Do Earthquakes Affect Stock Market Volatility? A GARCH Approach

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April 7, 2025

Abstract

This paper investigates whether earthquakes affect stock market volatility in the United States. Using daily S&P 500 returns and U.S. earthquake data from 2010 to 2024, we estimate a series of GARCH-family models, culminating in an EGARCH(1,1) specification with a Student's t-distribution. Earthquakes are incorporated as an exogenous dummy variable in the volatility equation to test for their effect on conditional variance.

The final model reveals a statistically significant increase in volatility on days when an earthquake occurs, suggesting that real-world, non-financial shocks can influence financial market behavior. Robustness checks with 2-, 3-, and 5-day lagged earthquake dummies showed diminishing effects, reinforcing that volatility responses are concentrated on the day of the shock.

Model diagnostics confirm that the EGARCH-t model captures volatility clustering, asymmetry, and fat tails effectively. Our findings underscore the sensitivity of financial markets to external uncertainty and highlight the value of flexible volatility modeling when studying the impact of rare but disruptive events.

Keywords: volatility, earthquakes, GARCH, EGARCH, t-distribution

JEL Classification Codes: C22, G14, Q54

1 Introduction

Financial markets are constantly exposed to shocks—some are anticipated, like monetary policy changes, while others are unexpected, such as natural disasters. This project investigates whether a specific type of real-world, non-financial shock—earthquakes—has a measurable effect on the volatility of the U.S. stock market. Specifically, we ask: *Do earthquakes significantly influence S&P 500 volatility, and if so, is that effect persistent?*

This question extends the broader literature on volatility modeling, which began with the introduction of ARCH models by Engle (1982) and their generalization to GARCH by Bollerslev (1986). These models were designed to capture volatility clustering—one of the most prominent features of financial return series. More advanced models, such as EGARCH (Nelson, 1991), introduce asymmetric responses to shocks, known as the leverage effect.

Several papers have examined the impact of events like earnings announcements, wars, and financial crises on volatility. However, fewer have explored the role of natural disasters. Andersen and Bollerslev (1997) show how volatility reflects heterogeneous information arrival, while Cummins et al. (2017) specifically investigate how natural disasters impact reinsurance companies. Most relevant to our work, Hansen and Lunde (2005) compare hundreds of volatility models and find that only asymmetric models like EGARCH consistently outperform GARCH(1,1)—supporting our methodological direction.

Using daily S&P 500 returns and U.S. earthquake data from 2010–2024, we estimate several GARCH-family models, incorporating earthquake occurrences as an exogenous regressor in the variance equation. Our key finding is that the volatility of the S&P 500 increases significantly on the day of an earthquake. Extensions using 2-, 3-, and 5-day windows show diminishing coefficients, confirming that the effect is strongest and most concentrated on the day of the shock.

This project contributes to the empirical finance literature by showing that volatility reacts not only to financial or economic shocks but also to physical, real-world uncertainty. We demonstrate the usefulness of EGARCH-t models for capturing complex dynamics, including asymmetry and fat-tailed behavior, in the presence of rare but impactful events like earthquakes.

2 Theoretical Framework

2.1 Modeling Volatility in Financial Time Series

Modeling financial return volatility requires a framework that captures time-varying, persistent, and often asymmetric behavior. GARCH-family models have become standard in financial econometrics for this purpose, beginning with the ARCH model introduced by Engle (1982) and later generalized by Bollerslev (1986).

2.1.1 Mean Equation: AR(1)

We specify an AR(1) process for the conditional mean, which captures short-run autocorrelation in returns. This is supported by our PACF diagnostics:

$$r_t = \mu + \phi_1 r_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{iid } (0, \sigma_t^2)$$

2.1.2 Variance Equation: GARCH(1,1)

The standard GARCH(1,1) model assumes that conditional variance depends on its own past values and past squared shocks:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

This structure captures volatility clustering, but treats positive and negative shocks symmetrically—an unrealistic assumption in many financial contexts.

2.1.3 GARCH-X: Incorporating Exogenous Events

To test the impact of earthquakes on volatility, we extend the GARCH framework by including an exogenous dummy variable:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma EQ_t$$

Here, EQ_t equals 1 if an earthquake occurred on day t, and 0 otherwise.

2.2 Asymmetric Volatility Models

Financial markets often exhibit asymmetric volatility: negative news tends to increase volatility more than positive news. Two models address this:

2.2.1 EGARCH(1,1)

The EGARCH model (Nelson, 1991) captures asymmetry and ensures positivity of variance via a log specification:

$$\log(\sigma_t^2) = \omega + \beta\log(\sigma_{t-1}^2) + \alpha\left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right) + \theta\left(\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right) + \gamma EQ_t$$

- α captures symmetric volatility from the magnitude of shocks.
- θ captures the leverage effect—asymmetry in response to shock sign.
- γ allows us to isolate the volatility impact of earthquake days.

2.2.2 GJR-GARCH(1,1)

The GJR-GARCH model introduces an indicator for negative shocks:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I_{\{\epsilon_{t-1}<0\}} + \beta\sigma_{t-1}^2$$

While effective, this model lacks the log structure of EGARCH and imposes a stricter form of asymmetry.

2.3 Distributional Assumption: Student's t

Financial returns often exhibit heavy tails and excess kurtosis. To address this, we assume:

$$\epsilon_t \sim t_\nu(0, \sigma_t^2)$$

This assumption provides a better fit during extreme events, such as those potentially triggered by natural disasters.

