NTNU

Innovation and Creativity

TTK5 Kalman Filtering and Navigation
Lecture Notes for Module 1

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TTK5: Organization

- Lectures
- 2 compulsory lab assignments/computer exercises

Date	Time	
28th August	14:00-15:45	**Module 1**
4th September	14:00-16:45	
11th September	14:00-16:45	
18th September	14:00-16:00	
9th October	13:15-15:00	**Module 2:** Kjetil Bergh Ånonsen (FFI)
16th October	13:15-15:00	
30th October	13:15-16:00	
6th November	13:15-16:00	

Check for lecture schedule updates on the TTK5 webpage!

NTNU
Innovation and Creativity

Lecture Components

- Fundamentals of GNSS
  - Current and Future GNSS Overview
  - Principles of Operation
  - Signal Structure
  - GNSS Receivers
  - Errors and Vulnerabilities
  - GNSS Observables and Position Estimation
  - Differencing Concepts
- DGNSS, RTK and PPP

GNSS

Module 1 - Recommended Literature:

- Vik, B. (2014) *Integrated Satellite and Inertial Navigation Systems, TTK5 Lecture Notes, Engineering Cybernetics Dept., NTNU.*
- Misra, P. and P. Enge, *Global Positioning System: Signals, Measurements and Performance*. Ganga-Jamuna Press, P. O. Box 633, Lincoln, MA 01773, 2nd ed., 2006.
- Kaplan E. D. and C. J. Hegarty (2005), Eds., *Understanding GPS: Principles and Applications*, 2nd ed. Norwood, MA, USA: Artech House Publishers.
- Hofmann-Wellenhof B., Lichtenegger H. and E. Wasle ed.(2008), *GNSS - Global Navigation Satellite Systems - GPS, GLONASS, Galileo and more*. Springer, Wien NewYork.
- Parkinson, B. W. & Spilker Jr., J. J., ed. (1996), *Global Positioning System: Theory and Applications, Vol. 1, American Institute of Aeronautics and Astronautics, Washington DC.*
- Brown, R. G. and P.Y.C. Hwang (1997) *Introduction to Random Signals and Applied Kalman Filtering.* John Wiley & Sons.
- Forssell, B. (1991). *Radionavigation Systems.* Prentice Hall International Ltd. London, UK.

GNSS

Module 1 – Other useful information sources:

- http://kom.aau.dk/~borre/ an extensive library of .mat files written by Prof. Kai Borre, Aalborg University covering coordinate frame transformations, satellite coordinate computation procedure, user position computation, formation of double-difference code and phase observables and many others.
- http://www.rtklib.com/ open source program package for standard and precise positioning with GNSS.
- http://www.ngs.noaa.gov/gps-toolbox/exist.htm contains a list of various open source GNSS SW tools.

Applications

- Military Applications…
- Civil Applications:

Air Navigation
- Precision and non-precision approach and landing
- Domestic en route
- Oceanic en route
- Terminal
- Remote areas
- Helicopter operations
- Aircraft attitude
- Collision avoidance
- Air Traffic Control

Land Navigation
- Vehicle monitoring
- Schedule improvement
- Minimal routing
- Law enforcement
- Pedestrian navigation

Marine Navigation
- Oceanic
- Coastal
- Harbor/approach
- Inland waterways

Static Positioning and Timing
- Offshore resource exploration
- Hydrographic surveying
- Aids to navigation
- Time transfer
- Land surveying
- Geographical information systems

Search and Rescue
- Position reporting and monitoring
- Coordinated search
- Collision avoidance

Space
- Launch
- In-flight/orbit
- Reentry/landing
- Attitude measurement

B.W. Parkinson, Introduction and Heritage of NAVSTAR, the Global Positioning System, 1994

What is GNSS?

Global **N**avigation **S**atellite **S**ystem (**GNSS**)

- Definition of GNSS given by [ICAO Annex 10,Volume 1]: *"A worldwide position and time determination system that includes one or more satellite constellations, aircraft receivers and system integrity monitoring, augmented as necessary to support the required navigation performance for the intended operation."*

✓ Nowadays, GNSS refers more broadly to any satellite-based system providing global, all weather, continuous, real time positioning and timing service to the user (e.g. GPS or GLONASS alone).

- Global Positioning System (GPS): U.S. system
- GLONASS: Russian System

} Fully operational

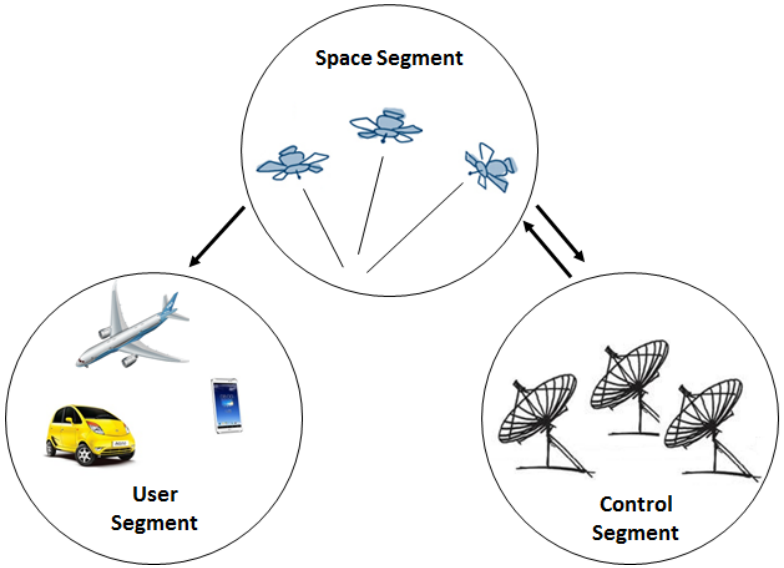
- Galileo : European system
- Compass/Beidou: Chinese system

} Under development

GNSS Architecture

- The **space segment** consists of GNSS satellites, orbiting about 20,000 km above the earth. Each GNSS has its own "constellation" of satellites, arranged in orbits to provide the desired coverage.

- The **control segment** consists of a ground-based network of master control stations, data uploading stations and monitor stations.
  - *monitor stations* installed over a broad geographic area monitor the satellites' signals and status, and relay this information to the master control station.
  - *master control station* analyses the signals then transmits orbit and time corrections to the satellites through data uploading stations.



- The **user segment** consists of equipment that processes the received signals from the GNSS satellites and uses them to derive and apply location and time information.

Current GNSS : GPS

Global **P**ositioning **S**ystem (**GPS**): owned and operated by the US government

- Nominal constellation: 24 satellites + 3 active spares

  Currently 32 satellites in orbit, 31 operational, 1 in maintenance.

- 6 orbital planes (4-6 satellites in each plane) spaced 60 ° around equator
- 11 hours 58 min orbital period

  - in a terrestrial frame, the constellation repeats every 23h 56min (1 siderial* day)

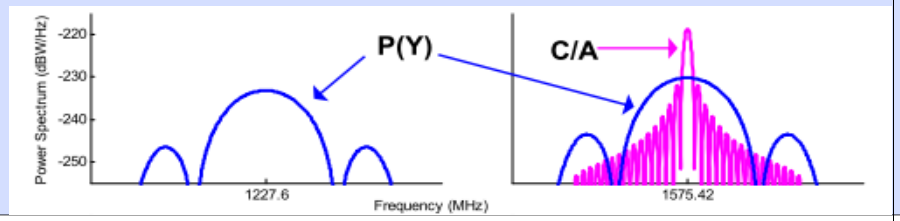
- 20,200 km + altitude
- Inclination: 55° with respect to the equatorial plane
- Code Division Multiple Access (CDMA) system

***A sidereal day is the rotation period of the Earth relative to the equinox and is equal to one calendar day (the mean solar day) minus approximately four minutes.**

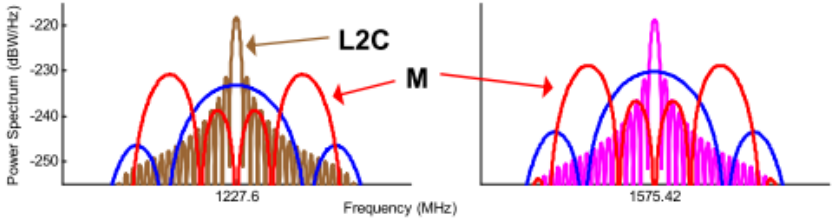
GPS Signals and Frequencies

L2 (*1227.60 MHz*) **L1 (*1575.42 MHz*)**

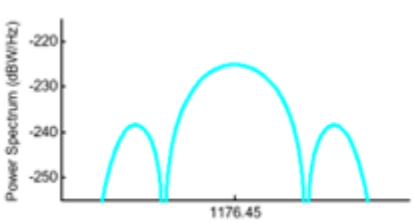
Legacy Signals:



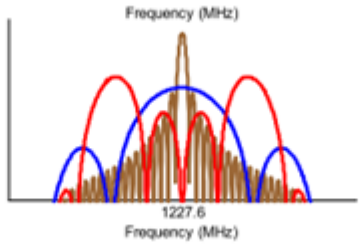
New signals:



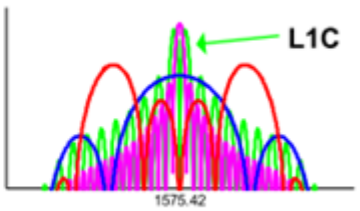
L5 (*1176.45 MHz*)



Full capability L5 -2020 (12 SVs in operation)



Full capability L2C - 2018 (18 SVs already in operation)



Full capability L1C - 2026

- New civil signal on L2 (L2C), designed to meet commercial needs
- New military signals on L1 and L2 (the M signals)

- New civil signal on L5 (L5) designed to meet transportation safety-of-life requirements

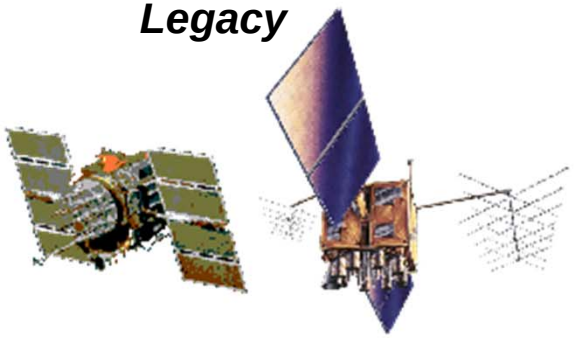
- Fourth civil signal L1C, designed for improved performance and GNSS interoperability

Space Segment: GPS

The GPS constellation is a mix of new and legacy satellites:

Block IIA/IIR

Legacy



- 12 (0/12) Block IIA / IIR satellites in the constellation
- (C/A) code navigation for civil users
- P(Y) code navigation for military users
- On-board clock monitoring (IIR)

C/A – Course Acquisition
P(Y) - Precision

Block IIR-M, IIF

Modernized

- 8 Block IIR-M satellites in the constellation (1 in maintenance)
- - 2nd civilian signal (L2C) for improved performance in commercial applications
- New military M-code
- More powerful signals

IIF: 12 in orbit. IIR-M capability plus 3rd civil signal (L5)

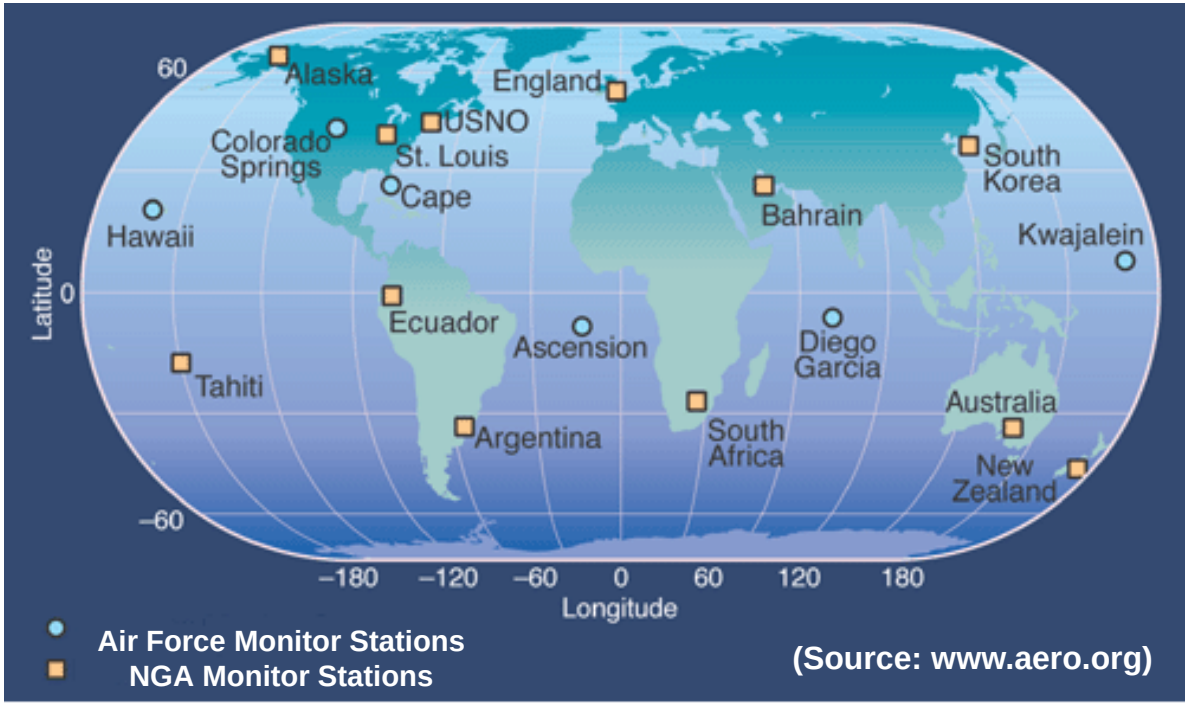
Block III

Future

- Currently under development
- 4th civilian GPS signal (L1C) for international interoperability
- More powerful signals
- 15-year design lifespan
- Signal Integrity
- Search and Rescue

[Crews, 2007]

Control Segment: GPS (1/2)



Original configuration:

Master Control Station (MCS):

-Schriever AFB in Colorado

5 AF Monitor Stations:

-Hawaii,
-Kwajalein
-Ascension Island
-Diego Garcia
-Colorado Springs

3 Ground Antennas:

-Ascension Island
-Diego Garcia
-Kwajalein

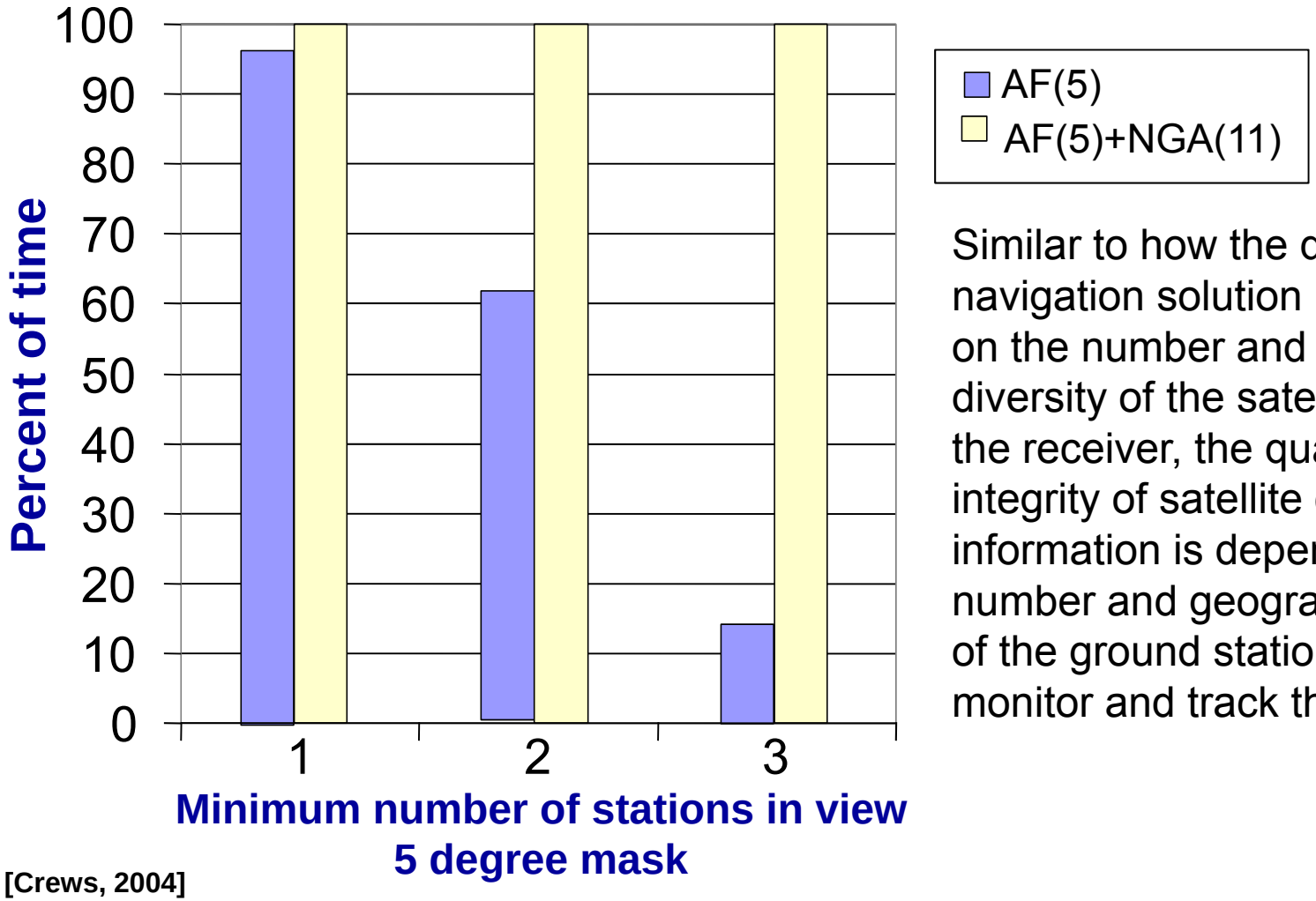
Basic functions:

-monitor satellite orbits
-monitor satellite health
-maintain GPS Time
-predict satellite ephemeris and clock parameters
-update satellite navigation messages

Original 6 stations augmented by 8 (eventually 11) NGA stations

National Geospatial-Intelligence Agency (NGA)

Control Segment: GPS (2/2)



[Crews, 2004]

Similar to how the quality of a navigation solution is dependent on the number and geometric diversity of the satellites visible to the receiver, the quality and integrity of satellite orbit and health information is dependent on the number and geographical diversity of the ground stations which monitor and track the satellites.

Result: more accurate orbit and clock information – improvement of User Ranging Error (URE)

GPS Levels of Service

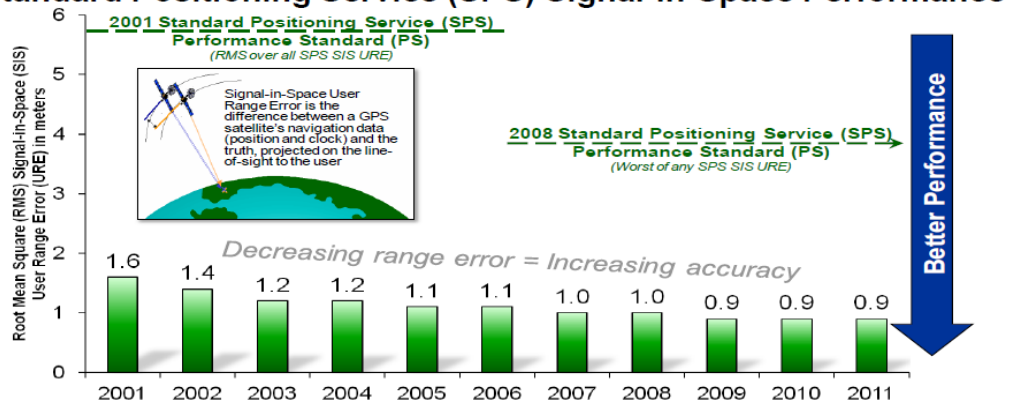
SPS – Standard Positioning Service

- Civilian service available for all users on a continuous, worldwide basis with no direct charge
- Currently access to only a single ranging code on GPS L1 frequency (note: 18 satellites with modernized signals, but not yet fully functional)
- Less accurate

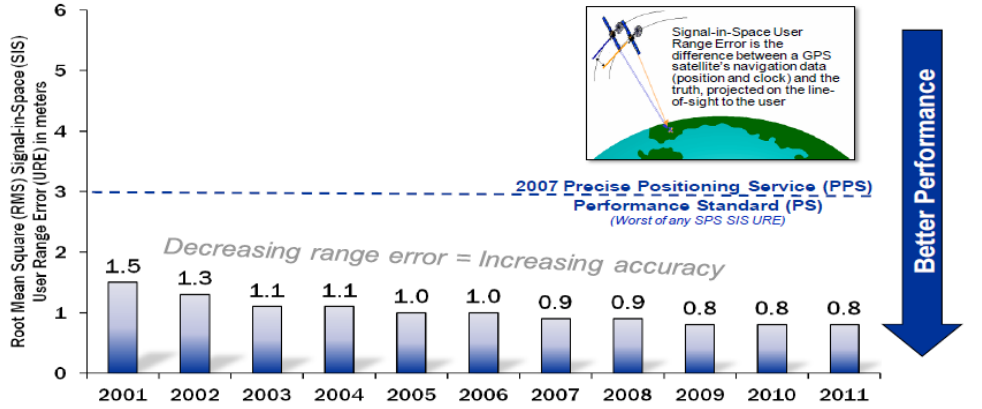
PPS – Precise Positioning Service

- Higher-accuracy service for authorized users (designed primarily for U.S. military use)
  High-bandwidth P(Y) ranging code on 2 frequencies (L1/L2)

Standard Positioning Service (SPS) Signal-in-Space Performance



Precise Positioning Service (PPS) Signal-in-Space Performance



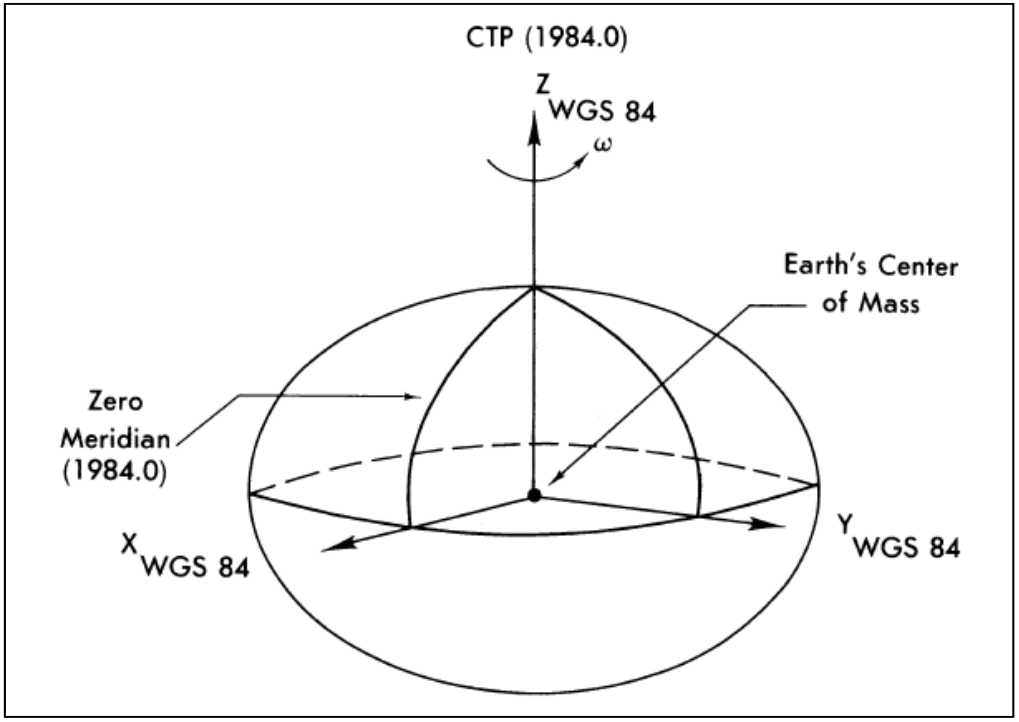
Note: Future service levels will be enhanced for all users by addition of new codes/frequencies

GPS Time

- GPS has its own time scale **GPS Time** which is based on the atomic time scale and is related to UTC (Universal Time Coordinated).
- UTC is maintained by the U.S. Naval Observatory (UTC (USNO). It is also based on an atomic time scale, with an offset of an integer number of seconds with respect to GPS system Time.
- GPS time is not corrected to match the rotation of the Earth, so it does not contain leap seconds or other corrections that are periodically added to UTC. GPS time was set to match UTC in 1980, but has since diverged.
  - last leap second occurred on December 31, 2016. There have been **27** leap seconds added to UTC since the current system began in 1972.
  - **GPS** is now ahead of UTC by **18** seconds.
- The GPS clock corrections are transmitted in terms of clock offset, clock frequency offset, and clock frequency rate (ageing), to allow the calculation of the difference between the individual GPS satellite's time and the GPS system time.
- GPS Time varies between **0 – 604799** seconds, each Saturday at midnight the time is reset to zero and the GPS Week number is incremented by one.

WGS-84

- GPS uses the World Geodetic System 1984 (WGS-84) as a reference.
  - WGS-84 is a realization of a Conventional Terrestrial System (CTS).
  - z-axis is defined by the Conventional Terrestrial Pole (CTP), x-axis is defined by Greenwich meridian, and y-coordinate is set to give a right-handed system.
  - Is consistent with International Terrestrial Reference Frame (ITRF) at ~2 cm level.



Notation and value	Fundamental parameters
a = 6378137.0 meters	Semi-major axis
$1/f$ = 298.257223563	Flattening Factor of the Earth
ω_e = 7292115 x10^{-11} rad/s	Angular Velocity of the Earth
GM = 3.9860050 x10^{14} m^3/s^2	Geocentric Gravitational Constant (Mass of Earth's Atmosphere Included)

Current GNSS: GLONASS

GLObal NAvigation Satellite System (**GLONASS**): System owned by the Russian Federation

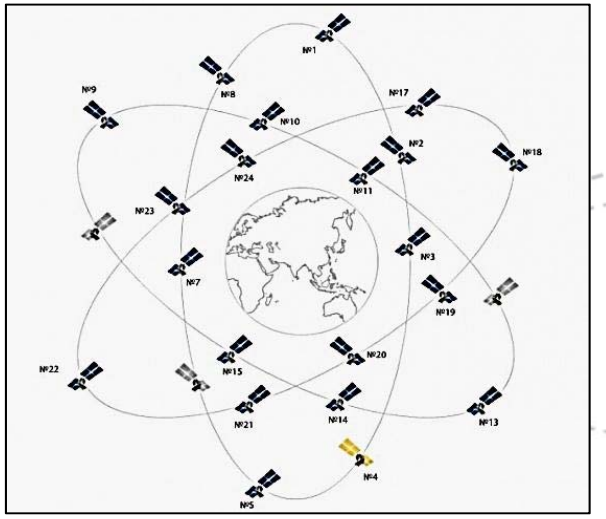
- Nominal constellation: 21 satellites + 3 active spares

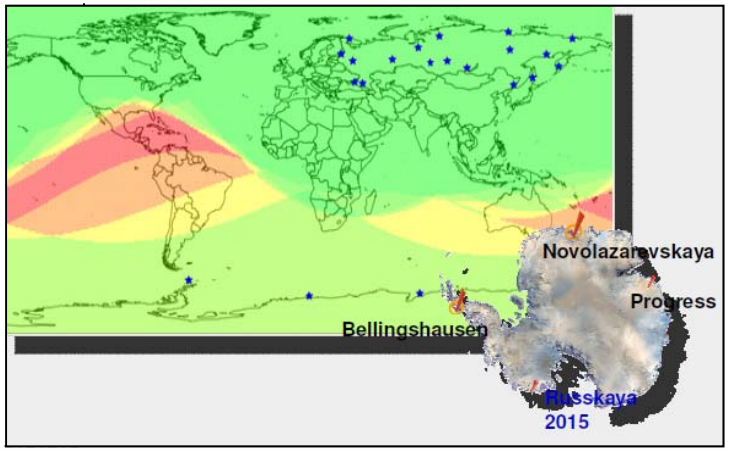
  Currently 27 satellites in orbit, 24+2 operational, 1 in the flight test phase.

- 3 orbital planes spaced by 120 °, 8 SV per orbit
- 11 hours 15 min 44 sec orbital period
- 19,140 km + altitude
- Inclination: 64.8°
- Originally an FDMA system, now transitioning to CDMA

- Ground Segment
  - 18 stations in Russia

    - reduced accuracy in other parts of the world
  - 3 stations in Antarctica
  - 1 station in Nicaragua (2017)





GLONASS Signals and Frequencies

- to provide better accuracy in addition to interoperability with GPS and future GNSSs, GLONASS satellites will transmit CDMA signals in addition to the system's traditional FDMA.
- GLONASS satellites transmit two types of signal: open standard-precision signal OF, and obfuscated high-precision signal SF.

	SV series and status	1602 + n×0.5625 MHz **(L1, FDMA)**	1246 + n×0.4375 MHz **(L2, FDMA)**	1202 MHz **(L3, CDMA)**	1600 MHz **(L1, CDMA)**	1237-1248 MHz **(L2, CDMA)**
1982 . . . 2003	**GLONASS-M In service**	L1OF, L1SF	L2OF, L2SF			
2011 . . .	**GLONASS-K (K1) Design maturation based on in-orbit validation**	L1OF, L1SF	L2OF, L2SF	L3OC		L2OC, L2SC
2020	**GLONASS-K2 In development**	L1OF, L1SF[1]	L2OF, L2SF[2]	L3OC	L1OC, L1SC	L2OC, L2SC

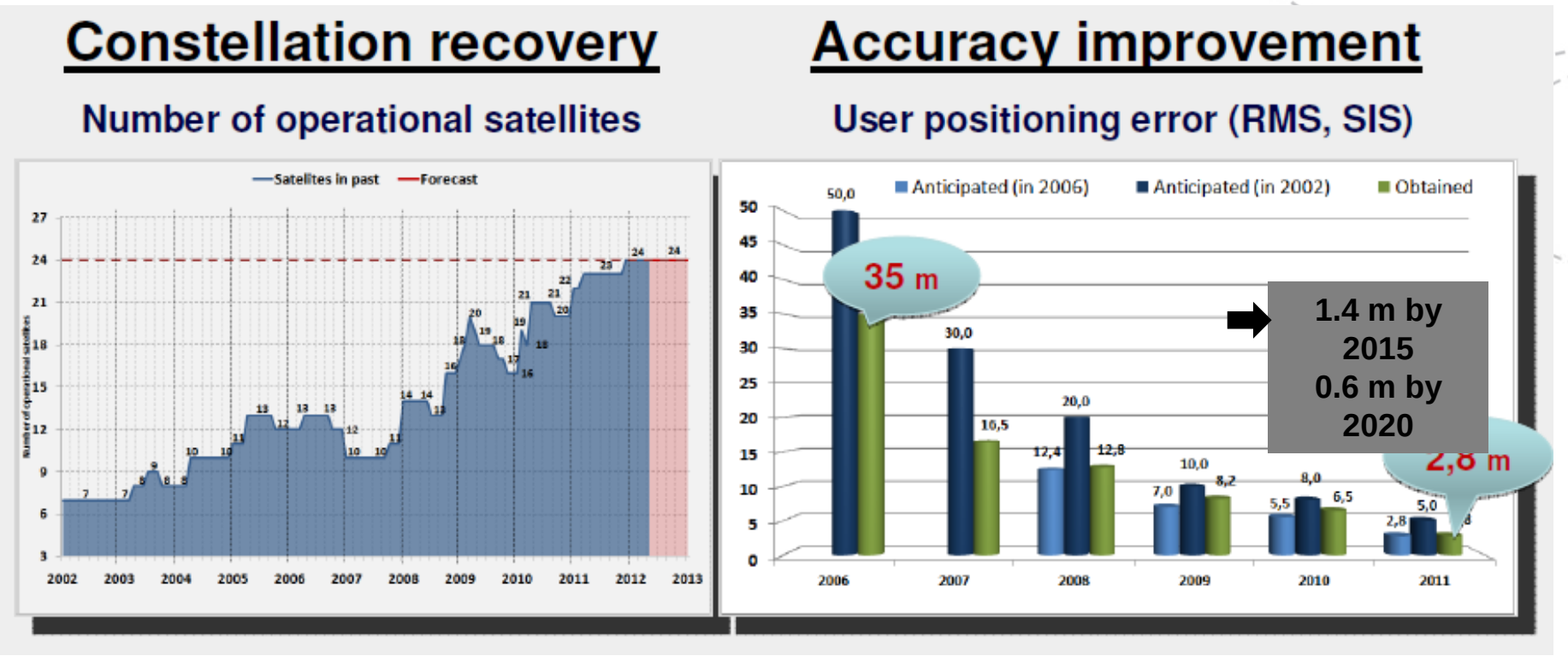
$n=-7,-6,-5,\ldots,6$

F- FDMA
C- CDMA

[1] - Glonass-K2 series plan to use 1592 MHz for the L1SF signal
[2] - Glonass-K2 series plan to use 1237 MHz for the L2SF signal

GLONASS Service Levels

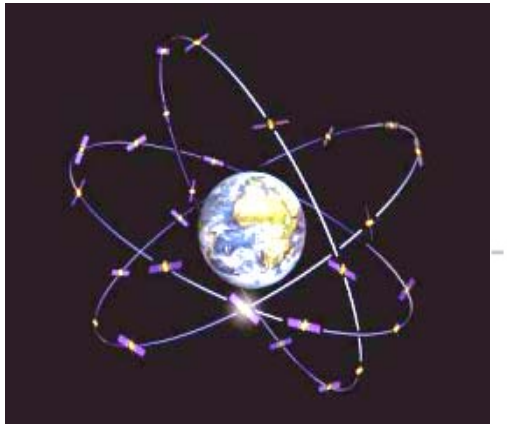
Equivalent to the Standard Positioning Service (SPS) and the Precise Positioning Service (PPS) of GPS, GLONASS provides a standard precision (SP) navigation signal and a high precision (HP) navigation signal, namely the Channel of Standard Accuracy (CSA) and Channel of High Accuracy (CHA).

