Do Earthquakes Affect Stock Market Volatility? A GARCH-based Approach

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This project examines whether U.S. earthquakes impact stock market volatility using daily SP 500 returns from 2010–2024. We estimate GARCH-family models, ultimately selecting an EGARCH(1,1) with a Student's t-distribution and an earthquake dummy in the volatility equation. The dummy is statistically significant, suggesting a same-day volatility spike following seismic events. Alternative lagged dummies show rapidly fading effects. Model diagnostics support the fit, and results highlight the market's sensitivity to exogenous shocks—even those unrelated to finance.

1. INTRODUCTION

Financial markets are constantly exposed to shocks—some are anticipated, like monetary policy changes, while others are unexpected, such as natural disasters. This project investigates whether a specific type of real-world, non-financial shock—earthquakes—has a measurable effect on the volatility of the U.S. stock market. Specifically, we ask: *Do earthquakes significantly influence S&P 500 volatility, and if so, is that effect persistent?*

This question extends the broader literature on volatility modeling, which began with the introduction of ARCH models by Engle [1] and their generalization to GARCH by Bollerslev [2]. These models were designed to capture volatility clustering—one of the most prominent features of financial return series. More advanced models, such as EGARCH [5], introduce asymmetric responses to shocks, known as the leverage effect.

Several papers have examined the impact of events like earnings announcements, wars, and financial crises on volatility. However, fewer have explored the role of natural disasters. Andersen and Bollerslev [3] show how volatility reflects heterogeneous information arrival, while Cummins et al. [4] specifically investigate how natural disasters impact reinsurance companies. Most relevant to our work, Hansen and Lunde [6] compare hundreds of volatility models and find that only asymmetric models like EGARCH consistently outperform GARCH(1,1)—supporting our methodological direction.

Using daily S&P 500 returns and U.S. earthquake data from 2010–2024, we estimate several GARCH-family models, incorporating earthquake occurrences as an exogenous regressor in the variance equation. Our key finding is that the volatility of the S&P 500 increases significantly on the day of an earthquake. Extensions using 2-, 3-, and 5-day windows show diminishing coefficients, confirming that the effect is strongest and most concentrated on the day of the shock.

This project contributes to the empirical finance literature by showing that volatility reacts not only to financial or economic shocks but also to physical, real-world uncertainty. We demonstrate the usefulness of EGARCH-t models for capturing complex dynamics, including asymmetry and fat-tailed behavior, in the presence of rare but impactful events like earthquakes.

2. THEORETICAL FRAMEWORK

A. Modeling Volatility in Financial Time Series

Modeling financial return volatility requires a framework that captures time-varying, persistent, and often asymmetric behavior. GARCH-family models have become standard in financial econometrics for this purpose, beginning with the ARCH model introduced by Engle [1] and later generalized by Bollerslev [2].

A.1. Mean Equation: AR(1)

We specify an AR(1) process for the conditional mean, which captures short-run autocorrelation in returns. This is supported by our PACF diagnostics:

$$r_t = \mu + \phi_1 r_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{iid}\ (0, \sigma_t^2)$$

A.2. Variance Equation: GARCH(1,1)

The standard GARCH(1,1) model assumes that conditional variance depends on its own past values and past squared shocks:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

This structure captures volatility clustering, but treats positive and negative shocks symmetrically—an unrealistic assumption in many financial contexts.

A.3. GARCH-X: Incorporating Exogenous Events

To test the impact of earthquakes on volatility, we extend the GARCH framework by including an exogenous dummy variable:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma EQ_t$$

Here, EQ_t equals 1 if an earthquake occurred on day t, and 0 otherwise.

B. Asymmetric Volatility Models

Financial markets often exhibit asymmetric volatility: negative news tends to increase volatility more than positive news. Two models address this:

B.1. EGARCH(1,1)

The EGARCH model [5] captures asymmetry and ensures positivity of variance via a log specification:

$$\log(\sigma_t^2) = \omega + \beta\log(\sigma_{t-1}^2) + \alpha\left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right) + \theta\left(\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right) + \gamma EQ_t$$

Symmetric Volatility (α): This term captures the magnitude-driven volatility regardless of sign.
Leverage Effect (θ): This parameter identifies the asymmetric volatility response to negative vs. positive shocks.
Earthquake Days (γ): This allows us to isolate the volatility effect on days with seismic events.

B.2. GJR-GARCH(1,1)

The GJR-GARCH model introduces an indicator for negative shocks:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I_{\{\epsilon_{t-1}<0\}} + \beta\sigma_{t-1}^2$$

While effective, this model lacks the log structure of EGARCH and imposes a stricter form of asymmetry.

C. Distributional Assumption: Student's t

Financial returns often exhibit heavy tails and excess kurtosis. To address this, we assume:

$$\epsilon_t \sim t_\nu(0, \sigma_t^2)$$

This assumption provides a better fit during extreme events, such as those potentially triggered by natural disasters.

