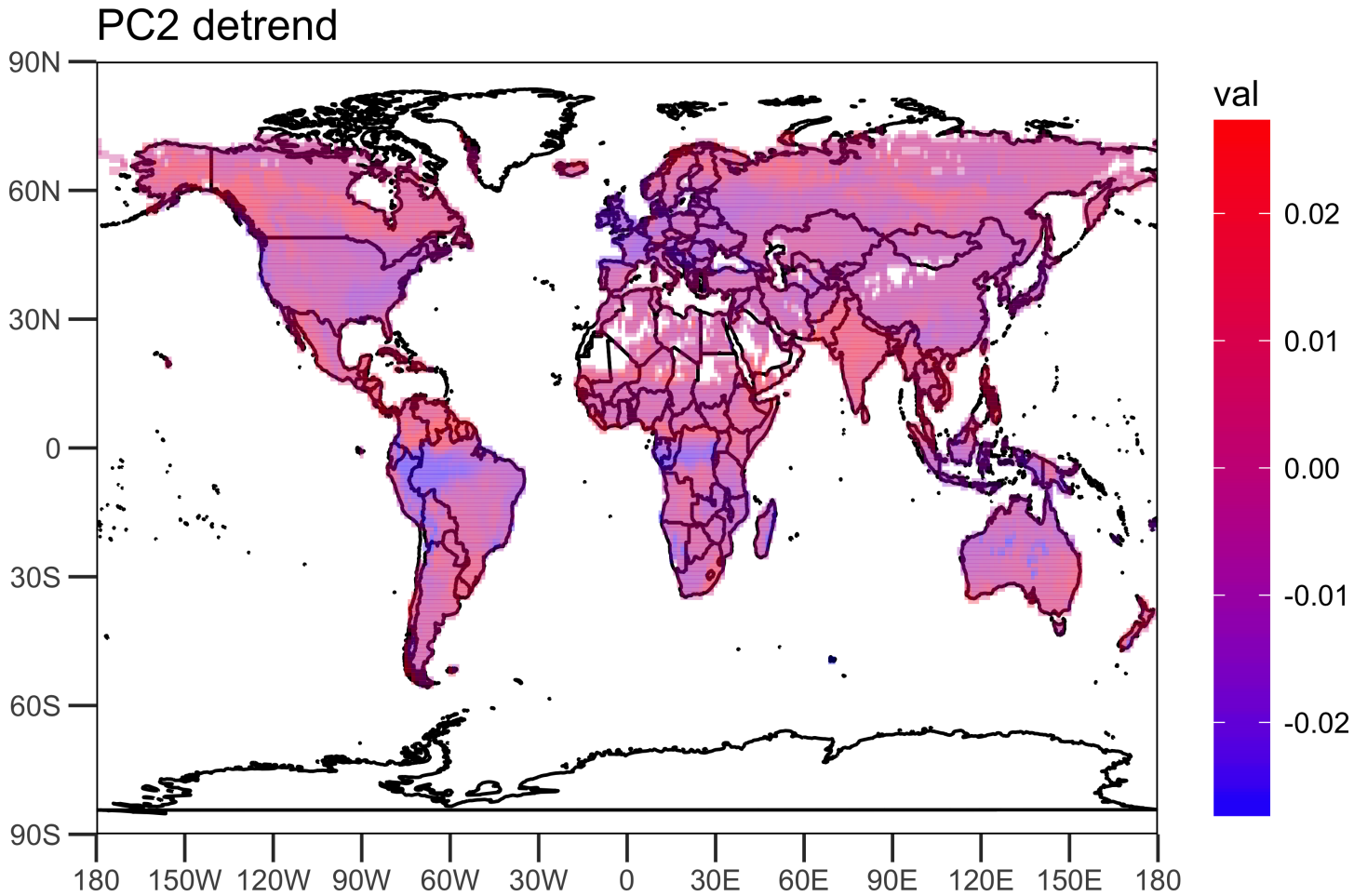


Figure: Loadings of Principal Component 2

Principal Component Analysis

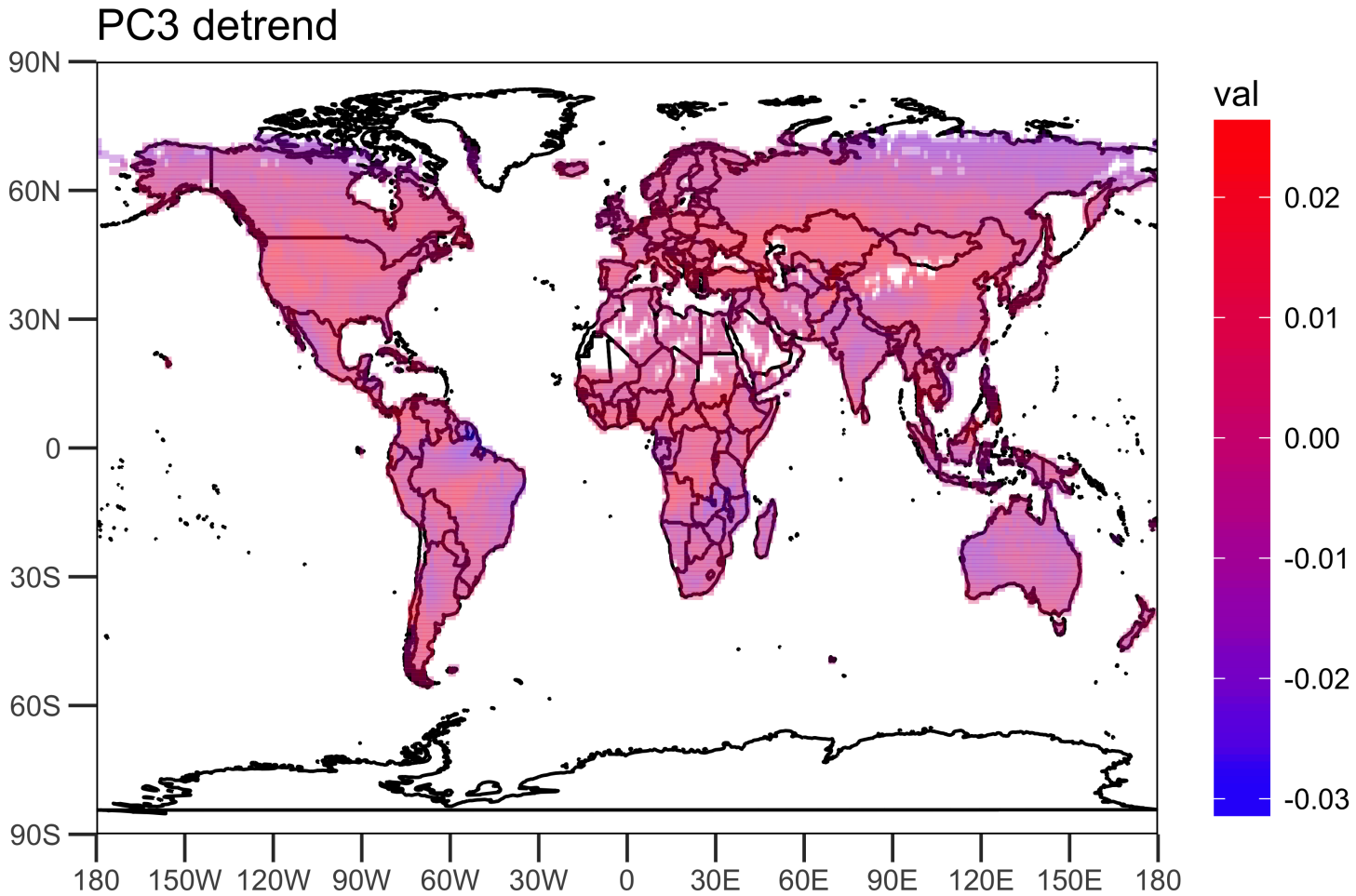


Figure: Loadings of Principal Component 3

Principal Component Analysis