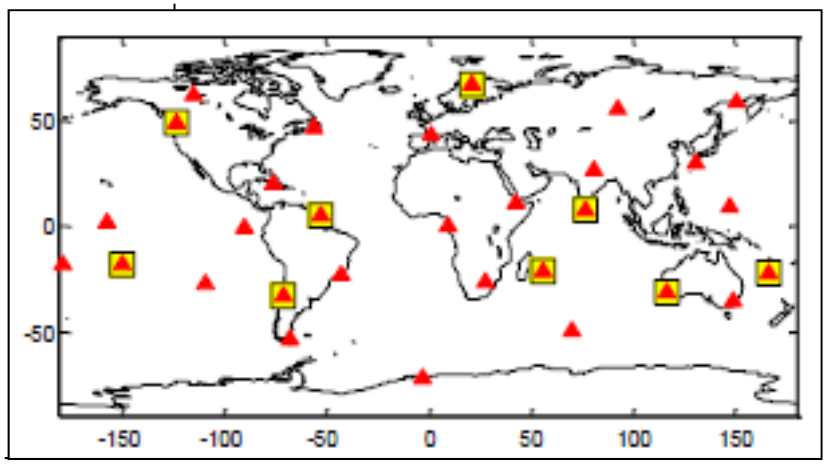


Future GNSS: Galileo

Galileo: Joint effort of the European Union (EU) and European Space Agency (ESA)

- Nominal constellation: 30 satellites (24+6)
- Currently 16 satellites in orbit:
  - 11 operational* satellites
  - 2 satellites in a wrong orbit, under test
  - 2 satellite under commissioning
  - 1 with failed transmitter
  - Constellation of 20 SV expected by 2018, contracts awarded for another 8 satellites. FOC expected in 2020.
- 3 orbital planes
- 14 hours 22 min orbital period
- 23,222 km + altitude
- Inclination: 56°
- CDMA system
- Ground segment
  30 monitoring stations,
  5 command antennas,
  2 control stations





Galileo: A Service-Based GNSS

- Open Service, (OS)
  - Free; comparable to GPS performance

- Commercial Service, (CS)
  - Fee-based, intended for professional applications
  - High precision and accuracy, authentication

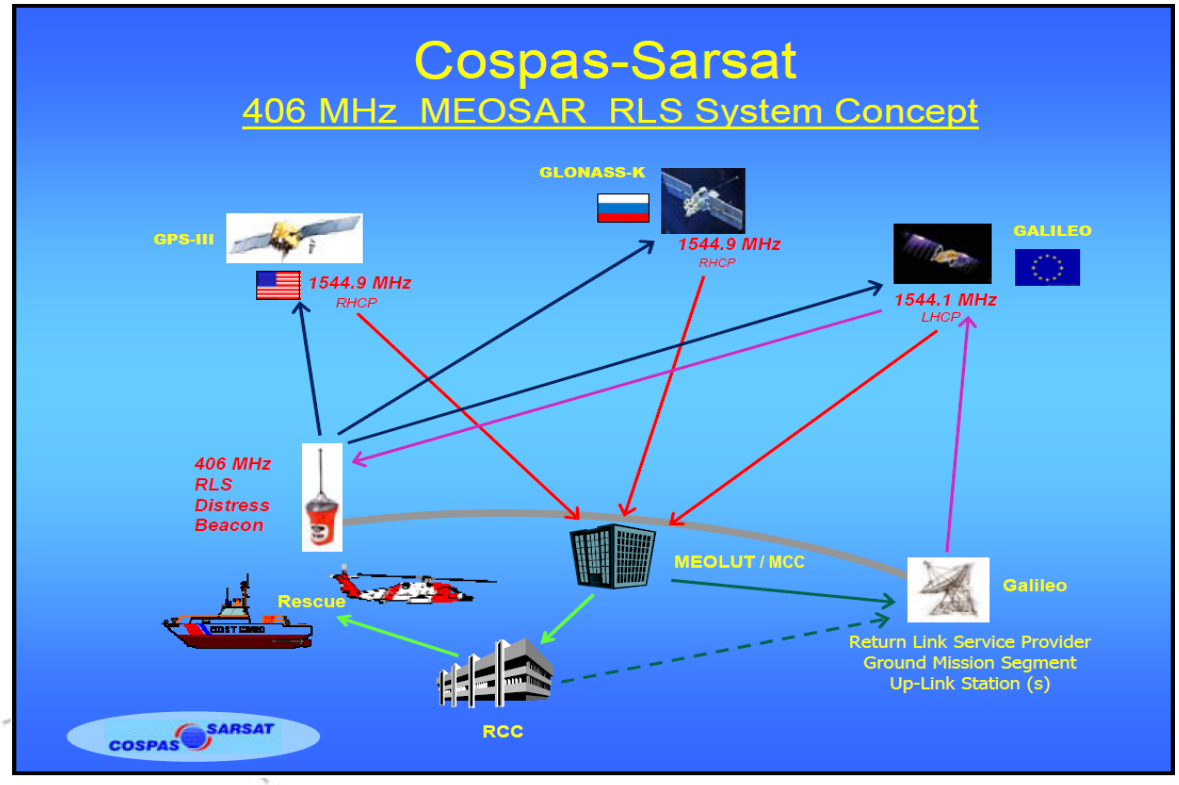
- Public Regulated Service, (PRS)
  - High continuity with controlled access
  - Signals with encrypted ranging (reserved to public authorities, national security and law enforcement)

- Search and Rescue, (SAR)

*Search and Rescue, (SAR)

Beginning in 2000 various studies determined that medium-earth orbiting (MEO) satellites could provide an improved space-based distress alerting and locating system.

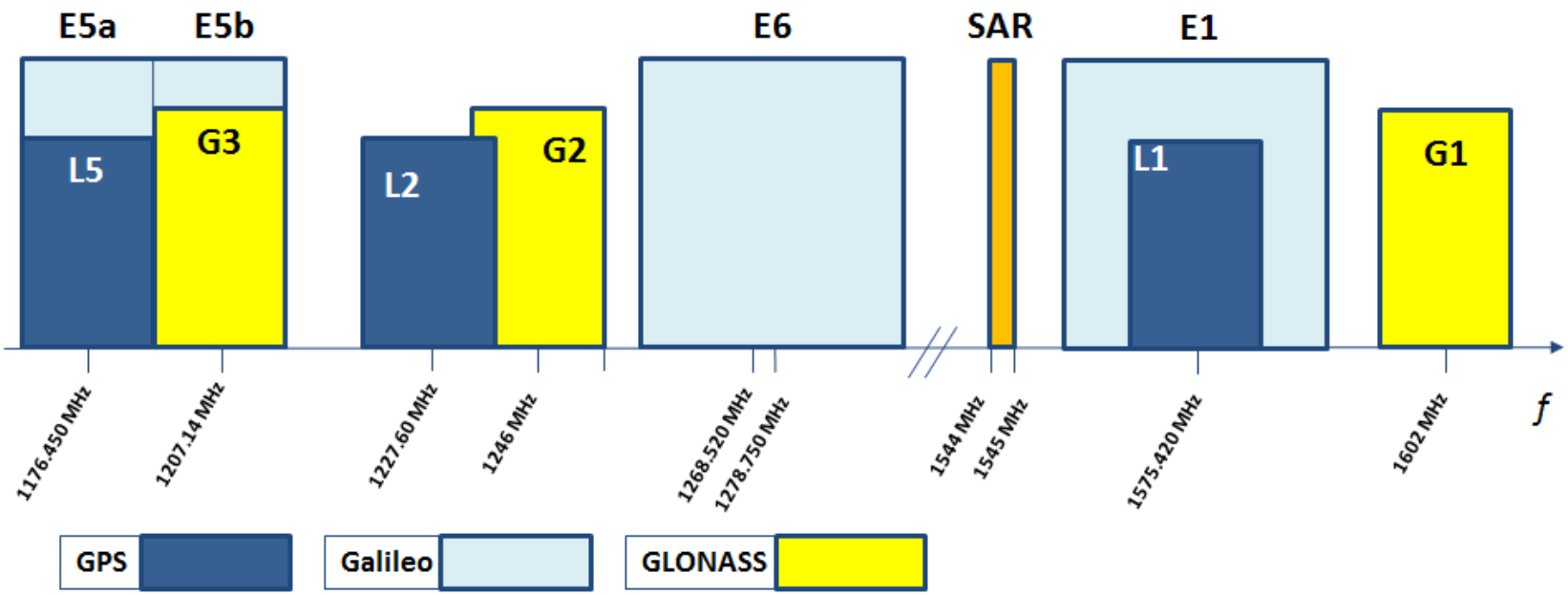
Russia (SAR/GLONASS), USA (SAR/GPS) and ESA/EC (SAR/Galileo) are working to incorporate the SAR capabilities on the new GNSS satellites.



- **Benefits:**
  - True global and continuous coverage
  - Return link service
  - Near real-time alert localization and message detection
  - Galileo's SAR service also provides a reply to the distress signal

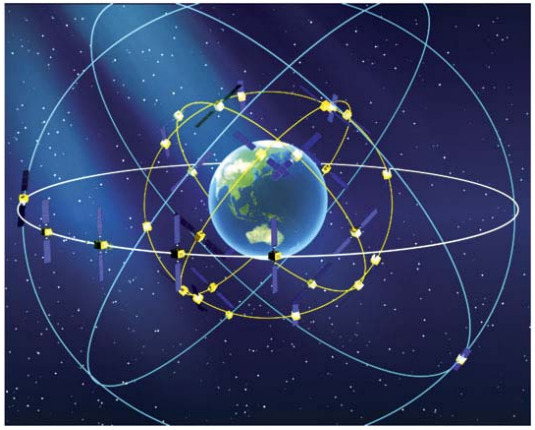
Galileo Signals and Frequencies

- Galileo will use 3 frequency bands.
- In total 10 signals on 4 carriers achieved through the use of different code structures
- Matching or overlaying other GNSS frequency bands solved by using effective. modulation techniques (e.g. CBOC on E1 OS, AltBOC on E5a and b).



Future GNSS : COMPASS/BeiDou

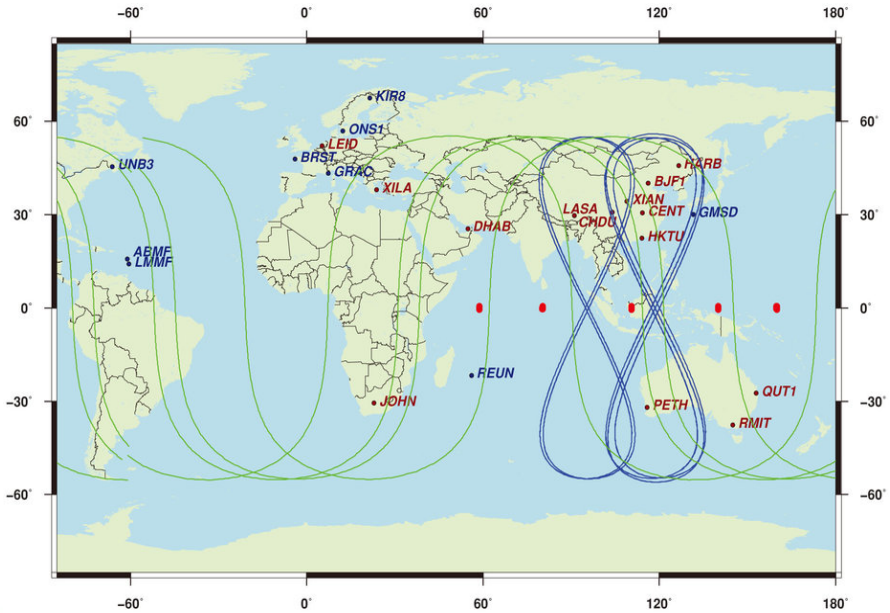
COMPASS: System owned and operated by the government of China



- Planned constellation: in total 35 satellites
  - 5 geostationary (GEO) satellites
  - 27 medium Earth orbit (MEO) satellites (3 planes with 8 +1 spare SV on each)

    21,500 km + altitude, Inclination: ≈55°

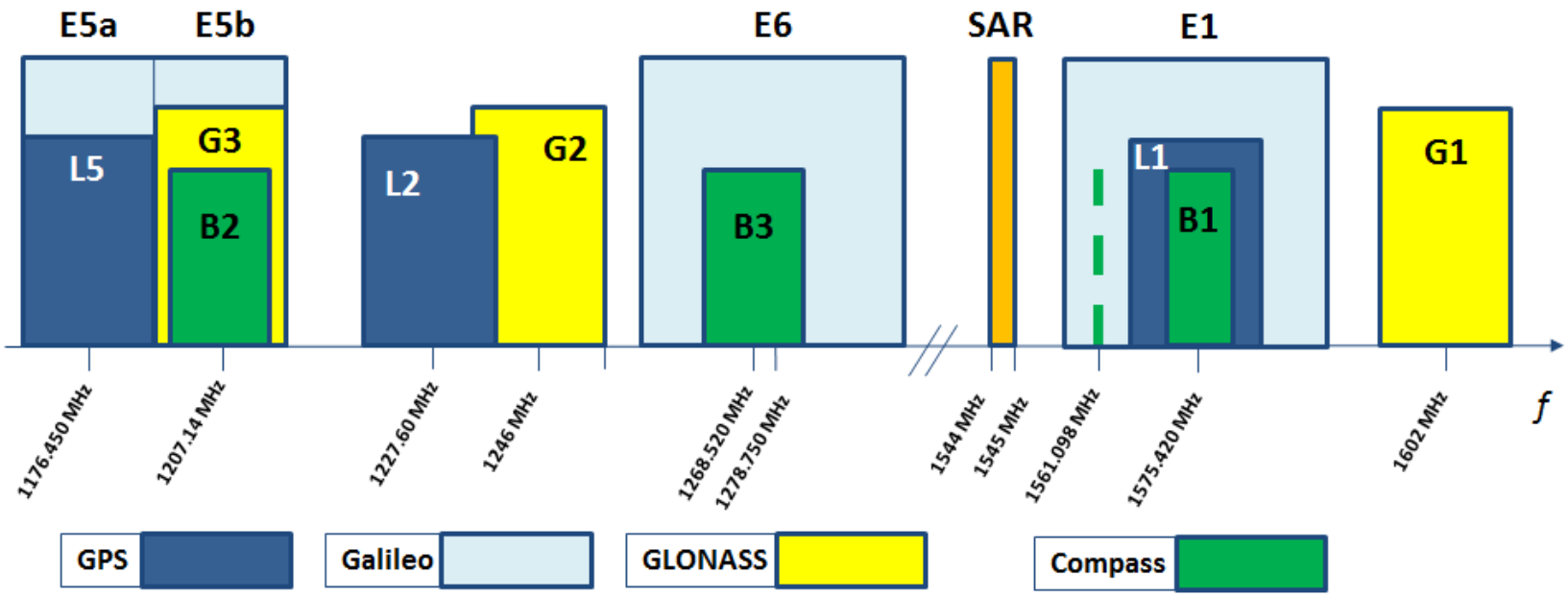
    Orbital period: 12h 53 min 24 sec
  - 3 satellites in Inclined Geosynchronous Orbit (IGSO), inclined at 55°



- Current constellation: 23 satellites - 7 MEO, 7 GEO and 8 IGSO.
- Full Operational Capability (FOC) is expected to be reached by 2020.
- CDMA system.
- Ground segment: 30 monitoring, 1 control and 2 uplink stations.

Compass Signals and Frequencies

- Current status: 3 frequency bands (B1, B2 and B3) either overlaying or matching the bands of other GNSSs. 5 signals.
- At FOC phase up to 10 signals expected.
- A draft of signal interface control document (ICD) released, but the details about the navigation message contents are not yet disclosed for general public.



Compass Services

- in line with other GNSS, there will be two levels of positioning service: open and restricted. In addition to global ones, there will be 2 regional services.

  - Open service for civilian users with positioning accuracy of within 10 m, velocity accuracy of within 0.2 m/s, timing accuracy 20 ns.

  - A licensed service with higher accuracy (within 10 cm) will be offered to authorized and military users.

- Since 2011 the system is on a trial basis offering positioning and navigation services to the Asia-Pacific region only.

- Global coverage is expected by 2020.

COMPASS coverage per 2012.

GNSS Limitations

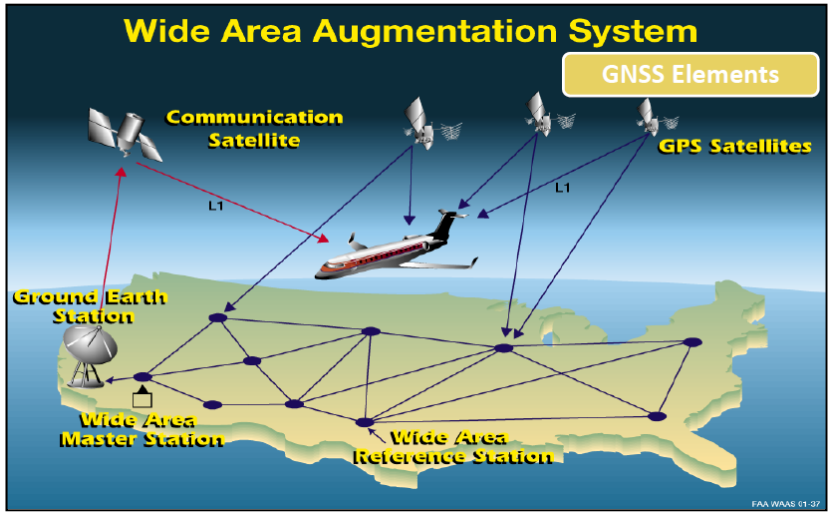
Current GNSSs Provide Position, Velocity and Time solution, but No Integrity and with Limited Accuracy

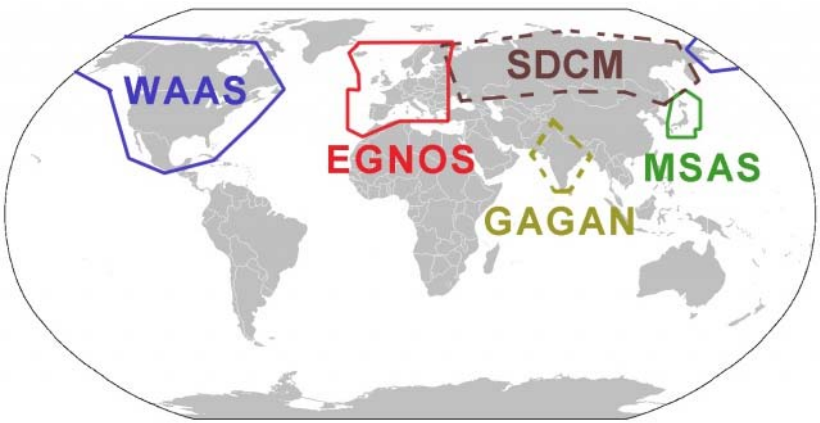
– Precision applications require accuracy
– Safety applications require integrity

Errors are Measurable, Locally-Correlated, but not Predictable
Solution – a system that can reliably measure the errors and make available to users - **<u>GNSS Augmentation</u>**

Space Based Augmentation System (SBAS)

Objective: Enhance GNSS accuracy and integrity





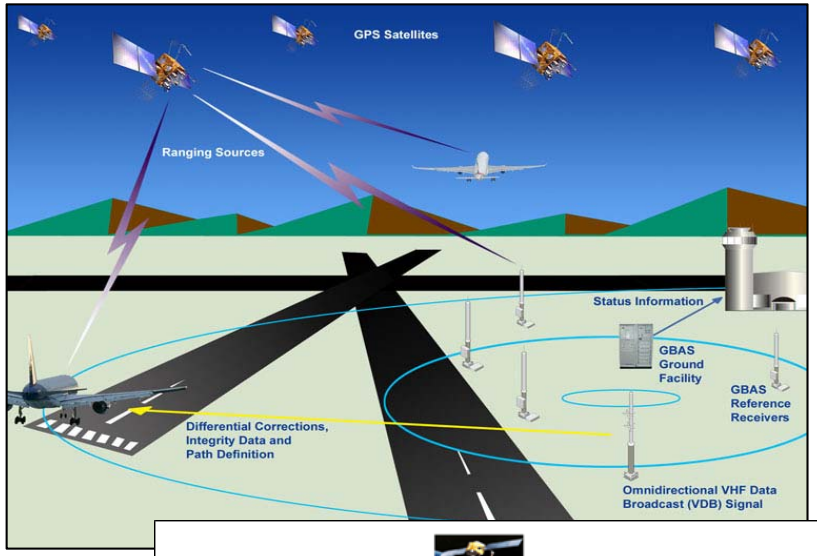
- Network of ground stations takes local measurements
- Control stations determine errors and transmits satellite specific corrections and an ionospheric "map" to satellites
- Satellite transmits corrections, integrity data to users

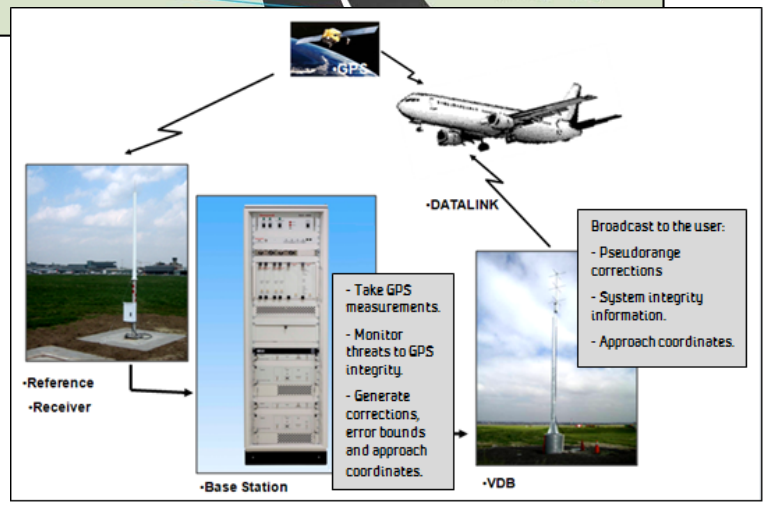
Operational:

- **WAAS** – Wide Area Augmentation System (US, operational since 2003)
- **EGNOS** - European Geostationary Navigation Overlay Service (EU, operational since 2011)
- **MSAS** - Multifunctional Satellite Augmentation System (Japan, operational since 2007)
- **SDCM** – System of Differential Correction and Monitoring (Russia, operational 2014)
- **GAGAN** – GPS Aided Geostationary Augmented Navigation (India, operational 2014)

- **QZSS** – Quazi-Zenith Satellite System (Japan, under development).

Ground Based Augmentation System GBAS/LAAS





- Ground station determines local errors
- Ground station broadcasts local corrections, provides integrity (aviation - also approach path information for precision landing).

Ground Subsystem:

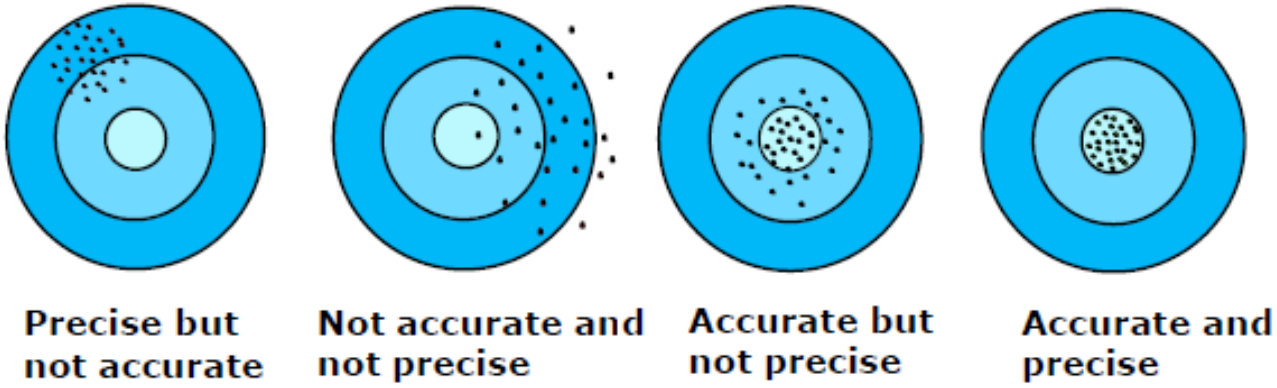
- 4 Reference receivers
- GBAS Ground Facility/processing unit
- 1 or 2 VHF data broadcast (VDB) transmitters

- Up to 45 km operation radius
- intended support of ground management operations.

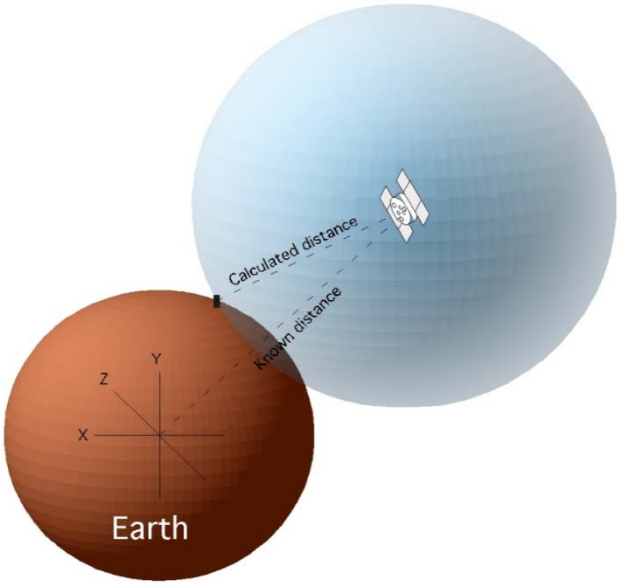
Performance Measures

- **Accuracy:** Measure of the navigation output deviation from truth under fault-free conditions (often specified in terms of 95% performance).
- **Integrity:** Ability of a system to provide timely alerts to users when the system should not be used for navigation.
- **Continuity:** Likelihood that the navigation system supports accuracy and integrity requirements for the duration of intended operation.
- **Availability:** Fraction of time that the navigation function is usable as determined by its compliance with the accuracy, integrity, and continuity requirements.

Accuracy and Precision



Basic Principle of GNSS Positioning



Receiver is somewhere on this sphere

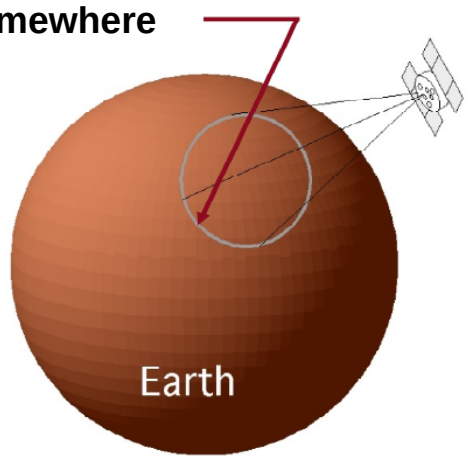


Image Source: [1].

- Position of each satellite is known, as well as its distance from the center of the Earth. Each satellite constantly broadcasts this information.

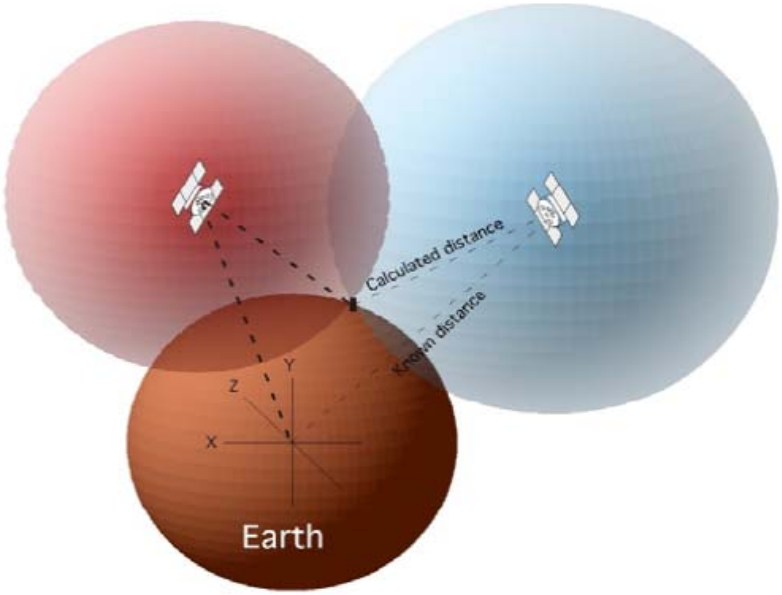
- The distance/range is computed by measuring the "**time of flight**" of an electromagnetic signal from the satellite:

$$\rho_i = c \cdot t_i = c \cdot \left(t_i^{rx} - t_i^{tx}\right)$$

- With this information and the calculated range, the receiver calculates its position by **trilateration**.

Given range measurement only to one satellite, users' location can be anywhere on the intersection of two spheres, which is a circle.

Basic Principle of GNSS Positioning



- Use of the second satellite narrows the location to two points on the Earths surface.
- Range measurements from three satellites allow to determine which point is the true user location.
- Two primary differences from terrestrial trilateration:
  - Satellites are moving at ~ 4 km/s
  - Geometry is changing as a function of time

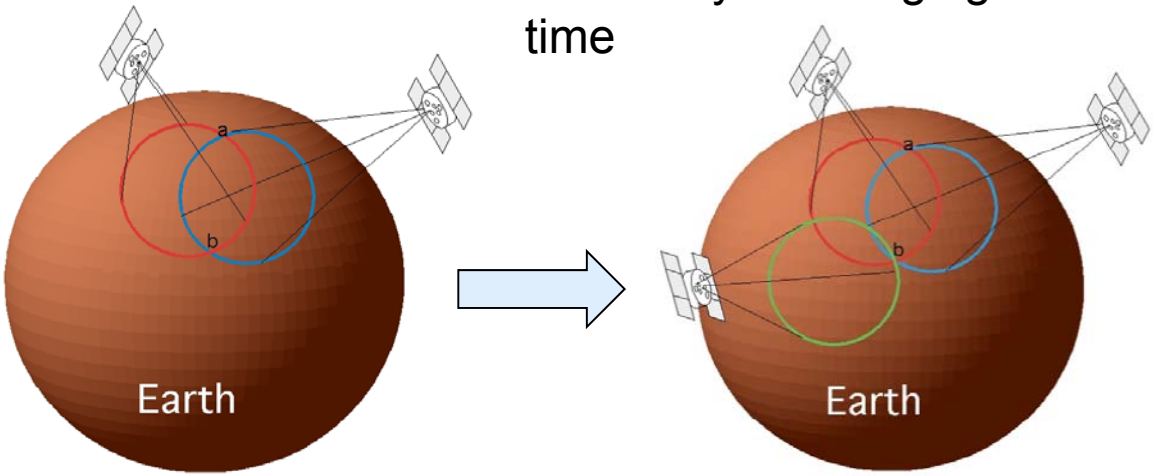


Image Source: [1].

Basic Principle of GNSS Positioning

- Knowing the distance (range) to 4 objects at known locations, the user position can be unambiguously calculated in 3 dimensions.

  4 equations and 3 unknowns:

  - one unknown per dimension (x_u, y_u, z_u)
  - one equation per range ρ_i

$$\rho_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} = \|\boldsymbol{r_u} - \boldsymbol{r_i}\|$$

  - Each equation is non-linear.
  - 3 equations lead to two valid solutions for 3 unknowns.
  - Intersection of 3 spheres.
  - Typically only one solution is near the earth's surface, so we can usually use 3 equations (Earth acts like 4th sphere).