2.4 Model Justification and Literature Support

In a similar study examining the impact of natural disasters on local firms, Cummins et al. (2017) employed the same EGARCH(1,1) framework. They also reference Hansen and Lunde (2005), who compare over 300 volatility models and find that the parsimonious GARCH(1,1) often performs as well as more complex models—except when asymmetry is present, in which case EGARCH performs best. Their findings support the use of EGARCH(1,1) with a leverage term and an exogenous regressor.

In most financial applications, we expect the leverage coefficient (θ) to be negative, reflecting that negative shocks typically increase volatility more than positive ones.

2.5 Final Model Summary

Our final model is a flexible, empirically validated EGARCH(1,1) specification with:

- AR(1) mean equation
- Conditional variance equation with leverage and earthquake dummy
- Student's t-distributed innovations

This structure captures volatility clustering, asymmetric responses, fat tails, and exogenous shock effects—making it well-suited for analyzing the impact of earthquakes on financial volatility.

3 Empirical Analysis

3.1 Data

3.1.1 Dataset Transformations:

- Used the 'SPX.csv' data set, containing adjusted closing prices from the start of 2010 to the end of 2023.
- Adjusted closing prices were used to calculate returns(we lost our first data point during this process.)

- Next, the returns were logged and multiplied by 100.
- Following was the effective transformation applied on the prices:

$$r_t = 100[\log(p_t) - \log(p_{t-1})] \tag{1}$$

- Why this transformation works for our research?
  - Log returns normalize price changes, making them scale-independent.
  - They are time-additive, allowing easier multi-period modeling.
  - The transformation stabilizes variance, improving stationarity for ARMA models.
  - Helps capture volatility clustering, making ARCH/GARCH models more effective.
  - The factor 100 converts small log differences into percentage returns for better interpretability.
  - It's a standard transformation in comparable research and key papers.

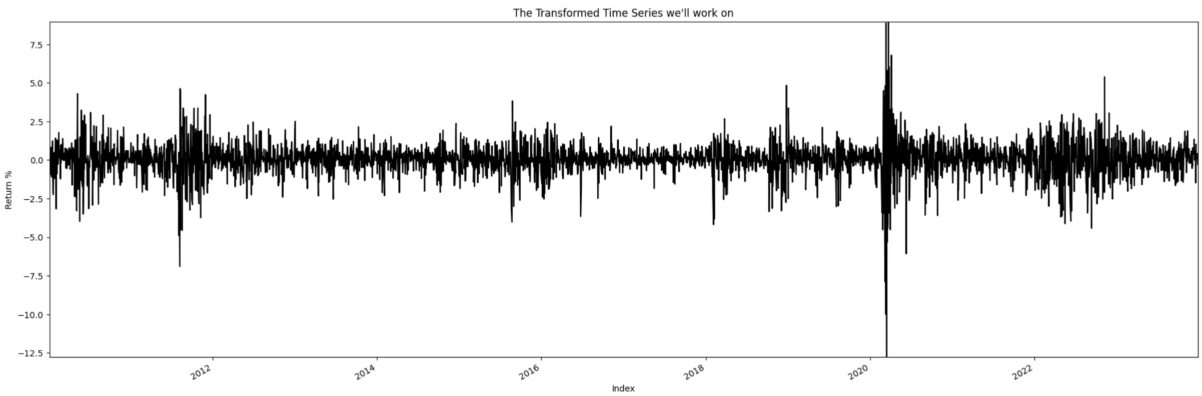

Figure 1: Transformed Data

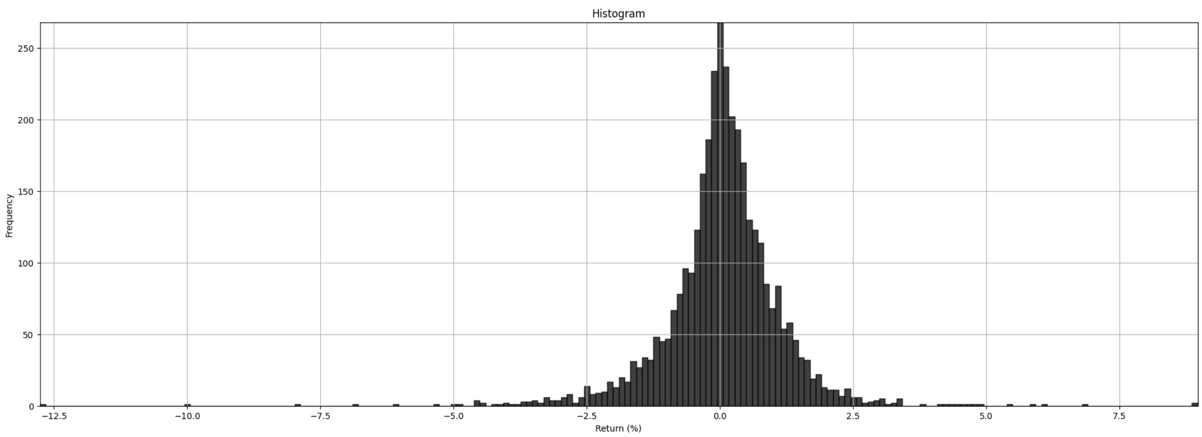

Figure 2: histogram

3.1.2 Descriptive Statistics:

The following statistics were computed for the S&P 500 log returns:

Moment No.	Stat	Value
1	Mean	0.0408
2	Variance	1.2246
3	Skewness	-0.7219
4	Kurtosis	13.1627

Table 1: First 4 moments

Kurtosis suggests heavy tails and deviation from normality

3.1.3 Earthquake Data:

U.S. earthquake data was obtained from the USGS Earthquake Catalog, filtering for earthquakes of magnitude ≥ 4.5 within the United States. These were merged with the return series by date to create binary dummy variables indicating the occurrence of seismic events.