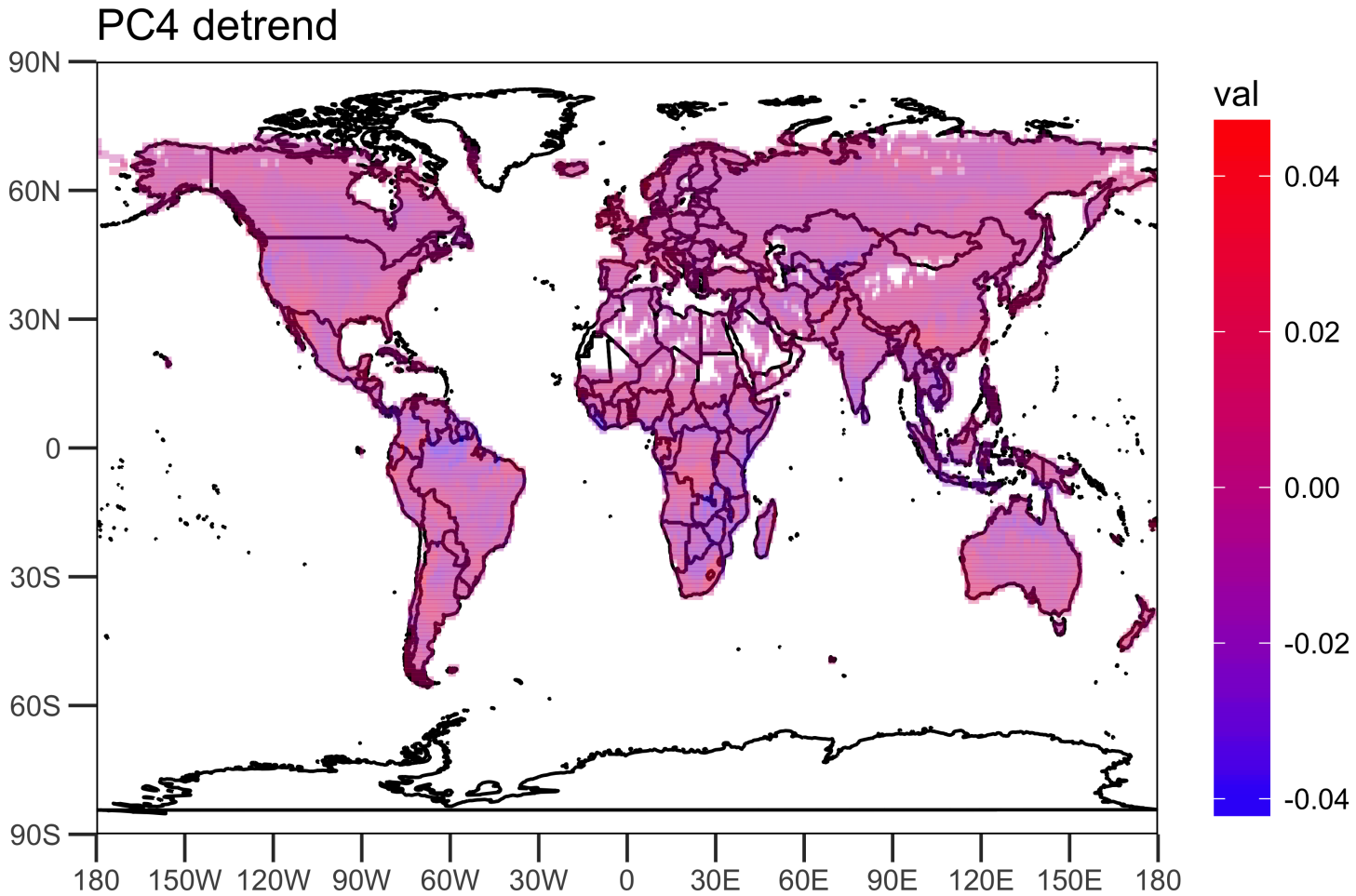


Figure: Loadings of Principal Component 4

Principal Component Analysis

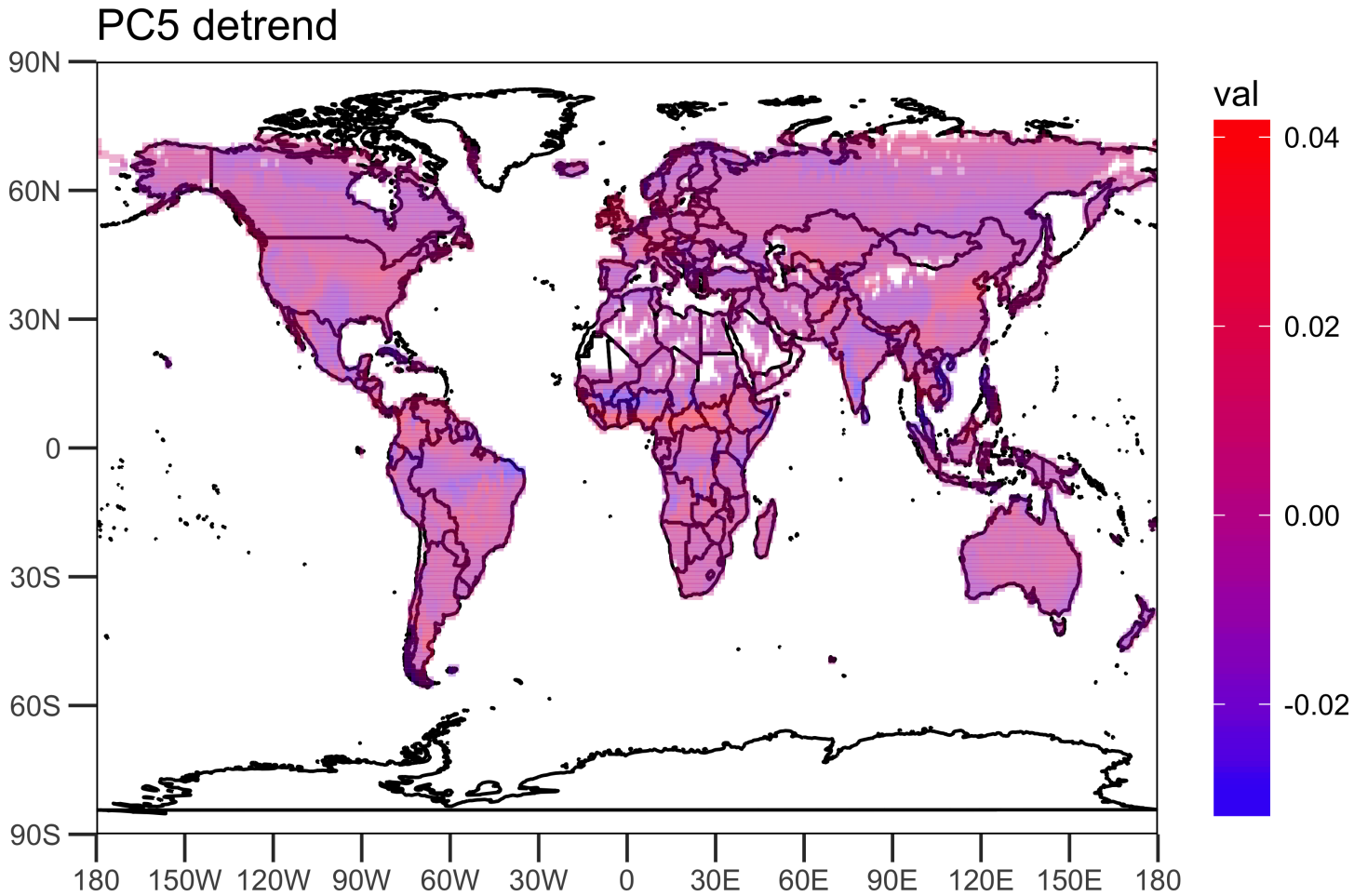


Figure: Loadings of Principal Component 5