Basic Principle of GNSS Positioning

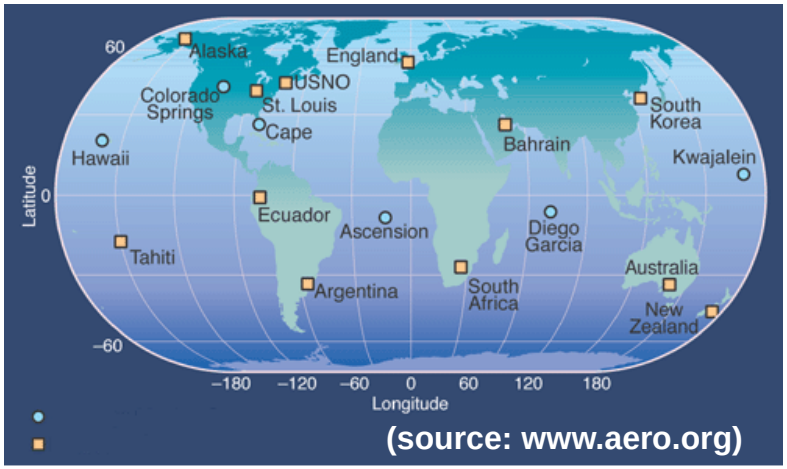
More challenges to deal with:

- **Accurately** determine the satellite positions $(\boldsymbol{x_i}, \boldsymbol{y_i}, \boldsymbol{z_i})$ at a **precise** time t_i^{tx}
- **Accurately** determine the **precise** time at which the signal is received t_i^{rx}

  Timing errors are scaled by the speed of light: 1 ns ≈ 30 cm!

Accurate Satellite Positions and Time

To solve the problem of accurate satellite positioning each GNSS contains a ground segment consisting of monitoring stations distributed around the globe.



GPS ground segment.

- Satellite positions and clocks are constantly monitored.
- Orbit model and clock error model parameters are uploaded to the satellites (at least daily).
- These parameters are retransmitted by the satellites.
- User computes satellite position using these parameters.

Accurate Receiver Time

- On-board clock of GNSS receiver is typically of low quality and is not capable to provide precise measurement of time at which the signal is received t^{rx}.
- It is not syncronized with the satellite clocks.
- The range measurement includes therefore a time varying error:

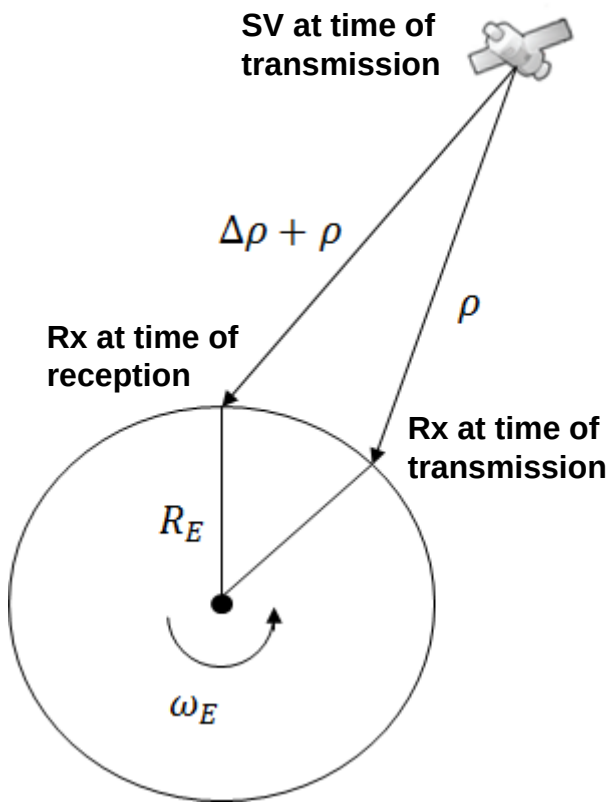
$$t^{rx} = t^{sys} + t_u^b$$

- Range measurements will be biased by $c \cdot t_u^b$.
- This is solved by treating the receiver clock bias as the **4th** parameter to be estimated in addition to x_u, y_u, z_u.
- Thus, in order to determine position the receiver must track a minimum of four satellites, representing the four unknowns of 3-D position and time: x_u, y_u, z_u, t_u^b.
- The measurement is therefore called **pseudorange**, and is defines as follows:

$$P_i = \rho_i + c \cdot \left(t_u^b - t_i^b\right),$$

where ρ_i is the true range to the satellite *i*, t_u^b is the receiver clock bias, and t_i^b is satellite clock bias.

Geometric Range



The Sagnac effect.

- Due to rotation of the Earth during the time of signal transmission, a relativistic error is introduced – **Sagnac effect.**
- During the propagation time of the SV signal transmission, a clock on the surface of the Earth will experience a finite rotation.
- Corrections for the Sagnac effect are often reffered to as Earth rotation corrections:

$$\boldsymbol{r}_i = \begin{bmatrix} cos(\omega_E\tau) & sin(\omega_E\tau) & 0 \\ -sin(\omega_E\tau) & cos(\omega_E\tau) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boldsymbol{r}_i$ - original estimate for satellite position vector.

$\tau = \left(t_i^{rx} - t_i^{tx}\right)$**,** and is the approximate transmission delay.

GNSS Signals

Practical considerations for signal properties:

- Multiple access capability: multiple signals from the same system must be visible at the same time. Need to limit interference between these signals.

  Solution: Code Division Multiple Access (CDMA) – each satellite has its own unique code. In order to decode the signal, the receiver must be aware of the codes for each satellite.

- Satellites from multiple systems will be visible at the same time. Need to limit inter-system interference.

  Solution: Effective modulation techniques (e.g. Binary Offset Carrier (BOC)).

- Ability to provide ionospheric delay measurements.

  Solution: use of dual/multiple frequencies.

- Signals must pass through the earth's atmosphere. This limits the choice of frequencies.

GNSS Signals (1/2)

- The general model of GNSS signal is as follows:

$$\boxed{y_i(t) = \sqrt{2 \cdot P_i} \cdot d_i\,(t) \cdot s_i\,(t) \cdot c_i(t) \cdot cos(2\pi f_i t + \varphi_i)}$$

Carrier power

Navigation message

Secondary code

PRN code

Centre frequency

Relative phase

- **Navigation Data:** Contains satellite orbit and clock parameters, plus other information.
- **Secondary Code:** Not always present. Used to aid synchronization.
- **Pseudo Random Noise (PRN) Code:** Provides timing and **spreads** the spectrum of the data signal.
- The information transmitted by the satellite is coded as a phase modulation of the carrier frequency.

Signal Multiplexing

- All GNSS transmit multiple signals. In addition, different services are provided on each signal.

- Signals on the same frequency must be combined (multiplexed). Some possible approaches include:

  - **Orthogonal carriers**: two signals can be muliplexed by transmitting them with a 90° phase offset, one in-phase (I), the other in quadrature (Q). This approach is used for the legacy GPS and GLONASS L1 signals.

  - **Time Multiplexing**: one signal is set to zero while the other transmits and vice versa (e.g. used for the GPS L2 Civil signal).

  - In the cases where three or more signals are combined, more advanced techniques are adopted (e.g. Galileo Alternative BOC (AltBOC) on E5a and E5b).

Pseudorandom Sequences

- Pseudorandom sequence is the basis for forming the pseudorange measurement:

$$\rho = c \cdot (t^{rx} - t^{tx})$$

- The transmitted sequence must be synchronized with the satellite clock, such that a given sequence element/phase corresponds to a precise time of transmission.
- At a given time the receiver measures the phase of the received sequence (giving t^{tx}), and records time according to the receiver clock, t^{rx}.

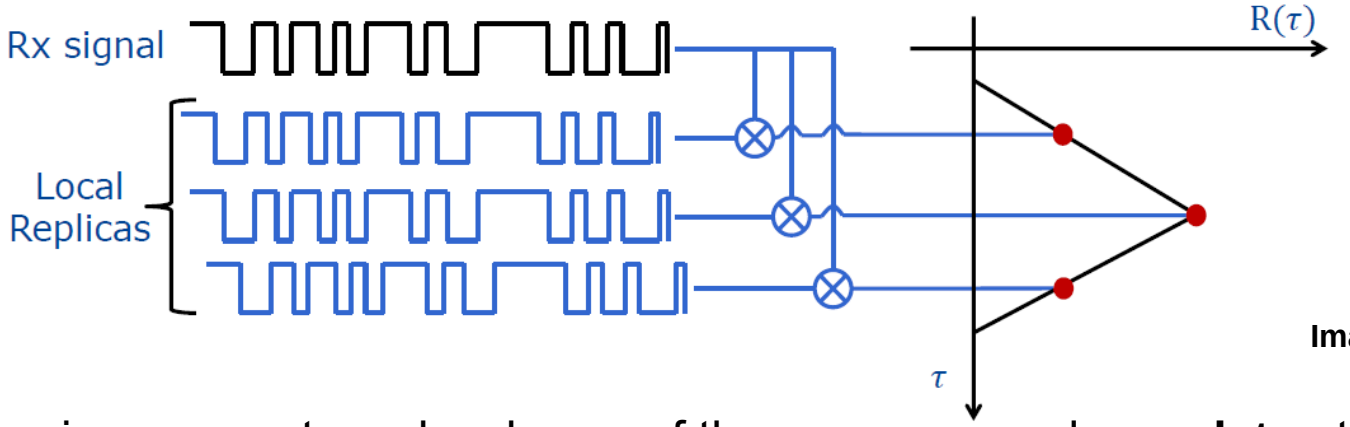
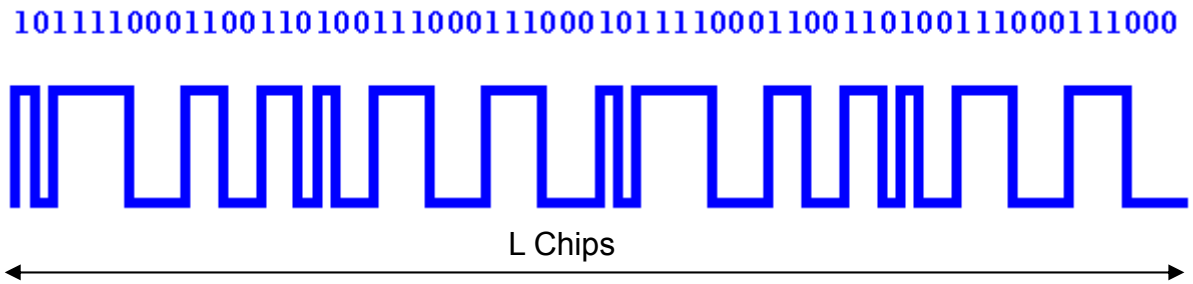


Image Source: [2].

- The receiver generates a local copy of the sequence and **correlates** this local replica with the received signal. Maximum correlation gives an estimate of the transmit time, t^{tx}.

Pseudorandom Sequences

Terminology:



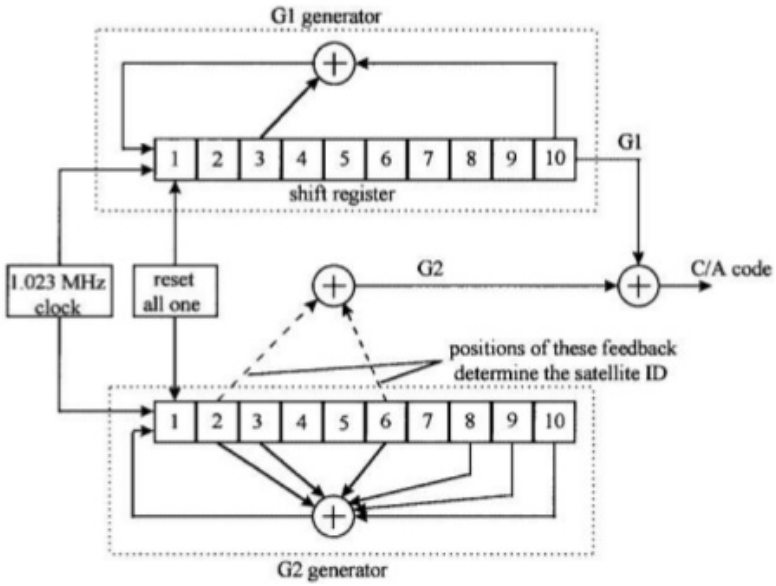
- Each element in a binary sequence is referred to as a **chip.**
- The chip duration is T_{chip} and the duration of the code is $L \cdot T_{chip}$ seconds.
- The frequency is referred to as the **chip rate** and is denoted $f_c = 1/T_{chip}$.
- The entire sequence is usually referred to as the **PRN code, spreading code**, or simply the **code**.
- The phase of the sequence is called the **code phase.**

Pseudorandom Sequences

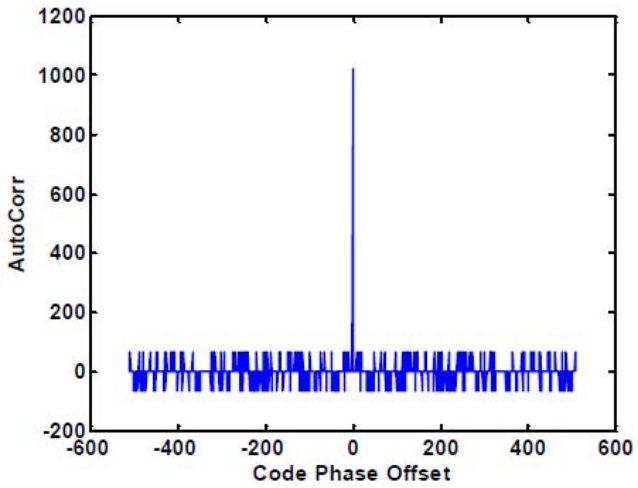
- **Gold Codes**: Used in GPS and Galileo.

  Sub-optimal auto-correlation, but excellent cross-correlation for a large family of codes.

  - Easy to generate (XOR of two m-sequences).



C/A code generator.



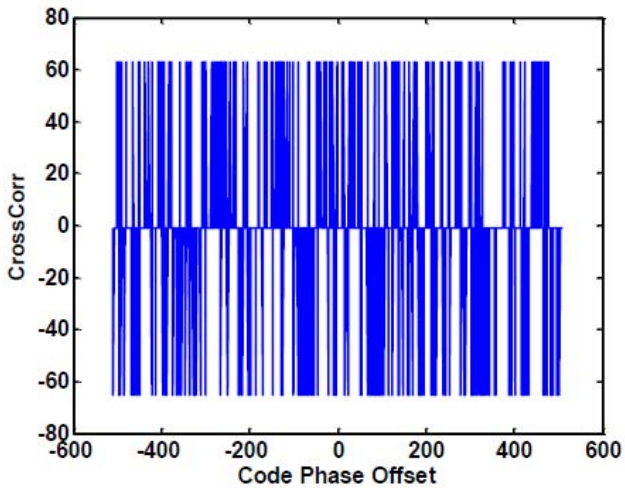


Image source [2].

Pseudorandom Sequences

- **M-Sequences**: optimal binary sequences in terms of autocorrelation. Used in GLONASS (signals from different satellites are broadcast on different frequencies – FDMA).
  - Easy to generate.
  - Excellent auto-correlation properties.
  - Generally poor cross-correlation properties.

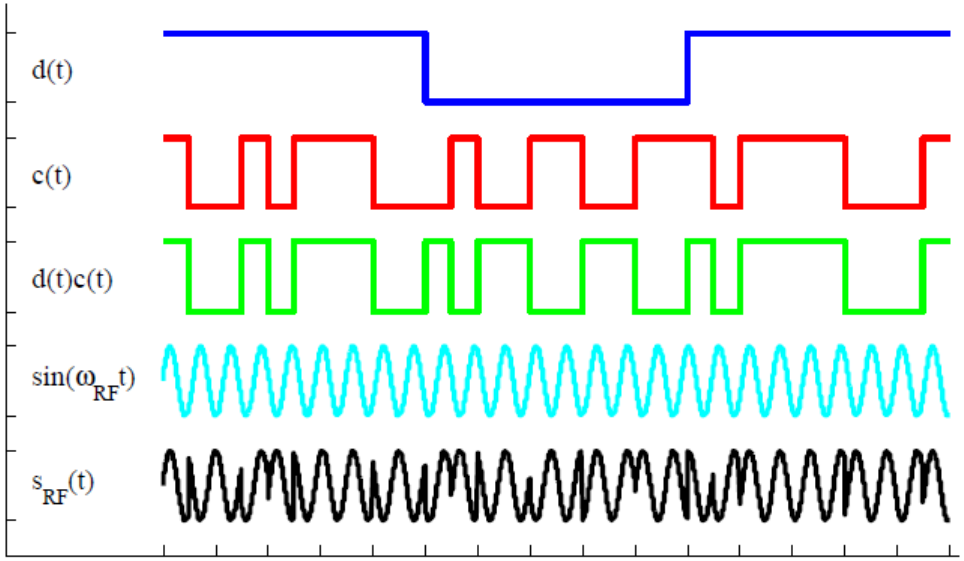
Legacy GPS Signals

- The legacy GPS signals include, the civil Coarse/Acquisition (**C/A**) signal on **L1** frequency (1575.42 MHz) and military Precise (P) signal on both **L1** and **L2** (1227.60 MHz) frequencies.
- L1 and L2 signals fundamental frequency (produced by the atomic clocks of GPS satellites) is $f_0 = 10.23\ MHz$. Derived from it:
  - $L1 = 154 \cdot f_0 = 1575.42\ MHz$, *wavelength 19 cm*
  - $L2 = 120 \cdot f_0 = 1227.60\ MHz$, *wavelength 24.4 cm*
- Signal Structure:

$$L_1(t) = A_1 \cdot P(t) \cdot d(t) \cdot cos(f_1 t) + \sqrt{2} A_1 \cdot C(t) \cdot d(t) \cdot \sin(f_1 t)$$
$$L_2(t) = A_2 \cdot P(t) \cdot d(t) \cdot cos(f_2 t)$$

- $A_{1,2}$ = Amplitude of L1 or L2.
- $P(t)$ = P code modulated on both L1 and L2.
- $d(t)$ = Navigation data modulated on both L1 and L2.
- $C(t)$ = C/A code modulated on L1 in quadrature with P code.

GPS Signals: L1



	C/A	P(Y)
Sub-Carrier	BPSK(1) [chip rate f_0 /10= 1.023 MHz]	BPSK(10) [chip rate f_0 = 10.23 MHz]
PRN	Gold	Combned m-sequences
L (chips)	1023	6.2×10^{12}
Data rate	50 bps	50 bps
Secondary code	none	none

- Combination of the C/A and/or P code chips and the navigation data is performed via modulo-2 addition (using XOR).
- The result is then modulated using the Binary Phase-Shift Keying (BPSK) technique on to the carrier.
- BPSK modulation:

  Logical 0 = 180°
  Logical 1 = 0°

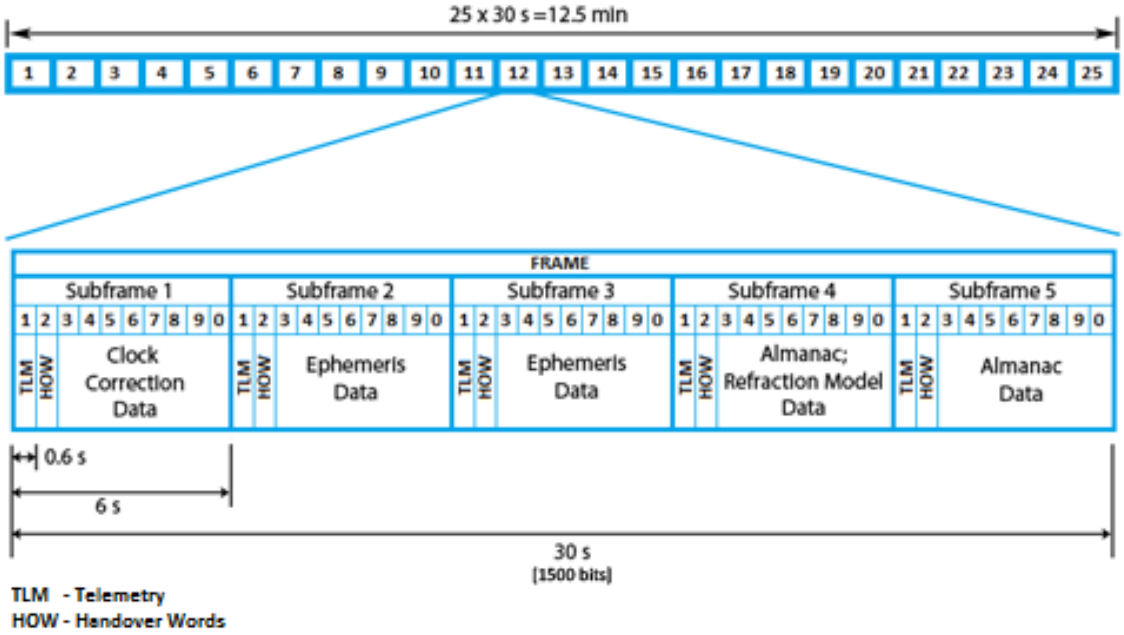
- C/A code: 1 ms code repeat. 1 chip = 300 meters.
- P(Y) code: 267 day (each satellite gets 1 week section).
- Navigation data is 20 ms long – contains 20 C/A codes.

GPS Navigation Message Structure

- provides the user with the information necessary to compute the precise locations of each visible satellite and time transmission for each navigation signal.

<u>GPS L1 C/A:</u>

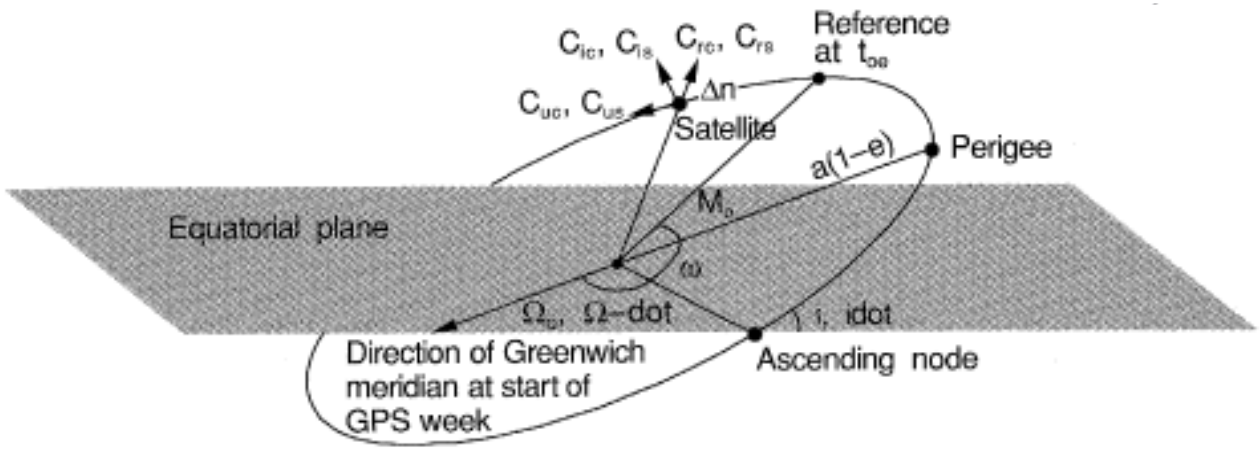
- Chip rate = 50 bps.
- 25 frames of 1500 bits each, divided into five 300 bits subframes.
- 50 bps ⇒ 300/50 = 6 sec to transmit one subframe, 6 x 5 x 25 = 750 sec (= 12.5 min) to transmit an entire navigation message.



- Each subframe is time-stamped, allowing the determination of the transmit time.
- **TLM** - Telemetry word. Contains the preamble for frame syncronization.
- **HOW** - Handover Word. Contains the Time of the Week (TOW) word (GPS time).

GPS Navigation Message Contents

- Satellite ephemeris parameters (Keplerian and perturbation parameters)
  - **Ephemeris** - *precise* orbital parameters for each satellite required for accurate positioning. Is considered to be valid for up to 2 hours.
- Satellite almanac parameters
  - **Almanac** - *coarse* orbital parameters. Allows the receiver to calculate approximate satellite positions to assist acquisition. Is updated every 6 days.
- Each satellite transmits its own clock correction and ephemeris parameters (sub-frames 1, 2 and 3) and almanac data for all the satellites (sub-frames 4 and 5). Information broadcasted in sub-frames 4 and 5 is common to all satellites – the almanac data for all satellites in orbit can be obtained from one tracked satellite.
- Each time the receiver is powered on, it must ensure that it is using up-to-date ephemeris and clock data for the satellites it is using in its position solution.



Ephemeris parametrs in the GPS navigation message	
$\sqrt{A}$	Square root of the semi-major axis
e	Eccentricity
i_0	Inclination angle at reference time
Ω_0	Longitude of right ascension node at weekly epoch of the GPS week
ω	Argument of perigee
M_0	Mean anomaly at reference time
t_{0e}	Ephemeris reference time
Δn	Correction to the computed mean motion
$i\ (i-dot)$	Rate of change of inclination with time
$\dot{\Omega}$	Rate of change of right ascension with time
C_{uc}, C_{us}	Amplitudes of harmonic correction terms for the computed argument of latitude
C_{rc}, C_{rs}	Amplitudes of harmonic correction terms for the computed orbit radius
C_{ic}, C_{is}	Amplitudes of harmonic correction terms for the computed inclination angle

GPS Navigation Message Contents

- Satellite clock corrections (sub-frame 1)

Clock parameters	
t_{0c}	Reference time
a_{f0}, a_{f1}, a_{f2}	Polynomial coefficients for clock correction (bias, drift and drift-rate)
T_{GD}	Total group delay

$$dT(t) = a_{f0} + a_{f1}(t - t_{0c}) + a_{f2}(t - t_{0c})^2 + \Delta t_{rel} + \Delta t_{GD}$$

Relativity correction (time runs slower at higher gravitational potential:

$$\Delta t_{rel} = \frac{-2\sqrt{\mu \cdot A \cdot e \cdot \sin E}}{c^2}$$

Group Delay correction:

$$\Delta t_{GD} = \begin{cases} -T_{GD} & (L1) \\ -\gamma T_{GD} & (L2), \\ 0 & (LC) \end{cases} \qquad \gamma = f_1^2 / f_2^2$$

- Satellite health indicators, age of data (sub-frame 1),
- Ionosphere model parameters, UTC data (sub-frame 4).

New GPS Signals

- New and future signals include **L2C** (1227.60 MHz), **L5** (1176.45 MHz), and **L1C** (1575.42 MHz).

Signal/Band	Minimum reception power [dBW]	Signal Component	Modulation	Chipping rate [Mcps]	Primary PRN code length	Data rate [bps]
L1 (C/A)	-158.5	Data	BPSK(1)	1.023	1023	50
L1C	-157	Data	TMBOC	1.023	10230	50
		Pilot		1.023 & 6.138		-
L2C	II/IIA/IIR: -164.5 IIR-M: -161.5 IIF: -161.5	Data	BPSK(1) 2 PRN sequences are chip-by – chip multiplexed	511.5 individually (1.023 after multiplexing)	CM: 10230	IIR-M: 25 with FEC IIF: 50
		Pilot			CL: 767250	-
L5	-157.9	Data	BPSK(10)	10.23	10230	50
		Pilot		10.23	10230	-

Time-multiplexed BOC (TMBOC)
FEC – Forward Error Correction

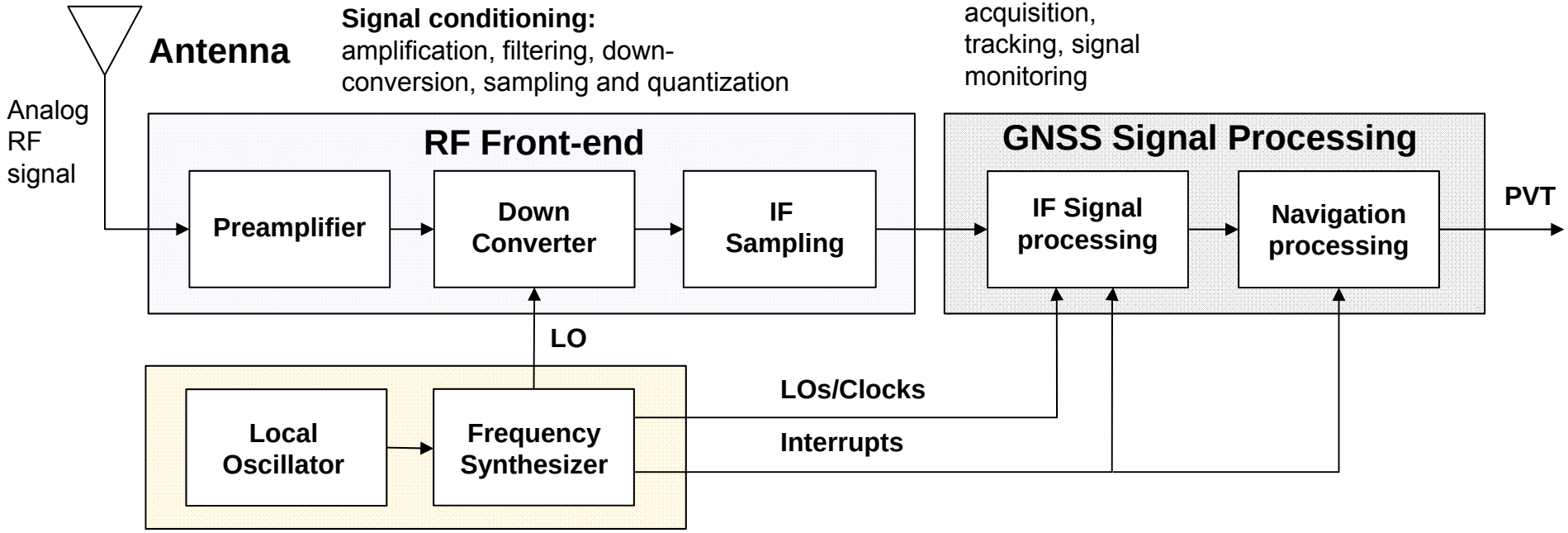
New GPS Signals

Summary:

- Longer spreading codes:
  - Improved auto- and cross-correlation.
- Most of the new signals will have pilot/data-less channel syncronized with the data channel.
  - Higher tracking sensitivity and lower cycle slip rate. (use of optimal discriminators, longer coherent integration).
- Use of secondary codes:
  - Better interference rejection (narrowband RFI).