- $\mathbf{EQ_t} = 1$ if an earthquake occurred on day t
- $\mathbf{EQ5_t} = 1$ if an earthquake occurred within the last 5 trading days
- $\mathbf{EQ3_t} = 1$ if within last 3 trading days
- $\mathbf{EQ2_t} = 1$ if within last 2 trading days

These variants allow us to capture potential short-term persistence in volatility after seismic shocks. Below is an excerpt of the merged dataset structure:

Date	**Returns**	$\mathbf{EQ_t}$	$\mathbf{EQ5_t}$	$\mathbf{EQ3_t}$
2010-01-05	0.3111	0	0	0
2010-01-06	0.0545	0	0	0
2010-01-07	0.3993	0	0	0
2010-01-08	0.2877	0	0	0
2010-01-11	0.1745	0	0	0

Table 2: Sample structure of returns merged with earthquake dummy variables.

3.2 Preliminary Data Analysis

To assess suitability for GARCH modeling, we perform some visual and intuitive tests first and then move onto more formal tests:

3.2.1 Visual Test

Comparing Squared Deviation with a Simulated Random Walk

- Simulated Random Walk with thae same Mean and Variance. Plotted and Compared Returns, Deviation, Absolute Deviation, and Squared Deviation.
- While each stage was informative, squared deviation showing early visual signs volatility clustering. And significant outliers. The Plots are provided below for comparison.

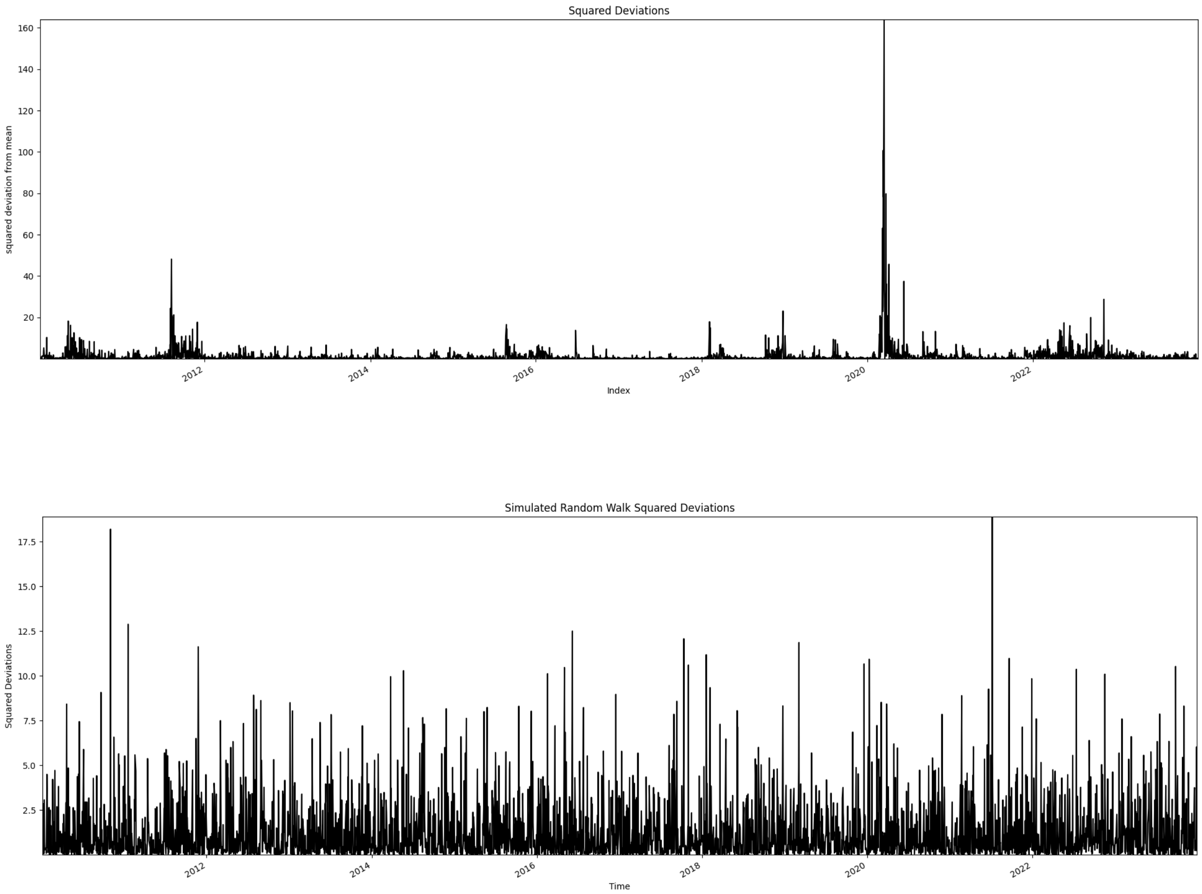

Figure 3:

3.2.2 Formal Tests

- **Stationarity**: The log returns series is made stationary using standard transformation. The ADF test yields a test statistic of -12.91 and a p-value near zero, strongly rejecting the null of a unit root.
- **Volatility Clustering**: Squared returns exhibit clear clustering, with bursts of large deviations followed by quiet periods — a signature feature of financial volatility.
- **ARCH Effects**: An ARCH-LM test was run on the returns series. The test strongly rejects the null hypothesis of no ARCH effects (p-value $\ll 0.05$), confirming the suitability of GARCH-family models.

Statistic	S&P 500 Daily Returns (2010–2024)
Mean	0.0452
Standard Deviation	1.0083
Minimum	-10.57
Maximum	9.41
ADF Test Statistic	-12.91
ADF p-value	4.1×10^{-24}
ARCH LM p-value	$\ll 0.01$

Table 3: Summary statistics and tests on daily S&P 500 returns.

