Tree cover variability increases from 2005 to 2100 in Sub-Saharan Africa

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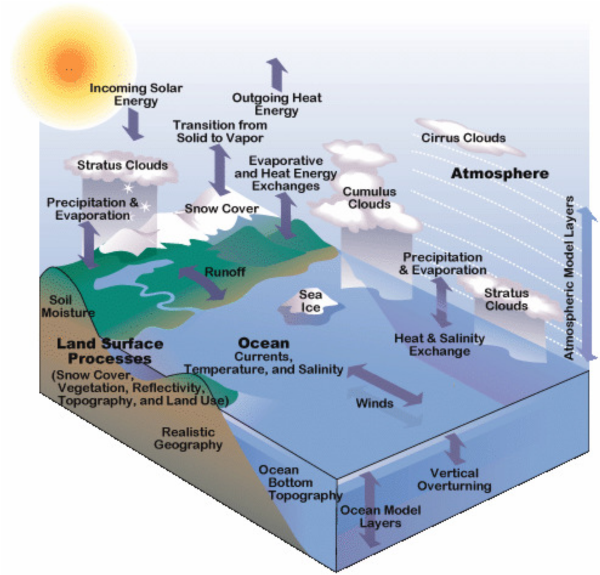
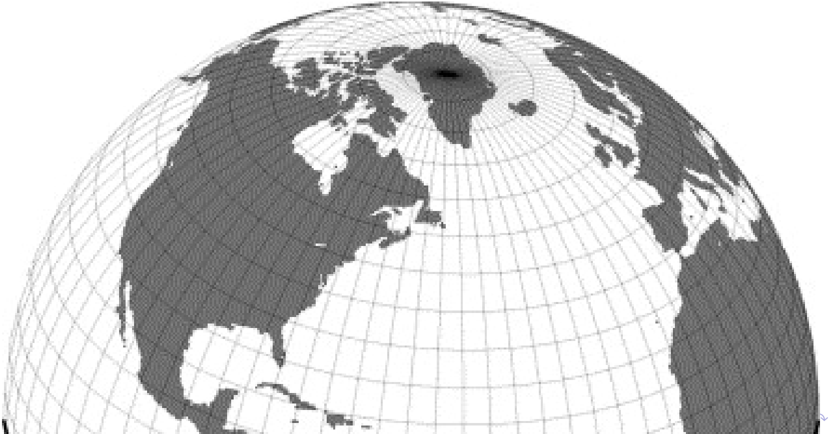
Introduction: climate system



https://www.ucar.edu/communications/CCSM/overview.html

Introduction: climate system modeling

- Climate change is driven by anthropogenic carbon forcing
- IPCC has developed representative carbon forcing trajectories
- Computer simulations are run predicated on particular forcing pathways
- These simulations are realizations of gridded meteorological PDEs

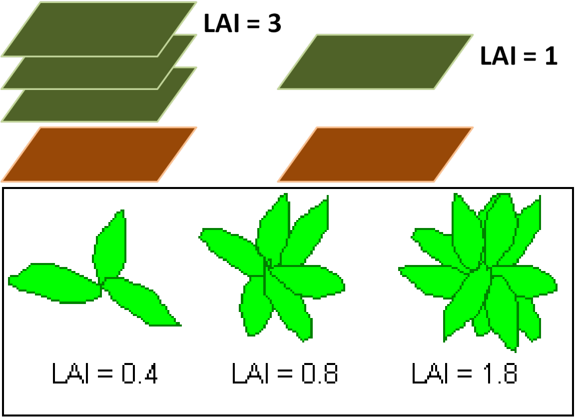


Introduction: climate system modeling

- The Community Climate Model System has 5 components and a coupler: atmosphere, sea, land, sea ice, and land ice.
- We focus on the land system output from a single ensemble under the RCP4.5 experiment.
- $\theta \in \Theta, \{X_t | t \in \mathbb{Z}\}$ where $X_t \sim Proc_\theta$
  $X_t = [X_t^{(l)}]$ for $l \in L = lat \times lon$

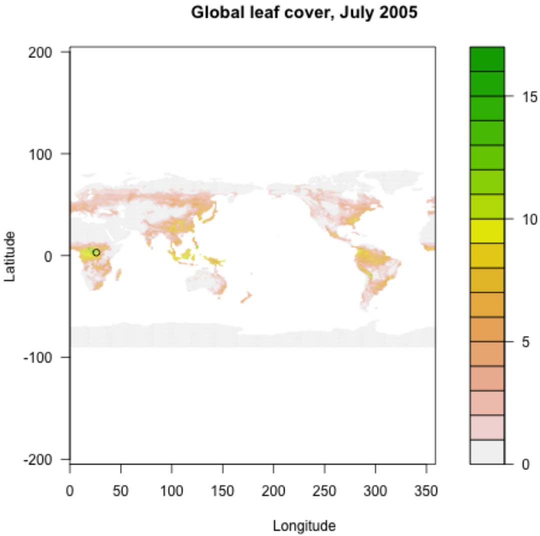
Introduction: leaf area index (LAI)