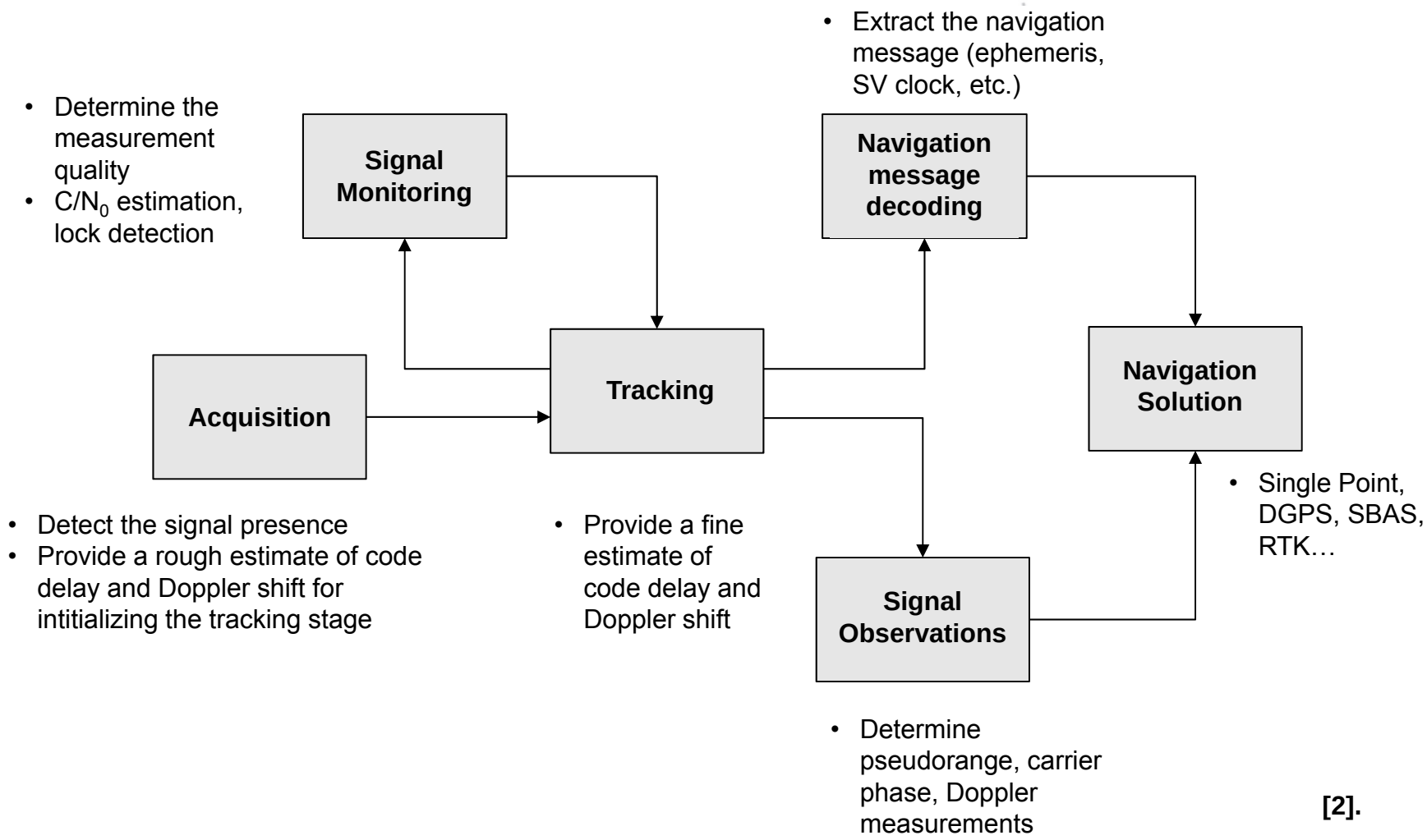
GNSS Receiver

High level architecture:



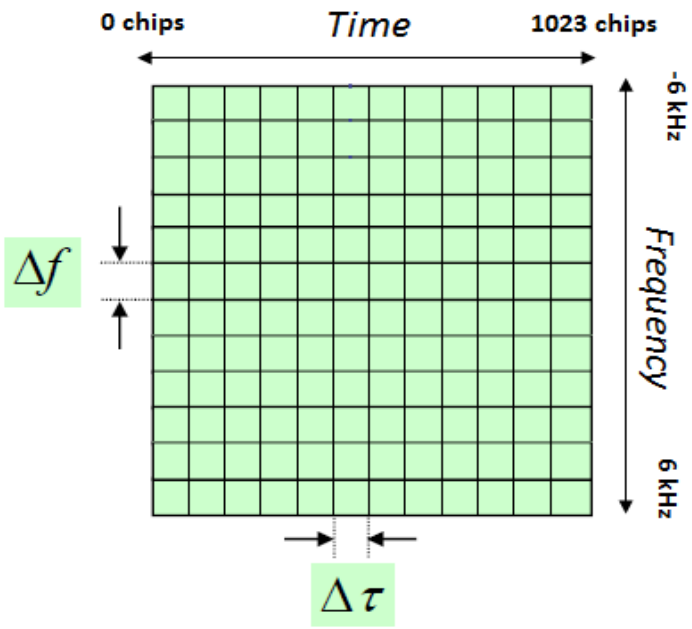
- Modern GNSS receivers are equipped with antennas capable of collecting signals on multiple frequencies.
- In a multi frequency case, the navigation processing block combines measurements from all frequencies.
- The local oscillator plays a significant role in the receiver operations:
  - used for generating the carriers for the signal down-conversion
  - local clock

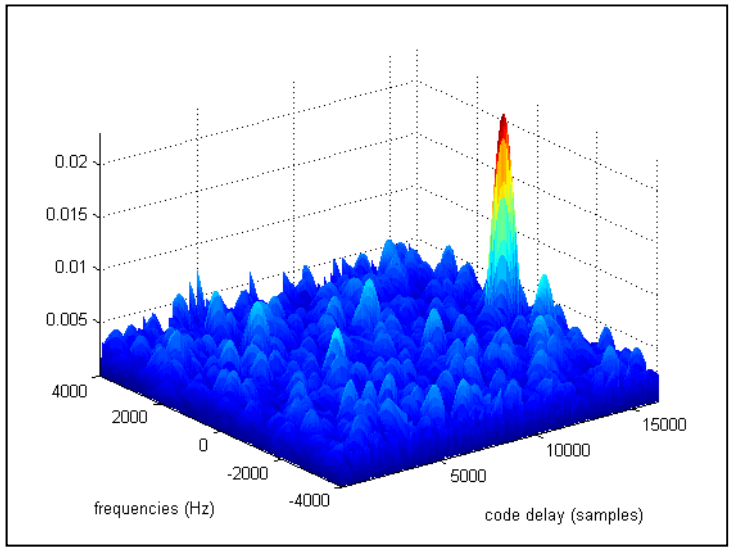
Signal Processing Stages



[2].

Acquisition Overview (1/2)



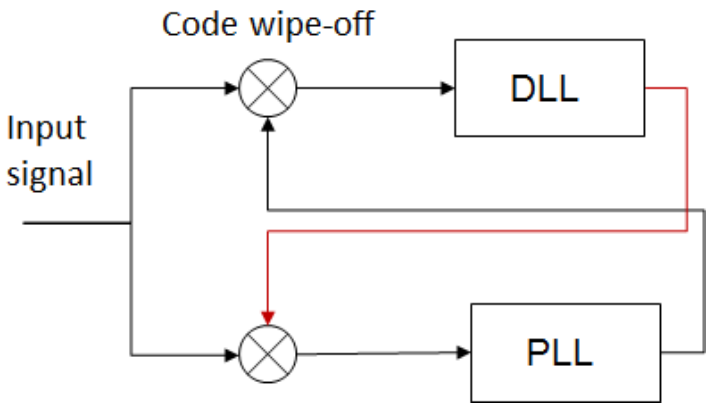


- Acquisition is a 2 dimentional search over frequency (Doppler) and time (code chips):
- The time search is required to identify the code chip phase which indicates the relative time delay of the signal reaching the receiver.
- The frequency search is required because the signal has a Doppler component due to the relative motion between the satellite and the receiver.
- Decision variable: correlator outputs with frequency and code delay. If the decision variable passes a threshold – the signal is declared present.
- A-priori knowledge of receiver position, time and satellite almanac limits the search to only the visible satellites restricting the range of the frequency and time bins to search. This significantly reduces the time necessary to acquire the satellite signal.

Signal Tracking Overview

- The Doppler frequency and code delay estimated by the acquisition block are too rough for being used for positioning and navigation. The phase information is not considered.

- **Signal Tracking:**
  - Estimation of fine code phase and Doppler frequency.
  - Phase estimation.
  - Continuous tracking of the signal parameters changes.



- Signal tracking is performed by generating precise replicas of the incoming code and carrier. The generated replicas are aligned with the input signal.

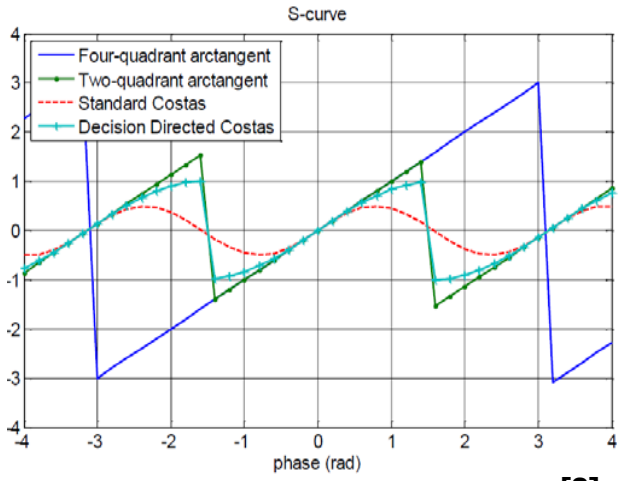
- Two separate tracking loops are typically used for:
  - the local code: code delay → Delay Lock Loop (**DLL**)
  - the local carrier: phase and frequency → Phase Lock Loop (**PLL**)

- The two loops are coupled → the DLL (PLL) requires precise carrier (code) wipe-off for operating correctly.

Tracking Loop Structure

Each tracking loop is made of:

- One or more correlators (**Integrate & Dump filters**). To measure of how similar the local and incoming signals are.
- A **non-linear discriminator** for extracting the quantity to be tracked (phase and delay). It is responsible for evaluating an error function of the quantity to be tracked. The loop tries to drive to zero the error evaluated by the discriminator (minimization/maximization of the cost function).
- A **loop filter** for improving the estimate of the quantity to be tracked. Loop filters are aimed at reducing the noise present at the discriminator output.
- A carrier/code numerically controlled oscillator (**NCO**), used to generate the local code and carrier.



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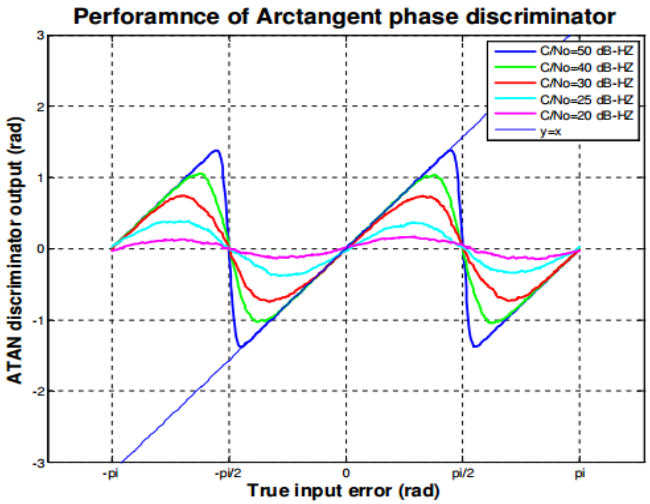
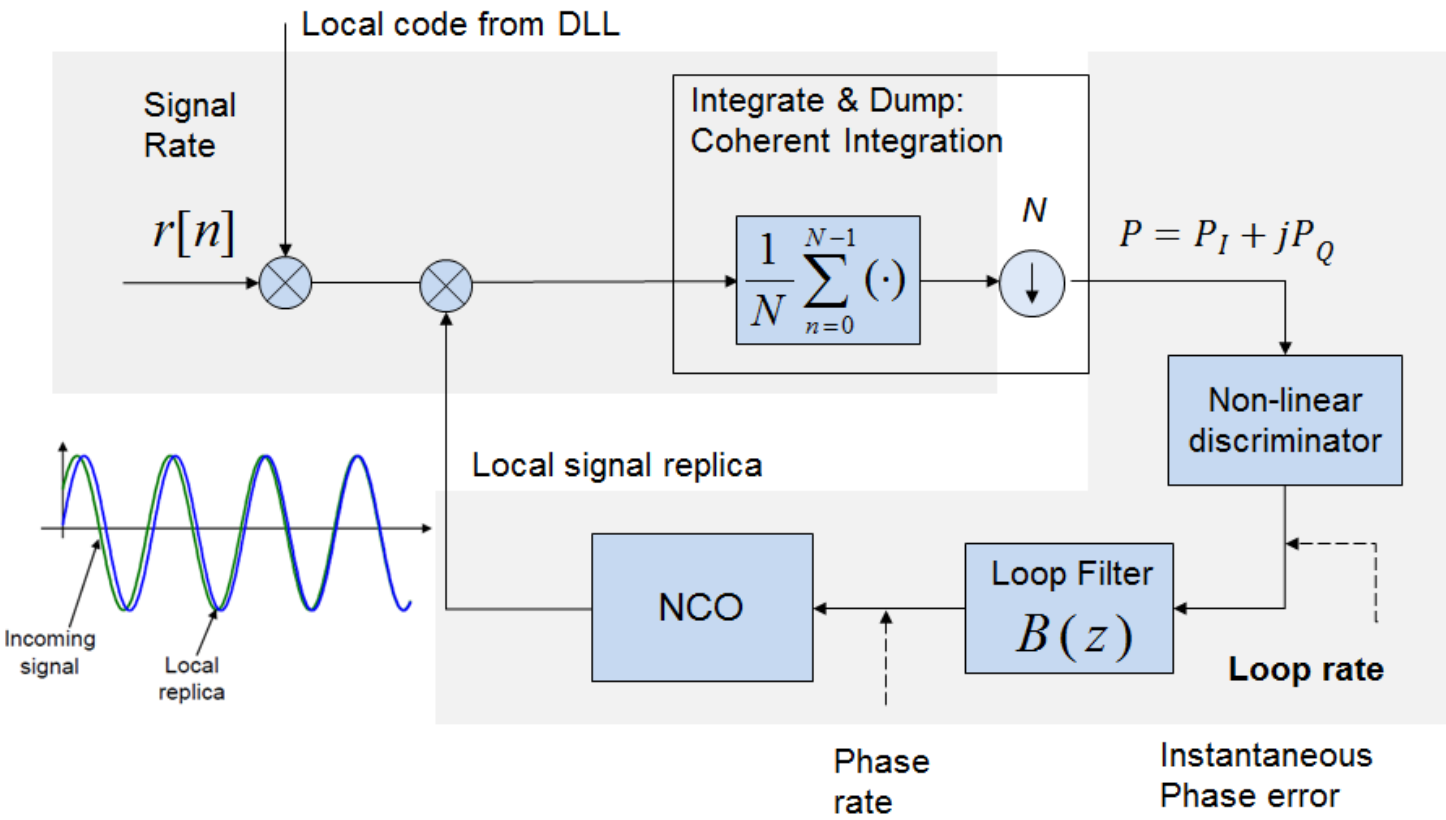


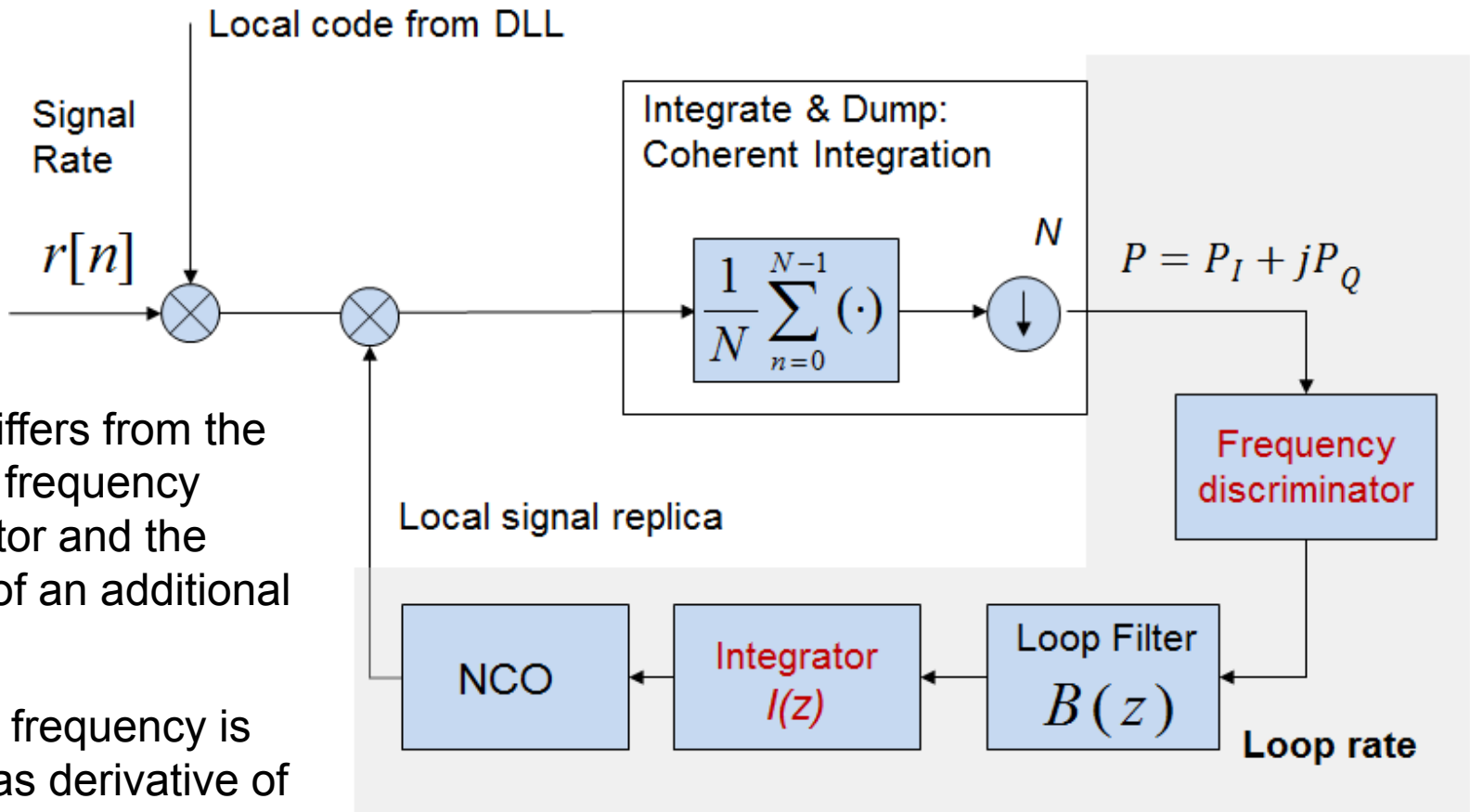
PLL

- **PLL** tasks:
  - generation of a local carrier aligned in phase and frequency with the input signal.
  - phase alignment obtained adjusting the <u>phase rate</u> measured at the loop filter output.
  - the phase rate (frequency) of the local replica is increased or decreased in order to recover the initial phase difference.



[2].

FLL

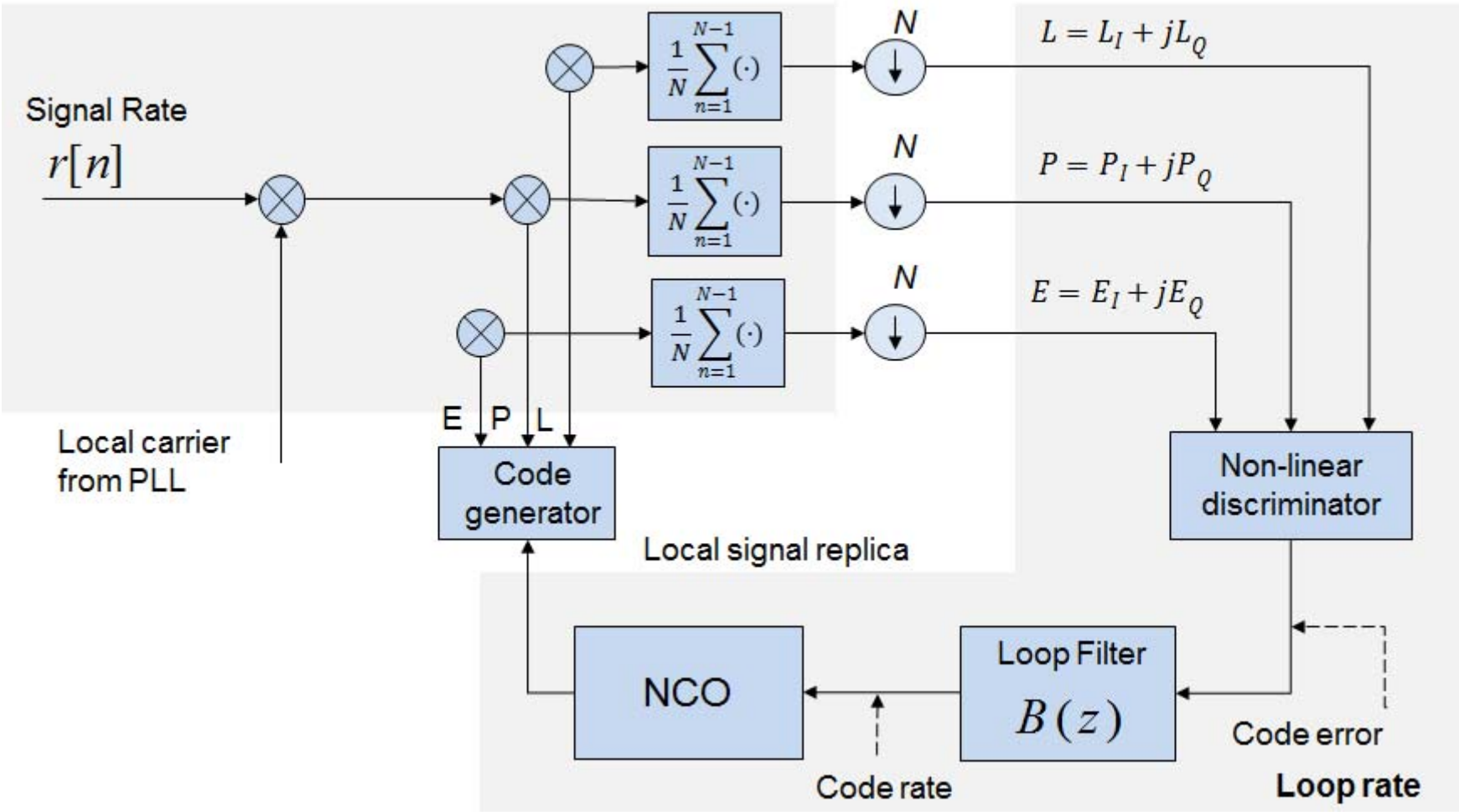


FLL:

- The FLL differs from the PLL in the frequency discriminator and the presence of an additional Integrator.
- The signal frequency is extracted as derivative of the phase.
- FLL and PLL can be jointly used to better track the signal dynamics.

[2].

DLL

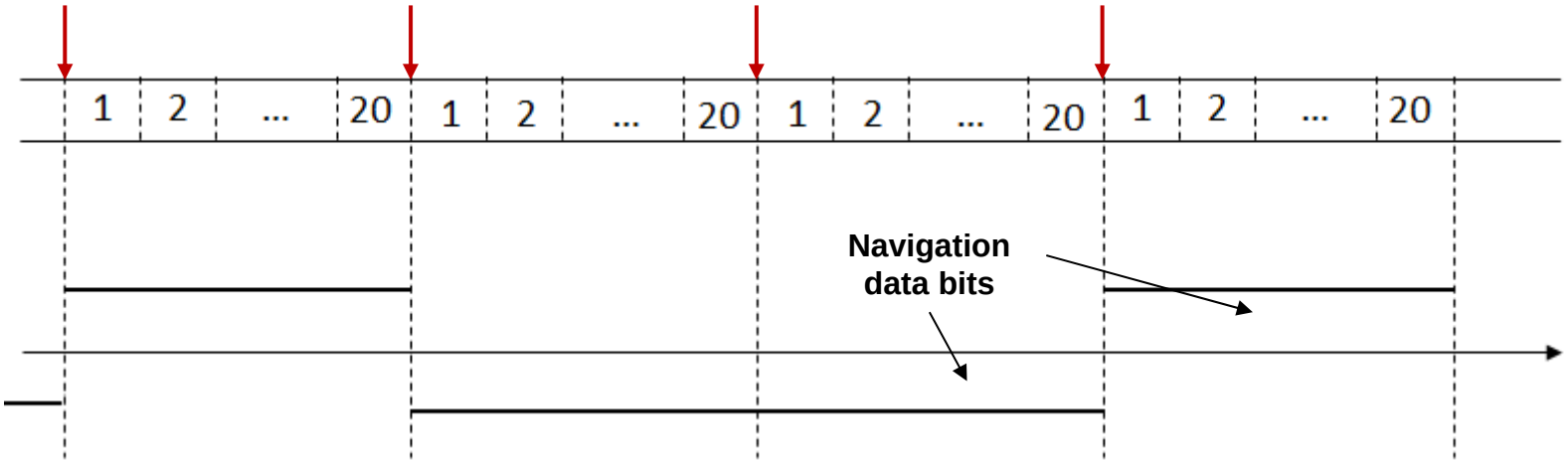

Signal Rate
$r[n]$
$\frac{1}{N}\sum_{n=1}^{N-1}(\cdot)$
N
$L = L_I + jL_Q$
$P = P_I + jP_Q$
$E = E_I + jE_Q$
E
P
L
Local carrier from PLL
Code generator
Non-linear discriminator
Local signal replica
NCO
Loop Filter
$B(z)$
Code error
Code rate
Loop rate

[2].

Bit Syncronization

- determination of the beginning of a data bit (a 20 element block characterized by the same polarity).

- Is neccessary to:
  - Extend the coherent integration time (up to 20 ms).
  - Start the data demodulation process.



- Each data bit lasts 20 ms, (20 primary C/A code periods).
- The primary codes are grouped in blocks of 20 elements, where each group is characterized by the same polarity (data bit).
- Acquisition and tracking recover only the boundary of the primary PRN code.

GNSS Measurements: Overview

The following measurements are typically available from each receiver channel:

- Code Pseudorange
- Doppler
- Carrier-phase measurement
- Carrier-to-Noise density (C/N_0)

} Raw measurements

Measurement rate varies from receiver to receiver (1-20 Hz)

Note: not all commersial GPS/GNSS receivers output the raw measurements. Most of the receivers generate only the navigation solution:

- position
- velocity
- heading, etc.

Code Pseudorange

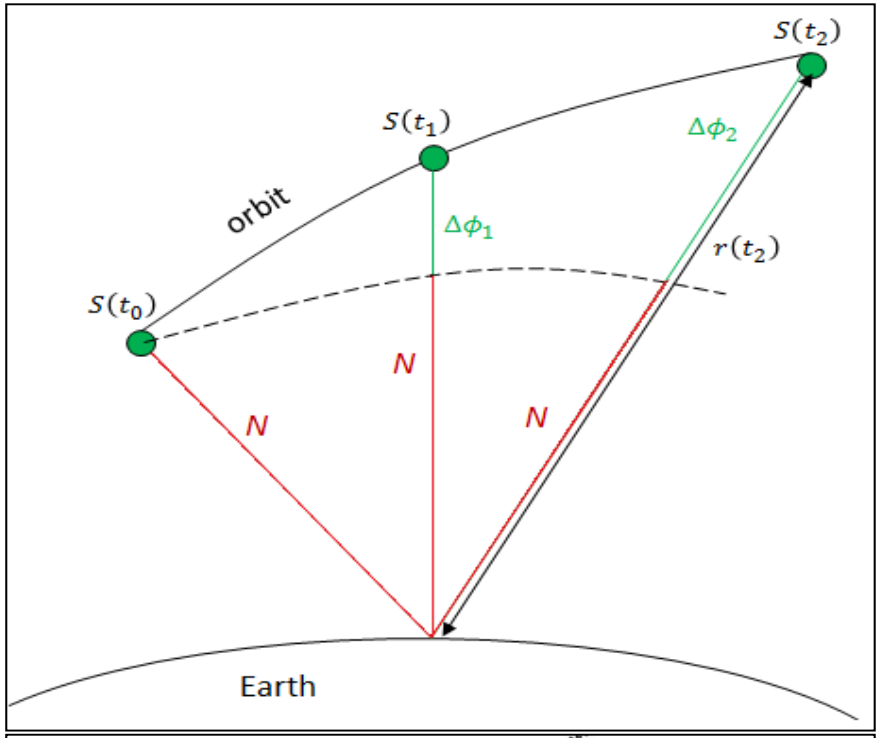
- Pseudorange observation equation (in m):

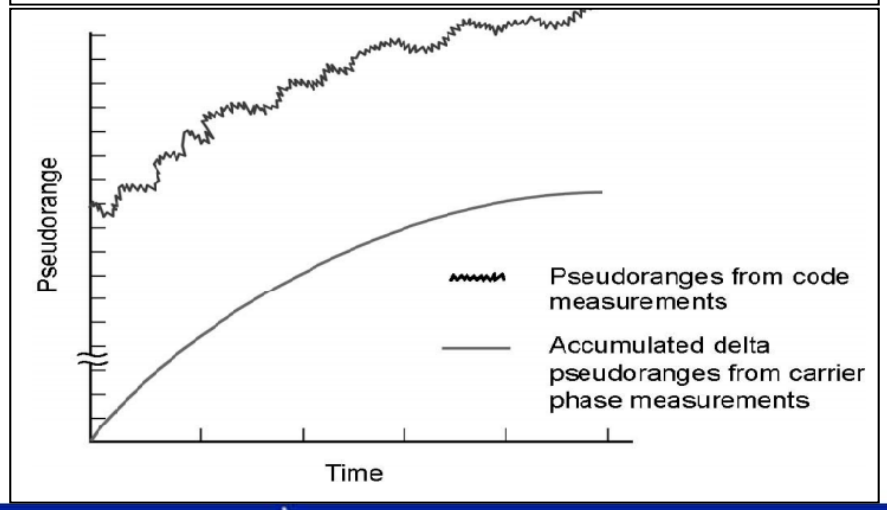
$$P = \rho + d\rho + c(dt - dT) + d_I + d_T + \varepsilon_P$$

- where

- P - receiver pseudorange measurement,
- ρ - geometric range, $\left\{\rho_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} = \|\boldsymbol{r_u} - \boldsymbol{r_i}\|\right\}$
- $d\rho$ - orbital errors,
- dt, dT - satellite and receiver clock errors,
- d_I - ionospheric delay,
- d_T - tropospheric delay,
- ε_P - measurement noise (miscellanious unmodelled range error).

Carrier Phase (1/2)





- When a satellite is locked (at t_0), the receiver starts tracking the incoming phase.
- It counts the (real) number of cycles as a function of time → $\Delta\phi(t)$.
- But the initial number of cycles N at t_0 is unknown.
- This initial unknown number of cycles is called the **carrier phase integer ambiguity**.
- However, if no loss of lock, N is constant over an orbit arc. And can be determined by applying an ambiguity resolution technique.
- It is calculated by integrating (accumulating) the Doppler measurements available from the receivers PLL.

Carrier Phase (2/2)

- Carrier phase observation equation (in m):

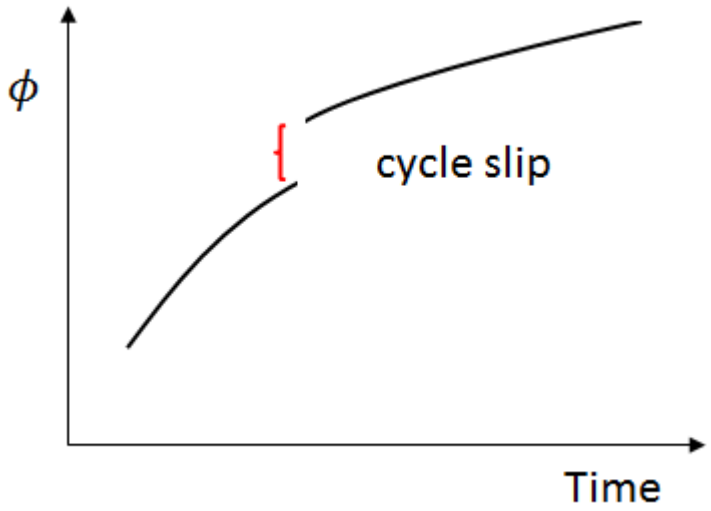
$$\phi = \rho + d\rho + c(dt - dT) + \lambda N - d_I + d_T + \varepsilon_\phi$$

- where

- ϕ - receiver carrier phase measurement,
- ρ - geometric range, $\left\{\rho_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} = \|\boldsymbol{r_u} - \boldsymbol{r_i}\|\right\}$
- $d\rho$ - orbital errors,
- dt, dT - satellite and receiver clock errors,
- N - cycle ambiguity (integer number),
- d_I - ionospheric delay,
- d_T - tropospheric delay,
- ε_ϕ - measurement noise (miscellanious unmodelled phase error).

Cycle Slip Overview

- Cycle slip – sudden jump of an integer number of cycles in the carrier phase observable caused by the loss of lock of a receiver phase lock loop.
- Can occur of multiple reasons:
  - receiver tracking problems,
  - interruption in the antenna's reception of satellite signals caused by e.g. temporary signal blockage.



- A slip of only a few cycles can bias measurements enogh to make centimetre-level positioning or navigation difficult → cycle slip detection critical.
- More difficult for kinematic positioning than static due to additional Doppler shift that is difficult to predict.

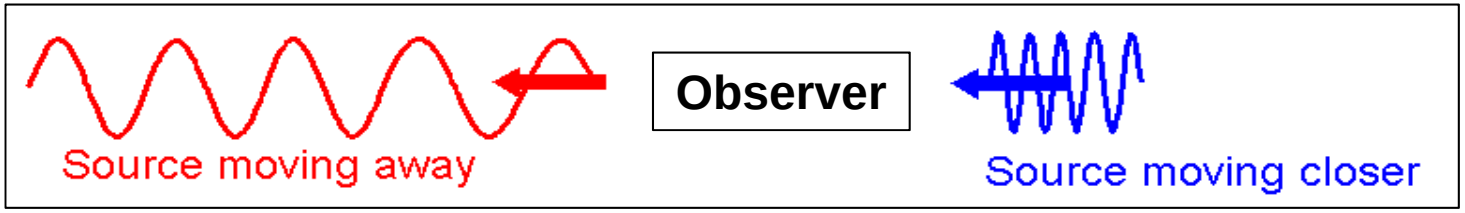
Cycle Slip Detection

- Several cycle slip detection methods are available, depending on the mode (static/kinematic) and receiver (single vs dual frequency).
- Kinematic Single Frequency Cycle Slip Detection: **Phase Velocity Trend Method**
  - Carrier phase measurement, ϕ_k, is predicted using the measurement taken at the previous time epoch, ϕ_{k-1}, and the phase rates at the current and previous time epochs:

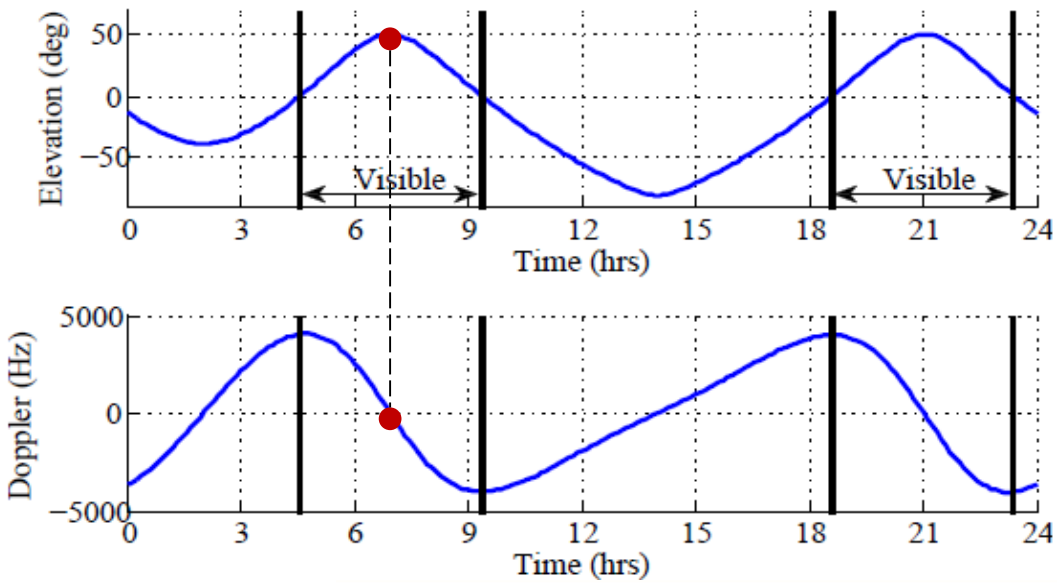
$$\hat{\phi}_k = \phi_{k-1} + \frac{\dot{\phi}_k + \dot{\phi}_{k-1}}{2}\Delta t$$

  - If $\left|\hat{\phi}_k - \phi_k\right|$ < present threshold, no cycle slip has occurred.
  - Threshold is a function of Δt and receiver dynamics (we are assuming that phase velocity is constant over Δt). Usually valid within 1 cycle if Δt= 1 s
  - Note: this method is usable in kinematic mode, but the threshold might be larger than 1 cycle at high dynamics.