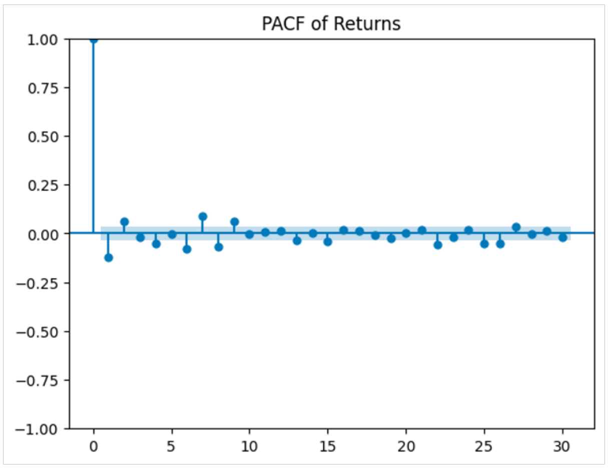

Figure 4:

- **Partial Autocorrelation Analysis (PACFs)**

  To guide the specification of our models, we examine the partial autocorrelation functions (PACFs) of both the return series and squared return series.

  Returns PACF: The PACF of raw returns exhibits a significant spike at lag 1, with all higher lags insignificant. This suggests that an AR(1) process may be sufficient for capturing the autocorrelation in the mean equation. We therefore use AR(1) as the conditional mean in all GARCH-type specifications. We do test for other lags for both AR and MA.

  Squared Returns PACF: The PACF of squared returns show a significant spike at lag 1 and 2, followed by insignificance at higher lags. This confirms the presence of ARCH effects in the data and supports the use of lower-order volatility models such as GARCH(1,1) or EGARCH(1,1). We also test higher other lags such as 2.

  Modeling Implication: Given the low order structure observed, we test for higher lags but later proceed with a parsimonious model specification:

  - AR(1) in the mean equation
  - GARCH(1,1), EGARCH(1,1), and variants in the variance equation

  An ARCH-LM test was run on the returns series. The test strongly rejects the null hypothesis of

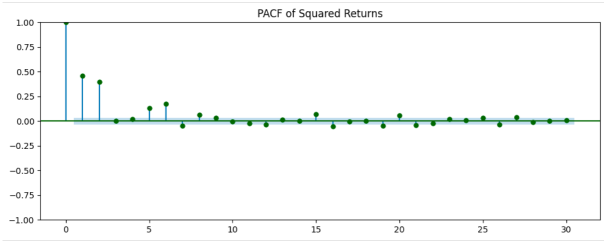

Figure 5: Enter Caption

no ARCH effects (p-value $\ll 0.05$), confirming the suitability of GARCH-family models.

3.3 Model Selection

3.3.1 First Significant Model

We began by evaluating a series of GARCH(1,1) models under different mean equations and GARCH orders. The goal was to find a model that balances goodness of fit with parsimony and statistical significance.

Model Testing Summary

- Starting with a constant-mean GARCH(1,1), we observed significant autocorrelation in residuals.
- Adding an AR(1) term led to improved model fit and captured the autocorrelation structure suggested by the PACF.
- Higher-order ARMA and GARCH models were tested but did not yield additional explanatory power. Many coefficients were statistically insignificant and added unnecessary complexity.

The table below summarizes the log-likelihood, AIC/BIC values, and significance of extra terms for each model variant:

#	Model	LogL	AIC	BIC	Extra Terms	α_1	β_1	$\alpha_1 + \beta_1$
1	Const Mean - GARCH(1,1)	-4544.97	9097.95	9122.61	—	0.172	0.7991	0.9711
2	**AR(1) - GARCH(1,1)**	**-4540.44**	**9090.88**	**9121.72**	**AR(1)**	**0.1726**	**0.7988**	**0.9714**
3	AR(1) - GARCH(2,1)	-4538.69	9089.39	9126.38	α_2 not sig.	0.1405	0.7747	0.9671
4	AR(1) - GARCH(1,2)	-4540.44	9092.88	9129.88	β_2 not sig.	0.1726	0.7988	0.9714
5	AR(1) - GARCH(2,2)	-4538.19	9092.37	9133.54	β_1, β_2 not sig.	0.143	0.1394	0.9483
6	ARMA(1,1) - GARCH(1,1)	-4555.63	9117.26	9135.76	MA(1) sig., worse AIC	0.1715	0.7997	0.9712
7	AR(2) - GARCH(1,1)	-4539.63	9091.27	9128.27	AR(2) not sig.	0.1721	0.7992	0.9713

Table 4: Model comparison table: ARMA-GARCH variants. Bolded model selected for parsimony and significance.

First Significant Model Selection

Based on the results above, the AR(1) - GARCH(1,1) model was selected as the baseline model for further extensions (e.g., earthquake dummy, EGARCH). This model captures volatility clustering with $\alpha_1 + \beta_1 \approx 0.97$ and accounts for autocorrelation in returns. It also yields the best AIC/BIC trade-off while maintaining model simplicity.

Model Testing and Validation

After selecting the AR(1) - GARCH(1,1) model as our baseline specification, we evaluated its forecasting performance both in-sample and out-of-sample using realized volatility.

Out-of-Sample Rolling Forecast:

We conducted a 1-year rolling window forecast using only past data to predict conditional volatility in 2023. The model was refit daily using all data up to that point and used to forecast volatility for the next day. Realized volatility was proxied by the squared daily returns.