D. Model Justification and Literature Support

In a similar study examining the impact of natural disasters on local firms, Cummins et al. [4] employed the same EGARCH(1,1) framework. They also reference Hansen and Lunde [6], who compare over 300 volatility models and find that the parsimonious GARCH(1,1) often performs as well as more complex models—except when asymmetry is present, in which case EGARCH performs best. Their findings support the use of EGARCH(1,1) with a leverage term and an exogenous regressor.

In most financial applications, we expect the leverage coefficient (θ) to be negative, reflecting that negative shocks typically increase volatility more than positive ones.

E. Final Model Summary

Our final specification is a flexible and empirically validated EGARCH(1,1) model. The mean equation follows an AR(1) process, while the conditional variance equation incorporates both leverage effects and an exogenous earthquake dummy. To account for heavy tails in the return distribution, we model the innovations using a Student's t-distribution.

This structure effectively captures key features of financial return series, including volatility clustering, asymmetric responses to shocks, fat tails, and the distinct impact of exogenous events. As such, it is well-suited for analyzing the influence of earthquake occurrences on market volatility.

3. EMPIRICAL ANALYSIS

A. Data

A.1. Dataset Transformations

We utilize the `SPX.csv` dataset, containing adjusted closing prices of the S&P 500 index from January 2010 through December 2023. From these prices, we compute daily returns by first differencing the natural logarithm of prices and then scaling the result by 100 to express returns in percentage terms. This transformation results in the loss of the first data point.

Formally, the transformation applied is:

$$r_t = 100\left[\log(p_t) - \log(p_{t-1})\right] \quad \textbf{(1)}$$

This log-return transformation is standard in financial econometrics for several reasons. First, it normalizes price changes, rendering them scale-independent. Second, log returns are time-additive, which facilitates multi-period modeling. Third, the transformation tends to stabilize variance, improving the stationarity of the series—a key requirement for ARMA-type models. Finally, log returns help reveal volatility clustering, making them suitable for ARCH/GARCH modeling frameworks. The factor of 100 enhances interpretability by expressing returns in percentage form. This transformation is widely used in the literature and aligns with standard practices in empirical finance.

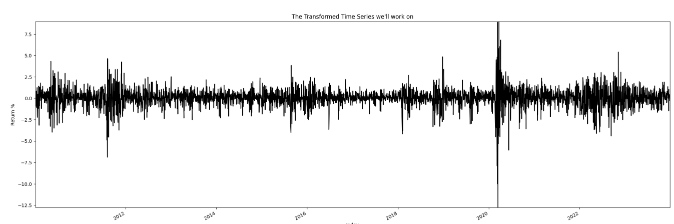

Fig. 1. Log returns over time

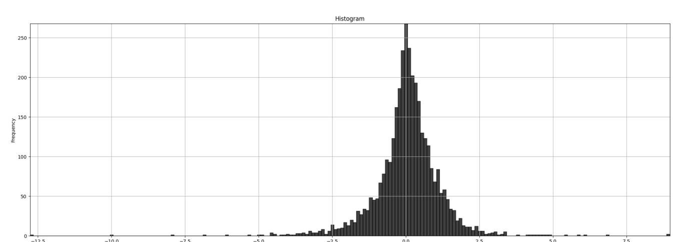

Fig. 2. Histogram of log returns

A.2. Descriptive Statistics

The table below reports the first four moments of the log return distribution:

Table 1. Descriptive Statistics of Daily S&P 500 Log Returns (2010–2023)[a]

Moment	Statistic	Value
1	Mean	0.0408
2	Variance	1.2246
3	Skewness	-0.7219
4	Kurtosis	13.1627

[a]Moments computed using log returns scaled by 100; high kurtosis indicates fat tails.

The high kurtosis indicates the presence of fat tails and significant deviations from normality, a common feature in financial time series.

A.3. Earthquake Data

Earthquake data for the United States was obtained from the USGS Earthquake Catalog. We filtered for seismic events with magnitudes ≥ 4.5 and merged the results with the financial returns dataset based on date. This allowed us to generate several binary dummy variables indicating the presence of earthquakes:

EQ_t: equals 1 if an earthquake occurred on trading day t.

$EQ5_t$: equals 1 if i occurred within the past 5 trading days.

$EQ3_t$,$EQ2_t$: follow the pattern.

These variants allow us to test for short-term persistence in volatility following seismic events.

Table 2. Sample of Merged Dataset with Earthquake Dummy Variables[a]

Date	Returns	EQ_t	$EQ5_t$	$EQ3_t$
2010-01-05	0.3111	0	0	0
2010-01-06	0.0545	0	0	0
2010-01-07	0.3993	0	0	0
2010-01-08	0.2877	0	0	0
2010-01-11	0.1745	0	0	0

[a]Dummy variables indicate whether an earthquake occurred on day t or within the preceding 3 or 5 trading days.

