Off-policy value function approximation

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Two sided problems

- Off-policy target value

behavior → data

↑ Tabular case

target π

$G_t \rightarrow \rho G_t$

ρ = IS

- Off-policy target state distribution

$p^b \rightarrow p^\pi$

① Ignore

② correct

$\lambda = E_{s \sim p^\pi(s)}\left[\frac{1}{2}(G - V_\theta)^2\right]$

TD + semi-grad

$\lambda = E_{s \sim p^b(s)}$

Target value problem

Target value problem

- If the target is on-policy, it needs correction
  - MC, N-step need importance sampling
- Some targets are off-policy, no need for correction
  - Expected SARSA
  - Deterministic policies
  - Q-learning
  - Tree-backup

A bird eye view of on/off-policy

Algorithm	V/Q value	Make it off-policy	Variance	Bias
Monte Carlo	V	IS	High	Low
	Q	IS	High	Low
One-step SARSA	V	IS	Lower	High
	Q	IS	Lower	High
One-step Expected SARSA	V	IS	Lower	High
	Q	Already	Low	High
One-step TD with Deterministic Policy (including Q-learning)	V	IS	Lower	High
	Q	Already	Low	High
N-step SARSA (including lambda)	V	IS	Medium	Medium
	Q	IS	Medium	Medium
Tree backup	V	Already	Low	High
	Q	Already	Low	High

Semi-gradient with correction

- Off-policy N-step

$$g_{t:t+n} = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \dots \gamma^n v(s_{t+n})$$

$$v(s_t) \leftarrow v(s_t) + \alpha \rho_{t:t+n-1} \left[G_{t:t+n} - v(s_t) \right]$$

- Off-policy N-step semi-gradient

$$\theta \leftarrow \theta + \alpha \rho_{t:t+n-1} \left(g_{t:t+n} - v_\theta(s_t) \right) \nabla_\theta v_\theta(s_t)$$

Q-learning with approximation

- No need for off-policy correction

$$q(s,a) \leftarrow$$
$$q(s,a) + \alpha\left[r(s,a) + \max_{a'} q(s',a') - q(s,a)\right]$$

for until q_θ is stable **do**
  take action according to $q_\theta(s,a)$
  collect (s,a,r,s')
  $\delta = r + \gamma \max_{a'} q_\theta(s',a') - q_\theta(s,a)$
  $\theta \leftarrow \theta + \alpha\delta\nabla_\theta q_\theta(s,a)$
end for

Summary

- Target value problem is straightforward
- Make sure that the target value is corrected
- Only **one part** of the problem