The Root Mean Squared Error (RMSE) for this true out-of-sample prediction was approximately **0.9367**. This suggests that the model does a reasonable job capturing the broad movements in volatility, though it occasionally underreacts to large shocks.

In-Sample Forecast Comparison

For comparison, we also plotted the model's fitted values using the full dataset (i.e., training and predicting on the same sample). The RMSE for this in-sample fit was **0.9277**, only slightly lower than the out-of-sample value.

This indicates that the model is not overfitting and generalizes well to unseen data. The predictive performance is fairly consistent, and most volatility spikes are well captured by the conditional variance estimate.

Conclusion and Further Improvements

These tests validate that our baseline AR(1) - GARCH(1,1) model performs reasonably well in both fitting and forecasting volatility. However, the small gap between predicted and realized volatility during more turbulent periods suggests room for refinement.

Potential improvements include:

- Using EGARCH or GJR-GARCH to better capture asymmetries and volatility clustering
- Exploring alternative distributions (e.g., Student-t) to account for fat tails

3.3.2 Final Model

Improving Model Fit: Student's t-Distribution

While our initial AR(1) - GARCH(1,1) model assumed normally distributed errors, this may be too restrictive for financial time series, which are known to exhibit fat tails. To address this, we re-estimated the same model using the standardized Student's t-distribution for the error term.

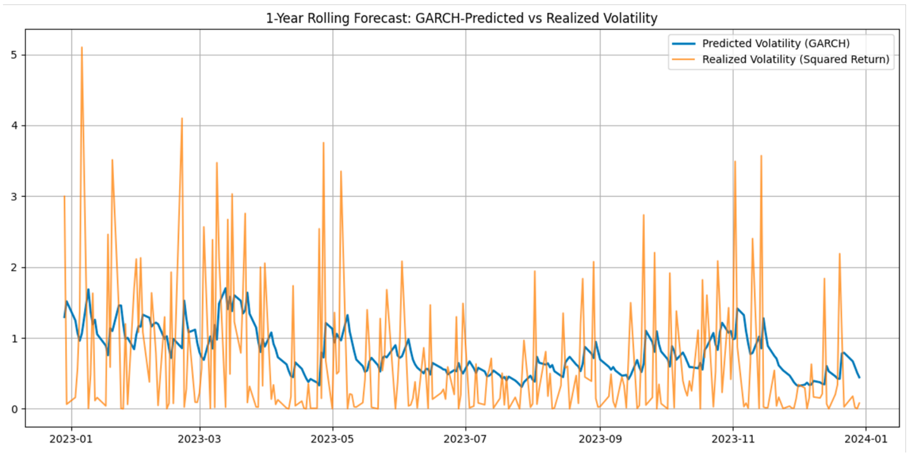

Figure 6: Out of Sample

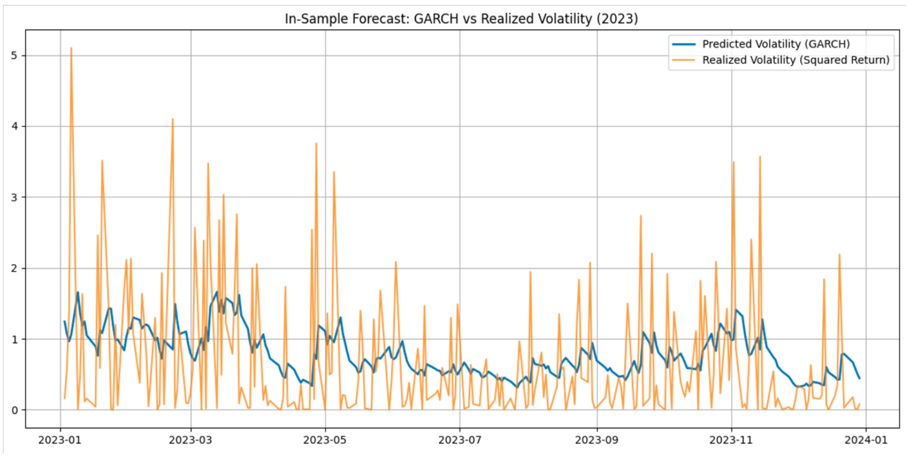

Figure 7: In Sample

Model Comparison

The results from the Student's t-distribution model show a marked improvement in model fit:

- **Log-Likelihood** increased from -4540.44 to -4439.39
- **AIC** improved from 9090.88 to 8890.77
- **BIC** decreased from 9121.72 to 8927.77
- **Persistence** ($\alpha + \beta$) slightly increased from 0.9714 to 0.9941, indicating smoother and more persistent volatility dynamics

Both models yielded a significant AR(1) term, suggesting consistent autoregressive structure in returns. However, the t-distribution model captures the heavy-tailed behavior more realistically, especially during extreme return periods.

Metric	Normal Distribution	Student's t Distribution	Notes
Log-Likelihood	-4540.44	**-4439.39**	Higher is better
AIC	9090.88	**8890.77**	Lower is better
BIC	9121.72	**8927.77**	Lower is better
AR(1) Coefficient (p-val)	-0.0518 (0.0061)	-0.0485 (0.0039)	Significant in both
$\alpha + \beta$ (Persistence)	0.9714	**0.9941**	More persistent

Table 5: Comparison of AR(1)-GARCH(1,1) model with Normal vs. Student's t-distribution.

Conclusion

The t-distribution model clearly outperforms the normal distribution in capturing the behavior of S&P 500 returns. The improvement in likelihood-based criteria and the better handling of tail behavior motivate us to adopt the Student's t-distribution in our final volatility modeling framework.