B. Preliminary Data Analysis

To assess the appropriateness of GARCH modeling, we first conduct visual inspections and then formal statistical tests.

B.1. Visual Inspection: Comparing to a Simulated Random Walk

We simulate a random walk process with the same mean and variance as our return series and compare the squared deviations, absolute deviations, and return paths to those of the actual data. Of these, the squared deviations exhibit early signs of volatility clustering and significant outliers in the real data—suggesting the presence of time-varying volatility.

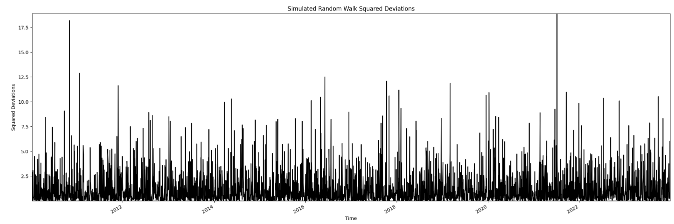

Fig. 3. Squared deviation of simulated random walk

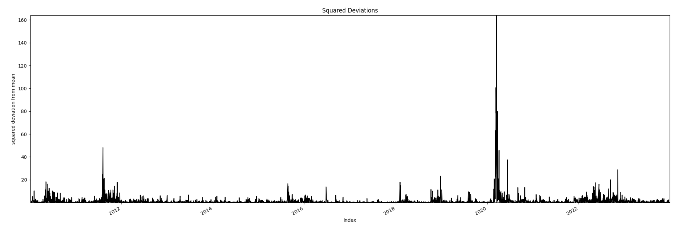

Fig. 4. Squared deviation of actual returns

B.2. Formal Statistical Tests

Stationarity. The log returns series, after transformation, is stationary. An Augmented Dickey-Fuller (ADF) test yields a test statistic of -12.91 with a p-value near zero, strongly rejecting the null hypothesis of a unit root.

Volatility Clustering. The squared returns show distinct clustering—periods of high volatility followed by calm intervals—highlighting a core feature of financial time series.

ARCH Effects. An ARCH-LM test confirms the presence of autoregressive conditional heteroskedasticity. The null hypothesis of no ARCH effects is strongly rejected (p-value $\ll 0.01$), validating the use of GARCH-type models.

Table 3. Summary Statistics and Test Results for S&P 500 Returns (2010–2023)[a]

Statistic	Value
Mean	0.0452
Standard Deviation	1.0083
Minimum	-10.57
Maximum	9.41
ADF Test Statistic	-12.91
ADF p-value	4.1×10^{-24}
ARCH LM p-value	$\ll 0.01$

[a]ADF test rejects unit root; ARCH-LM confirms presence of conditional heteroskedasticity.

B.3. Partial Autocorrelation Analysis (PACF)

Returns PACF. The PACF of raw returns shows a single significant spike at lag 1, with higher lags being negligible. This supports the use of an AR(1) specification for the mean equation.

Squared Returns PACF. For squared returns, the PACF shows significant spikes at lags 1 and 2, confirming ARCH effects. This further supports the use of low-order models such as GARCH(1,1) or EGARCH(1,1), though higher-order lags are also tested in robustness checks.