- LAI is unitless quantity (leaf area (m^2))/(ground area (m^2))
- "Comparative Physiological Studies on the Growth of Field Crops" Watson 1947 Annals of Botany
- Bounded below by 0, above by physiological limits

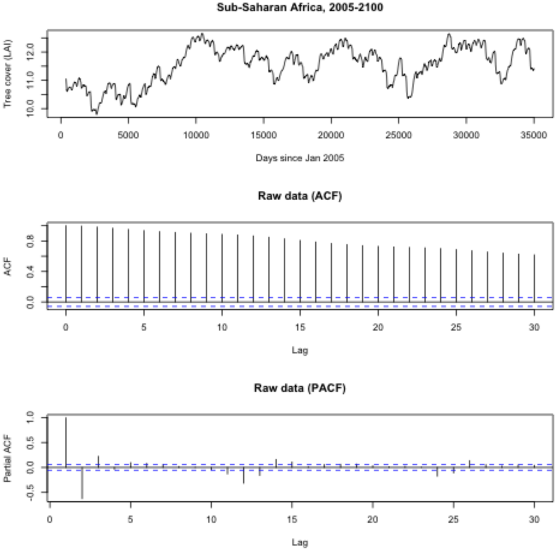


Introduction: Sub-Saharan African LAI

- Initially, use only one location $l = l_0$.
- Depicted under black circle on plot

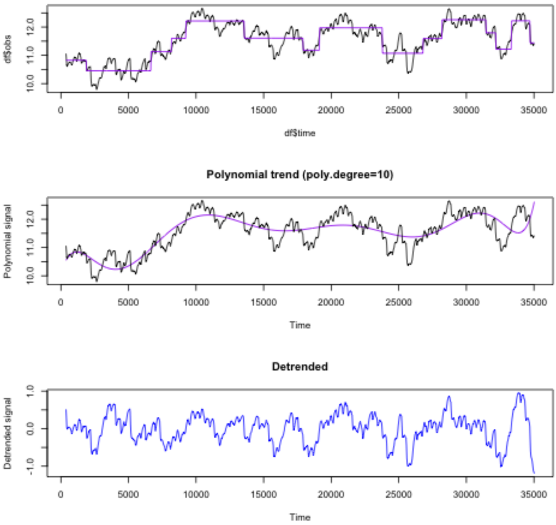


Raw data

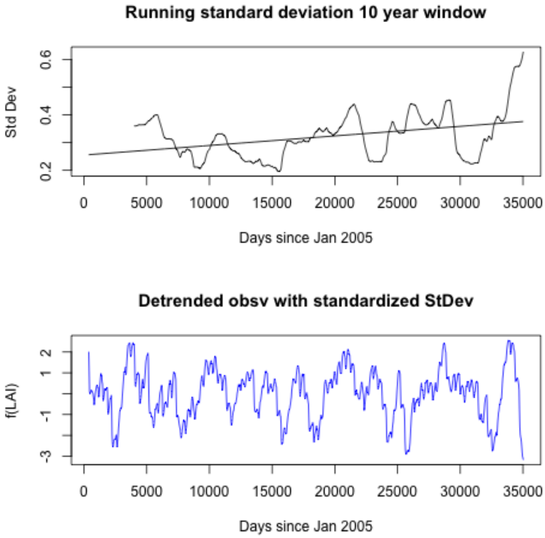


Sub-Saharan Africa, 2005-2100
Tree cover (LAI)
10.0
11.0
12.0
0
5000
10000
15000
20000
25000
30000
35000
Days since Jan 2005
Raw data (ACF)
ACF
0.0
0.4
0.8
0
5
10
15
20
25
30
Lag
Raw data (PACF)
Partial ACF
-0.5
0.0
0.5
1.0
0
5
10
15
20
25
30
Lag