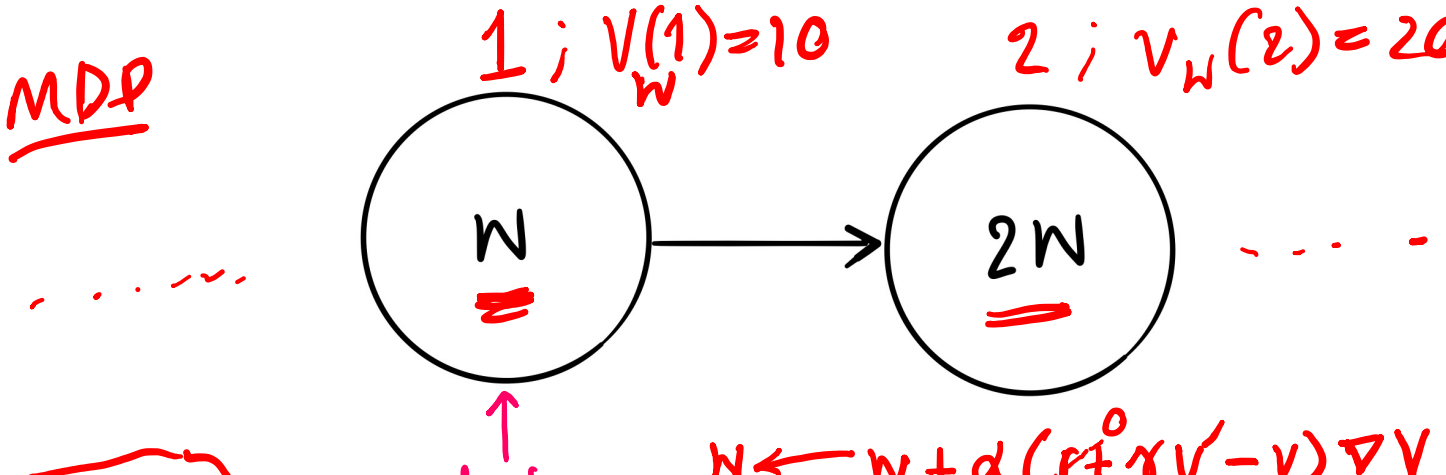
Target distribution problem

$p^b \Rightarrow p^\pi$

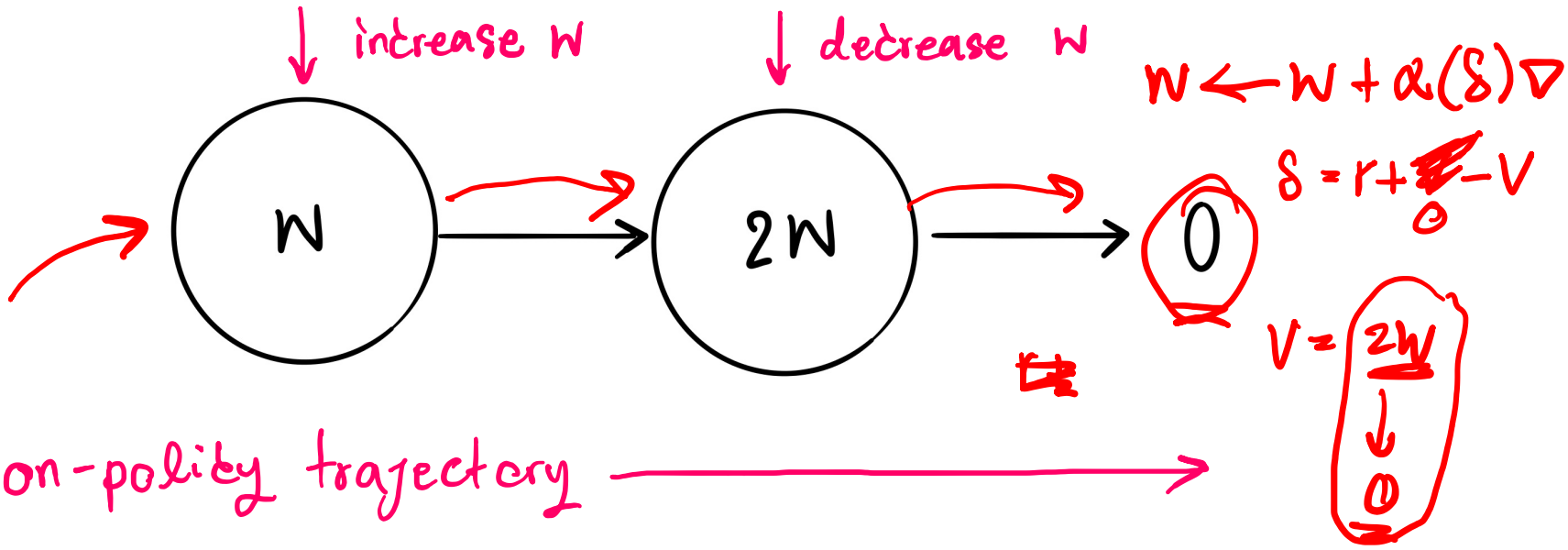
Target distribution problem

- Update distribution is "crucial" for convergence of semi-gradient
  - Off-policy data = off-policy distribution
    - Convergence guarantee only on-policy distribution
  - No convergence guarantee

Example of divergence



On-policy is more reasonable



linear case : $2w \rightarrow 0$ in one-update ($\alpha = 1$)

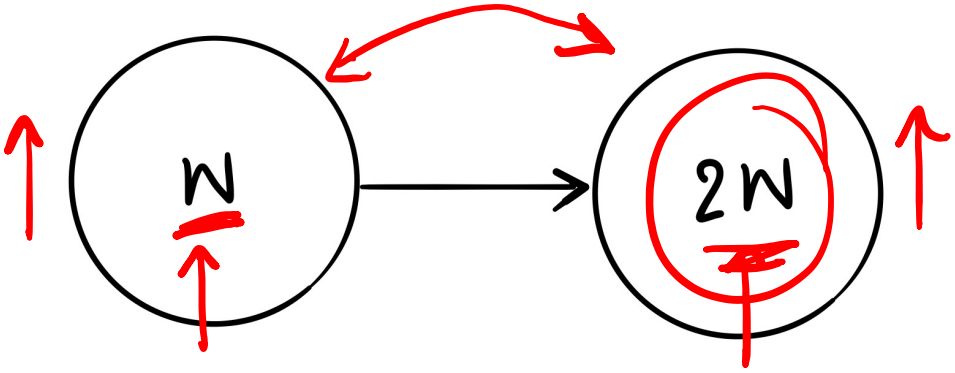
non-linear : not possible ! (needs multiple updates)