Principal Component Analysis

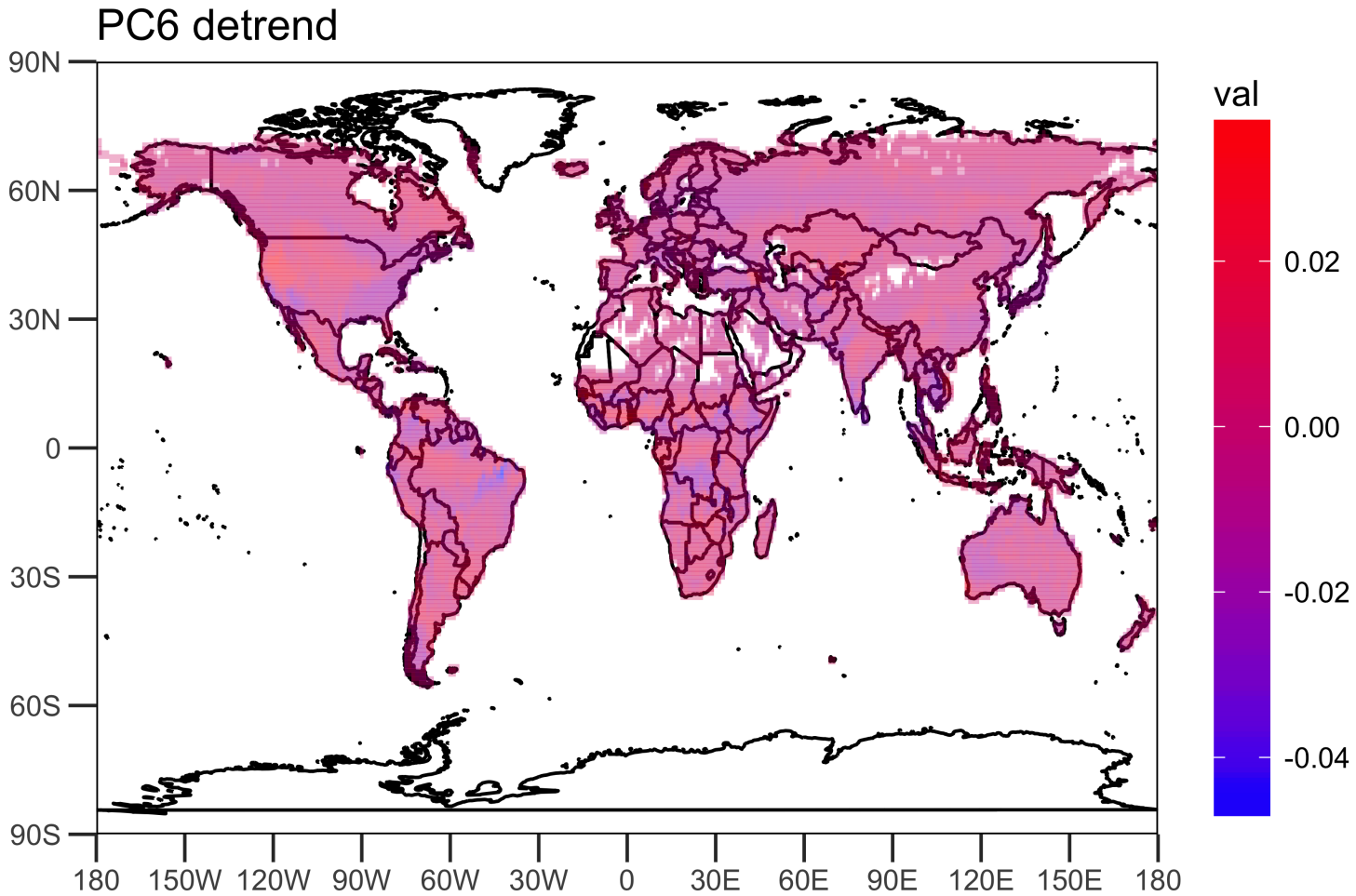


Figure: Loadings of Principal Component 6

Principal Component Analysis

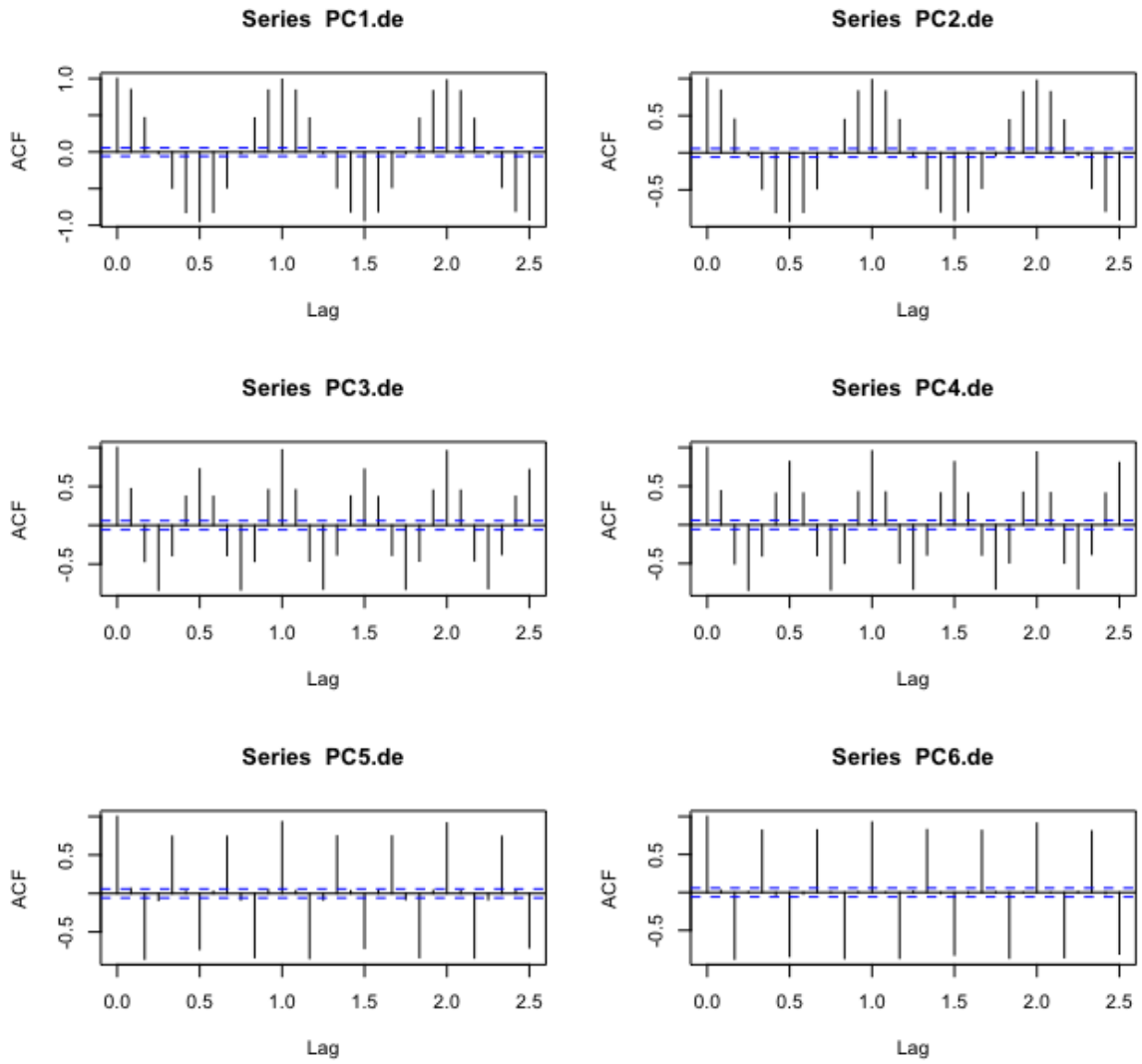


Figure: Auto-Covariance Function