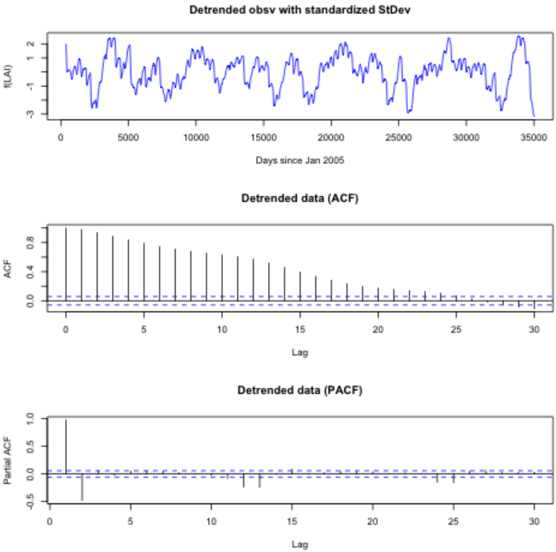
Detrended data



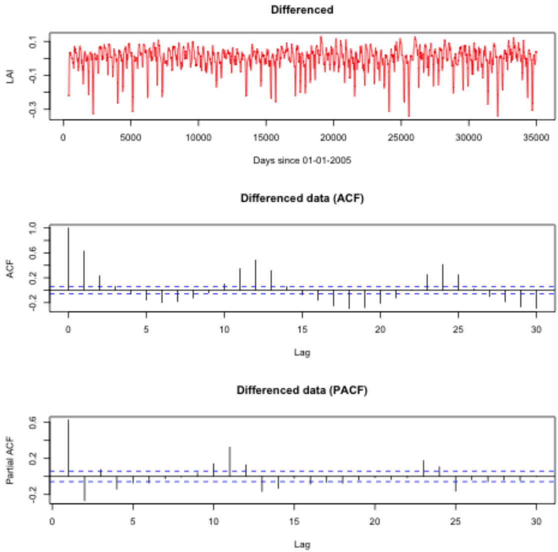
Standardize variance



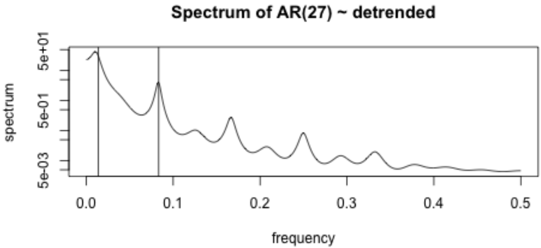
Standardized variance

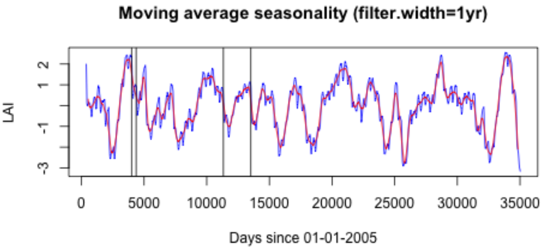


Differenced data

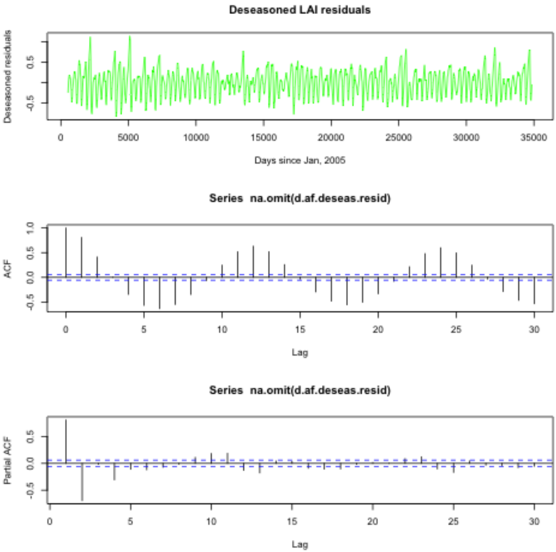


Deseasonized data

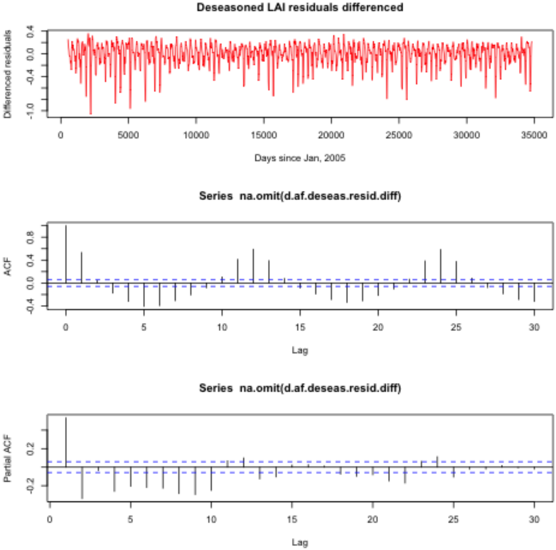




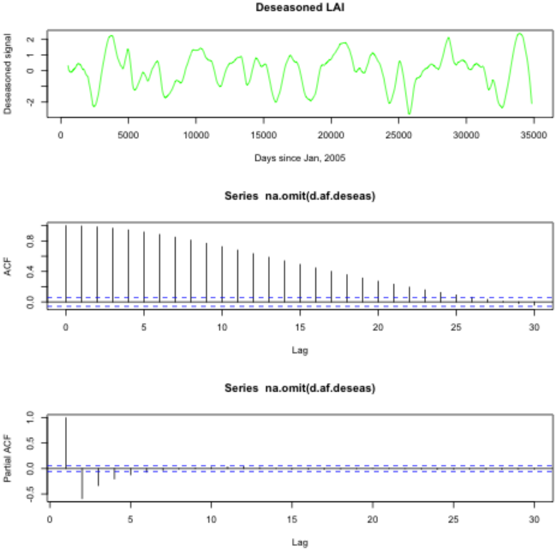
Deseasonized data



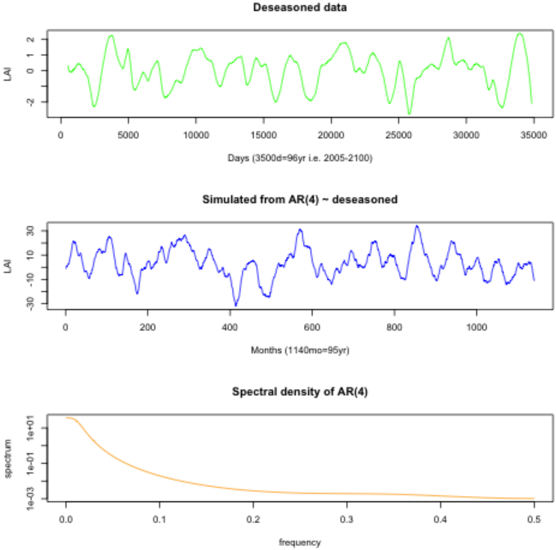
Deseasonized data



Deseasonized data



AR(4)


Deseasoned data
LAI
-2
0
1
2
0
5000
10000
15000
20000
25000
30000
35000
Days (3500d=96yr i.e. 2005-2100)
Simulated from AR(4) ~ deseasoned
LAI
-30
-10
10
30
0
200
400
600
800
1000
Months (1140mo=95yr)
Spectral density of AR(4)
spectrum
1e-03
1e-01
1e+01
0.0
0.1
0.2
0.3
0.4
0.5
frequency

Changepoint Analysis

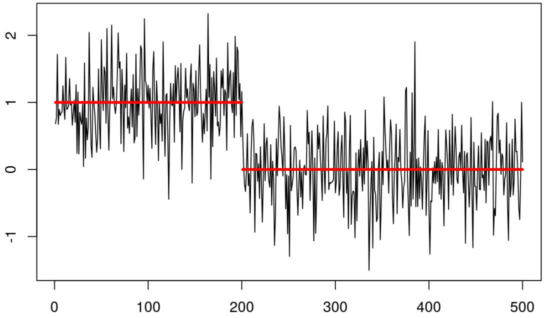


Figure: Example of a Changepoint

http://members.cbio.mines-paristech.fr/ thocking/change-tutorial/RK-CptWorkshop.html

Changepoint Analysis

- Conceptually, for data $Z_1, ..., Z_n$, a changepoint τ is a point in time, such that $Z_1, ..., Z_\tau$ differ from $Z_\tau + 1, ..., Z_n$
- How do we detect where a changepoint occurs, mathematically?
- We focus on the simple case where there is at most one changepoint to illustrate the method