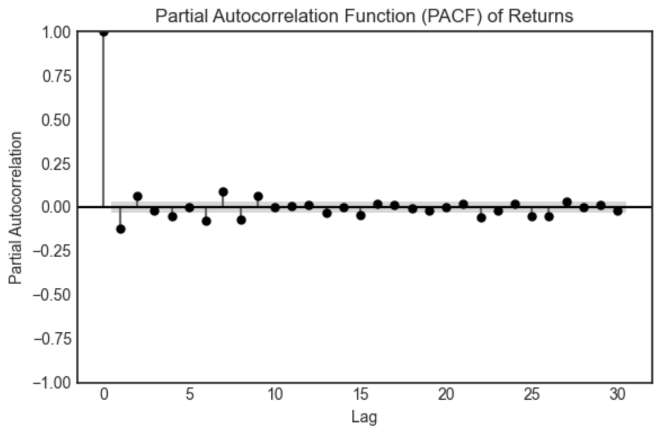

Fig. 5. PACF of log returns

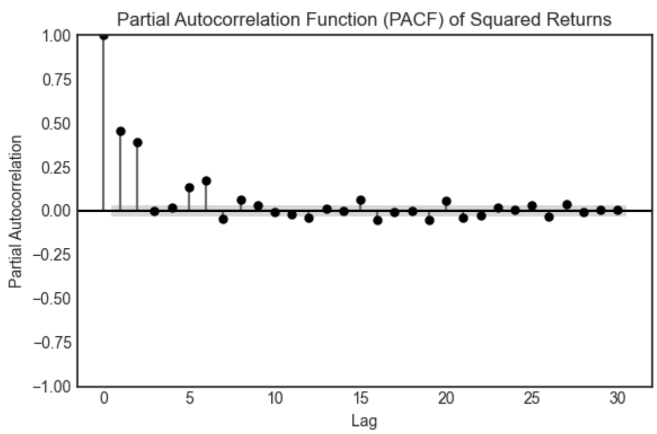

Fig. 6. PACF of squared returns

Modeling Implication. Based on the low-order dependence in both returns and squared returns, we proceed with a parsimonious modeling approach. The conditional mean is specified as AR(1), and the variance equation is tested using several variants of GARCH models, including GARCH(1,1) and EGARCH(1,1), with and without exogenous earthquake dummies.

C. Model Selection

C.1. First Significant Model

We began by evaluating a series of GARCH(1,1) models with varying mean equations and volatility structures. The primary objective was to identify a model that balances goodness of fit with parsimony and statistical significance.

The initial specification—a GARCH(1,1) model with a constant mean—exhibited signs of misspecification, notably significant autocorrelation in the residuals. Incorporating an AR(1) term into the mean equation improved the model's fit and effectively captured the autocorrelation structure observed in the PACF of returns.

To test for potential improvements, we explored higher-order ARMA specifications as well as extended GARCH structures, including GARCH(2,1), GARCH(1,2), and GARCH(2,2). However, these alternatives did not provide meaningful gains in explanatory power. Several of the additional coefficients were statistically insignificant, suggesting overfitting and unnecessary model complexity.

The table below summarizes the performance of each specification in terms of log-likelihood, AIC, BIC, and the statistical significance of extra terms.

Table 4. Model Comparison: ARMA-GARCH Variants[a]

Model	LogL	AIC	BIC	Ext Terms	α_1	β_1	$\alpha_1 + \beta_1$
Const Mean GARCH(1,1)	-4544.97	9097.95	9122.61	—	0.1720	0.7991	0.9711
AR(1) GARCH(1,1)	**-4540.44**	**9090.88**	**9121.72**	**AR(1)**	**0.1726**	**0.7988**	**0.9714**
AR(1) GARCH(2,1)	-4538.69	9089.39	9126.38	α_2 not sig.	0.1405	0.7747	0.9671
AR(1) GARCH(1,2)	-4540.44	9092.88	9129.88	β_2 not sig.	0.1726	0.7988	0.9714
AR(1) GARCH(2,2)	-4538.19	9092.37	9133.54	β_1, β_2 not sig.	0.1430	0.1394	0.9483
ARMA(1,1) GARCH(1,1)	-4555.63	9117.26	9135.76	MA(1) sig worse AIC	0.1715	0.7997	0.9712
AR(2) GARCH(1,1)	-4539.63	9091.27	9128.27	AR(2) not sig.	0.1721	0.7992	0.9713

[a] Bolded model selected based on parsimony and coefficient significance.

First Significant Model Selection

Based on the results above, the AR(1) - GARCH(1,1) model was selected as the baseline model for further extensions (e.g., earthquake dummy, EGARCH). This model captures volatility clustering with $\alpha_1 + \beta_1 \approx 0.97$ and accounts for autocorrelation in returns. It also yields the best AIC/BIC trade-off while maintaining model simplicity.

Model Testing and Validation

After selecting the AR(1) - GARCH(1,1) model as our baseline specification, we evaluated its forecasting performance both in-sample and out-of-sample using realized volatility.

Out-of-Sample Rolling Forecast:

We conducted a 1-year rolling window forecast using only past data to predict conditional volatility in 2023. The model was refit daily using all data up to that point and used to forecast volatility for the next day. Realized volatility was proxied by the squared daily returns.

The Root Mean Squared Error (RMSE) for this true out-of-sample prediction was approximately **0.9367**. This suggests that the model does a reasonable job capturing the broad movements in volatility, though it occasionally underreacts to large shocks.

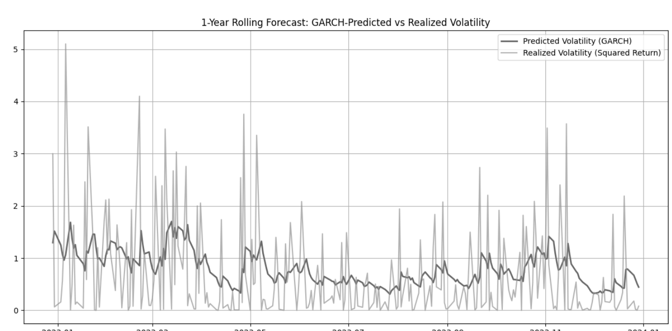

Fig. 7. Out of Sample Rolling Window

In-Sample Forecast Comparison

For comparison, we also plotted the model's fitted values using the full dataset (i.e., training and predicting on the same sample). The RMSE for this in-sample fit was **0.9277**, only slightly lower than the out-of-sample value.