Doppler Effect



- The user received frequency, f_R, is a Doppler shifted version of the transmitted frequency, f_S.
- This is a frequency dependent frequency shift → higher frequencies are shifted by larger amounts for the same relative velocity.
- Wavelength in space is not affected.



- Considering GPS L1 frequency, Doppler shift is approximately ±5 kHz for a static receiver, and ±10 kHz for a moving receiver under high dynamics.
- Doppler is a measure of relative velocity or range rate between the satellite and the receiver.

Doppler Frequency Measurement (1/2)

- Doppler shift, Δf, can be expressed as follows:

$$\Delta f = f_R - f_S$$

where the recieved GPS signal frequency, f_R, is:

$$f_R = f_{R_meas}\left(1 + \dot{\delta t}_{Rx}\right)$$

  - $f_{R_{meas}}$ - measured received frequency (Hz),
  - $\dot{\delta t}_{Rx}$ - receiver clock drift rate (s/s).

- Thus, the actual measurements are formed by differencing the measured received frequency and the transmit frequency:

$$\Delta f = f_{R_meas} - f_S$$

- **This expression is derived by approximating the received GPS frequency using a standard Doppler equation for EM waves:**

$$\boldsymbol{f_R = f_S\left(1 - \frac{\overline{v_R}\cdot\overline{a}}{c}\right),}$$

- **where:**
- $\boldsymbol{f_R}$ **- received frequency in Hz,**
- $\boldsymbol{f_S}$ **- transmitted frequency in Hz,**
- $\boldsymbol{\overline{v_R}}$ **- satellte-to-user relative velocity vector (m/s),**
- $\boldsymbol{\overline{a}}$ **- unit vector pointing along the line-of-site from user to satellite,**
- ***c* – speed of light.**

Doppler Frequency Measurement (2/2)

- Doppler frequency observation equation (in m/s):

$$\dot{\phi} = \dot{\rho} + \dot{d\rho} + c\left(\dot{dt} - \dot{dT}\right) - \dot{d}_I + \dot{d}_T + \varepsilon_{\dot{\phi}}$$

- where

- $\dot{\phi}$ - receiver Doppler measurement,
- $\dot{\rho}$ - geometric range rate,
- $\dot{d\rho}$ - orbital error drift,
- $\dot{dt}, \dot{dT}$ - satellite and receiver clock error drift,
- $\dot{d}_I$ - ionospheric delay drift,
- $\dot{d}_T$ - tropospheric delay drift,
- $\varepsilon_{\dot{\phi}}$ - measurement noise.

Carrier-to-Noise Density Ratio (C/N_0)

- C/N_0 is a measure of the ratio of the amplitude of the recovered GPS carrier signal to the noise → measure of the signal quality.
- Each GNSS receiver **estimates** the C/N_0 of the signals → different estimates can be provided by different receivers.
- The **receiver sensitivity** is determined by the C/N_0 (or equivalently, signal power) of the weakest signal that the receiver is able to process.
- The C/N_0 is a ratio, but is usually expressed in log scale (dB-Hz):

$$\left(\left[\frac{C}{N_0}\right]\right)_{dB} = \left(\frac{[C]}{[N_0]}\right)_{dB} = \left(\frac{Watt}{Watt/Hz}\right)_{dB} = (Hz)_{dB} = dB\ Hz$$

where the noise Power Spectral Density can be calculated as follows:

$$N_0 = k_B T_0$$

$k_B = 1.38 \cdot 10^{-23}$ (Jules/Kelvin), is the Boltzmann constant, and T_0 is the noise temperature.

Example: Assuming T_0 = 290 K, N_0 = -204 dB W/Hz. For the GPS L1 C/A signal, the received power is approximately C = -160 dBW, leading to a nominal C/N_0 of about 44 dB-Hz.

Carrier Smoothed Pseudorange

- Pseudorage measurements are noisy, but 'absolute', whereas carrier phase measurements are much more accurate, but 'relative'.
- Carrier-smoothing is an approach to obtain an alternative to pure pseudorange observations. Is used in most of the receivers firmware.
- **Methodology:**
  - Recursive filter to progressively increase weight on carrier while decreasing weight on pseudorange:

$$\tilde{P}_k = \alpha \cdot P_k + (1-\alpha) \cdot \left(\tilde{P}_{k-1} + \frac{\lambda}{2\pi}(\phi_k - \phi_{k-1}) \right)$$

- $\tilde{P}_k$ - smoothed pseudo-range,
- $\tilde{P}_{k-1}$ - smoothed pseudo-range at the previous time epoch,
- P - raw pseudo-range measurement,
- λ - L1 wavelength,
- ϕ_k - carrier phase measurement,
- ϕ_{k-1} - carrier phase measurement at the previous time epoch,
- α - filter weighting function equal to the sample interval divided by the smoothing time constant.

Propagation Channel

Propagation channel model includes:

- Satellite payload impariments
- Satellite antenna
- Propagation medium (vacuum plus atmosphere close to the Earth)
- Obstacles including reflectors
- Interference (intra-system interference, inter-system interference (GPS, Galileo, GLONASS, SBAS, etc.), external interference.

Received min GPS RF signal strength in dBW (0 dBic antenna).

	L1		L2	
	min	max	min	max
C/A	-160	-156	n/a	n/a
P	-163	-159	-166	-164

- GNSS signals are extremely weak, approximately 0.000 000 000 000 0001 Watts at antenna
  - They do not pierce concrete
  - They do not pierce tunnels
  - Signals can be significantly degraded by tree foliage

RF Propagation Terminology

Propagation of radio waves is characterized by several factors:

- **Free-Space Loss or geometric spreading**. Decrease of EM field strength due to EM wave energy spreading over larger areas at increased distances from transmitting source. The received power, P_r, at a distance r is equal to the transmitted power, P_t, divided by the surface area of the sphere with radius r:

$$P_r = \frac{P_t}{2\pi r^2}$$

- **Attenuation:** decerase in the intensity of the EM wave as a result of absorption of energy (not including the geometric spreading). Is typically expressed in dB, and can be expressed as:

$$[dB]_{Gain} = 10\log_{10}\frac{P_t}{P_r} = 20\log_{10}\frac{V_t}{V_r} = 20\log_{10}\frac{I_t}{I_r}$$

V_t - transmitted voltage, V_r - received voltage,

I_t - transmitted current, I_r - received current.

- **Examples:**

-1 dB ⟹ $10^{(-1/10)}$ ⟹ 79 % of original power remains

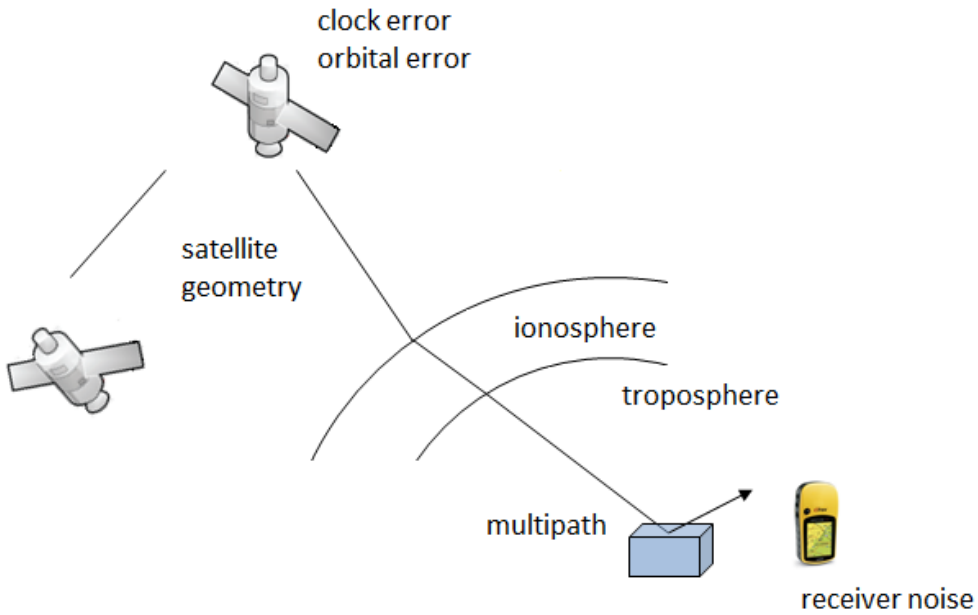
-3 dB ⟹ $10^{(-3/10)}$ ⟹ 50 % of original power remains

Attenuation of Building Materials

1500 MHz band:

Material	Attenuation [dB]
Dry wall	1
Plywood	1 – 3
Glass	1 – 4
Painted glass	10
Wood	2 – 9
Iron mat	2 – 11
Roofing tiles/Bricks	5 – 31
Concrete	12 – 43
Ferro-concrete	29 - 33

Error Sources



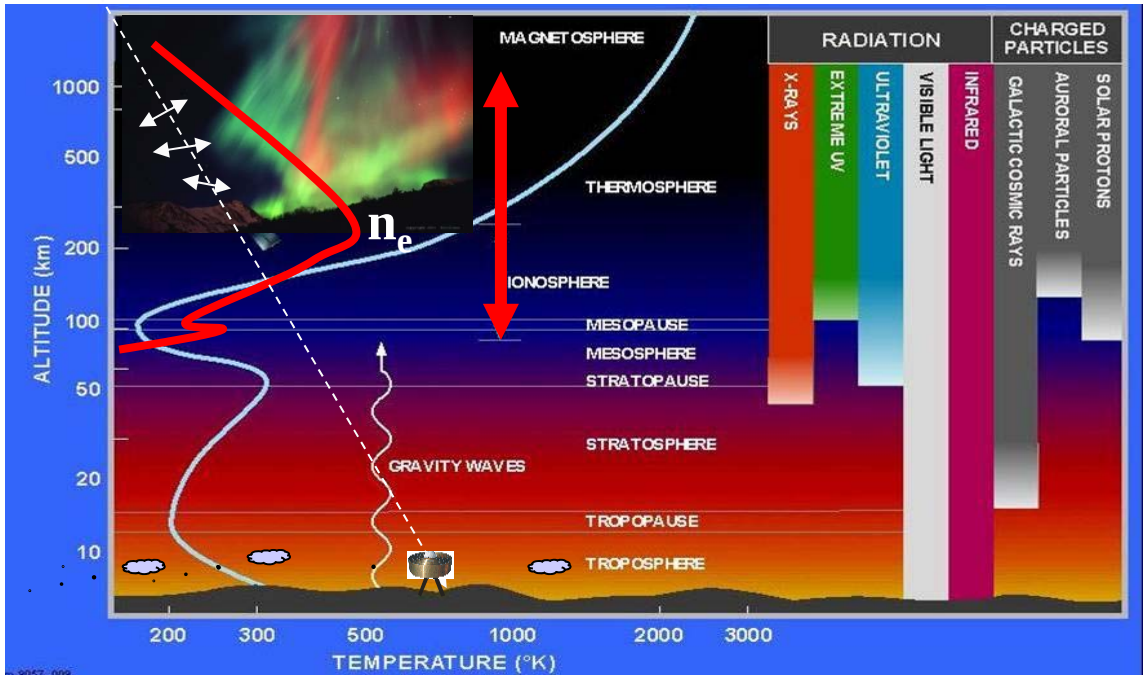
- Measurement errors strongly depend on the <u>environment</u> and <u>receiver performance</u>.
- **Important error characteristics:**
  - Magnitude
  - Variability
    - Temporal correlation
    - Spatial correlation

Dominant GNSS Error Sources:

- Satellite-based errors:
  - Satellite Clock
  - Orbit error
- Propagation errors:
  - Ionosphere
  - Troposphere
  - Multipath
  - Interference
- Receiver based errors:
  - Antenna effects
  - Receiver clock
  - Noise
- Poor geometry of satellites increases these errors

Ionospheric Delay

Ionosphere- region of free electrons in the upper atmosphere, from 70 to about 1000 km above the Earth.



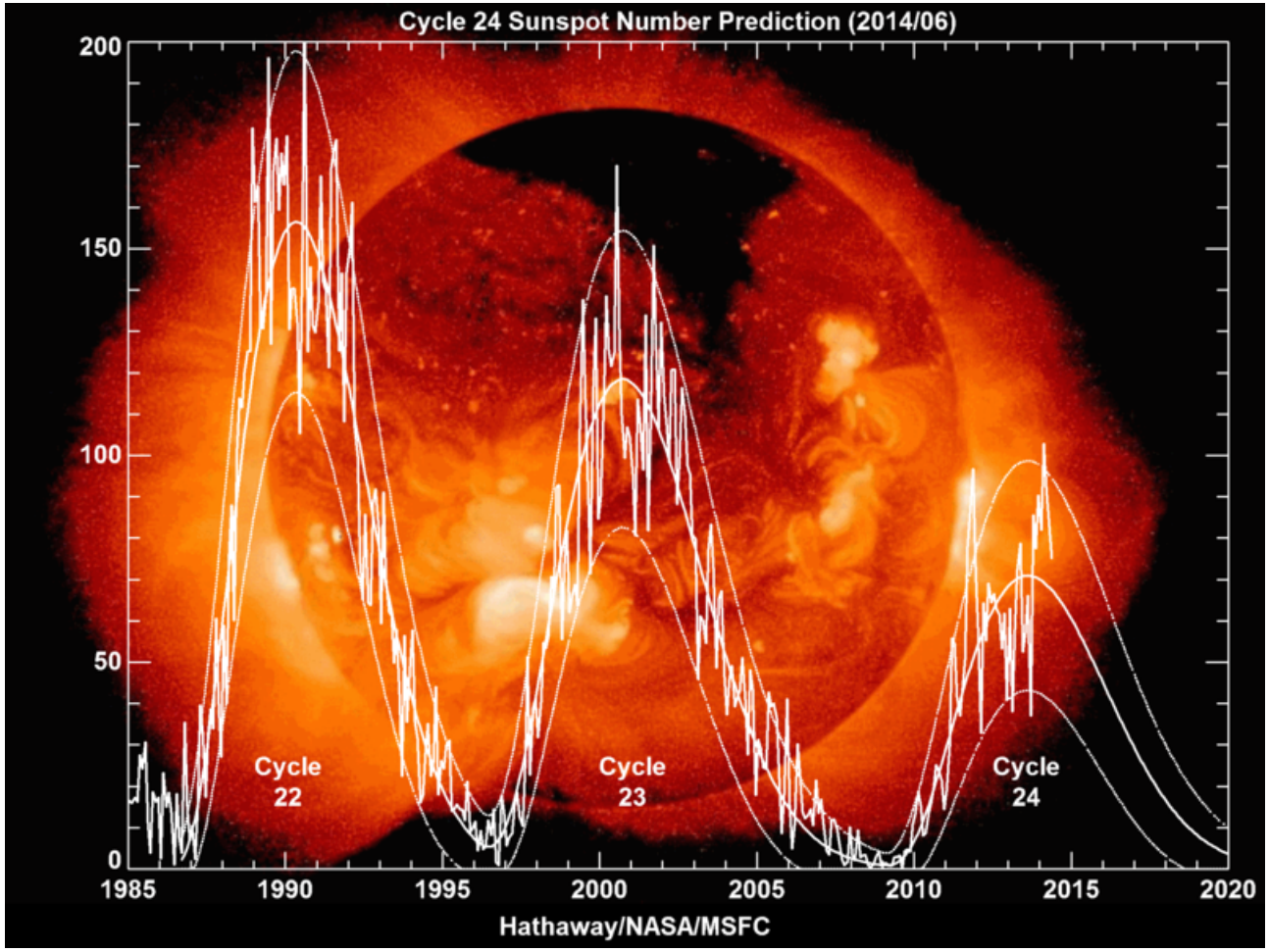
[4].

- Formed mostly by Ultra Violet (UV) radiation from the sun.
- Degree of ionization varies with lattitude, time of the day, time of the year and level of the solar activity.
- The amount of electrons (electron desity) is directly related to the effects on RF signal propagation in medum.
- Total Electron Content: electron density along the path length.

$$TEC = \int_{SV}^{User} n_e dl$$

n_e - variable electron density along the signal path.

Solar Cycle



- The solar magnetic activity cycle is the nearly periodic **11 year** change in the Sun's activity.
- The changes on the sun cause effects in space, in the atmosphere and on the Earth's surface. While it is the dominant variable in solar activity, aperiodic fluctuations also occur.

Image source: http://solarscience.msfc.nasa.gov/images/Cycle22Cycle23Cycle24big.gif

Ionospheric Delay

- Ionosphere is **frequency dispersive** (refractive index is a function of frequency)
  - measurements on L2 and L5 frequencies will be more affected than L1.
  - **phase advance** and **group delay:**

$$\Delta S_{iono,p} = \frac{-40.3 \cdot TEC}{f^2},$$

$$\Delta S_{iono,g} = \frac{40.3 \cdot TEC}{f^2}$$

- 1 TECU = 10^{16} electron/m^2
- The relationship between the signal delay due to the ionosphere, ϵ_{iono}, and the total electron content (TEC) is:

$$\epsilon_{iono} = \frac{1}{sin(\phi')} \frac{40.3 \cdot TEC}{f^2},$$

- Where f^2 is the signal frequency, and ϕ', the elevation angle at ionospheric pierce point (IPP).

Ionospheric Compensation (1/2)

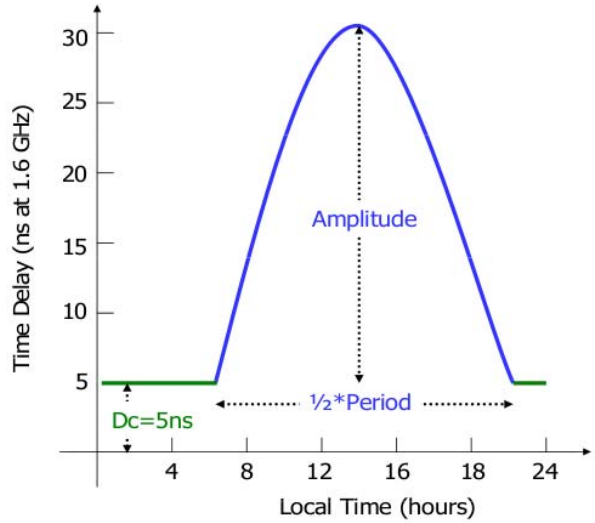
Broadcast ionosphere model:

- GPS: Klobuchar model [6] or "half-cosine model":
  - Parameters are transmitted in the GPS navigation message.
  - Assumes that the vertical ionospheric delay can be approximated by half a cosine function of the local time during daytime, and constant level during the nighttime.

$$d_{iono}^{Night} = DC = 5\ ns$$

$$d_{iono}^{Day} = DC + A \cdot cos[2\pi(t - 50400)/P]$$

  - *A* and *P* are functions of the broadcast parameters $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2, \beta_3)$.
  - Removes ~50 % of ionospheric delay at mid-latittudes.
- Galileo: NeQuick model [7].



$$A = \sum_{n=0}^{3} \alpha_n \phi^n$$

$$P = \sum_{n=0}^{3} \beta_n \phi^n$$

ϕ - geomagnetic latitude

Ionospheric Compensation (2/2)

- Carrier-code divergence

- Due to frequency dispersive nature use of two frequencies allows compensation of the ionospheric delay:

pseudorange measurements at L1 and L2

$$\Delta S^{iono} = \left(\frac{f_2^2}{f_2^2 - f_1^2}\right)(\rho_1 - \rho_2)$$

  - this approach requires a dual-frequency receiver.
  - most accurate.

- Differencing (common effect eliminated).

  - requires two receivers.

Ionospheric Scintillation (1/2)

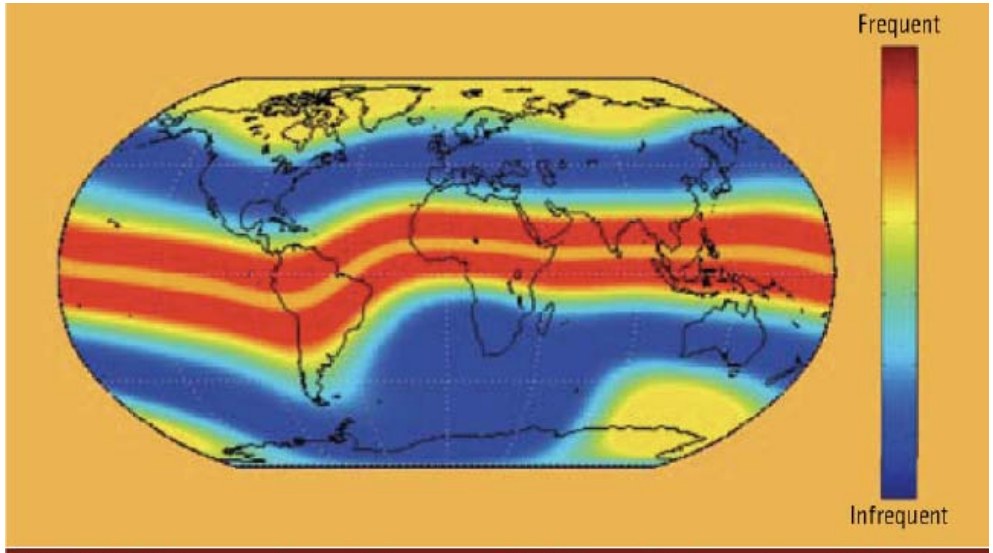


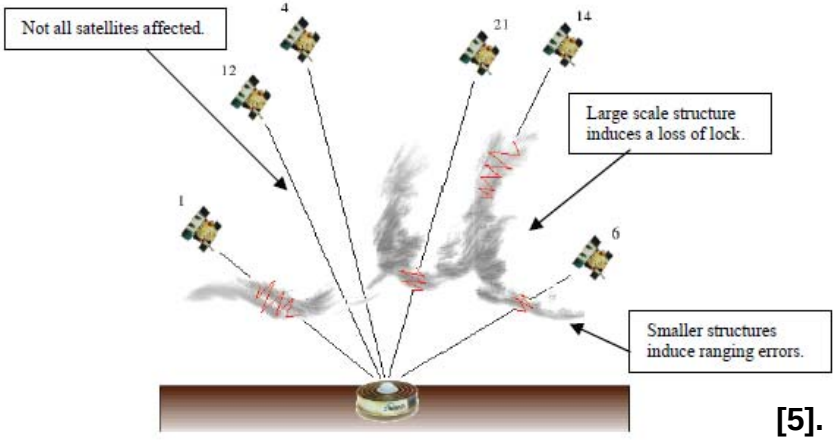
FIGURE 1 Scintillation map showing the frequency of disturbances at solar maximum. Scintillation is most intense and most frequent in two bands surrounding the magnetic equator, up to 100 days per year. At poleward latitudes, it is less frequent and it is least frequent at mid-latitude, a few to ten days per year.

[5].

- varies with geographic location, local time, season, magnetic activity, and the 11-year solar cycle.
- occurs most often when the ionosphere is disturbed by solar storms (anomalous ionosphere activity).

Additional Ionospheric Effects:

Scintillation: rapid variation in electron density in the ionosphere causes interference pattern on the ground. Like passing signal through a diffraction grating.

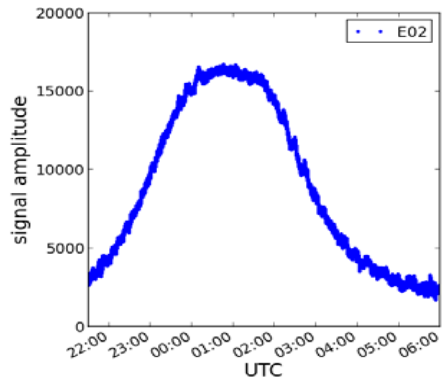


[5].

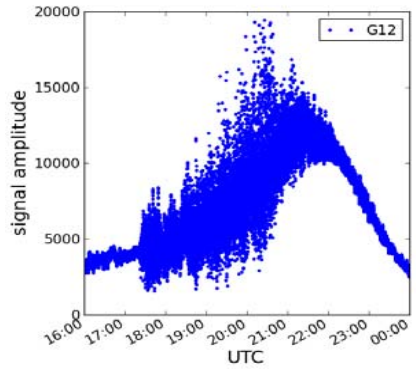
Ionospheric Scintillation (2/2)

- <u>Amplitude scintillation/short term fading</u>
  can create deep signal fades that interfere with a user's ability to receive GNSS signals (signal levels can drop below the tracking sensitivity of the receiver forcing it to reacquire).

Nominal data

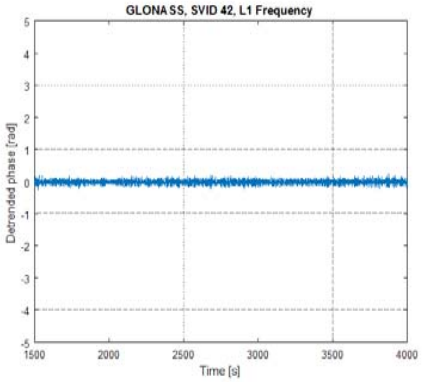


Scintillation event

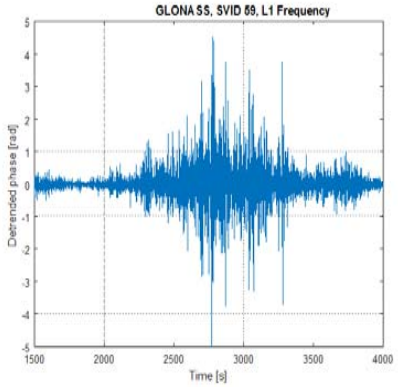


- <u>Phase scintillation</u> - rapid fluctuations in the observed carrier phase obtained from the receiver's phase lock loop (PLL). Can cause increased phase noise, cycle slips, and even loss of lock if the phase fluctuations are too rapid for the receiver to track.

Nominal data



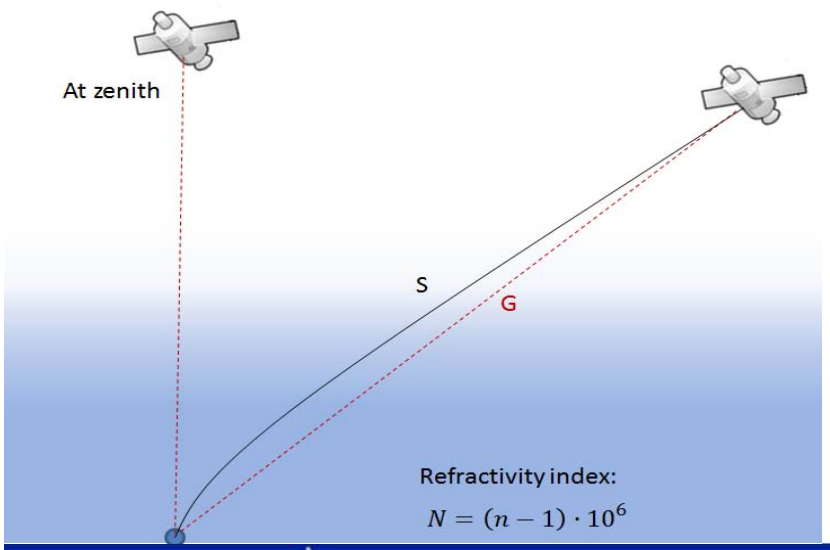
Scintillation event



Tropospheric Delay

- Troposphere is the atmospheric layer between Earth's surface and an altitude of about 50 kilometres. The effect of the troposphere on the GNSS signals appears as an extra delay in the measurement of the signal traveling from the satellite to receiver. For GNSS frequencies, it is not frequency dispersive.
- The excess propagation time (distance) is given as:

$$\Delta\rho(h) = 10^{-6} \cdot \int_S N(h) \cdot dh \qquad (m)$$



- The total tropospheric refractivity index, *N*, is usually divided into two parts: **dry** and **wet**
  - Dry component accounts for about 90 % of the tropospheric delay: stable. Is a function of:
    - Surface pressure (*P*)
    - Surface temperature (*T*)
  - Wet component is highly variable, hard to predict. Consists of:
    - Partial pressure of water vapor (*e*)
    - Surface temperature (*T*)

Troposphere Models

- Generalized formulation:

$$N = \overbrace{\boxed{776.4 \cdot \frac{P}{T}}}^{\text{Dry}} + \overbrace{\boxed{3.73 \cdot 10^6 \cdot \frac{e}{T^2}}}^{\text{Wet}}$$

  where *P* is given in kPa, and *T* in Kelvin.

- Left uncompensated can result in a ranging error of ~2-3 meters at zenith (larger error at low elevation angles).

- A number of models available [8]:
  - Saastamoinen
  - Modified Hopfield
  - Black and Fisher
  - etc.

Satellite Clock and Orbital Error

- Ephemeris information is predicted from previous measurements of satellite motion and knowledge of the Earth's gravity field.
- Even though the satellites are positioned in very precise orbits, slight shifts are possible due to:
  - gravitational influences of the sun and the moon,
  - solar radiation pressure,
  - Non-spherical gravitational potential of the Earth,
  - etc.
- The satellite clock corrections included in the GPS navigation message are not perfect as well, and a small error is resulting in the pseudorange measurement.
- More accurate GPS ephemeris can be obtained online from e.g. International GNSS Service (IGS) (sp3 files). Several products are available including rapid, ultra-rapid and final precise orbital data. The difference is in the level of accuracy and update latency.

Satellite Clock and Orbital Errors

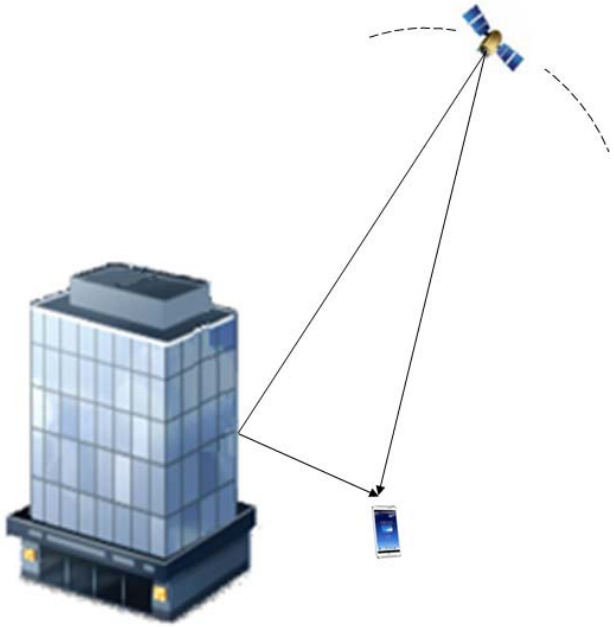
- Comparison of broadcast and IGS orbit information.

Ephemeris/ Clock	Accuracy	Latency	Update
Broadcast	~ 2 m / 7 ns	Real-time	2 hour
IGS1 (Ultra rapid)	0.1 m / 5 ns	Real-time	4/day
IGS2 (Rapid)	0.05 m / 0.1 ns	3 hours	4/day
IGS (Fast)	0.05 m / 0.1 ns	17 hours	1/day
IGS (Final)	0.05 m / 0.1 ns	13 days	1/week

Source: IGS.