- One approach is to use a Likelihood Ratio Test.

Changepoint Analysis: Simple Example

- Assume the parametric form $Z_t|\theta_t \;\, N(\theta_t, 1)$
- We state the hypotheses as:
- $H_0 : \theta_t = \theta \;\forall t$ vs. $H_1 : \theta_t = \theta_1, \;\, t \leq \tau; \;\, \theta_t = \theta_2, \;\, t > \tau$
- To use a likelihood ratio method, we need to maximize the likelihood under the null and alternative
- Under H_0, the maximum log-likelihood is $\log p(y_{1:n}|\hat{\theta})$
- Under H_1 with a changepoint τ, the maximum log-likelihood is $ML(\tau) = \log p(y_{1:\tau}|\hat{\theta}_1) + \log p(y_{\tau+1:n}|\hat{\theta}_1)$.
- $p(\cdot)$ is the probability density function associated with the distribution of the data and $\hat{\theta}$ is the maximum likelihood estimate of the parameters.

Changepoint Analysis: Test Statistic

- The log-LR statistic is then:

$$LR(\tau) = 2\left[\max_{\tau} ML(\tau) - \log p(y_{1:n}|\hat{\theta})\right]$$

- We compare this statistic with a cut-off value, λ. If $LR > \lambda$, then we reject H_0 and estimate the changepoint as

$$\hat{\tau} = \arg\max_{\tau} LR(\tau)$$

- There are many different ways to select λ. Selecting an optimal value of λ remains an open research topic.
- We use the package 'changepoint' to investigate if our LAI time series has a changepoint.