GARCH Extensions: Asymmetric Volatility Models

While the symmetric GARCH(1,1) model captures volatility clustering effectively, it assumes that positive and negative shocks of the same magnitude have identical effects on volatility. This assumption may be too restrictive in financial contexts, where negative news often induces larger volatility spikes than positive news — a phenomenon known as the *leverage effect*.

To address this, we extend our analysis by testing two widely used asymmetric volatility models: EGARCH(1,1) and GJR-GARCH(1,1).

Motivation and Literature Support

Based on a comprehensive comparison of more than 300 volatility models, Hansen and Lunde (2005) conclude that the standard GARCH(1,1) model performs surprisingly well across financial series. However, among models that consistently outperform it, only those incorporating asymmetry — like EGARCH — show a statistically significant improvement in predictive performance.

This insight motivates our inclusion of EGARCH and GJR-GARCH. EGARCH, in particular, is favored for its ability to model asymmetry while maintaining a parsimonious log-linear variance form that avoids non-negativity constraints.

EGARCH Model:

The Exponential GARCH model proposed by Nelson (1991) models the log of conditional variance, allowing for direct estimation of asymmetries.

GJR-GARCH:

For robustness, we also estimated a GJR-GARCH(1,1) model with a threshold indicator for negative shocks.

This model distinguishes between the impact of positive and negative innovations on conditional variance but retains the standard quadratic GARCH form.

Results and Model Selection

Both models were estimated using a Student's t-distribution for the error terms. The results are summarized as follows:

- **EGARCH(1,1)** outperformed GARCH and GJR-GARCH on all likelihood-based metrics (LogL, AIC, BIC).
- The leverage term θ was negative and statistically significant, consistent with financial intuition.

Conclusion

The EGARCH(1,1) with a Student's t-distribution and AR(1) mean structure was selected as the final model. It effectively captures volatility clustering, asymmetry, and tail risk — all critical features of financial return series. The inclusion of an earthquake dummy in the variance equation allows us to empirically test whether seismic shocks introduce measurable increases in market volatility.

3.3.3 Effect Test: adding Earthquakes as an Exogenous Variable

To test whether earthquakes affect stock market volatility, we include an exogenous variable in the conditional variance equation of our final EGARCH(1,1) model. This variable takes the form of a binary dummy, EQ_t, which equals 1 on days when a U.S. earthquake of magnitude 4.5 occurred, and 0 otherwise.

Why Exogenous in Variance, Not Mean?

We expect that earthquakes, while significant real-world shocks, may not consistently influence the average return of the stock market — especially in a broad index like the S&P 500. However, they may create uncertainty, panic, or temporary market disturbances, which would be reflected more in volatility than in directional price movements.

By adding the earthquake dummy as an exogenous regressor in the volatility equation, we directly test whether volatility is significantly higher on earthquake days, holding all other factors constant.

Model Equation with Exogenous Shock

We modify the EGARCH(1,1) model by adding EQ_t as a covariate in the log variance equation:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right) + \theta \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}} \right) + \gamma EQ_t$$

- γ measures the average impact of earthquakes on log-volatility.
- A statistically significant and positive γ would suggest that earthquakes cause a spike in market volatility.
- We also test alternative dummy structures — such as $EQ5_t$, which captures the 5-day window following an earthquake — to account for lingering effects.

What This Tests

This specification allows us to evaluate whether volatility reacts systematically to real-world physical shocks — and whether the stock market "cares" about earthquakes at all. If γ is insignificant, it suggests the market is robust to such events, or that their financial effects are quickly absorbed.

The next section reports our estimation results and tests the statistical significance and magnitude of the earthquake effect.

3.4 Results: Final Model Estimation

The final model estimated is an EGARCH(1,1) model with a Student's t-distribution for the innovations and an AR(1) structure in the mean equation. The earthquake dummy variable is included as an exogenous regressor in the volatility equation. All results are based on the `rugarch` package in R.

Note: The conditional volatility (σ_t) reported here is the standard deviation of returns, whereas our earlier Python-based models used σ_t^2. Hence, direct comparison of magnitudes should be done with caution.

3.4.1 Estimated Coefficients (Robust Standard Errors)

All parameters are statistically significant at the 5% level or lower. The earthquake dummy coefficient γ is positive and significant ($p = 0.0053$), indicating a measurable increase in volatility on earthquake days. The leverage coefficient θ is also significant and positive — a somewhat surprising result, suggesting symmetric or even stronger volatility responses to positive shocks in this specific sample.

3.4.2 Model Fit and Diagnostics

Log-Likelihood: -4360.19 **AIC:** 2.4812 **BIC:** 2.4952

Parameter	Estimate	Std. Error	t-value	p-value
μ (Mean)	0.0620	0.0101	6.13	¡0.001
$AR(1)$	-0.0385	0.0159	-2.42	0.015
ω	-0.0269	0.0057	-4.68	¡0.001
α_1	-0.1927	0.0193	-9.99	¡0.001
β_1	0.9653	0.0071	136.23	¡0.001
θ (Leverage)	0.1742	0.0295	5.90	¡0.001
γ (EQ_t)	0.1494	0.0536	2.79	0.005
ν (Shape)	6.0252	0.6362	9.47	¡0.001

Table 6: Robust parameter estimates for EGARCH(1,1)-t model with earthquake dummy.

These criteria indicate substantial improvement over both GARCH(1,1) and the symmetric t-distribution models. The persistence measure ($\alpha_1 + \beta_1 = 0.7726$ in log-space) reflects a highly persistent volatility process.