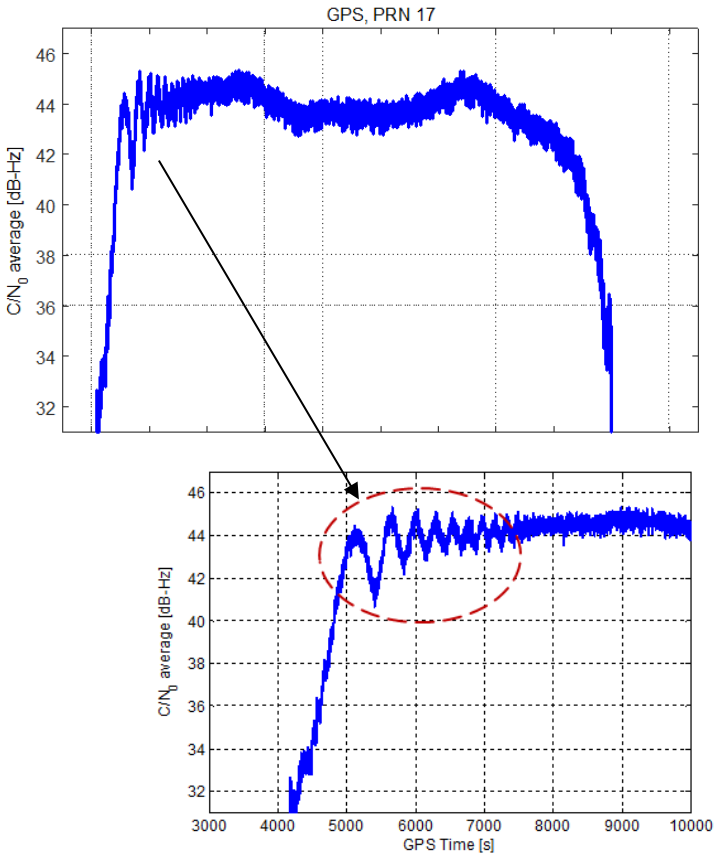
Multipath

- Local phenomenon → must be handled by receivers (multipath rejecting antennas, use of narrow correlator/multiple-correlator technigues, etc.).



- the signal can reach the receiver through **multiple paths → multipath**
- this results in constructive/destructive interference between the line-of-sight (LOS) and the reflections.
- as the geometry changes, the relative phase between LOS and reflections changes as well.
- Multipath is one of the largest error sources for most GNSS receivers.
- The range measurement error due to multipath depends on the <u>strength</u> of the reflected signal and the <u>delay</u> between direct and reflected signals.

Multipath



Example: multipath effect on C/N_0 measurements. Static receiver. Sinusoidal nature of the effect can be clearly observed for the satellite raising period.

- If reflecting geometry is constant over time, the periodic nature of the satellite orbits will cause the effect to repeat.

Pseudorange

- Can cause the measured range to appear either too short, or too long depending on the phase of the reflected signal.
- Maximum 150 m using one chip correlator spacing and 15 m using 0.1 chip correlator spacing.
- Not reduced through differential processing.

Carrier Phase

- Can reach magnitudes of 0.25λ (~4.75 cm at L1).
- Not reduced through differential processing.

Radio Frequency Interference (RFI)

Jamming:

- prevention of receiver operation
- degradation of the receiver performance
- discontinuity in receiver operations (frequent loss of lock)

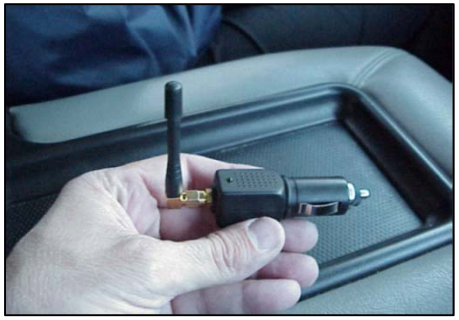
Spoofing:

- attempt to deceive a GPS receiver by broadcasting a more powerful signal than that received from the satellites, structured to resemble a set of normal GPS signals
- causes the receiver to determine its position to be somewhere other than where it actually is.

Cheap in-car/handheld GPS jammers

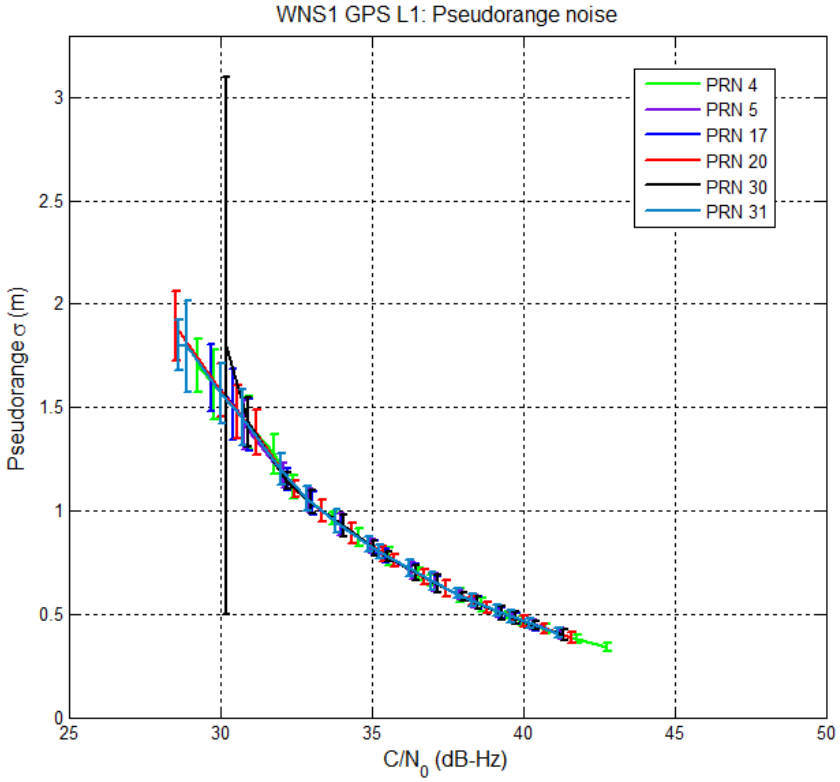
Used for prevention of being tracked by a fleet management or theft protection system

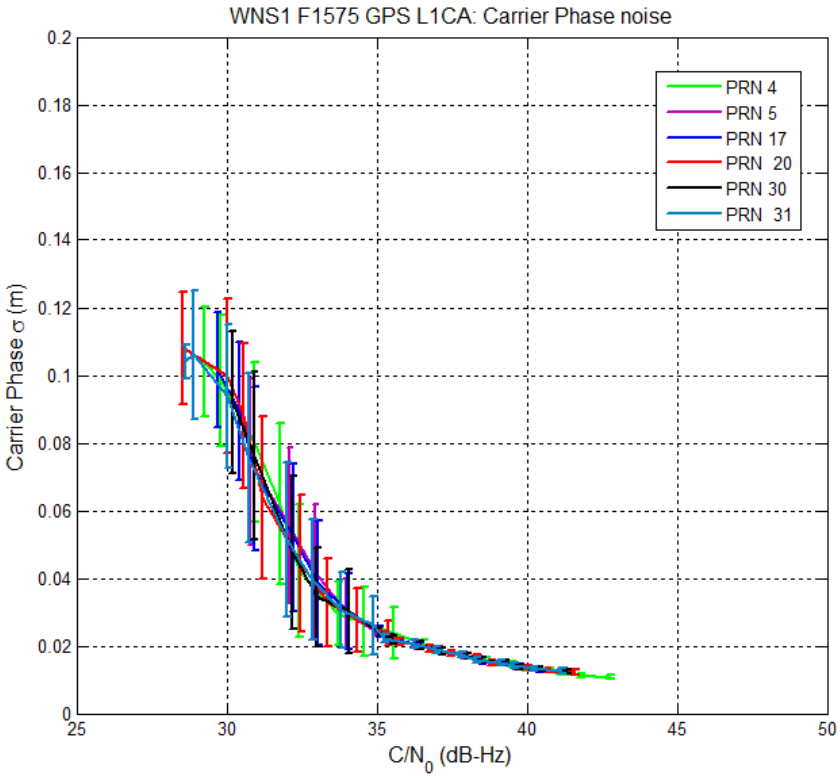
a fraud attempt against a distance based charging system, etc.



Radio Frequency Interference (RFI)

- Example: Level of measurement quality degradation and uncertainty due to RFI impact on the GPS L1 observables (pseudorange and carrier phase).
- Interference signal type - wideband noise.



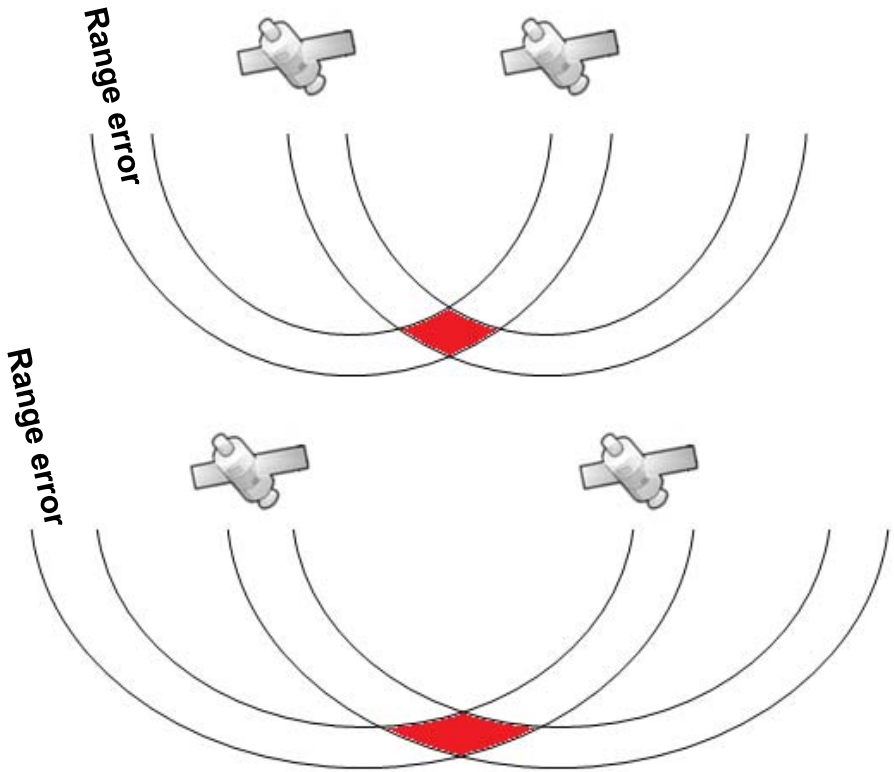


Receiver Noise

- The code and carrier measurements are affected by random measurement noise, called receiver noise. It depends on receiver technology (i.e. sequential or block processing architecture, tracking loop structure, tracking loop parameters, etc.), received signal power and dynamic stress.

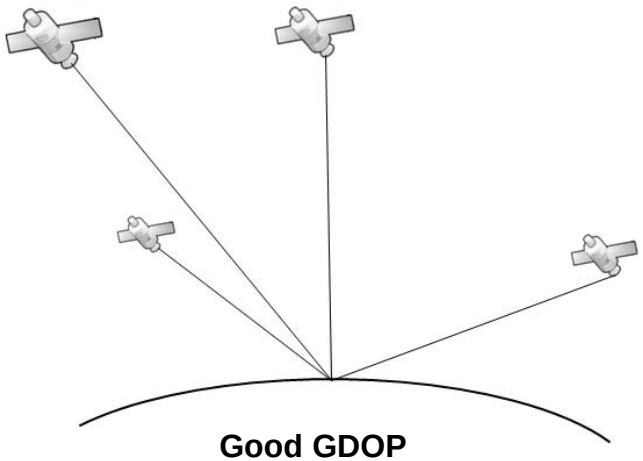
  - code : decimeter level
  - carrier phase : mm level

Satellite Geometry

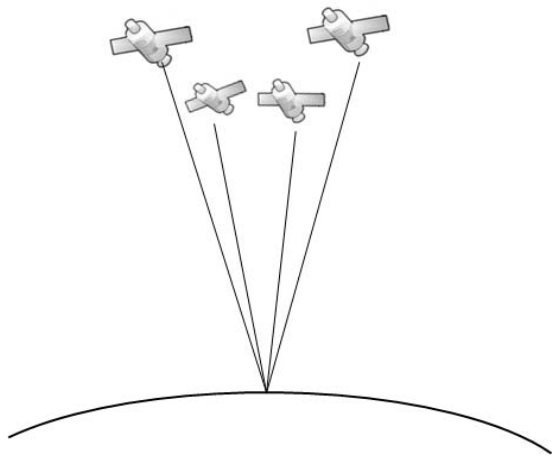


- Satellite Geometry refers to the relative position of the satellites at any given time.
- Distance measurement errors give an error volume with spherical boundary surfaces.
- Desirable satellite geometry - satellites located at wide angles relative to each other.
- Poor geometry - satellites are located in a line, or are tightly clustered.
- Ideal situation - one satellite in zenith, other three at zero elevation and equally distributed in azimuth (i.e. 120°).
- Satellite geometry error is not measureable, it tends to enhance other errors.

Satellite Geometry and Accuracy Measures



Good GDOP



Poor GDOP

- A metric normally used for measuring this impact is Dilution of Precision (**DOP**), which represents the degree to which satellite-user geometry dilutes the accuracy of the solution.
- DOP can be computed from the cofactor matrix $\boldsymbol{H} = (\boldsymbol{G}^T\boldsymbol{G})^{-1}$, that is calculated given the user-to-satellite LOS unit vector:

$$\boldsymbol{h}^{(i)} = \frac{\boldsymbol{r}^{(i)} - \boldsymbol{r}_u}{\|\boldsymbol{r}_u - \boldsymbol{r}_i\|}$$

where r_i and r_u are the position of the *i*th satellite and the user, and the geometry matrix ***G***:

$$\boldsymbol{G} = \begin{bmatrix} \left(-\boldsymbol{h}^{(1)}\right)^T & 1 \\ \left(-\boldsymbol{h}^{(2)}\right)^T & 1 \\ \vdots & \vdots \\ \left(-\boldsymbol{h}^{(i)}\right)^T & 1 \end{bmatrix}$$

Dilution of Precision

$$\boldsymbol{H} = (\boldsymbol{G}^T\boldsymbol{G})^{-1} = \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} & q_{xt} \\ q_{xy} & q_{yy} & q_{yz} & q_{yt} \\ q_{xz} & q_{yz} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{bmatrix}$$

There are several DOP values commonly used in GNSS:

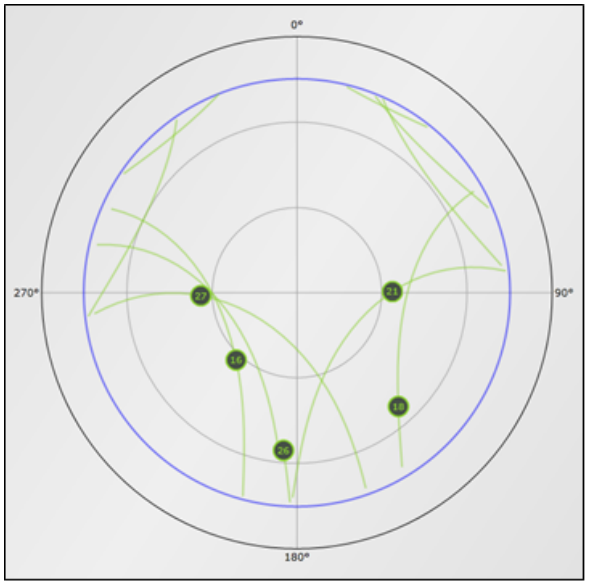
- Geometric DOP (3D position+time): $GDOP = \sqrt{q_{xx} + q_{yy} + q_{zz} + q_{tt}}$
- Position DOP (3D position): $PDOP = \sqrt{q_{xx} + q_{yy} + q_{zz}}$
- Time DOP (time dimension only): $TDOP = \sqrt{q_{tt}}$

- Horizontal DOP (2D position – horizontal plane): $HDOP = \sqrt{q_{xx} + q_{yy}}$
- Vertical DOP (1D position – height): $VDOP = \sqrt{q_{zz}}$

Dilution of Precision

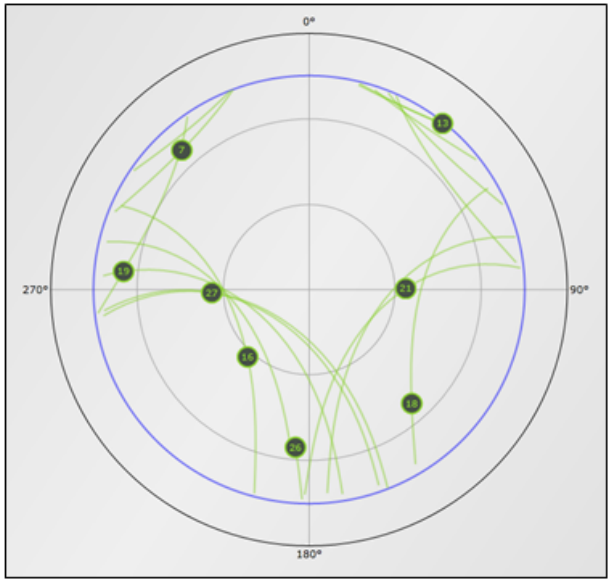
- Example of Geometrical and Vertical DOP for a location at 63°N, 10°E.

5 GPS SVs



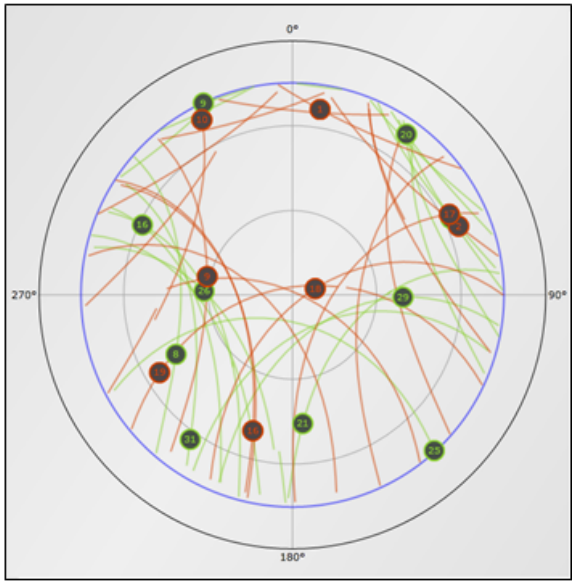
GDOP=7
VDOP=5

8 GPS SVs



GDOP=2.5
VDOP=1.6

9 GPS SVs
8 GLONASS SVs



GDOP=1.9
VDOP=1.2

User Equivalent Range Error (UERE)

- Quadratic sum of errors affecting the accuracy of measured pseudoranges:

$$UERE = \sqrt{d\rho^2 + dt^2 + d_I^2 + d_T^2 + \varepsilon_{multip.}^2 + \varepsilon_P^2}$$

Source	**Single-Point**		**DGPS (BL=100 km)**	
Ephemeris error	1.0 m		0.1 m	
SV Clock Error			0 m	
Ionospheric Error	1.5 m		0.2 m	
Tropospheric Error	0.3 m		0.1 m	
Multipath	1.0 m		1.2 m	
Rx Noise	0.3 m		0.3 m	
UERE	3 m		1.8 m	
HDOP/VDOP	1.5	2.5	1.5	2.5
Horisontal/Vertical RMS Error	**4.5 m**	**7.5 m**	**2.7 m**	**4.5 m**

- measure of the accuracy of single pseudorange measurements (or associated standard deviation, σ_ρ.

- relationship between σ_ρ and associated standard deviation of position is given as:

$$\sigma = DOP \cdot \sigma_\rho$$

Single-Point Position Estimation

- Unknown parameters:
  - x_u, y_u, z_u coordinates of the user $(\rho_i = \|\boldsymbol{r_u} - \boldsymbol{r_i}\|)$
  - receiver clock offset, $c \cdot dT$

- The standard approach for estimating the receiver position and clock offset is first to linearize the pseudorange measurements around a rough guess of the receiver position and clock bias and then to iterate until the difference between the guess and the measurements approaches zero.

- The rough guess of the receiver position can be very coarse, e.g. the centre of the Earth is a good choice. We can the denote the assumed user position as $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$, and the offset between the true user position and the assumed one as $(\Delta x_u, \Delta y_u, \Delta z_u)$.

- Recall from earlier: $P_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} + ct_u =$
$$= f(x_u, y_u, z_u, t_u)$$

- Similarly, an approximate pseudorange can be calculated as a function of assumed user position and clock error $\left(f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)\right)$.

Single-Point Position Estimation

- The unknown user position can be considered to consist of an approximate and an incremental component:

$$f(x_u, y_u, z_u) = f(\hat{x}_u + \Delta x_u, \hat{y}_u + \Delta y_u, \hat{z}_u + \Delta z_u, \hat{t}_u + \Delta t_u)$$

- this function can be expanded about the approximate point and predicted receiver clock offset using a Taylor series:

$$f(\hat{x}_u + \Delta x_u, \hat{y}_u + \Delta y_u, \hat{z}_u + \Delta z_u, \hat{t}_u + \Delta t_u)$$
$$= f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{x}_u}\Delta x_u + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u}\Delta y_u + \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u}\Delta z_u$$
$$+ \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{t}_u}\Delta t_u + \cdots$$

truncated to include only the 1st order partial derivatives to eliminate non-linear terms.

Single-Point Position Estimation

- Partial derivative of pseudorange to satellite i with respect to the unknowns:

$$\frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{x}_u} = -\frac{x_i - \hat{x}_u}{\hat{\rho}_i} \qquad \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{y}_u} = -\frac{y_i - \hat{y}_u}{\hat{\rho}_i}$$

$$\frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{z}_u} = -\frac{z_i - \hat{z}_u}{\hat{\rho}_i} \qquad \frac{\partial f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u)}{\partial \hat{t}_u} = c$$

- where $\rho_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2}$

$$\Delta P_i = \hat{P}_i - P_i = \frac{x_i - \hat{x}_u}{\hat{\rho}_i}\Delta x_u + \frac{y_i - \hat{y}_u}{\hat{\rho}_i}\Delta y_u + \frac{z_i - \hat{z}_u}{\hat{\rho}_i}\Delta z_u - \mathrm{c}\Delta t_u$$

Single-Point Position Estimation

- Considering pseudorange measurements to four or more satellites, this relationship can be put in matrix form as follows:

Measurement vector

$$\Delta \boldsymbol{P} = \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \end{bmatrix}$$

Geometry matrix

$$\boldsymbol{G} = \begin{bmatrix} \frac{x_1 - \hat{x}_u}{\widehat{\rho_1}} & \frac{y_1 - \hat{y}_u}{\widehat{\rho_1}} & \frac{z_1 - \hat{z}_u}{\widehat{\rho_1}} & 1 \\ \frac{x_2 - \hat{x}_u}{\widehat{\rho_2}} & \frac{y_2 - \hat{y}_u}{\widehat{\rho_2}} & \frac{z_2 - \hat{z}_u}{\widehat{\rho_2}} & 1 \\ \frac{x_3 - \hat{x}_u}{\widehat{\rho_3}} & \frac{y_3 - \hat{y}_u}{\widehat{\rho_3}} & \frac{z_3 - \hat{z}_u}{\widehat{\rho_3}} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_n - \hat{x}_u}{\widehat{\rho_n}} & \frac{y_n - \hat{y}_u}{\widehat{\rho_n}} & \frac{z_n - \hat{z}_u}{\widehat{\rho_n}} & 1 \end{bmatrix}$$

Parameter vector

$$\Delta \boldsymbol{x} = \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ -c\Delta t_u \end{bmatrix}$$

- We get the following measurement equation: $\Delta \boldsymbol{P} = \boldsymbol{G} \cdot \Delta \boldsymbol{x}$.
- In reality, the true user-to-satellite measurements are corrupted by independant errors (e.g. measurement noise, multipath, etc.) $\rightarrow \Delta \boldsymbol{P} = \boldsymbol{G} \cdot \Delta \boldsymbol{x} + \widetilde{\boldsymbol{\varepsilon_P}}$.
- Given the correction terms broadcasted in the navigation message, the user can estimate and compensate for most of the error terms (dt_i, d_I, d_T). The term, $\widetilde{\boldsymbol{\varepsilon_P}}$, includes the remaining residuals and other unmodelled effects.

Single-Point Position Estimation

- The estimate in the Least Squares (LS) sense for the parameter vector is then given by:

$$\Delta \boldsymbol{x} = (\boldsymbol{G}^T \boldsymbol{G})^{-1} \boldsymbol{G}^T \Delta \boldsymbol{P} + \widetilde{\boldsymbol{\varepsilon}_{\boldsymbol{P}}}$$

- The geometry matrix, $\boldsymbol{G}$, is used to calculate the dilution of precision (DOP) values, which give a good indication of the estimated position accuracy.
- The errors in the residual error vector, $\widetilde{\boldsymbol{\varepsilon}_{\boldsymbol{P}}}$, are assumed to be zero-mean and uncorrelated.
- Iterative process updates the approximate initial guess each round:

$$\widehat{\boldsymbol{x}}_0 = (0,0,0,0)^T$$

$$\widehat{\boldsymbol{x}}_{i+1} = \widehat{\boldsymbol{x}}_i + \Delta \boldsymbol{x}$$

- The convergence of the iterative solution depends on the geometry of the receiver-satellites system, which, in turn, affects the rank of matrix $\boldsymbol{G}$.
  - Problems when all the satellites lie in or are very close to the same plane in a three-dimensional space.

Single-Point Position Estimation

- When the pseudorange measurements from different satellites are not of the same quality, the estimation procedure can be weighted. The Weighted Least Squares solution for this case is:

$$\Delta \boldsymbol{x} = (\boldsymbol{G}^T \boxed{\boldsymbol{R}^{-1}} \boldsymbol{G})^{-1} \boldsymbol{G}^T \boxed{\boldsymbol{R}^{-1}} \Delta \boldsymbol{P} + \widetilde{\boldsymbol{\varepsilon}_{\boldsymbol{P}}}$$

- $\boldsymbol{R}$ is the covariance matrix assosiated with the pseudorange measurements:

$$\boldsymbol{R} = \boldsymbol{C}_{\Delta \boldsymbol{P} \Delta \boldsymbol{P}} = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_n^2 \end{bmatrix}$$

- Different approaches can be used for setting the $\boldsymbol{R}$ matrix:
  - use the user equivalent range errors (UERE)
  - define the variance as a function of satellite elevation angle
  - define the variance as a function of C/N_0
  - etc.

Observation Weighting

- Identical variance:

$$var_{(ID)} = \sigma^2$$

  - all observations of the same type have an equal variance. UERE can be used as an approximation.
  - valid only in an open sky area for high elevation satellites.

- Elevation dependent variance:

$$var_{(ELV)} = \sigma^2 \frac{1}{sin^2(El)}$$

  - σ is the std of the observations at zenith, and *El* is the satellite elevation angle.
  - suitable model for an open sky area where multipath and fading effects are minor.

- Signal Power dependent variance:

$$var_{(ELV)} = \sigma^2 \frac{C/N_0(zenith)}{C/N_0}$$

  - σ is the std of the observations at zenith, and C/N_0 is the signal-to-noise ratio for a satellite at zenith in nominal environment.
  - suitable for use in degraded signal environments.

Single-Point Position Estimation

Covariance matrix of estimated parameters:

- To calculate the parameter estimate covariance, the covariance matrix of the measurements, $\boldsymbol{R}$, is needed.
- Propagation of covariance can be applied to the weighted least squares problem:

Recall from earlier: $\Delta \boldsymbol{x} = (\boldsymbol{G}^T \boldsymbol{R}^{-1} \boldsymbol{G})^{-1} \boldsymbol{G}^T \boldsymbol{R}^{-1} \Delta \boldsymbol{P} + \widetilde{\boldsymbol{\varepsilon}_P}$

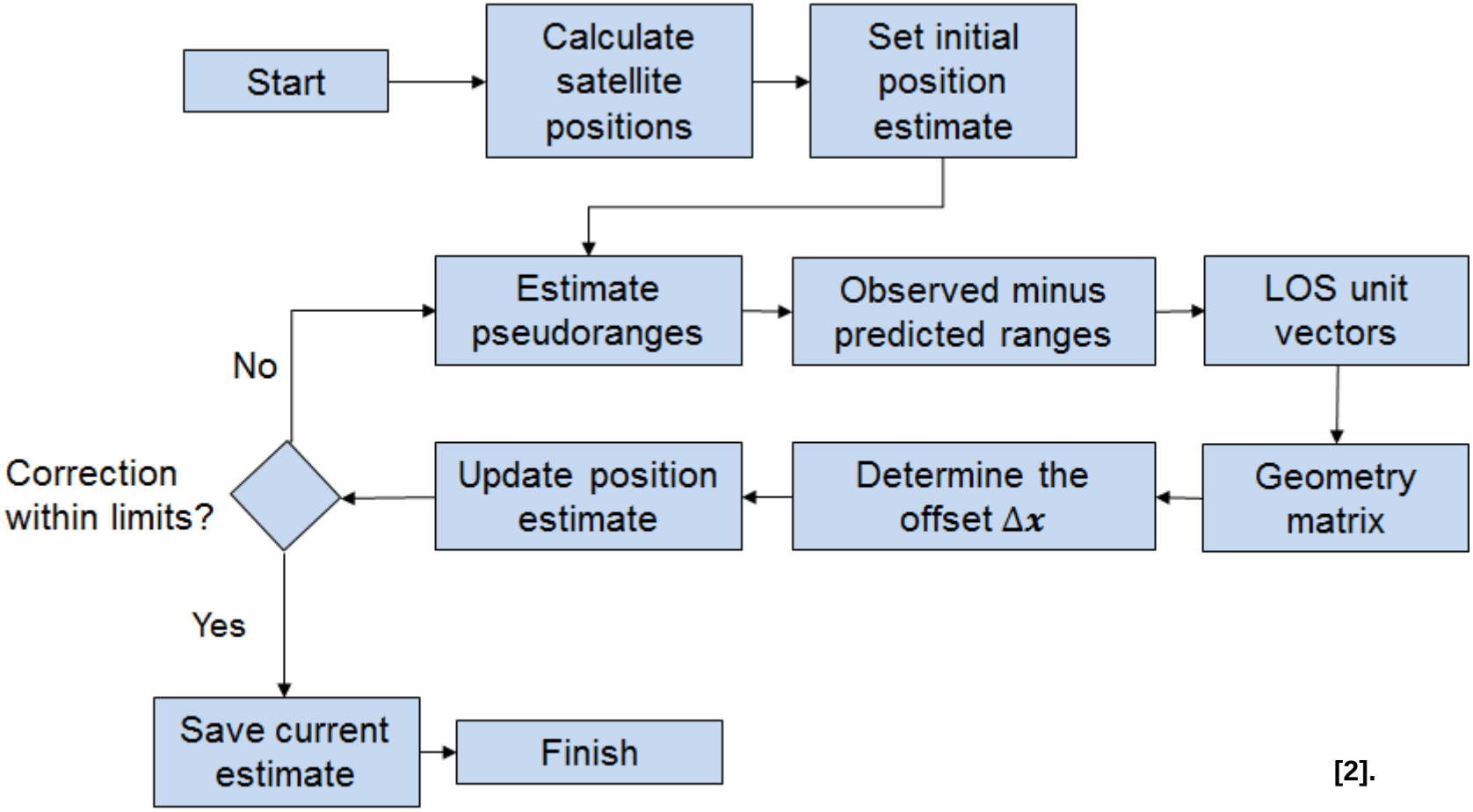
The covariance matrix of $\Delta \boldsymbol{x}$ can be computed as:

$$\langle \Delta \boldsymbol{x} \Delta \boldsymbol{x} \rangle = (\boldsymbol{G}^T \boldsymbol{R}^{-1} \boldsymbol{G})^{-1} \boldsymbol{G}^T \boldsymbol{R}^{-1} \langle \Delta \boldsymbol{P} \Delta \boldsymbol{P} \rangle \boldsymbol{R}^{-1} \boldsymbol{G} (\boldsymbol{G}^T \boldsymbol{R}^{-1} \boldsymbol{G})^{-\mathbf{1}} =$$

$$\boxed{= \boldsymbol{C}_{\Delta \boldsymbol{x} \Delta \boldsymbol{x}} = (\boldsymbol{G}^T \boldsymbol{R}^{-1} \boldsymbol{G})^{-1}}$$

Single-Point Position Estimation

Position computation steps:



[2].

LOS unit vector: $\boldsymbol{h}^{(i)} = \frac{\boldsymbol{r}^{(i)} - \boldsymbol{r}_u}{\|\boldsymbol{r}_u - \boldsymbol{r}_i\|}$.

Velocity Estimation

- The observation equations relate the observed range rate to the user velocity and form the basis for estimating the user velocity from GPS observations. Velocity computation can be performed using either Doppler frequency measurements (filtered or raw), or time differenced carrier phase observations.
- Recall the equation for the Doppler frequency observation:

$$\dot{\phi} = \dot{\rho} + \dot{d\rho} + c\left(\dot{dt} - \dot{dT}\right) - \dot{d}_I + \dot{d}_T + \varepsilon_{\dot{\phi}}$$

- Following similar principle as with position computation we obtain:

$$\dot{\phi}^{(i)} = \left(\boldsymbol{v}^{(i)} - \boldsymbol{v}_u\right) \cdot \boldsymbol{h}^{(i)} + \dot{dT} + \varepsilon_{\dot{\phi}}^{(i)}$$

where $\boldsymbol{v}^{(i)}$ and $\boldsymbol{v}_u$ stand for the satellite and user velocity vectors, and $\boldsymbol{h}^{(i)}$ is a user-to-satellite LOS unit vector, $\boldsymbol{h}^{(i)} = \frac{\boldsymbol{r}^{(i)} - \boldsymbol{r}_u}{\|\boldsymbol{r}_u - \boldsymbol{r}_i\|}$.