Changepoint Analysis on LAI TS

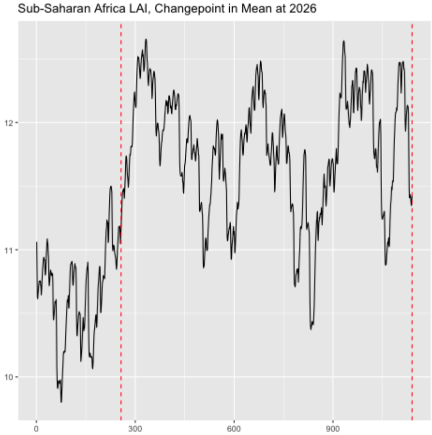


Figure: LAI TS

- Result: $\hat{\tau} = 237$ months after Jan. 2005, i.e. 2026 is when LAI experiences a changepoint in mean.

Changepoint Analysis: Interpretation

- This data is simulated with carbon emissions to peak in 2020. Even so, the changepoint occurs in 2026, implying that the chain of events that cause the shift may have been set into motion years before the changepoint actually occurs.
- If this is the case, it seems that even the optimistic outcome of curbing carbon emissions by 2020 may be too late. The domino may have already been pushed.

Spectral Analysis: Periodogram

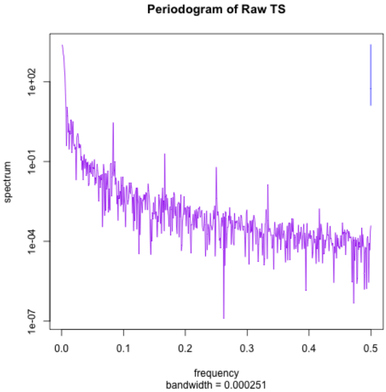


Figure: LAI TS

Peaks at $\omega = \frac{1}{12}, \frac{2}{12}, \frac{3}{12}$...etc., indicating annual, semiannual, quarterly...etc. periods. We investigate these periods further after noticing some curious behavior in our principal component analysis.

Principal Component Analysis

- 12818 locations, 1140 time points
- $\mathbf{Z}(s,t) = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^T$
- $\mathbf{Z}(s,t) =$
$$\begin{pmatrix} z(s_1,t_1) & z(s_2,t_1) & z(s_3,t_1) & \dots & z(s_{12818},t_1) \\ z(s_1,t_2) & z(s_2,t_2) & z(s_3,t_2) & \dots & z(s_{12818},t_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z(s_1,t_{1140}) & z(s_2,t_{1140}) & z(s_3,t_{1140}) & \dots & z(s_{12818},t_{1140}) \end{pmatrix}$$

Principal Component Analysis

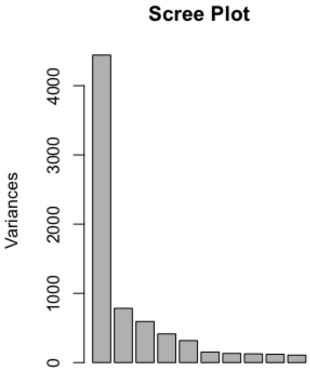


Figure: Raw Data

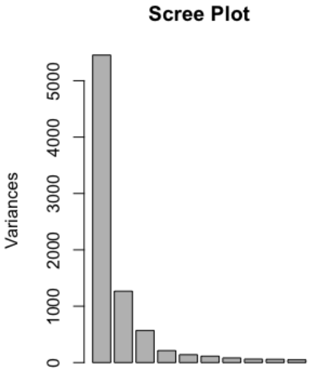


Figure: Detrend Data

Principal Component Analysis

	PC1	PC2	PC3	PC4	PC5	PC6
StDiv	66.650	28.003	24.356	20.348	17.832	12.301
Prop. of Var.	0.347	0.061	0.046	0.032	0.025	0.012
Cum. Prop.	0.347	0.408	0.454	0.486	0.511	0.523

Table: Raw Data

	PC1	PC2	PC3	PC4	PC5	PC6
StDiv	73.857	35.586	23.852	14.581	11.859	10.669
Prop. of Var.	0.426	0.099	0.044	0.017	0.011	0.009
Cum. Prop.	0.426	0.524	0.569	0.585	0.596	0.605