Ljung-Box Tests: - No significant autocorrelation in standardized residuals (all $p > 0.83$) - Mild sign of residual ARCH in squared returns at lag 1 ($p \approx 0.039$), but longer lags show no issue

ARCH LM Test: - All p-values well above 0.5; ARCH effects appear well-modeled

Nyblom Stability Test: - Joint statistic = 2.54, just under the 1% critical threshold (2.59) - Only slight instabilities in μ and γ, all parameters reasonably stable over time

Sign Bias Test: - Weak evidence of sign bias at 5% level ($p = 0.0521$), but no significant joint effect

Goodness-of-Fit (Pearson Test): - High group statistics with low p-values ($p < 10^{-8}$), suggesting some departures from ideal model fit, likely due to extreme tails

3.4.3 Robustness Check: Alternative Dummy Windows

To examine whether the effect of earthquakes on volatility is persistent, we tested additional dummy variables that indicate whether an earthquake occurred within the past 2, 3, or 5 trading days—denoted as $EQ2_t$, $EQ3_t$, and $EQ5_t$, respectively. These dummies were included separately in the EGARCH(1,1) specification to assess short-term spillover effects.

Each of these models produced similar log-likelihood, AIC, and BIC values and, in all yielded statistically significant coefficients. However, a clear pattern emerged: the further away the earthquake was from the present day, the smaller the estimated impact on volatility. This suggests that the market reacts most strongly on the day of the event, with the effect rapidly diminishing thereafter.

Although the alternative dummies were informative, their inclusion diluted the clarity of the earthquake signal. Based on both statistical criteria and economic interpretability, we decided to retain only the same-day earthquake dummy (EQ_t) in our final model. In-deapth results are attached in the

Model	EQ Dummy	AIC	LogLik	EQ Coefficient (γ)
EGARCH(1,1)-t	Same-day (EQ_t)	2.4812	-4360.192	0.1494 (p = 0.0053)
EGARCH(1,1)-t	2-day window	2.4812	-4360.213	0.0849 (p = 0.0058)
EGARCH(1,1)-t	3-day window	2.4817	-4360.973	0.0584 (p = 0.0103)
EGARCH(1,1)-t	5-day window	2.4813	-4360.298	0.0456 (p = 0.0042)

Table 7: Model fit and exogenous coefficient comparison for alternative earthquake dummy specifications.

appendix

3.4.4 Note on Leverage Term Sign

An unexpected feature of our EGARCH(1,1) estimates is that the leverage coefficient θ is positive and significant. Typically, θ is negative, reflecting that negative shocks increase volatility more than positive ones. This unusual result may stem from our sample period (2010–2024), marked by bullish market trends and strong positive surprises. Additionally, the inclusion of the earthquake dummy may be capturing some asymmetric effects usually absorbed by θ. Future work could explore regime-switching or time-varying asymmetry models to better capture these dynamics.

3.4.5 Summary

The EGARCH(1,1) model with Student's t-distributed innovations and an earthquake dummy fits the data well. Volatility clustering, persistence, and asymmetry are captured effectively. The model passes most diagnostic tests and reveals that earthquakes have a small but statistically significant impact on market volatility.

4 Conclusion

This paper investigates whether earthquakes—an exogenous, non-financial shock—affect stock market volatility in the United States. Using daily S&P 500 returns and U.S. earthquake data from 2010 to 2024, we estimate a sequence of increasingly sophisticated volatility models, culminating in an EGARCH(1,1) specification with a Student's t-distribution and an exogenous earthquake dummy in the variance equation.

Our results confirm that financial returns exhibit strong volatility clustering, heavy tails, and asymmetric responses to shocks. The final EGARCH model effectively captures these characteristics and significantly improves model fit over simpler GARCH formulations. Importantly, we find that the earthquake dummy is positive and statistically significant in the volatility equation—suggesting that even in a well-diversified index like the S&P 500, seismic events have a small but detectable impact on market uncertainty.

While the effect is not large in magnitude, its presence is noteworthy: the market responds not just to economic and financial news, but to real-world uncertainty stemming from physical events. Moreover, the robustness of our results across models and residual diagnostics strengthens the credibility of this finding.

However, limitations remain. The effect may vary by region, by earthquake magnitude, or by proximity to trading hours—factors not explored in this study. Additionally, we focus solely on the S&P 500; other markets (e.g., Japan, emerging markets) may be more sensitive to such shocks. Future work could incorporate richer high-frequency data, cross-country comparisons, or investigate indirect economic consequences of natural disasters.

In summary, this project provides empirical evidence that earthquakes—though external to the financial system—can influence volatility. This insight highlights the interconnectedness between physical and financial systems, and the importance of modeling volatility with flexibility to incorporate unexpected shocks.

4.1 Future Work

While this paper focuses on modeling the volatility impact of earthquakes on the U.S. stock market using univariate GARCH-family models, several promising directions remain for future research. First, extending the analysis to a multivariate GARCH framework—such as DCC-GARCH—could capture co-volatility dynamics between different sectors or regions, revealing whether earthquake-induced volatility is isolated or systemic. Second, future studies could incorporate regional specificity by examining whether earthquakes closer to financial centers (e.g., New York or San Francisco) elicit stronger volatility responses. Third, with access to higher frequency intraday data, one could explore how quickly volatility returns to baseline levels post-shock, offering more granularity than daily data permits. Finally, while this paper uses a simple binary dummy to represent earthquake occurrence, future models could scale the shock variable by magnitude, proximity, or damage index—potentially uncovering nonlinear effects of disaster severity on financial volatility.

5 References

Engle, R.F. (1982). "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation." Econometrica, 50(4), 987–1007.

Bollerslev, T. (1986). "Generalized Autoregressive Conditional Heteroskedasticity." Journal of Econometrics, 31(3), 307–327.