This indicates that the model is not overfitting and generalizes well to unseen data. The predictive performance is fairly consistent, and most volatility spikes are well captured by the conditional variance estimate.

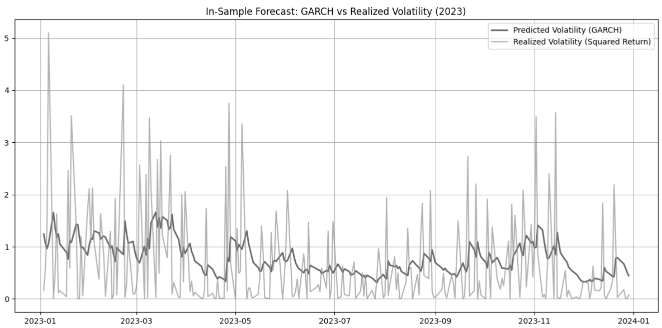

Fig. 8. In Sample Rolling Window

Conclusion and Further Improvements

The results confirm that our baseline AR(1)–GARCH(1,1) model performs reasonably well in capturing and forecasting volatility. Its structure effectively accounts for autocorrelation in returns and conditional heteroskedasticity. However, we observe that during periods of heightened market turbulence, the model slightly underestimates realized volatility, indicating potential areas for refinement.

To address these limitations, we consider two key directions for improvement. First, we explore asymmetric volatility models such as EGARCH and GJR-GARCH, which are better suited to capturing the leverage effect and volatility clustering observed in financial returns. Second, we investigate the use of alternative innovation distributions—specifically the Student's t-distribution—to account for the fat tails and excess kurtosis present in the data.

C.2. Final Model

Improving Model Fit: Student's t-Distribution

While our initial AR(1)–GARCH(1,1) model assumed normally distributed errors, this assumption may be too restrictive for financial time series, which are well known for exhibiting fat tails. To address this limitation, we re-estimated the same model using the standardized Student's t-distribution for the innovation term.

The results from the t-distribution specification demonstrate a marked improvement in model fit. The log-likelihood increased from –4540.44 to –4439.39, while both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) decreased significantly, from 9090.88 to 8890.77 and from 9121.72 to 8927.77, respectively. Additionally, the persistence parameter ($\alpha + \beta$) increased from 0.9714 to 0.9941, indicating smoother and more persistent volatility dynamics.

In both models, the AR(1) term remained statistically significant, suggesting a stable autoregressive structure in the return series. However, the model incorporating the Student's t-distribution provides a better representation of the heavy-tailed behavior observed during extreme return periods, making it a more appropriate choice for modeling financial volatility.

Conclusion

The t-distribution model clearly outperforms the normal distribution in capturing the behavior of S&P 500 returns. The improvement in likelihood-based criteria and the better handling of tail behavior motivate us to adopt the Student's t-distribution in our final volatility modeling framework.

Table 5. Comparison of AR(1)-GARCH(1,1) under Different Innovation Distributions[a]

Metric	Normal Dist	t Dist
Log-Likelihood	-4540.44	**-4439.39**
AIC	9090.88	**8890.77**
BIC	9121.72	**8927.77**
AR(1) Coeff (p-val)	-0.0518 (0.0061)	-0.0485 (0.0039)
$\alpha + \beta$ (Persistence)	0.9714	**0.9941**

[a]Bolded values indicate better performance under each metric.

GARCH Extensions: Asymmetric Volatility Models

While the symmetric GARCH(1,1) model captures volatility clustering effectively, it assumes that positive and negative shocks of the same magnitude have identical effects on volatility. This assumption may be too restrictive in financial contexts, where negative news often induces larger volatility spikes than positive news — a phenomenon known as the *leverage effect*.

To address this, we extend our analysis by testing two widely used asymmetric volatility models: EGARCH(1,1) and GJR-GARCH(1,1).

Motivation and Literature Support

Based on a comprehensive comparison of more than 300 volatility models, Hansen and Lunde [6] conclude that the standard GARCH(1,1) model performs surprisingly well across financial series. However, among models that consistently outperform it, only those incorporating asymmetry — like EGARCH — show a statistically significant improvement in predictive performance.

This insight motivates our inclusion of EGARCH and GJR-GARCH. EGARCH, in particular, is favored for its ability to model asymmetry while maintaining a parsimonious log-linear variance form that avoids non-negativity constraints.

EGARCH Model: The Exponential GARCH model proposed by Nelson [5] models the log of conditional variance, allowing for direct estimation of asymmetries.