Table: Detrend

Principal Component Analysis

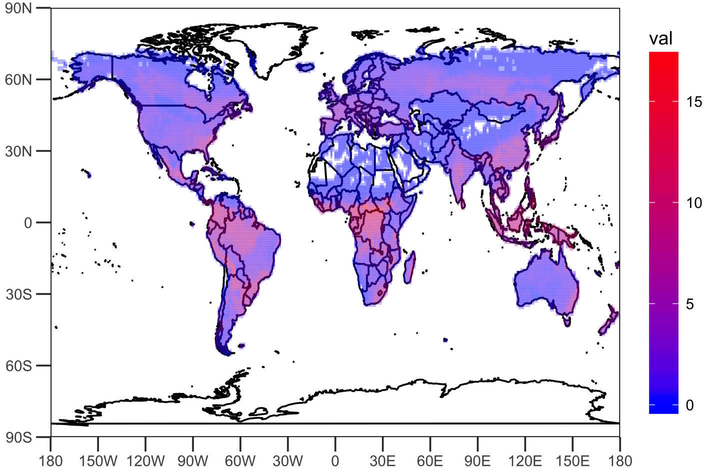


Figure: Global Leaf Index Jan 2005

Principal Component Analysis

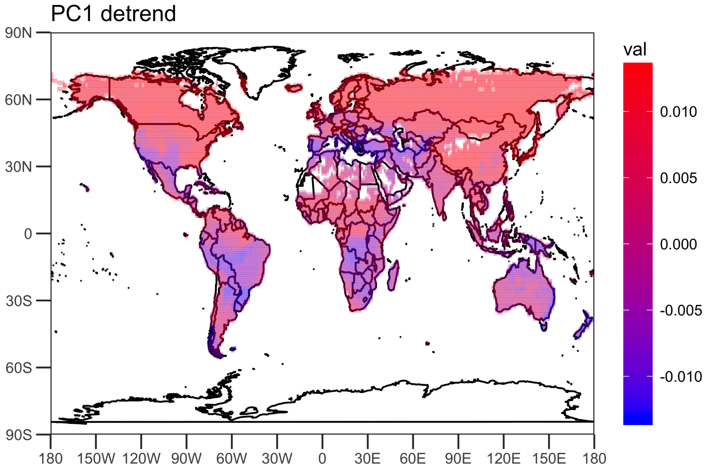


Figure: Loadings of Principal Component 1

Principal Component Analysis

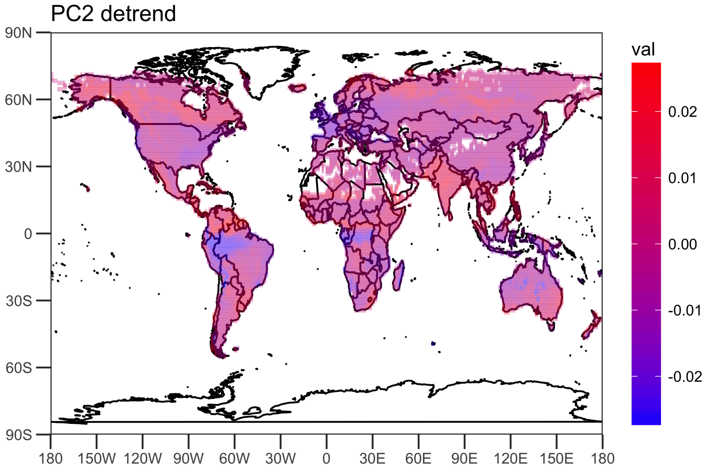


Figure: Loadings of Principal Component 2

Principal Component Analysis

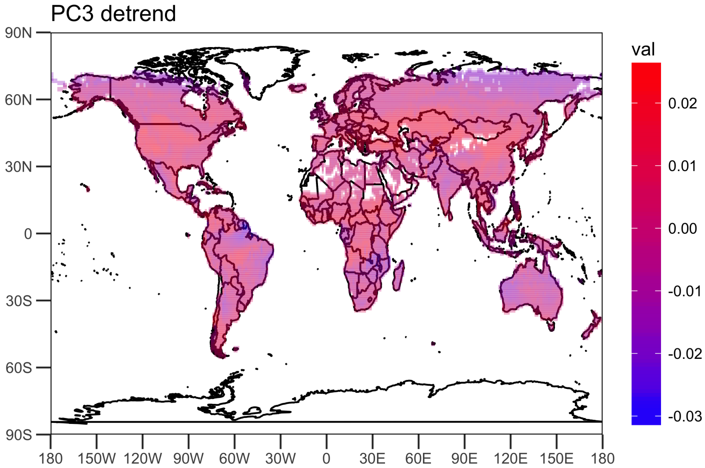


Figure: Loadings of Principal Component 3

Principal Component Analysis

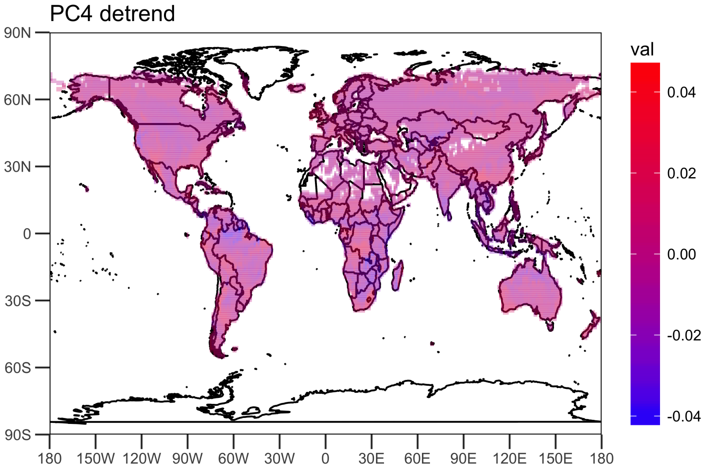


Figure: Loadings of Principal Component 4

Principal Component Analysis

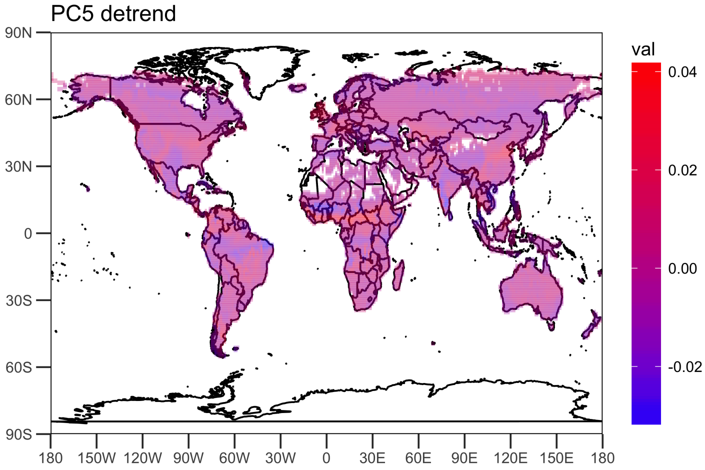