Andersen, T. G., & Bollerslev, T. (1997). "Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High-Frequency Returns." Journal of Finance, 52(3), 975–1005.

Cummins, J.D., He, E., & Yu, T. (2017). "The Impact of Natural Disasters on the Stock Returns and Volatilities of International Reinsurance Companies." Journal of Risk and Insurance, 84(2), 789–823.

Nelson, D.B. (1991). "Conditional Heteroskedasticity in Asset Returns: A New Approach." Econometrica, 59(2), 347–370.

Hansen, P. R., & Lunde, A. (2005). *A forecast comparison of volatility models: Does anything beat a GARCH(1,1)?* Journal of Applied Econometrics, 20(7), 873–889.

6 Appendix

6.1 Full Estimation Output: EGARCH(1,1) with Student-t Distribution

Model Summary:

- Mean Model: AR(1)
- Variance Model: EGARCH(1,1)
- Distribution: Standardized Student's t
- Log-Likelihood: -4360.192

Information Criteria:

Akaike Information Criterion (AIC)	2.4812
Bayesian Information Criterion (BIC)	2.4952
Shibata	2.4812
Hannan-Quinn	2.4862

Parameter Estimates (Robust Standard Errors):

Residual Diagnostics (Ljung-Box Tests):

ARCH LM Test (Selected Lags):

Nyblom Stability Test:

- Joint Statistic: 2.5401
- Critical Values (10%, 5%, 1%): 1.89, 2.11, 2.59

Sign Bias Test:

Adjusted Pearson Goodness-of-Fit Test:

Parameter	Estimate	Std. Error	t-Value	p-Value
μ	0.0620	0.0101	6.13	¡0.0001
$AR(1)$	-0.0385	0.0159	-2.42	0.0154
ω	-0.0269	0.0057	-4.68	¡0.0001
α_1	-0.1927	0.0193	-9.99	¡0.0001
β_1	0.9653	0.0071	136.23	¡0.0001
θ (leverage)	0.1742	0.0295	5.90	¡0.0001
γ (EQ$_t$)	0.1494	0.0536	2.79	0.0053
ν (shape)	6.0252	0.6362	9.47	¡0.0001

Table 8: EGARCH(1,1) model estimates using robust standard errors.

Test	Statistic	p-Value
Lag 1 Residuals	0.0159	0.8996
Lag 2 Residuals	0.1240	0.9998
Lag 5 Residuals	1.5570	0.8325

Table 9: Ljung-Box tests on standardized residuals.

6.2 EGARCH Model Results with Alternative Dummy Specifications

We estimated three additional EGARCH(1,1) models with Student's t-distributed errors using 2-day, 3-day, and 5-day earthquake dummies. The following tables report their robust parameter estimates and key diagnostics.

6.2.1 Model Comparison Summary

6.2.2 Key Residual Diagnostics (All Models)

- **Ljung-Box tests:** No serial correlation detected in residuals or squared residuals ($p > 0.80$ in most cases).
- **ARCH LM tests:** No significant ARCH effects in residuals for lags 3, 5, or 7 across models ($p > 0.67$).
- **Sign Bias tests:** Slight evidence of sign bias (p-values near 0.05), but no significant joint effect.
- **Nyblom stability tests:** Joint test statistic close to but below the 1% critical threshold in all models.

Lag	Statistic	Shape	Scale	p-Value
3	0.1774	0.500	2.000	0.6737
5	1.0106	1.440	1.667	0.7298
7	1.2984	2.315	1.543	0.8607

Table 10: Weighted ARCH LM test results.

Parameter	Individual Statistic
μ	0.5032
$AR(1)$	0.3456
ω	0.6938
α_1	0.2583
β_1	0.0860
θ	0.5337
γ	0.3899
ν	0.2761

Table 11: Nyblom individual parameter stability statistics.

- **Goodness-of-fit tests:** Slight misfit detected in Pearson tests (p-values $< 10^{-8}$), likely due to fat-tailed behavior.

6.2.3 Interpretation

While each of the extended dummy models performed reasonably well and returned statistically significant earthquake coefficients, the size of the coefficient declined as the window expanded. This suggests that the volatility impact is most concentrated on the day of the earthquake. Hence, we retain the same-day dummy (EQ_t) in our final model specification.

Note: Full parameter estimates for these models (including robust standard errors, t-values, and p-values) are available upon request.

Acknowledgement of AI Assistance

Portions of the writing, organization, and formatting were aided by OpenAI's ChatGPT. All empirical analysis, modeling choices, and conclusions are my own.

Component	t-Value	p-Value
Sign Bias	1.9429	0.0521
Negative Sign Bias	1.2954	0.1953
Positive Sign Bias	0.4982	0.6184
Joint Effect	4.1888	0.2418

Table 12: Sign bias test results.

Groups	Statistic	p-Value
20	74.56	1.58×10^{-8}
30	105.30	1.36×10^{-10}
40	109.59	1.26×10^{-8}
50	122.41	3.25×10^{-8}

Table 13: Goodness-of-fit statistics across different bin sizes.

Model	**EQ Dummy**	**AIC**	**LogLik**	**EQ Coefficient** (γ)
EGARCH(1,1)-t	Same-day (EQ_t)	2.4812	-4360.192	0.1494 (p = 0.0053)
EGARCH(1,1)-t	2-day window	2.4812	-4360.213	0.0849 (p = 0.0058)
EGARCH(1,1)-t	3-day window	2.4817	-4360.973	0.0584 (p = 0.0103)
EGARCH(1,1)-t	5-day window	2.4813	-4360.298	0.0456 (p = 0.0042)

Table 14: Model fit and exogenous coefficient comparison for alternative earthquake dummy specifications.