Principal Component Analysis

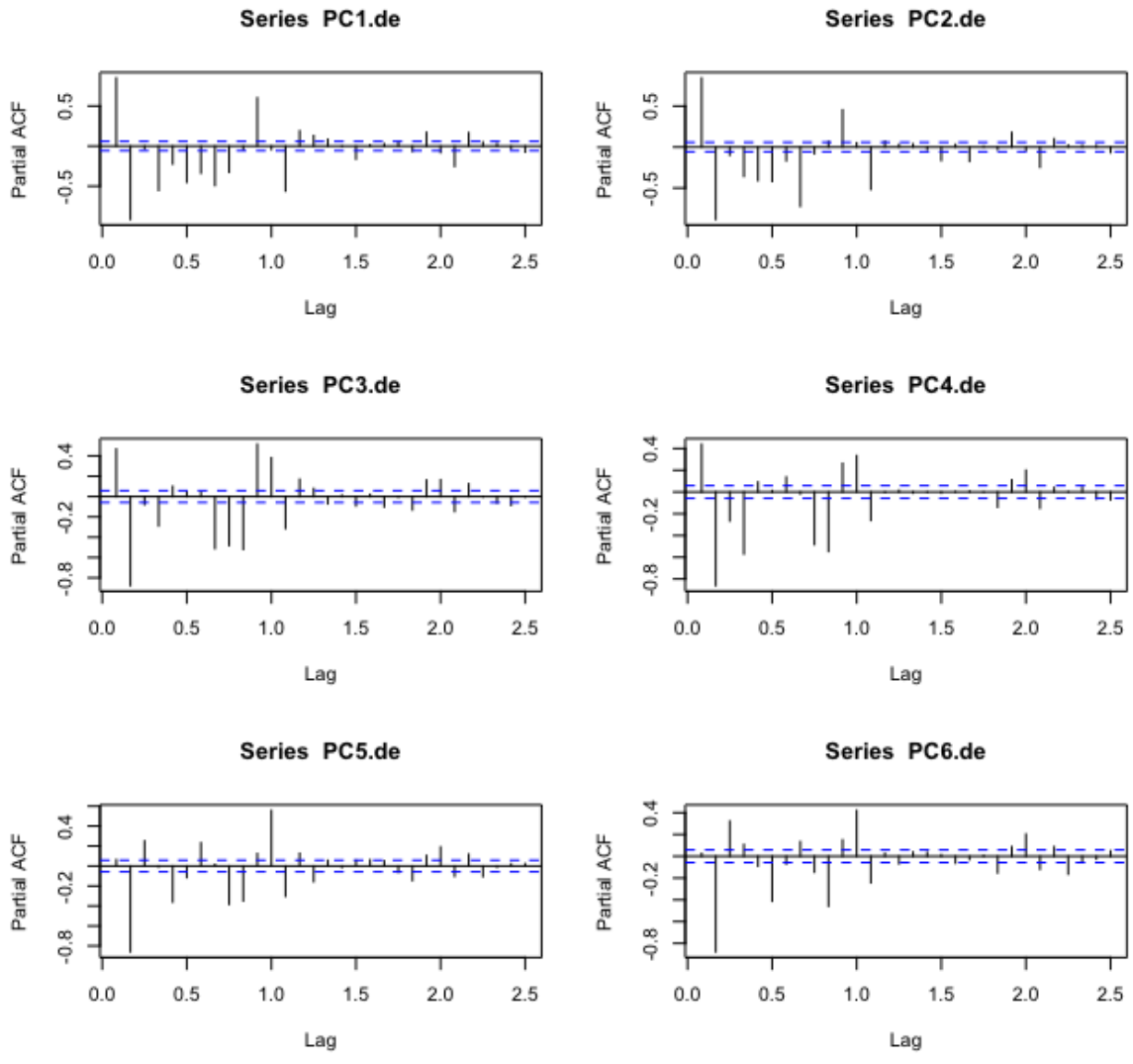


Figure: Partial ACF

Principal Component Analysis

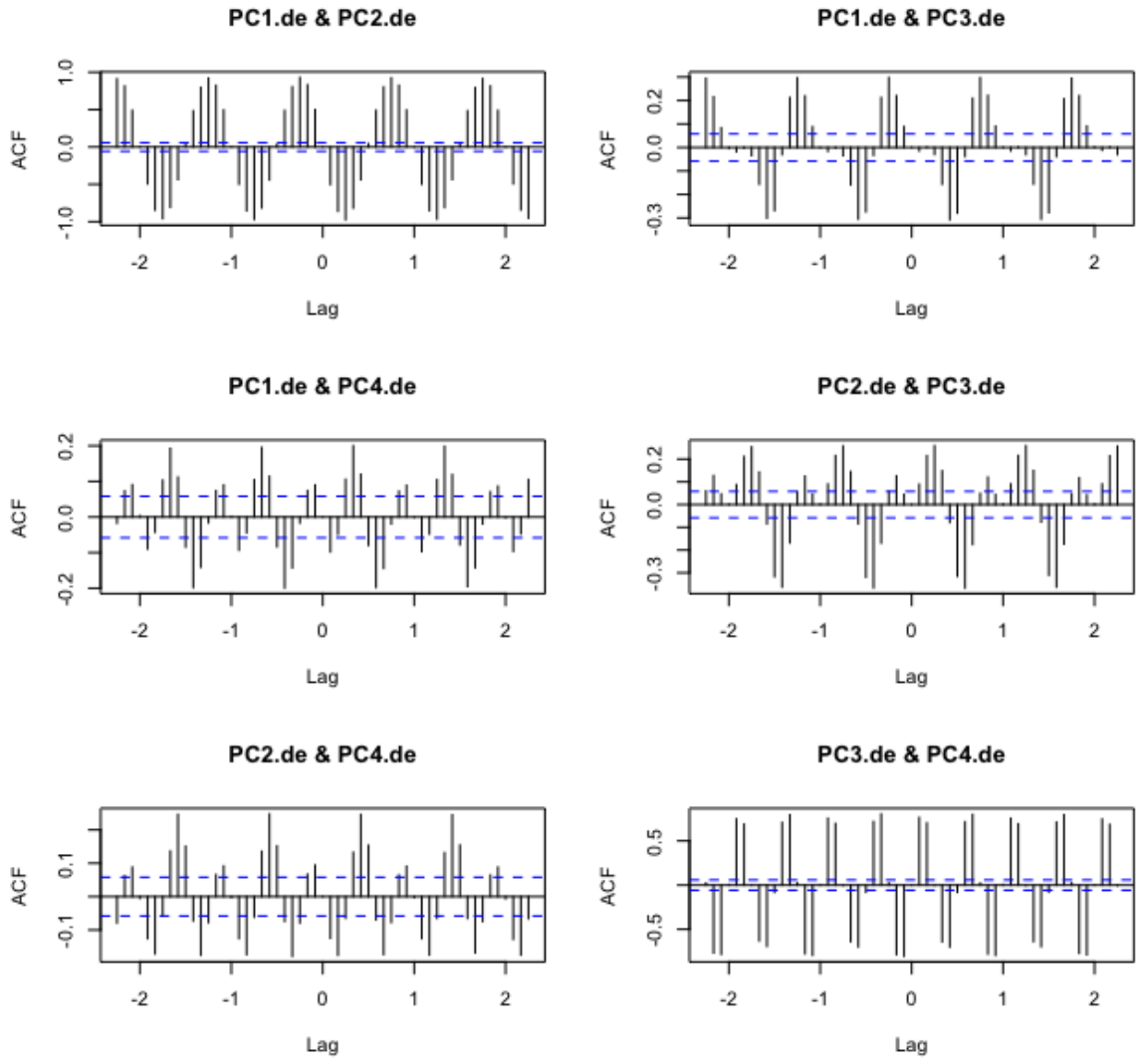


Figure: Cross-Covariance Function

Principal Component Analysis