GJR-GARCH: For robustness, we also estimated a GJR-GARCH(1,1) model with a threshold indicator for negative shocks.

This model distinguishes between the impact of positive and negative innovations on conditional variance but retains the standard quadratic GARCH form.

Results and Model Selection

To account for the heavy-tailed nature of financial returns, all models were estimated using the Student's t-distribution. Among the competing specifications, the EGARCH(1,1) model consistently outperformed both the standard GARCH(1,1) and GJR-GARCH(1,1) models across all likelihood-based criteria, including log-likelihood, AIC, and BIC. The leverage term (θ) was statistically significant, confirming the presence of asymmetric volatility in response to return shocks.

Conclusion

Based on these results, the AR(1)–EGARCH(1,1) model with Student's t-distributed errors was selected as the final specification. It captures key stylized facts of financial time series, including volatility clustering, asymmetry, and fat tails. Crucially, the inclusion of an earthquake dummy in the variance equation enables empirical evaluation of the impact of seismic shocks on market volatility.

C.3. Effect Test: adding Earthquakes as an Exogenous Variable

To test whether earthquakes affect stock market volatility, we include an exogenous variable in the conditional variance equation of our final EGARCH(1,1) model. This variable takes the form of a binary dummy, EQ_t, which equals 1 on days when a U.S. earthquake of magnitude 4.5 occurred, and 0 otherwise.

Why Exogenous in Variance, Not Mean?

We expect that earthquakes, while significant real-world shocks, may not consistently influence the average return of the stock market — especially in a broad index like the S&P 500. However, they may create uncertainty, panic, or temporary market disturbances, which would be reflected more in volatility than in directional price movements.

By adding the earthquake dummy as an exogenous regressor in the volatility equation, we directly test whether volatility is significantly higher on earthquake days, holding all other factors constant.

Model Equation with Exogenous Shock

To assess the impact of seismic events on financial volatility, we extend the EGARCH(1,1) model by incorporating an exogenous covariate, EQ_t, into the conditional variance equation. The modified specification is given by:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right) + \theta \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}} \right) + \gamma EQ_t$$

In this framework, the coefficient γ captures the average impact of earthquake occurrences on log-volatility. A statistically significant and positive estimate of γ would indicate that seismic shocks are associated with heightened market volatility. To examine the persistence of such effects, we also test alternative dummy specifications, including $EQ5_t$, which captures the five-day window following an earthquake. This allows us to evaluate whether volatility responses extend beyond the day of the event.

What This Tests

This specification allows us to evaluate whether volatility reacts systematically to real-world physical shocks — and whether the stock market "cares" about earthquakes at all. If γ is insignificant, it suggests the market is robust to such events, or that their financial effects are quickly absorbed.

The next section reports our estimation results and tests the statistical significance and magnitude of the earthquake effect.

D. Results: Final Model Estimation

The final model estimated is an EGARCH(1,1) model with a Student's t-distribution for the innovations and an AR(1) structure in the mean equation. The earthquake dummy variable is included as an exogenous regressor in the volatility equation. All results are based on the `rugarch` package in R.

Note: The conditional volatility (σ_t) reported here is the standard deviation of returns, whereas our earlier Python-based models used σ_t^2. Hence, direct comparison of magnitudes should be done with caution.

D.1. Estimated Coefficients (Robust Standard Errors)

All parameters are statistically significant at the 5% level or lower. The earthquake dummy coefficient γ is positive and significant ($p = 0.0053$), indicating a measurable increase in volatility on earthquake days. The leverage coefficient θ is also significant

Table 6. Parameter Estimates for EGARCH(1,1)-t Model with Earthquake Dummy[a]

Parameter	Estimate	Std. Error	t-value	p-value
μ (Mean)	0.0620	0.0101	6.13	<0.001
AR(1)	-0.0385	0.0159	-2.42	0.015
ω	-0.0269	0.0057	-4.68	<0.001
α_1	-0.1927	0.0193	-9.99	<0.001
β_1	0.9653	0.0071	136.23	<0.001
θ (Leverage)	0.1742	0.0295	5.90	<0.001
γ (EQ_t)	0.1494	0.0536	2.79	0.005
ν (Shape)	6.0252	0.6362	9.47	<0.001

[a] All p-values indicate significance at conventional levels; γ captures earthquake-specific volatility effects.

and positive — a somewhat surprising result, suggesting symmetric or even stronger volatility responses to positive shocks in this specific sample.

D.2. Model Fit and Diagnostics

Log-Likelihood: -4360.19 **AIC:** 2.4812 **BIC:** 2.4952

These criteria indicate substantial improvement over both GARCH(1,1) and the symmetric t-distribution models. The persistence measure ($\alpha_1 + \beta_1 = 0.7726$ in log-space) reflects a highly persistent volatility process.