Figure: Loadings of Principal Component 5

Principal Component Analysis

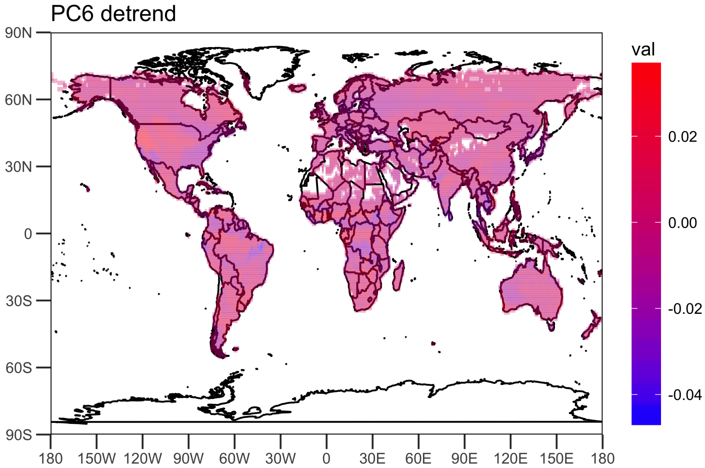


Figure: Loadings of Principal Component 6

Principal Component Analysis

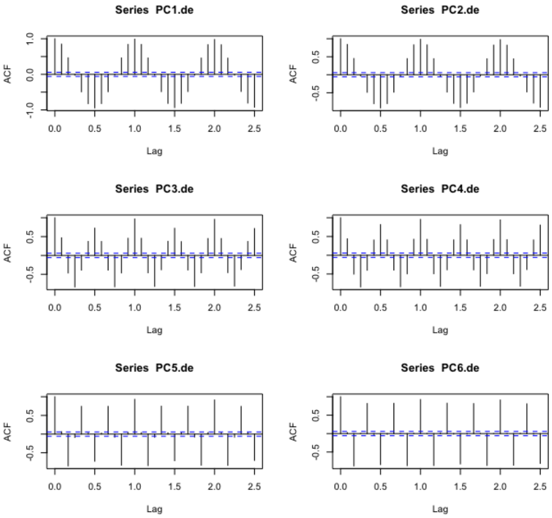


Figure: Auto-Covariance Function

Principal Component Analysis

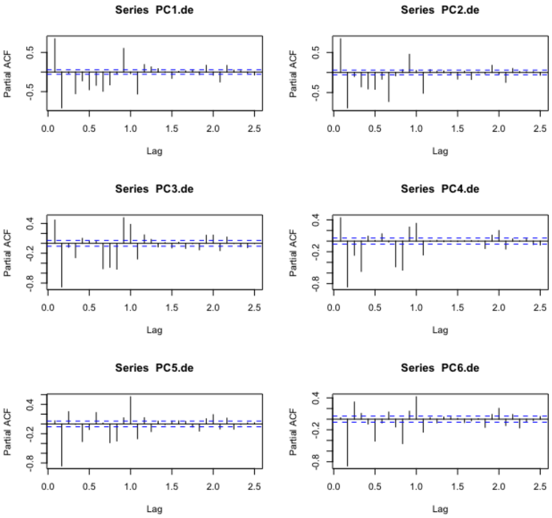


Figure: Partial ACF

Principal Component Analysis

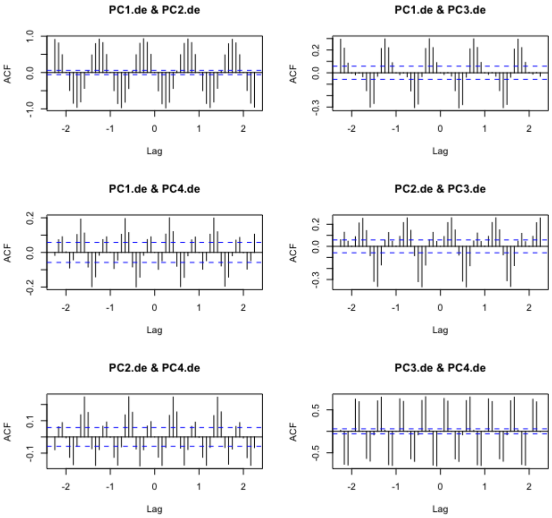


Figure: Cross-Covariance Function

Principal Component Analysis

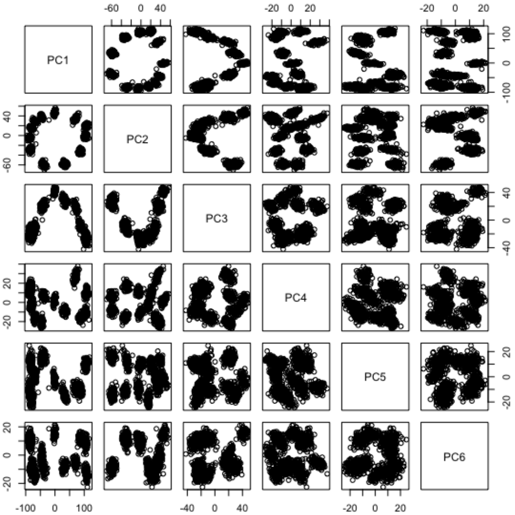


Figure: Pair Plot

PCA: Spectal Analysis

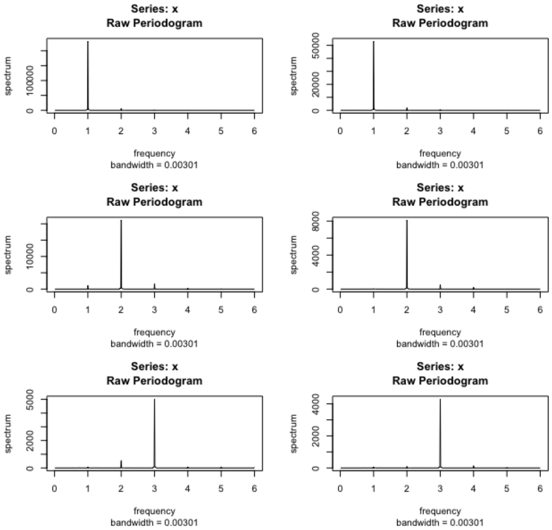


Figure: Periodogram

PCA: Spectal Analysis

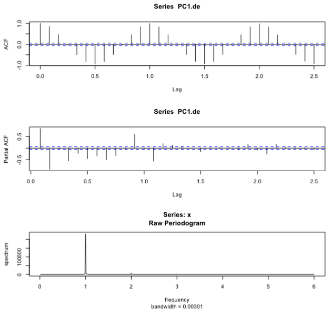


Figure: PC 1

$$X_t = A\cos(2\pi\frac{1}{12}t) + B\sin(2\pi\frac{1}{12}t)$$
$$= R\sin(2\pi\frac{1}{12} + \varphi)$$
$$\text{where } R^2 = A^2 + B^2,$$
$$\varphi = \arctan(\frac{A}{B})$$
$$\gamma(h) = \sigma^2\cos(2\pi\frac{1}{12}h)$$

PCA: Spectal Analysis

	Estimate	Std. Error	t value	Pr(>|t|)
cos(2 * pi/12 * 1:1140)	-32.6520	0.5834	-55.97	0.0000
sin(2 * pi/12 * 1:1140)	-97.1938	0.5834	-166.61	0.0000

- Residual standard error: 13.93 on 1138 degrees of freedom
- Multiple R-squared: 0.9645, Adjusted R-squared: 0.9644
- F-statistic: 1.545e+04 on 2 and 1138 DF, p-value: $< 2.2\text{e-}16$
- $\hat{X}_t = 102.5319 \sin(2\pi \frac{1}{12} + 0.3241)$

Final Remarks