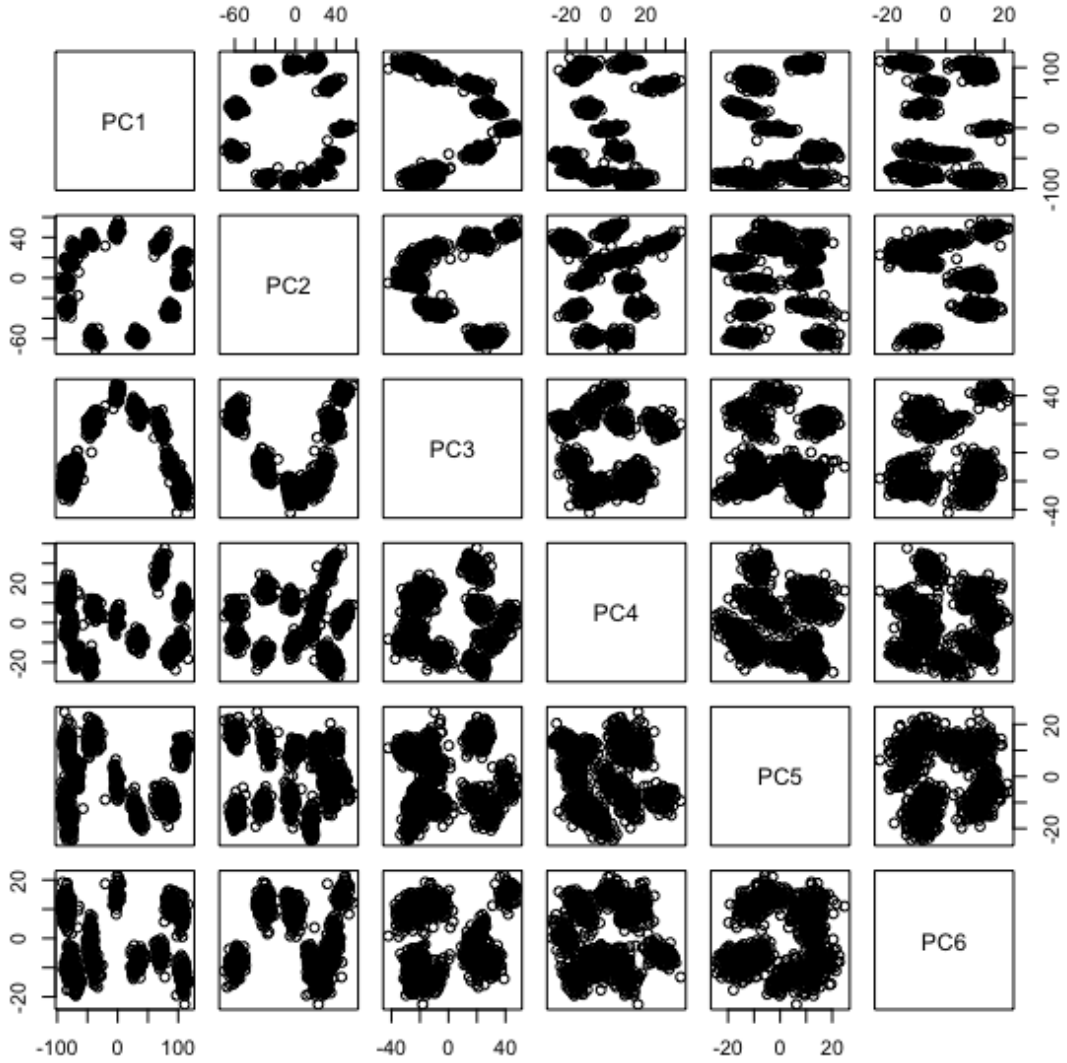


Figure: Pair Plot

PCA: Spectal Analysis

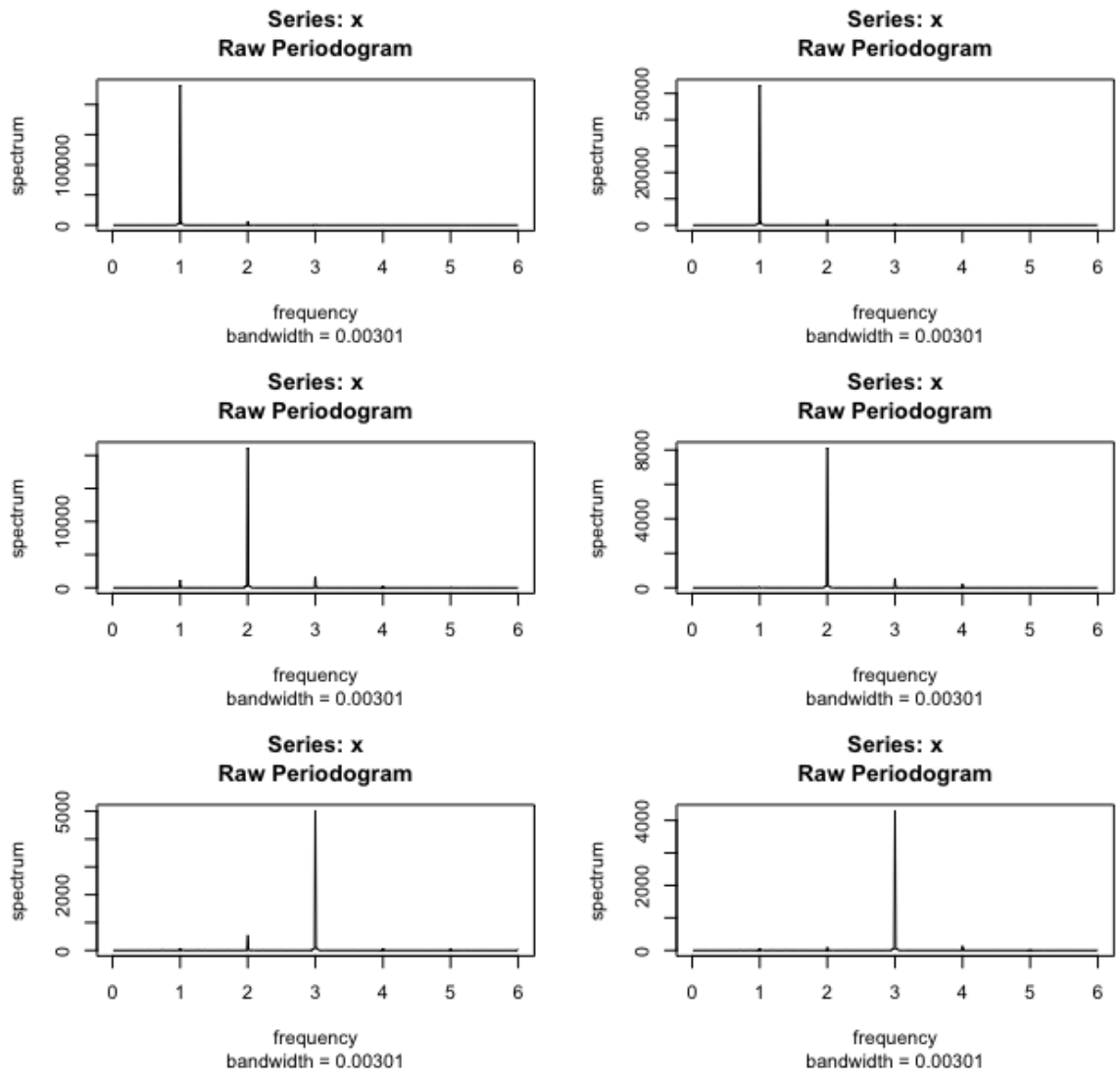


Figure: Periodogram

PCA: Spectal Analysis

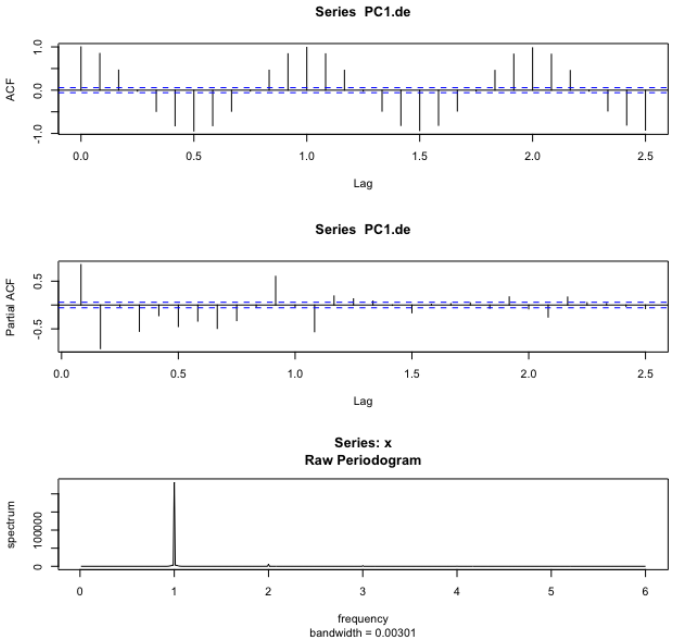


Figure: PC 1

$$X_t = A\cos(2\pi\frac{1}{12}t) + B\sin(2\pi\frac{1}{12}t)$$
$$= R\sin(2\pi\frac{1}{12} + \varphi)$$
$$\text{where } R^2 = A^2 + B^2,$$
$$\varphi = \arctan(\frac{A}{B})$$
$$\gamma(h) = \sigma^2\cos(2\pi\frac{1}{12}h)$$

PCA: Spectal Analysis

	Estimate	Std. Error	t value	Pr(>|t|)
cos(2 * pi/12 * 1:1140)	-32.6520	0.5834	-55.97	0.0000
sin(2 * pi/12 * 1:1140)	-97.1938	0.5834	-166.61	0.0000

- Residual standard error: 13.93 on 1138 degrees of freedom
- Multiple R-squared: 0.9645, Adjusted R-squared: 0.9644
- F-statistic: 1.545e+04 on 2 and 1138 DF, p-value: ¡ 2.2e-16
- $\hat{X}_t = 102.5319 \sin(2\pi \frac{1}{12} + 0.3241)$

Final Remarks