Ljung-Box Tests: - No significant autocorrelation in standardized residuals (all $p > 0.83$) - Mild sign of residual ARCH in squared returns at lag 1 ($p \approx 0.039$), but longer lags show no issue

ARCH LM Test: - All p-values well above 0.5; ARCH effects appear well-modeled

Nyblom Stability Test: - Joint statistic = 2.54, just under the 1% critical threshold (2.59) - Only slight instabilities in μ and γ, all parameters reasonably stable over time

Sign Bias Test: - Weak evidence of sign bias at 5% level ($p = 0.0521$), but no significant joint effect

Goodness-of-Fit (Pearson Test): - High group statistics with low p-values ($p < 10^{-8}$), suggesting some departures from ideal model fit, likely due to extreme tails

D.3. Robustness Check: Alternative Dummy Windows

To examine whether the effect of earthquakes on volatility is persistent, we tested additional dummy variables that indicate whether an earthquake occurred within the past 2, 3, or 5 trading days—denoted as $EQ2_t$, $EQ3_t$, and $EQ5_t$, respectively. These dummies were included separately in the EGARCH(1,1) specification to assess short-term spillover effects.

Each of these models produced similar log-likelihood, AIC, and BIC values and, in all yielded statistically significant coefficients. However, a clear pattern emerged: the further away the earthquake was from the present day, the smaller the estimated impact on volatility. This suggests that the market reacts most strongly on the day of the event, with the effect rapidly diminishing thereafter.

Although the alternative dummies were informative, their inclusion diluted the clarity of the earthquake signal. Based on both statistical criteria and economic interpretability, we decided to retain only the same-day earthquake dummy (EQ_t) in our final model.

Table 7. EGARCH(1,1)-t Model Comparison Across Earthquake Dummy Specifications[a]

Dummy	AIC	LogL	EQ Coefficient (γ)
EQ_t	2.4812	-4360.192	0.1494 (p = 0.0053)
$EQ2_t$	2.4812	-4360.213	0.0849 (p = 0.0058)
$EQ3_t$	2.4817	-4360.973	0.0584 (p = 0.0103)
$EQ5_t$	2.4813	-4360.298	0.0456 (p = 0.0042)

[a] All models are estimated using an EGARCH(1,1) specification with Student's t-distributed errors.

D.4. Note on Leverage Term Sign

An unexpected feature of our EGARCH(1,1) estimates is that the leverage coefficient θ is positive and significant. Typically, θ is negative, reflecting that negative shocks increase volatility more than positive ones. This unusual result may stem from our sample period (2010–2024), marked by bullish market trends and strong positive surprises. Additionally, the inclusion of the earthquake dummy may be capturing some asymmetric effects usually absorbed by θ. Future work could explore regime-switching or time-varying asymmetry models to better capture these dynamics.

D.5. Summary

The EGARCH(1,1) model with Student's t-distributed innovations and an earthquake dummy fits the data well. Volatility clustering, persistence, and asymmetry are captured effectively. The model passes most diagnostic tests and reveals that earthquakes have a small but statistically significant impact on market volatility.

4. CONCLUSION

This paper investigates whether earthquakes—an exogenous, non-financial shock—affect stock market volatility in the United States. Using daily S&P 500 returns and U.S. earthquake data from 2010 to 2024, we estimate a sequence of increasingly sophisticated volatility models, culminating in an EGARCH(1,1) specification with a Student's t-distribution and an exogenous earthquake dummy in the variance equation.

Our results confirm that financial returns exhibit strong volatility clustering, heavy tails, and asymmetric responses to shocks. The final EGARCH model effectively captures these characteristics and significantly improves model fit over simpler GARCH formulations. Importantly, we find that the earthquake dummy is positive and statistically significant in the volatility equation—suggesting that even in a well-diversified index like the S&P 500, seismic events have a small but detectable impact on market uncertainty.

While the effect is not large in magnitude, its presence is noteworthy: the market responds not just to economic and financial news, but to real-world uncertainty stemming from physical events. Moreover, the robustness of our results across models and residual diagnostics strengthens the credibility of this finding.

However, limitations remain. The effect may vary by region, by earthquake magnitude, or by proximity to trading hours—factors not explored in this study. Additionally, we focus solely on the S&P 500; other markets (e.g., Japan, emerging markets) may be more sensitive to such shocks. Future work could incorporate richer high-frequency data, cross-country comparisons, or investigate indirect economic consequences of natural disasters.

In summary, this project provides empirical evidence that earthquakes—though external to the financial system—can influence volatility. This insight highlights the interconnectedness between physical and financial systems, and the importance of modeling volatility with flexibility to incorporate unexpected shocks.