The equation can be rewritten as:

$$\boxed{\left(\dot{\phi}^{(i)} - \boldsymbol{v}^{(i)} \cdot \boldsymbol{h}^{(i)}\right)} = -\boldsymbol{h}^{(i)} \cdot \boldsymbol{v}_u + \dot{dT} + \varepsilon_{\dot{\phi}}^{(i)}$$

Observations corrected for the contribution of satellite motion

Velocity Estimation

- Denoting $\left(\dot{\phi}^{(i)} - \boldsymbol{v}^{(i)} \cdot \boldsymbol{h}^{(i)}\right) \Longrightarrow \dot{\phi}'^{(i)}$, we get:

$$\dot{\phi}'^{(i)} = G^{(i)}[\boldsymbol{v}_u^T \quad -c\dot{d}T] + \varepsilon_{\dot{\phi}}$$

where $G^{(i)}$ is the ith row of the $\boldsymbol{G}$ matrix. In the matrix form we have the following:

$$\begin{bmatrix} \dot{\phi}'^{(1)} \\ \dot{\phi}'^{(2)} \\ \vdots \\ \dot{\phi}'^{(n)} \end{bmatrix} = \boldsymbol{G}[\boldsymbol{v}_u^T \quad -c\dot{d}T] + \boldsymbol{\varepsilon}_{\dot{\phi}}$$

Geometry matrix

Residual error vector

Parameter vector

- **user velocity vector**
- **receiver clock error drift**

Measurement vector, *D*

- **Doppler measurements corrected for the satellite motion**

- Final solution obtained by solving:

$$\begin{bmatrix} \boldsymbol{v}_u \\ -c\dot{d}T \end{bmatrix} = (\boldsymbol{G}^T\boldsymbol{G})^{-1}\boldsymbol{G}^T\boldsymbol{D}$$

Extended Kalman Filter

- PVT estimation can as well be performed by using an Extended Kalman Filter (EKF)
  - Is an implementation of a Bayes estimator.
  - Basically, a weighted, recursive least squares estimator.
  - The outputs from an EKF will often be better than those from the Least Squares approach.
- EKF requires some knowledge of the receiver dynamics. Incorrect assumption will cause problems.
  - P
  - PV
  - PVA
- In the extended Kalman filter (EKF), the state transition and observation models need not be linear functions of the state.

Extended Kalman Filter

Notation and terminology:

- x_k is the state vector.
- P_k is the error covariance matrix associated with state vector at time t_k, it shows how confident the filter is in its solution.

- Φ_k is the state transition matrix from time t_k to time t_{k+1}.
- Q_k, is the process noise covariance matrix, accounts for uncertainty in the assumed process model (it is a matrix that you determine and that does not get updated by the filter).

- z_k is the measurement vector at t_k.
- R_k is the measurement noise spectral density matrix at time t_k, defining the confidence in each of the measurements.
- H_k is the geometry or design matrix at time t_k; provides a noiseless/ideal connection between the measurement and the state vectors.

- K_k - the Kalman gain matrix is a weighting factor, which indicates how much of the new information contained in the innovation sequence should be accepted by the system.

Extended Kalman Filter

- Equations in the Kalman filter algorithm fall under two groups:
  - The first group of equations **predicts** the state (and the associated covariance) of the system, based on the current state and the assumed system model, to obtain an *a priori* estimates for the next time step.
  - The second group of equations **updates** the predicted states and covariance estimates with the available measurements, in accordance with the measurement model.

Prediction

1. Predict the state ahead

$$\hat{x}_{k+1}^{-} = \Phi_{k+1,k} \cdot \hat{x}_k^{+} + w_k$$

2. Project the error covariance ahead

$$P_{k+1}^{-} = \Phi_{k+1,k} \cdot P_k^{+} \cdot \Phi_{k+1,k}^{T} + Q_k$$

Update

1. Compute the Kalman Gain

$$K_{k+1} = P_{k+1}^{-} H_{k+1}^{T} \left(H_{k+1} P_{k+1}^{-} H_{k+1}^{T} + R_{k+1} \right)^{-1}$$

2. Update estimate with measurement z_{k+1}

$$\hat{x}_{k+1}^{+} = \hat{x}_{k+1}^{-} + K_{k+1} (z_{k+1} - H_{k+1} \hat{x}_{k+1}^{-})$$

3. Update the error covariance

$$P_{k+1}^{+} = P_{k+1}^{-} - K_{k+1} H_{k+1} P_{k+1}^{-}$$

Extended Kalman Filter

Formulation example:

- The navigation error state vector components:

  δr - position error vector (ECEF coordinate frame),
  δv - velocity error vector (ECEF coordinate frame),

  t - clock offset error state,
  δt - clock drift error state.

- Position and velocity error dynamic model:

  $\dot{\delta r} = \delta v$
  $\dot{\delta v} = \eta_v$ (velocity errors are being modeled as random walk processes)

- Here the 'dot' represents the time derivative, and the symbol, η_v, is the process driving noise with spectral density q_v, (e.g. $1\ \frac{m}{s^2}/\sqrt{Hz}$ considering a land vehicle).

Extended Kalman Filter

- The clock error states are defined in the units of range and range rates, by multiplying it by the speed of light, for compatibility with position and velocity states. The dynamic model for these errors can be written as:

$$\dot{c\,t} = c\,\delta t + c\,\eta_t$$
$$c\,\dot{\delta t} = c\,\eta_{\delta t}$$

- where

η_t - is the clock error driving noise with spectral density, q_t.
$\eta_{\delta t}$ - is the clock drift error driving noise with spectral density, $q_{\delta t}$.

- For computing the clock error spectral densities, a standard clock stability model can be used where the spectral densities are computed using the following equations:

$$q_t = 2h_0$$
$$q_{\delta t} = 8\pi^2 2h_2$$

Timing standard	h_0	h_1	h_2
Temperature compensated crystal	$2\text{x}10^{-19}$	$7\text{x}10^{-21}$	$2\text{x}10^{-20}$

- h_0 and h_2 are Allan variance parameters describing the clock drift.

Extended Kalman Filter

- The final state/process model can be then expressed as:

$$\underbrace{\begin{bmatrix} \dot{\delta r} \\ \dot{\delta v} \\ \dot{t} \\ \dot{\delta t} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0_{3x3} & I_{3x3} & 0 & 0 \\ 0_{3x3} & 0_{3x3} & 0 & 0 \\ 0_{1x3} & 0_{1x3} & 0 & 1 \\ 0_{1x3} & 0_{1x3} & 0 & 0 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} \delta r \\ \delta v \\ t \\ \delta t \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0_{3x3} & 0 & 0 \\ I_{3x3} & 0 & 0 \\ 0_{1x3} & c & 0 \\ 0_{1x3} & 0 & c \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} (\eta_v)_{3x1} \\ \eta_v \\ \eta_{\delta t} \end{bmatrix}}_{w}$$

- Assuming that the dynamic matrix is time invariant over the transition time interval, the transition matrix, $\Phi_{k+1,k}$, can be obtained from the continuous time dynamic matrix:

$$\Phi = e^{F\Delta t} = \boxed{I + F\Delta t} + \frac{(F\Delta t)^2}{2} + \cdots$$

where *I* is the identity matrix, and Δ*t* is the transition time interval.

- The process noise covariance matrix, Q_k, which accounts for uncertainty in the assumed process model, can be obtained by the integral of the continuous time spectral density matrix, $Q(t)$:

$$Q_k = \int_{t_k}^{t_{k+1}} \Phi_{k+1,t} G(t) Q(t) G^T(t) \Phi_{k+1,\tau}^T \, dt$$

Extended Kalman Filter

- The corresponding process noise spectral density matrix:

$$Q(t) = \begin{bmatrix} (q_v)_{3x3} & 0 & 0 \\ 0_{1x3} & q_t & 0 \\ 0_{1x3} & 0 & q_{\delta t} \end{bmatrix}$$

- Measurement model:

$$\rho_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} + ct_u$$

$$\dot{\rho}_i = \frac{(x_u - x_i)(v_{u,x} - v_{i,x}) + (y_u - y_i)(v_{u,y} - v_{i,y}) + (z_u - z_i)(v_{u,z} - v_{i,z})}{\rho_i} + c\dot{t}_u$$

- These equations are non-linear, and thus need to be linearized to be used in the Kalman filter algorithm (linearization point is the predicted state estimate at a particular epoch).

Extended Kalman Filter

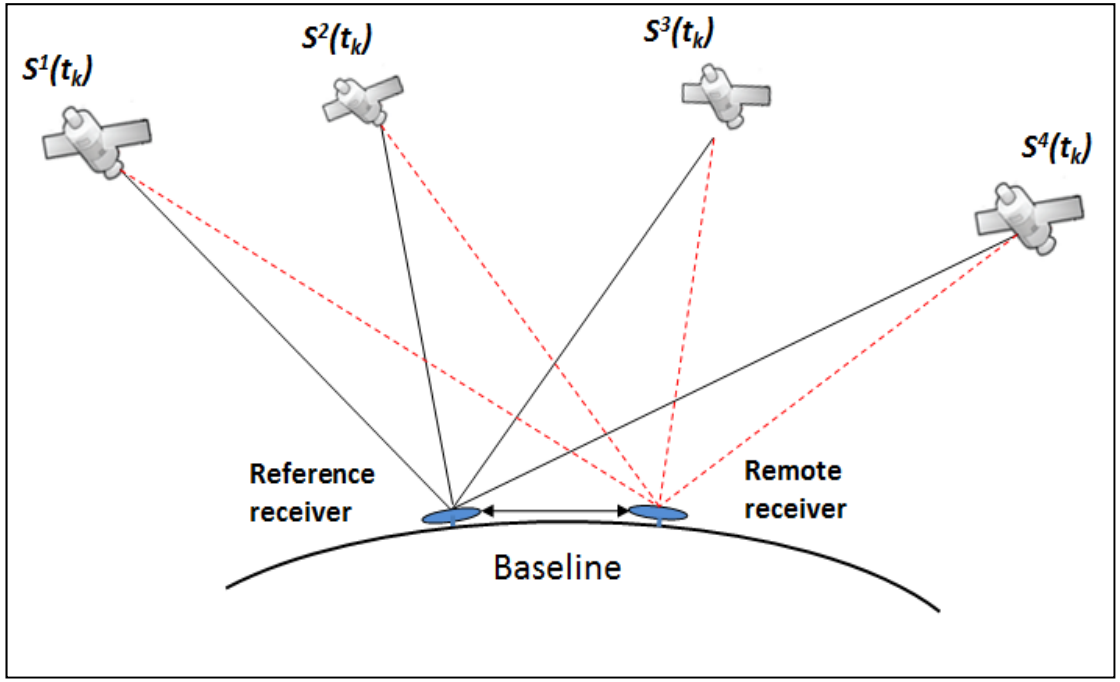
- The corresponding design matrices relating the pseudorange and Doppler observations to the error states are given as:

$$H_\rho = \begin{bmatrix} \frac{\partial \rho^1}{\partial x} & \frac{\partial \rho^1}{\partial y} & \frac{\partial \rho^1}{\partial z} & 0 & 0 & 0 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial \rho^N}{\partial x} & \frac{\partial \rho^N}{\partial y} & \frac{\partial \rho^N}{\partial z} & 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{m_1 \times 8}$$

$$H_{\dot\rho} = \begin{bmatrix} \frac{\partial \dot\rho^1}{\partial x} & \frac{\partial \dot\rho^1}{\partial y} & \frac{\partial \dot\rho^1}{\partial z} & \frac{\partial \dot\rho^1}{\partial v_x} & \frac{\partial \dot\rho^1}{\partial v_y} & \frac{\partial \dot\rho^1}{\partial v_z} & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial \dot\rho^N}{\partial x} & \frac{\partial \dot\rho^N}{\partial y} & \frac{\partial \dot\rho^N}{\partial z} & \frac{\partial \dot\rho^N}{\partial v_x} & \frac{\partial \dot\rho^N}{\partial v_y} & \frac{\partial \dot\rho^N}{\partial v_z} & 0 & 1 \end{bmatrix}_{m_2 \times 8}$$

- Final design matrix: $H = \begin{bmatrix} H_\rho \\ H_{\dot\rho} \end{bmatrix}$, and the measurement misclosure: $\delta z = \begin{bmatrix} \hat\rho \\ \hat{\dot\rho} \end{bmatrix} - \begin{bmatrix} \rho \\ \dot\rho \end{bmatrix}$

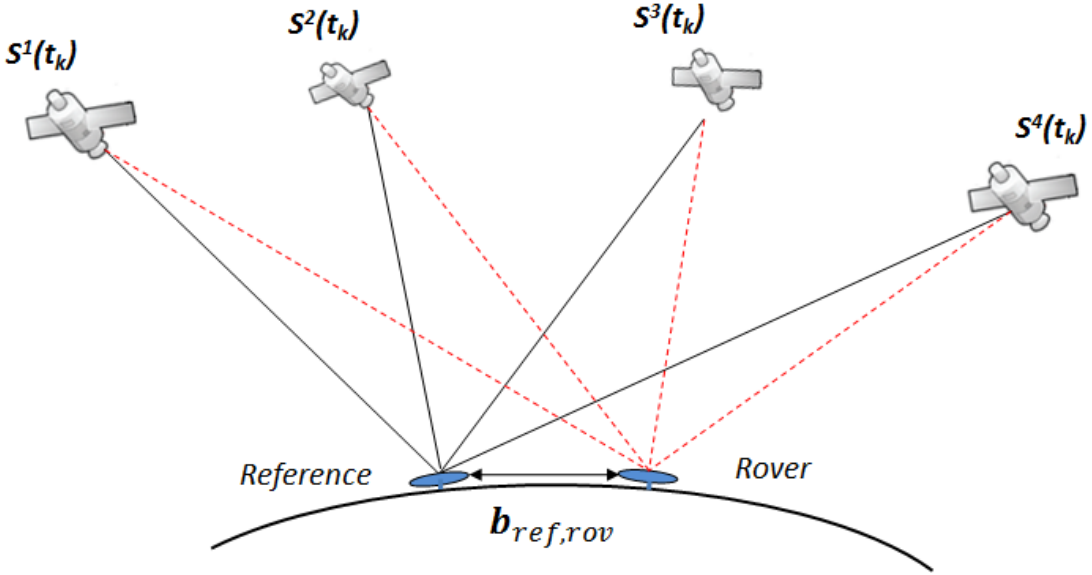
Relative/Differential GNSS Overview



- **Drawbacks**:
- Result in a loss of degrees of freedom → poorer geometry (higher Dilution of Precision).
- Introduce serial correlations among derived measurements. The measurement covariance matrix of the derived measurements is fully populated.

- Differential methods are commonly used eliminate or reduce errors that are spatially and/or temporally correlated.
- Measurements are assumed made at the same time epoch.
- Remote receiver can be either mobile or fixed.
- Position accuracy of the remote receiver with respect to that of the reference/base unit is improved due to error reduction/elimination.
- Site selection is important for minimizing environmental effects such as multipath.

Relative Positioning



- $\boldsymbol{X}_{rov} = \boldsymbol{X}_{ref} + \boldsymbol{b}_{ref,rov}$

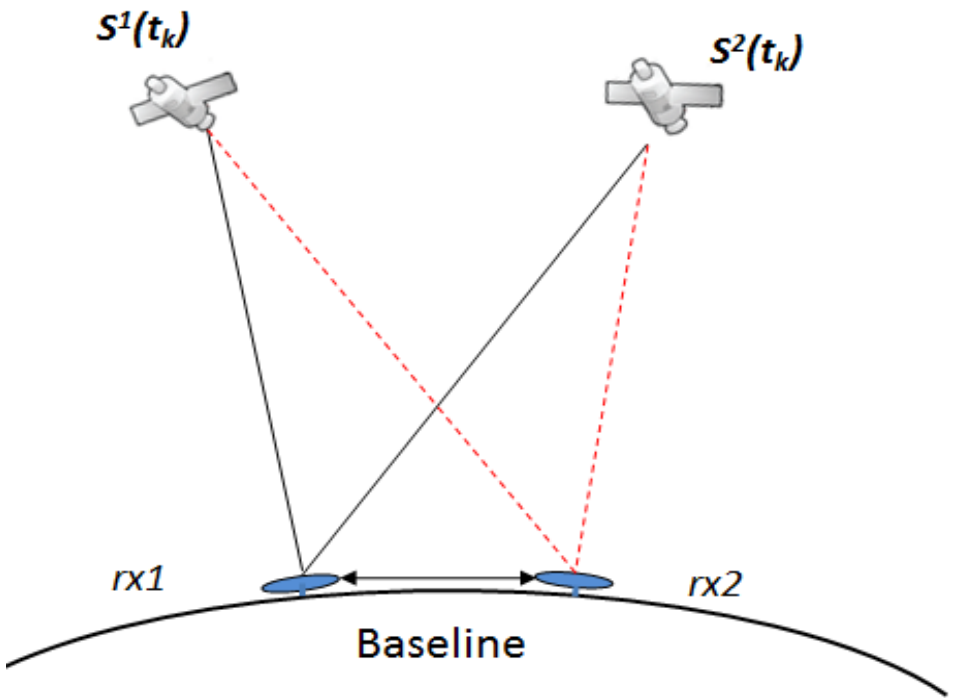
- $\boldsymbol{b}_{ref,rov} = \begin{bmatrix} x_{ref} - x_{rov} \\ y_{ref} - y_{rov} \\ z_{ref} - z_{rov} \end{bmatrix}$

- For some applications you do not need to know your precise position but you do need to know your position relative to another object or location.
  - need to determine where an object is located on earth with respect to another object's location.
  - determine the horizontal and vertical distances between one object to another object.

→ **determine the baseline**.

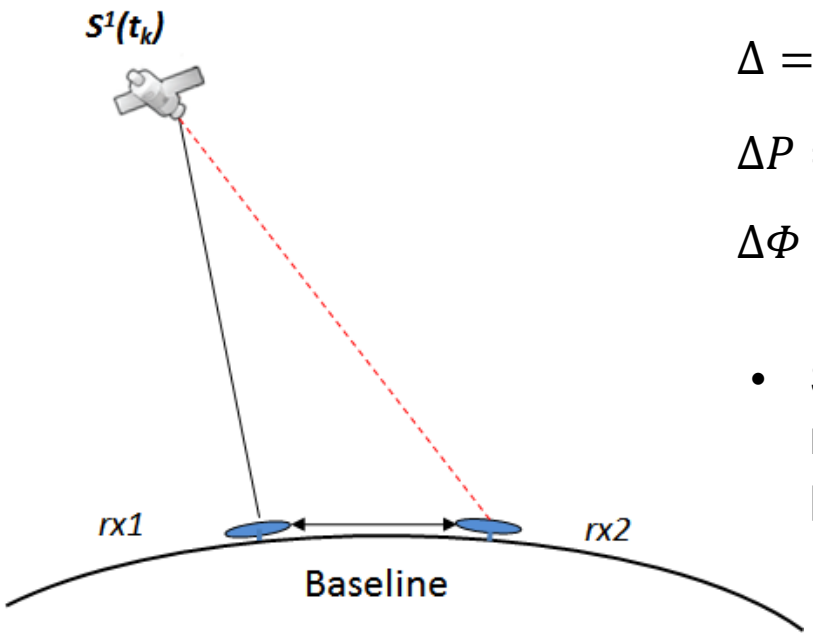
Relative/Differential GNSS Concept

- Linear combinations of observations between receivers, satellites and time are generated to reduce the error budget.



- **Errors due to orbit, satellite clock and atmospheric effects** are removed by differencing observations between **receivers**.
- $\Delta = (\cdot)_{r_2} - (\cdot)_{r_1}$
- Errors due to **receiver clock** can be removed by differencing observations between **satellites**.
- $\Delta = \left((\cdot)^{S_1} - (\cdot)^{S_2}\right)_{r_1}$

Between Receiver Single Difference



$$\Delta = (\cdot)_{r_2} - (\cdot)_{r_1}$$

$$\Delta P = \Delta\rho + \Delta d\rho - c\Delta dT + \Delta d_I + \Delta d_T + \varepsilon_{\Delta P}$$

$$\Delta\Phi = \Delta\rho + \Delta d\rho - c\Delta dT + \lambda\Delta N - \Delta d_I + \Delta d_T + \varepsilon_{\Delta\phi}$$

- Subtract pseudorange/carrier phase at reference station from pseudorange/carrier phase at the remote.

ΔP

- Reduces orbital and atmospheric errors.
- Eliminates satellite clock error, dt.
- Does not reduce the amount of noise, $\varepsilon_P = \sqrt{2} \cdot \varepsilon_P$

$\Delta\Phi$

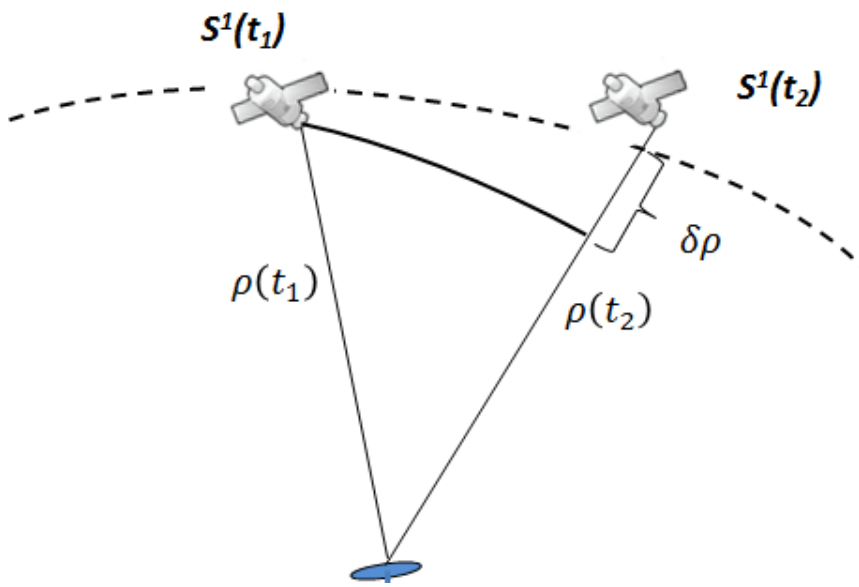
- Reduces orbital and atmospheric errors.
- Eliminates satellite clock error, *dt*.
- Does not eliminate receiver clock error, *dT*.
- Does not eliminate ambiguity, *N*.
- Increases noise by $\sqrt{2}$, compare to single phase approach.

Relative/Differential GNSS Concept

- It is a computation of the range difference over time: $\delta(\cdot) = (\cdot)_{t_2} - (\cdot)_{t_1}$
- **Ambiguities** are constant over time and can therefore be removed by differencing observations between **time epochs**.

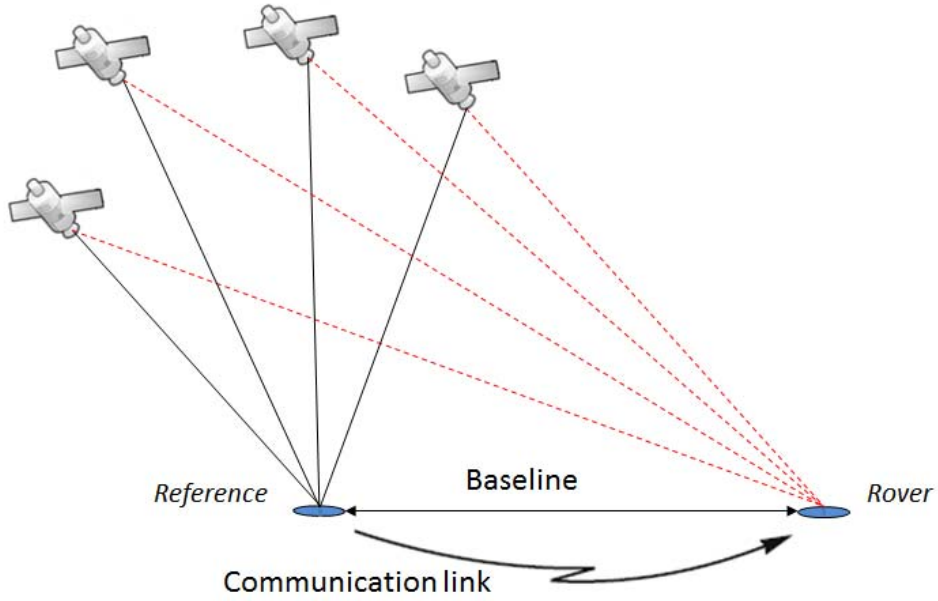
$$\delta P = \delta\rho + \delta d\rho + c(\delta dt - \delta dT) + \delta d_I + \delta d_T + \varepsilon_{\delta P}$$

$$\delta\Phi = \delta\rho + \delta d\rho + c(\delta dt - \delta dT) - \delta d_I + \delta d_T + \varepsilon_{\delta\Phi}$$



- Remaining errors: multipath, receiver noise.
- Receiver must maintain phase lock between t_1 and t_2.
- If applied alone, impact of orbital and atmospheric errors increases over time.

Differential GNSS (DGNSS)



- Position accuracy in the order of 1 meter (1 sigma).
- Based on pseudorange and/or carrier phase measurements.
- Real-time and/or post-mission processing.
- Position of the reference receiver is known (measured with high accuracy).

- The easiest approach is to take measurements by the reference receiver, compare the solution to the surveyed one, compute the coordinate differences, and transmit the differences to the rover receiver.
- **Limitations:**
  - both the reference and rover receivers are constrained to use the same set of satellites.
  - both the reference and rover receivers have to use exactly the same position solution computation techniques.

Differential GNSS (DGNSS)

- Most typical implementation is broadcast of the pseudorange and range rate corrections from the reference to the rover or rovers for each visible satellite.

DGNSS example with pseudoranges:

- Given the known position of the reference receiver and known satellite coordinates, the geometric range from the reference to the satellite can be computed:

$$R_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2}$$

- The reference station makes a measurement of the pseudorange, ρ_m^i, to the *i*th satellite:

$$\rho_m^i = R_i + \underbrace{\boxed{c\delta t_{u,m}}}_{\text{Reference Rx clock error}} + \underbrace{\boxed{\varepsilon_m}}_{\text{Pseudorange errors}}$$

- Differential correction is then formed at the reference:

$$\Delta\rho^i = R_i - \rho_m^i = -c\delta t_{ref,m} - \varepsilon_m$$

Differential GNSS (DGNSS)

- This correction is broadcast to the rover receiver, which uses it to correct its own pseudorange measurement to the same satellite:

$$\rho_{rov}^{i} + \Delta\rho^{i} = R_{rov}^{i} + c\delta t_{rov} + \varepsilon_{rov} + \left(-c\delta t_{u,m} - \varepsilon_{m}\right)$$

- Recall the "*between receiver single difference*" combination: to a significant extent, the rover receiver's pseudorange measurement error components will be common to the ones observed at the reference receiver (excluding the receiver noise and multipath).

- The corrected range can be expressed as:

$$\rho_{rov,cor}^{i} = R_{rov}^{i} + \boxed{\varepsilon_{rov,m}} + \boxed{c\delta t_{rov,m}}$$

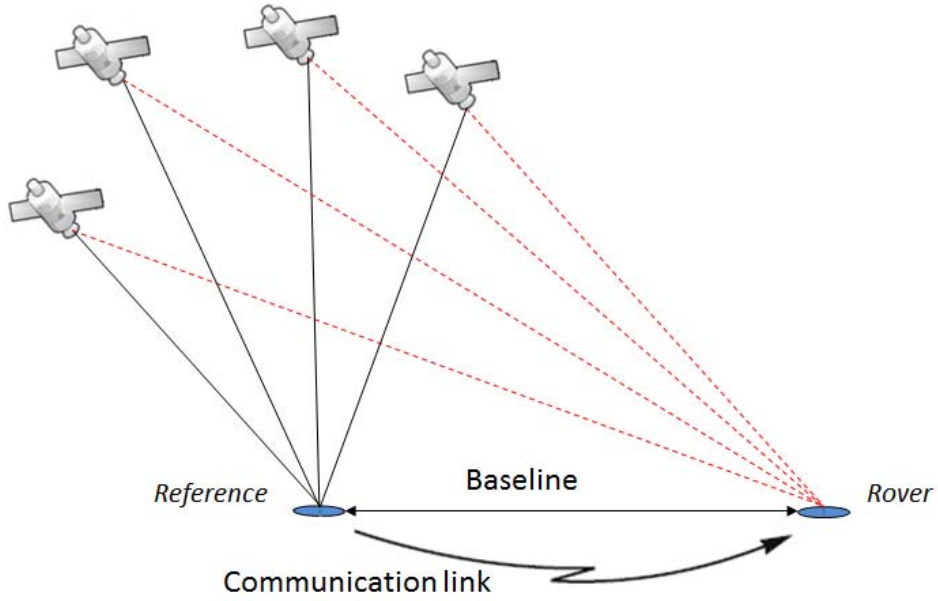
residual pseudorange error: $(\varepsilon_{rov} - \varepsilon_{m})$

residual clock error: $(\delta t_{rov} - \delta t_{m})$

- Corrected pseudoranges are then used to compute rover receiver's position and relative clock offset.

- In addition to the pseudorange corrections, the time derivative or range rate correction is determined at the base station.

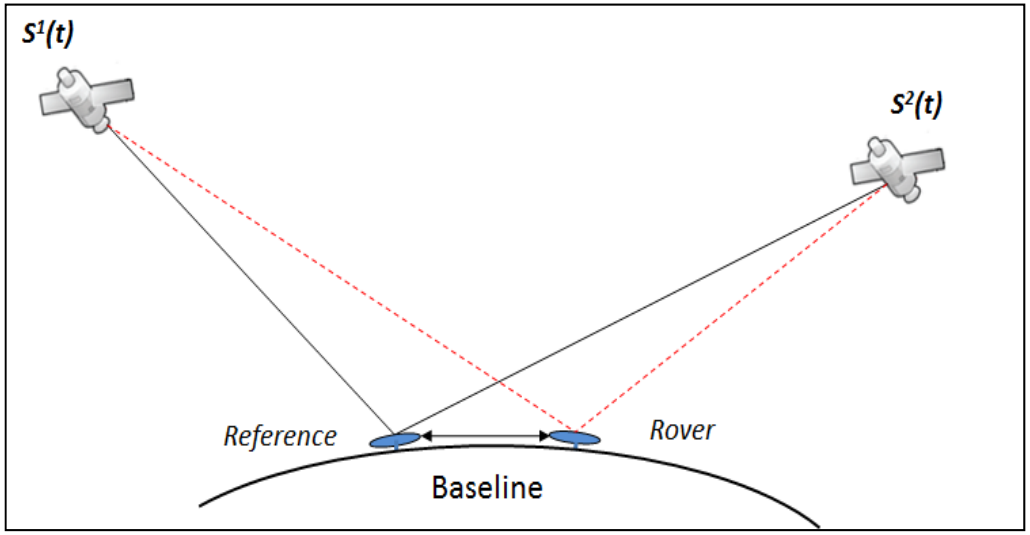
Real Time Kinematic (RTK)



- Baseline length/Performance
  - for long baselines correlated errors become significant
  - for long baselines correlated errors must be modelled or reduced to resolve the ambiguities

- Typical position accuracy < 0.1 m.
- Based on pseudorange and carrier phase measurements → requires carrier phase ambiguity resolution.
- Real-time.
- Reference receiver data transmitted to the rover via communication link (transmission of differential corrections or raw data from reference to remote receivers.
- Typical message format – **R**adio **T**echnical **C**ommission for **M**aritime Services, RTCM SC-104 → is used both for DGNSS/RTK.