A. Future Work

While this paper focuses on modeling the volatility impact of earthquakes on the U.S. stock market using univariate GARCH-family models, several promising directions remain for future research. First, extending the analysis to a multivariate GARCH framework—such as DCC-GARCH—could capture co-volatility dynamics between different sectors or regions, revealing whether earthquake-induced volatility is isolated or systemic. Second, future studies could incorporate regional specificity by examining whether earthquakes closer to financial centers (e.g., New York or San Francisco) elicit stronger volatility responses. Third, with access to higher frequency intraday data, one could explore how quickly volatility returns to baseline levels post-shock, offering more granularity than daily data permits. Finally, while this paper uses a simple binary dummy to represent earthquake occurrence, future models could scale the shock variable by magnitude, proximity, or damage index—potentially uncovering nonlinear effects of disaster severity on financial volatility.

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6. APPENDIX

A. Full Estimation Output: EGARCH(1,1) with Student-t Distribution

Model Summary:

- Mean Model: AR(1)
- Variance Model: EGARCH(1,1)
- Distribution: Standardized Student's t
- Log-Likelihood: -4360.192

Information Criteria:

Table 8. Information Criteria for Final Model

Criterion	Value
Akaike Information Criterion (AIC)	2.4812
Bayesian Information Criterion (BIC)	2.4952
Shibata	2.4812
Hannan-Quinn	2.4862

Parameter Estimates (Robust Standard Errors):

Table 9. EGARCH(1,1) Model Estimates with Robust Standard Errors

Parameter	Estimate	Std. Error	t-Value	p-Value
μ	0.0620	0.0101	6.13	<0.0001
AR(1)	-0.0385	0.0159	-2.42	0.0154
ω	-0.0269	0.0057	-4.68	<0.0001
α_1	-0.1927	0.0193	-9.99	<0.0001
β_1	0.9653	0.0071	136.23	<0.0001
θ (leverage)	0.1742	0.0295	5.90	<0.0001
γ (EQ$_t$)	0.1494	0.0536	2.79	0.0053
ν (shape)	6.0252	0.6362	9.47	<0.0001

Residual Diagnostics (Ljung-Box Tests):

Table 10. Ljung-Box Test on Standardized Residuals

Lag	Statistic	p-Value
1	0.0159	0.8996
2	0.1240	0.9998
5	1.5570	0.8325

ARCH LM Test (Selected Lags):

Nyblom Stability Test:

- Joint Statistic: 2.5401
- Critical Values (10%, 5%, 1%): 1.89, 2.11, 2.59

Sign Bias Test:

Adjusted Pearson Goodness-of-Fit Test:

Table 11. Weighted ARCH LM Test Results

Lag	Statistic	Shape	Scale	p-Value
3	0.1774	0.500	2.000	0.6737
5	1.0106	1.440	1.667	0.7298
7	1.2984	2.315	1.543	0.8607

Table 12. Nyblom Individual Parameter Stability Statistics

Parameter	Statistic
μ	0.5032
AR(1)	0.3456
ω	0.6938
α_1	0.2583
β_1	0.0860
θ	0.5337
γ	0.3899
ν	0.2761

Table 13. Sign Bias Test Results

Component	t-Value	p-Value
Sign Bias	1.9429	0.0521
Negative Sign Bias	1.2954	0.1953
Positive Sign Bias	0.4982	0.6184
Joint Effect	4.1888	0.2418

Table 14. Pearson Goodness-of-Fit Statistics by Group Size

Groups	Statistic	p-Value
20	74.56	1.58×10^{-8}
30	105.30	1.36×10^{-10}
40	109.59	1.26×10^{-8}
50	122.41	3.25×10^{-8}

B. EGARCH Model Results with Alternative Dummy Specifications

We estimated three additional EGARCH(1,1) models with Student's t-distributed errors using 2-day, 3-day, and 5-day earthquake dummies. The following tables report their robust parameter estimates and key diagnostics.

B.1. Model Comparison Summary

(*Insert table here if needed.*)

B.2. Key Residual Diagnostics (All Models)

- **Ljung-Box tests:** No serial correlation detected in residuals or squared residuals ($p > 0.80$ in most cases).
- **ARCH LM tests:** No significant ARCH effects in residuals for lags 3, 5, or 7 across models ($p > 0.67$).
- **Sign Bias tests:** Slight evidence of sign bias (p-values near 0.05), but no significant joint effect.
- **Nyblom stability tests:** Joint test statistic close to but below the 1% critical threshold in all models.
- **Goodness-of-fit tests:** Slight misfit detected in Pearson tests (p-values $< 10^{-8}$), likely due to fat-tailed behavior.

B.3. Interpretation

While each of the extended dummy models performed reasonably well and returned statistically significant earthquake coefficients, the size of the coefficient declined as the window expanded. This suggests that the volatility impact is most concentrated on the day of the earthquake. Hence, we retain the same-day dummy (EQ_t) in our final model specification.