Satellite Receiver Double Difference

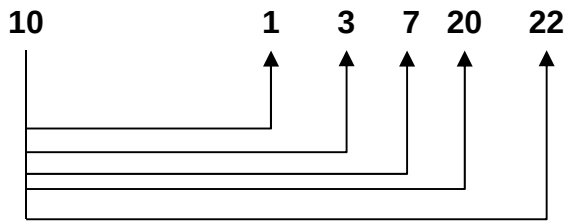


$$\Delta\nabla = \{(\cdot)_{S_1} - (\cdot)_{S_2}\}_{r_1} - \{(\cdot)_{S_1} - (\cdot)_{S_2}\}_{r_2}$$

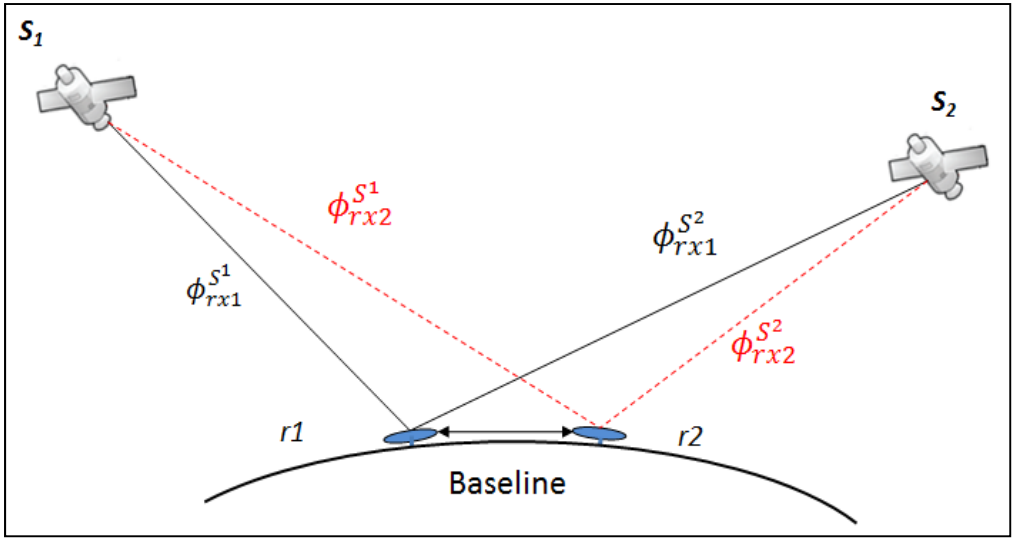
$$\Delta\nabla P = \Delta\nabla\rho + \Delta\nabla d\rho + \Delta\nabla d_I + \Delta\nabla d_T + \varepsilon_{\Delta\nabla P}$$

$$\boxed{\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d\rho + \lambda\Delta\nabla N - \Delta\nabla d_I + \Delta\nabla d_T + \varepsilon_{\Delta\nabla\phi}}$$

- Used for carrier phase ambiguity resolution.
- Assumes observations are taken at the same time epoch.
- Reduces orbital and atmospheric errors.
- Eliminates satellite and receiver clock errors.
- Compared to single measurements the noise level in the case of $\Delta\nabla$ is increase by a factor of **2**.
- formation of the double differences:



Double Difference Model



Float Solution (LSE):

$$\hat{\boldsymbol{x}} = \boldsymbol{x}_0 + (\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}\big(\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}_0)\big)$$

- State vector:

$$x = \left(x_{r_2}, y_{r_2}, z_{r_2}, N_{r_1r_2}^{S_2S_1}, N_{r_1r_2}^{S_3S_1}, \dots, N_{r_1r_2}^{S_iS_1}\right)^T$$

- clock terms are removed.

- Measurement vector at (t_k):

$$z_{t_k} = \left(\Phi_{r_1r_2\ t_k}^{S_2S_1}, \Phi_{r_1r_2\ t_k}^{S_3S_1}, \dots, \Phi_{r_1r_2\ t_k}^{S_iS_1}\right)^T$$

- Measurement error covariance:

- $$R_{t_k} = \begin{pmatrix} 4\sigma_\Phi^2 & 2\sigma_\Phi^2 & \cdots & 2\sigma_\Phi^2 \\ 2\sigma_\Phi^2 & 4\sigma_\Phi^2 & \cdots & 2\sigma_\Phi^2 \\ \vdots & \vdots & \ddots & \vdots \\ 2\sigma_\Phi^2 & 2\sigma_\Phi^2 & \cdots & 4\sigma_\Phi^2 \end{pmatrix}$$

- $$h_{t_k}(x) = \begin{bmatrix} \rho_{r_2,t_k}^{S_2S_1} - \rho_{r_1,t_k}^{S_2S_1} + \lambda N_{r_2,r_1}^{S_2S_1} \\ \rho_{r_2,t_k}^{S_3S_1} - \rho_{r_1,t_k}^{S_3S_1} + \lambda N_{r_2,r_1}^{S_3S_1} \\ \cdots \\ \rho_{r_2,t_k}^{S_iS_1} - \rho_{r_1,t_k}^{S_iS_1} + \lambda N_{r_2,r_1}^{S_iS_1} \end{bmatrix}$$

Integer Ambiguity Resolution

- Design matrix:

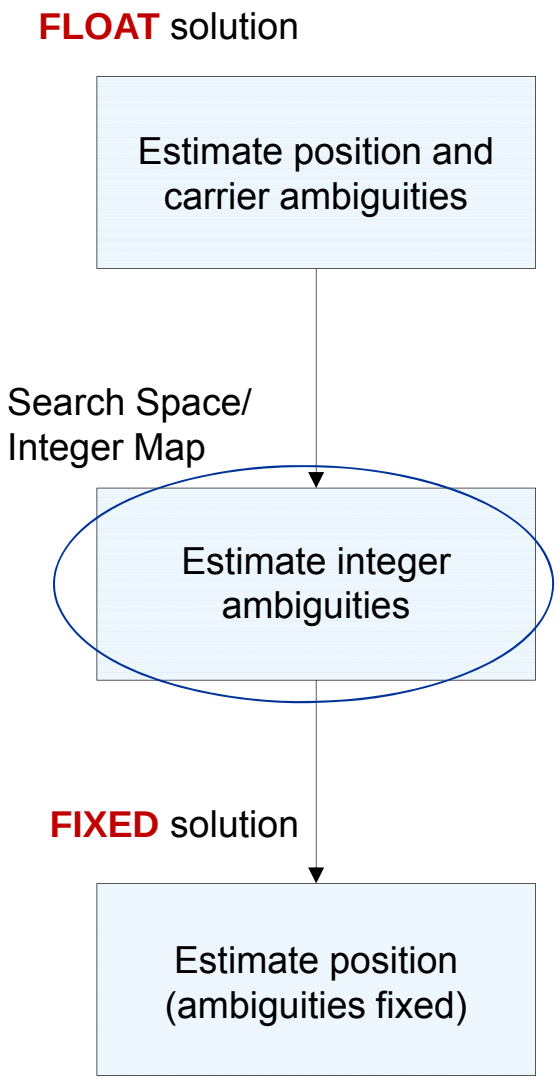
$$\boldsymbol{H} = \begin{pmatrix} \frac{\delta\Phi_{t_k}^{S_2S_1}}{\delta x_{r_1}} & \frac{\delta\Phi_{t_k}^{S_2S_1}}{\delta y_{r_1}} & \frac{\delta\Phi_{t_k}^{S_2S_1}}{\delta z_{r_1}} & 1 & 0 & \cdots & 0 \\ \frac{\delta\Phi_{t_k}^{S_3S_1}}{\delta z_{r_1}} & \frac{\delta\Phi_{t_k}^{S_3S_1}}{\delta z_{r_1}} & \frac{\delta\Phi_{t_k}^{S_3S_1}}{\delta z_{r_1}} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\delta\Phi_{t_k}^{S_iS_1}}{\delta x_{r_1}} & \frac{\delta\Phi_{t_k}^{S_iS_1}}{\delta y_{r_1}} & \frac{\delta\Phi_{t_k}^{S_iS_1}}{\delta z_{r_1}} & 0 & 0 & \cdots & 1 \end{pmatrix}$$

- Note:

$$\frac{\delta\Phi_{t_k}^{S_iS_j}}{\delta x_r} = \frac{\delta P^i}{\delta x_{r1}} - \frac{\delta P^j}{\delta x_{r1}}; \qquad \frac{\delta P^i}{\delta x_r} = \frac{\left(x_S^i - x_{r1}\right)}{\rho^i};$$

- Estimates of float ambiguities improve with time as the geometry accumulates.
- Cycle slips cause a reset in the estimation process and of accuracy.
- When processing the data based on double-differences (using e.g. the LS approach), the ambiguities are estimated as real or floating-point numbers, that is why the first double-difference solution is called the **float** ambiguity solution.

Search Space Definition



- Given the floating-point estimates and their covariance, a search space can be defined:

$$SearchSpace = \left[\nabla\Delta\hat{N} - k\sigma ; \nabla\Delta\hat{N} + k\sigma \right]$$

float ambiguity estimate

k is a scalar, e.g. 3 for 99.7 % probability

	$\nabla\Delta\hat{N}$ [cycles]	σ [cycles]	$\nabla\Delta\hat{N} - 3\sigma$ [cycles]	$\nabla\Delta\hat{N} + 3\sigma$ [cycles]	Total search space [cycles]
PRN3 / PRN9	21.1	2	15	27	12

- The number of ambiguity sets to be considered depends on the number of satellites tracked and the search range of the double-difference ambiguities.
- Example: 10 satellites tracked → 9 ambiguities. If the range for each ambiguity is 3 cycles, the number of combinations to search is: 3^9 = 19 683.

Integer Ambiguity Resolution

The next step is to try and minimize the search space if possible, and verify the integer values.

- Several different strategies exist:
  - Rounding
  - Bank of Kalman Filters
  - Ambiguity Function Method (AFM)
  - Fast Ambiguity Resolution Approach (FARA)
  - Least Square Ambiguity Search Technique (LSAST)
  - Choleski Search Technique
  - Fast Ambiguity Search Filter (FASF)
  - Least Squares Ambiguity Decorrelation Adjustment (LAMBDA)……

Least Squares Ambiguity Decorrelation Adjustment (LAMBDA)

- One of the most used AR techniques.
  - Teunissen, P.J.G. (1995) The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. *Journal of Geodesy,* Vol. 70, No. 1-2, pp. 65-82.
- **Step 1:** Reduce integer search space by decorrelation adjustment:

Decorrelate the second ambiguity from the first:

$$\begin{bmatrix} N_1 \\ N_{2''} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ int\left(-\sigma_{2,1}{\sigma_1}^{-2}\right) & 1 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

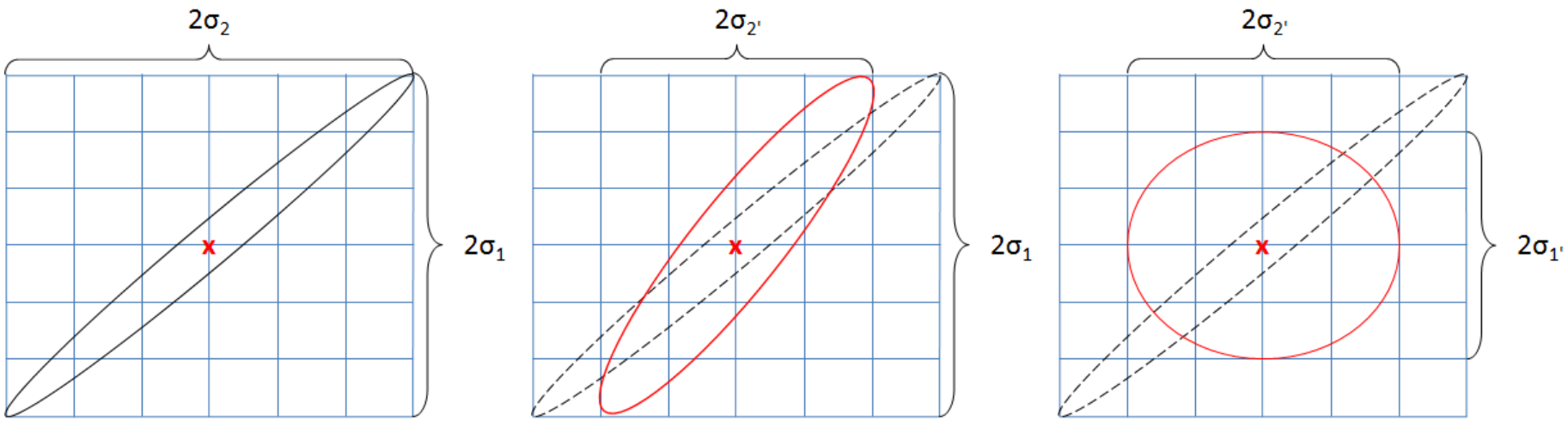
Decorrelate the first ambiguity from the decorrelated second:

$$\begin{bmatrix} N_{1''} \\ N_{2''} \end{bmatrix} = \begin{bmatrix} 1 & int\left(-\sigma_{1,2''}{\sigma_{2''}}^{-2}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} N_1 \\ N_{2''} \end{bmatrix}$$

- Here, the operator *int* performs the rounding to the nearest integer.
- The number of candidates is preserved, but they can be found much more efficiently.

LAMBDA Decorrelation

- The ambiguity search ellipsoid is extremely elongated → the search is very time consuming. The decorrelation process transforms the outstretched ellipsoid into an almost spheroid.



Decorrelate N_2 from N_1 **Decorrelate N_1 from $N_{2'}$**

- Assuming that each grid point of the search window is a possible candidate to be investigated, the advantage of the transformed search space becomes obvious.

LAMBDA Verification

- The measurement residuals are used in this step to assess the 'fit' of the integer ambiguities to the carrier phase data.
- Goal is to find integer set that minimizes Ω'_{total}.
- Using the residuals information, the Sum-of-Squared Residuals (SOS) which is an indicator of residual magnitude can be computed using the following relation:

$$\Omega'_{total} = (\nabla\Delta N_{float} - \nabla\Delta N_{\text{int}})^T C_N^{-1} (\nabla\Delta N_{float} - \nabla\Delta N_{\text{int}})$$

- $\Omega_{total} = \Omega_{float} + \Omega'_{total}$ is the Sum-of-Squared Residuals of the floating solution, and C_N is the covariance matrix of the measurements.
- In this solution both Ω_{total} and Ω'_{total} can be used to assess the correct integer set, which is done by performing the following test:

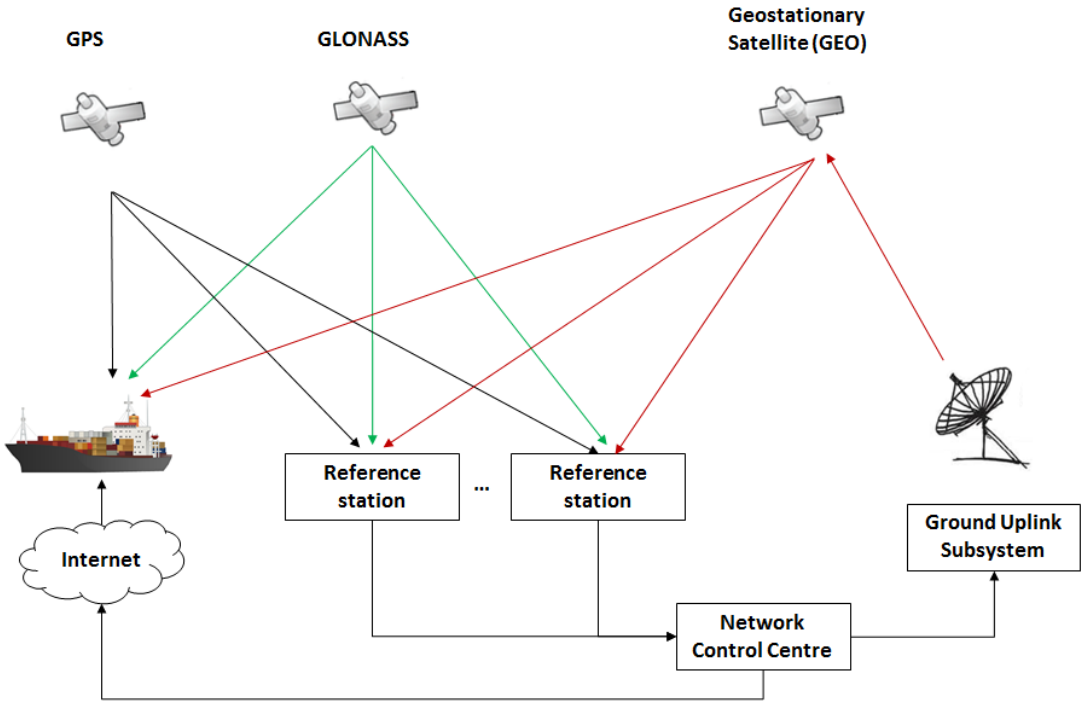
$$\frac{\Omega_{total2}}{\Omega_{total1}} \geq threshold$$

- Where Ω_{total1} is the smallest SOS result, and Ω_{total2} the second smallest.

Integer Ambiguity Resolution

- It is generally easier to resolve ambiguities in static mode.
- The closer the rover is to the base station, the easier it is to resolve ambiguities due to reduction of the correlated errors.
- Multipath can effect the ambiguity resolution process, even if the base station and rover receivers are close.
- Dual frequency receivers can resolve ambiguities faster than single frequency ones, by using linear combinations of carrier phase measurements (Wide Lane, Narrow Lane etc.).
- Another important aspect of ambiguity resolution is the satellite geometry.
  - In regard to length of time required to resolve ambiguities.
  - The more satellites tracked the better.

Precise Point Positioning (PPP)



Providers of PPP services:

- VERIPOS
- TerraStar
- OmniSTAR and SeaSTAR (Fugro)
- StarFire

- Positioning technique that removes or models GNSS system errors (satellite clock and orbital) to provide a high level of position accuracy from a **single** receiver.
- The PPP solution depends on satellite clock and orbit corrections, generated from a network of global reference stations.
- Corrections are delivered to the user via satellite or over the Internet.
- Data format

  Orbit: NGS SP3
  Clock: NGS SP3 or RINEX Clock Extension

Precise Point Positioning (PPP)

- **External correction data** includes satellite orbit and clock corrections. (E.g. in the case of TerraStar service, the corrections generated are broadcast for end-users by Inmarsat telecommunication satellites).

- The first-order **ionospheric delay** can be eliminated by using the combinations of **dual-frequency** GNSS measurements.

- **Residual tropospheric delay is estimated** when estimating position and other unknowns. Modeling is also used in the PPP receiver to correct the solid earth tides effect.

- Measurement model:

$$f_\Phi = \rho + cdT + \lambda N + M \cdot zpd + \varepsilon_\Phi$$

$$f_P = \rho + cdT + M \cdot zpd + \varepsilon_P$$

- Tropospheric path delay is modelled as a function of the zenith path delay (zpd) and mapping function (M).

Precise Point Positioning (PPP)

- An Extended Kalman Filter (EKF) is used for the PPP estimation. Position, receiver clock error, tropospheric delay and carrier-phase ambiguities are estimated EKF states.
- State vector:

$$x = \left(x_u, y_u, z_u, dT, zpd, N^j_{j=1,nsat}\right)^T$$

zpd– troposphere zenith path delay
N – non-integer carrier phase ambiguities

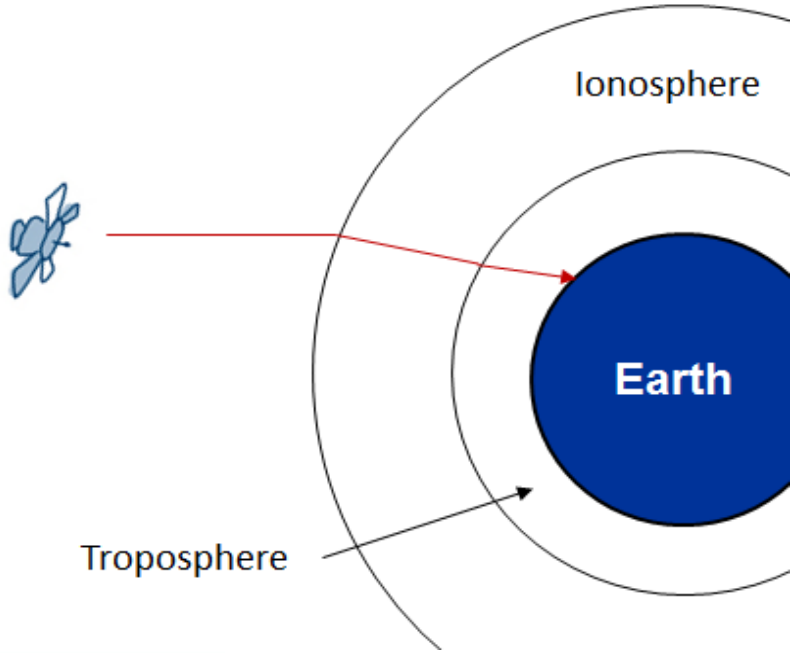
$$H = \begin{bmatrix} \frac{\delta P^i}{\delta x_u} & \frac{\delta P^i}{\delta y_u} & \frac{\delta P^i}{\delta z_u} & \frac{\delta P^i}{\delta t_u} & \frac{\delta P^i}{\delta zdt} & \frac{\delta P^i}{\delta N_i} \end{bmatrix}$$

$$\frac{\delta P^i}{\delta x_u} = -\frac{x_i - \hat{x}_u}{\hat{\rho}_i}, \quad \frac{\delta P^i}{\delta y_u} = -\frac{y_i - \hat{y}_u}{\hat{\rho}_i}, \quad \frac{\delta P^i}{\delta z_u} = -\frac{z_i - \hat{z}_u}{\hat{\rho}_i}, \quad \frac{\delta P^i}{\delta zdt} = M, \quad \frac{\delta P^i}{\delta N} = 1 \; or \; 0$$

- Typical convergence time of PPP to under 10 cm horizontal error is between 20 and 40 minutes, depending on the number of satellites available, satellite geometry, quality of the correction products, receiver multipath environment and atmospheric conditions [9].

RF Propagation Terminology*

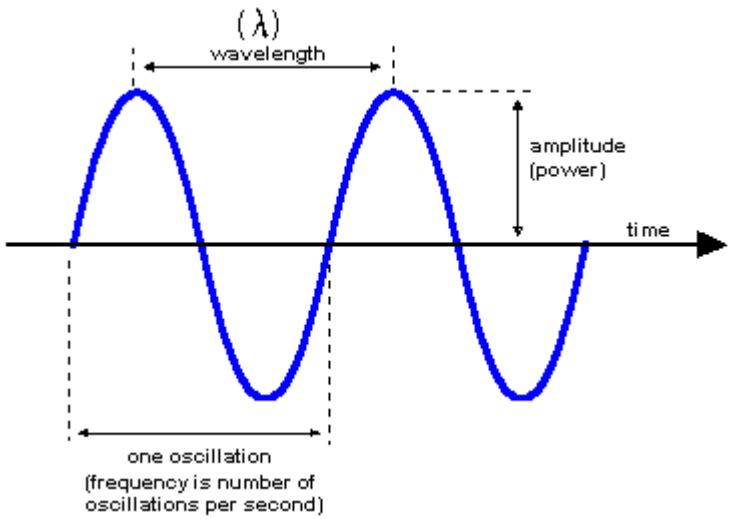
- **Refraction:** happens when a signal passes through an object and comes out at a different angle that it went in at.
- **Defraction:** process in which the direction of energy flow of a wave deviates when it passes an obstacle. Propagation path is not a straight line – leads to errors in the measurements.
  - In regards to GNSS, affected by the atmospheric factors such as e.g. frequency and free electron content, amount of water vapor.
- **Fading:** variation of the radio field strength caused by changes in the transmission path with time (caused by e.g. ionosphere).
- **Reflection:** An EM wave may be partially or completely reflected by the boundary of two meadia, and can result in **multipath**.

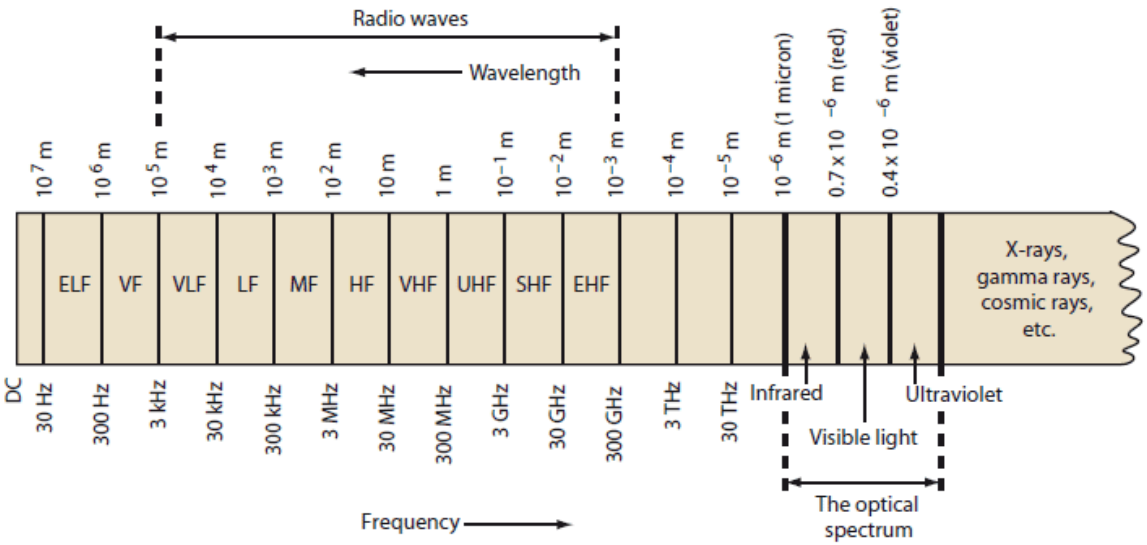


Radio Waves*

- electromagnetic waves with frequencies from 3 kHz – 300 GHz

- Frequency: f [1 Hz = 1 cycle per second]
- Wavelength (meters): $\lambda = c/f$
- Propagation speed (m/s): $c \approx 3 \cdot 10^8$
- Phase angle (radians): ϕ
- Phase (meters): $\Phi = \phi \cdot \lambda$
- Amplitude (V): A





GNSS Data Format Overview*

- RINEX
  - Raw GPS static data format for data processing and archive

- NMEA
  - Standard related to data communication between marine electronic devices.
  - Transmission of GPS position from GPS receiver to other devices (e.g. GPS receiver to PDA)
  - For real time positioning

- RTCM SC-104
  - Transmission of GPS correction from GPS reference station to GPS Rover
  - For DGPS/RTK surveys

Lab 1 Note:

- Start with defining your state vector (offsets in the user coordinates, and the offset in the receiver clock bias: $\Delta \boldsymbol{x} = \left[\Delta x_u, \Delta y_u, \Delta z_u, \Delta t_u\right]^T$, and the measurement noise matrix, R_k.

Calculate satellite positions — **Given**

Set initial position estimate — $\hat{\boldsymbol{x}}_u = \hat{\boldsymbol{x}}_0$ **= e.g. (0,0,0,0)**

Estimate pseudoranges

$$\hat{P}_i = \sqrt{(\boldsymbol{x_0} - \boldsymbol{x_i})^2 + (\boldsymbol{y_0} - \boldsymbol{y_i})^2 + (\boldsymbol{z_0} - \boldsymbol{z_i})^2} + ct_0,$$

Observed minus computed ranges

$$\Delta P_i = \hat{P}_i - \boxed{P_i}$$

Given

Set the measurement vector

Geometry matrix

$$\boldsymbol{G} = \begin{bmatrix} \frac{x_1 - \hat{x}_u}{\widehat{\rho_1}} & \frac{y_1 - \hat{y}_u}{\widehat{\rho_1}} & \frac{z_1 - \hat{z}_u}{\widehat{\rho_1}} & 1 \\ \frac{x_2 - \hat{x}_u}{\widehat{\rho_2}} & \frac{y_2 - \hat{y}_u}{\widehat{\rho_2}} & \frac{z_2 - \hat{z}_u}{\widehat{\rho_2}} & 1 \\ \frac{x_3 - \hat{x}_u}{\widehat{\rho_3}} & \frac{y_3 - \hat{y}_u}{\widehat{\rho_3}} & \frac{z_3 - \hat{z}_u}{\widehat{\rho_3}} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_n - \hat{x}_u}{\widehat{\rho_n}} & \frac{y_n - \hat{y}_u}{\widehat{\rho_n}} & \frac{z_n - \hat{z}_u}{\widehat{\rho_n}} & 1 \end{bmatrix}$$

Determine the offset Δx

$$\Delta \boldsymbol{x} = (\boldsymbol{G}^T \boldsymbol{R}^{-1} \boldsymbol{G})^{-1} \boldsymbol{G}^T \boldsymbol{R}^{-1} \Delta \boldsymbol{P}$$

Update the estimates

$$\hat{\boldsymbol{x}}_{i+1} = \hat{\boldsymbol{x}}_i + \Delta \boldsymbol{x}$$

$$\hat{t}_{i+1} = \hat{t}_i - \Delta t$$

Do not forget that this is an iterative process. Hint: 10 iterations per epoch can be appropriate.

Step 1: Define the elements

- Define your dynamics model. Keep in mind that the receiver was static during the data collection so that you can go for a simple Position model (P model) with $\dot{\delta r} = \eta_r$ (position errors modelled as random walk processes). This will be the example considered herein.

  As we deal with a static positioning problem it is simple to derive the model describing our system given by the linear state-space equation:

$$x_{k+1} = \Phi_k \cdot x_k + w_k$$

  where Φ_k is the state transition matrix and in our case it is equal to the identity matrix (because we are static). w_k is white Gaussian noise expressing the system noise.

- Define your state vector x_k. With the P model you should have 3 states for the receiver coordinate offsets, and one for receiver clock bias offset: $\Delta \boldsymbol{x} = \left[\Delta x_u, \Delta y_u, \Delta z_u, \Delta t_u\right]^T$. It should be initialized at whatever state you want to start at, e.g. all states = 0.

- Set up the error covariance matrix, P_k, associated with state vector. Recall: this matrix is to show how confident the filter is in its solution. It also has to be initialized, but when the filter runs it will become populated. The best way to do it, is to set it up as a diagonal matrix with each entry corresponding to the variance of the state in that row. So for our case, it can be initialized as:

$$P_k = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_t^2 \end{bmatrix}$$

- An example of the initial values to be used is 10^2 - 50^2 meters for the position errors, and 50^2 - 150^2 nano seconds for the receiver clock error.

- Define the process noise covariance matrix, Q_k. It is formed similar to the above except that it is a matrix that you determine and that does not get updated by the filter. An example of somewhat conservative values that can be used is 10^{-5}.

- The next matrix to set is the measurement noise matrix, R_k. It defines the confidence in each of the measurements and is diagonal as well, due to uncorrelated measure. Different approaches can be used to define it, with the simples being using equal uncertainty for all. See slides 102-103 for more details.

- Our measurement vector is the same as in the case of LS, $\boldsymbol{z_k} = \Delta\boldsymbol{P}$, where $\Delta P_i = \hat{P}_i - P_i$, and $\hat{P}_{\boldsymbol{i}} = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} + ct_u$, and P_i is the measurement provided by the receiver.

- Define the design matrix H_k to connect the measurement and the state vectors:

$$\boldsymbol{H} = \begin{bmatrix} \frac{x_1-\hat{x}_u}{\widehat{\rho_1}} & \frac{y_1-\hat{y}_u}{\widehat{\rho_1}} & \frac{z_1-\hat{z}_u}{\widehat{\rho_1}} & 1 \\ \frac{x_2-\hat{x}_u}{\widehat{\rho_2}} & \frac{y_2-\hat{y}_u}{\widehat{\rho_2}} & \frac{z_2-\hat{z}_u}{\widehat{\rho_2}} & 1 \\ \frac{x_3-\hat{x}_u}{\widehat{\rho_3}} & \frac{y_3-\hat{y}_u}{\widehat{\rho_3}} & \frac{z_3-\hat{z}_u}{\widehat{\rho_3}} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_n-\hat{x}_u}{\widehat{\rho_n}} & \frac{y_n-\hat{y}_u}{\widehat{\rho_n}} & \frac{z_n-\hat{z}_u}{\widehat{\rho_n}} & 1 \end{bmatrix}, \text{ where } \widehat{\rho_1} = \sqrt{(\hat{x}_u - x_i)^2 + (\hat{y}_u - y_i)^2 + (\hat{z}_u - z_i)^2}$$

Step 2: Implement the prediction and measurement update steps

- Notation: the 'hat' here will represent the estimated quantity, the superscript '-' will represent the predicted quantity (before measurement update), and the superscript '+' the updated quantity (after the measurement update).

- Recall from the lectures: the extended Kalman filter mechanism is divided into two steps – predict and update. So first we need to predict ahead the state vector and the error covariance:

1. Predict the state ahead

$$\hat{x}_{k+1}^{-} = \Phi_{k+1,k} \cdot \hat{x}_k^{+} + w_k$$

2. Project the error covariance ahead

$$P_{k+1}^{-} = \Phi_{k+1,k} \cdot P_k^{+} \cdot \Phi_{k+1,k}^{T} + Q_k$$

- Next comes the measurement update step where the filter incorporates the predicted values and the information from the measurements to improve the estimated position errors and clock bias. The measurement update filter equations are:

1. Compute the Kalman Gain

$$K_{k+1} = P_{k+1}^{-} H_{k+1}^{T} \left(H_{k+1} P_{k+1}^{-} H_{k+1}^{T} + R_{k+1} \right)$$

2. Update estimate with measurement z_{k+1}

$$\hat{x}_{k+1}^{+} = \hat{x}_{k+1}^{-} + K_{k+1} (z_{k+1} - H_{k+1} \hat{x}_{k+1}^{-})$$

3. Update the error covariance

$$P_{k+1}^{+} = P_{k+1}^{-} - K_{k+1} H_{k+1} P_{k+1}^{-}$$

- The final action at each time epoch is to update your position and receiver clock bias with the estimated values ($\hat{\boldsymbol{x}}_{\boldsymbol{k}+1}^{+}$).

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