↳ on-policy + non-linear

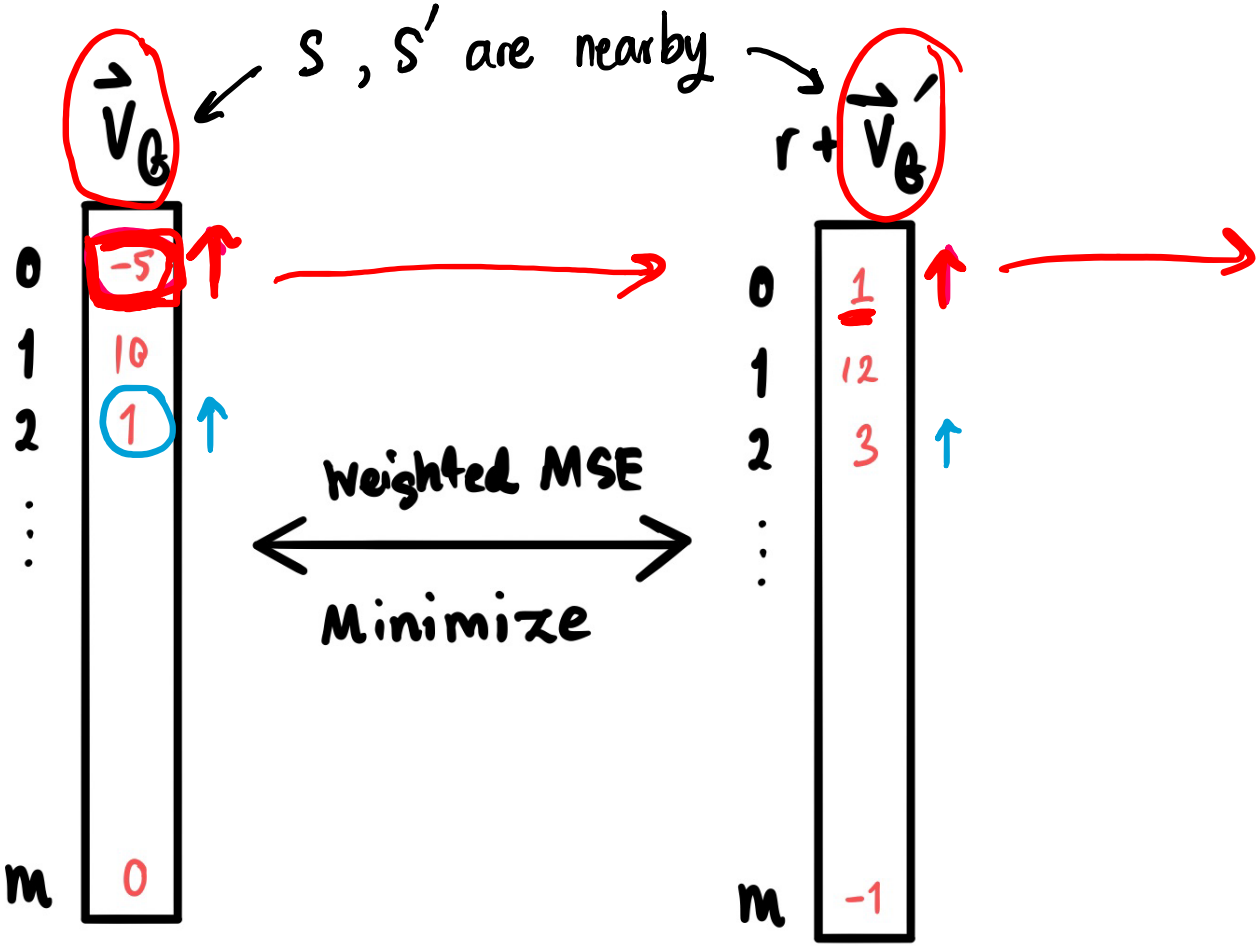
≠ convergence

Intuitive divergence

- Problem of generalization of nearby states



Intuitive divergence



The deadly triad

1. Off-policy
2. Bootstrapping
3. Approximation

TD (semi-gradient)

Having the three at the same time causes "instability"

(with semi-gradient)

Double is not deadly

1. **Off-policy + bootstrap:** + tabular

   Q-learning

2. **Off-policy + approximation:** + G_t

   Off-policy MC with approx.

3. **Bootstrap + approximation:** + on policy

   On-policy linear TD

All are stable.

When does divergence happen?

- When you use off-policy data
- You don't correct the target distribution
  - Even you have corrected the target value
- You use approximation
- **You use semi-gradient**
  - Imply bootstrapping

Bandages for semi-gradient

The deadly triad bandages

- Off-policy => **more on-policy** ①
- Bootstrapping => **less bootstrap** ②
- Approximation => **less approximate** ③

More on-policy

$p^b(s) \Rightarrow p^\pi(s)$

- On-policy state distribution correction
- Importance sampling

s_t

$$\theta \leftarrow \theta + \alpha \rho_{0:t-1} \rho_t \left(r_{t+1} + \gamma v_\theta(s_{t+1}) - v_\theta(s_t)\right) \nabla_\theta v_\theta(s_t)$$

Distribution correction

Target correction

$$\rho_{t:T-1} = \prod_{i=t}^{T-1} \frac{\pi(a_i|s_i)}{b(a_i|s_i)} = \frac{p^\pi(\tau)}{p^b(\tau)}$$

$s \sim p^b(s)$

$\rho \downarrow$

$s \sim p^\pi(s)$

- High variance

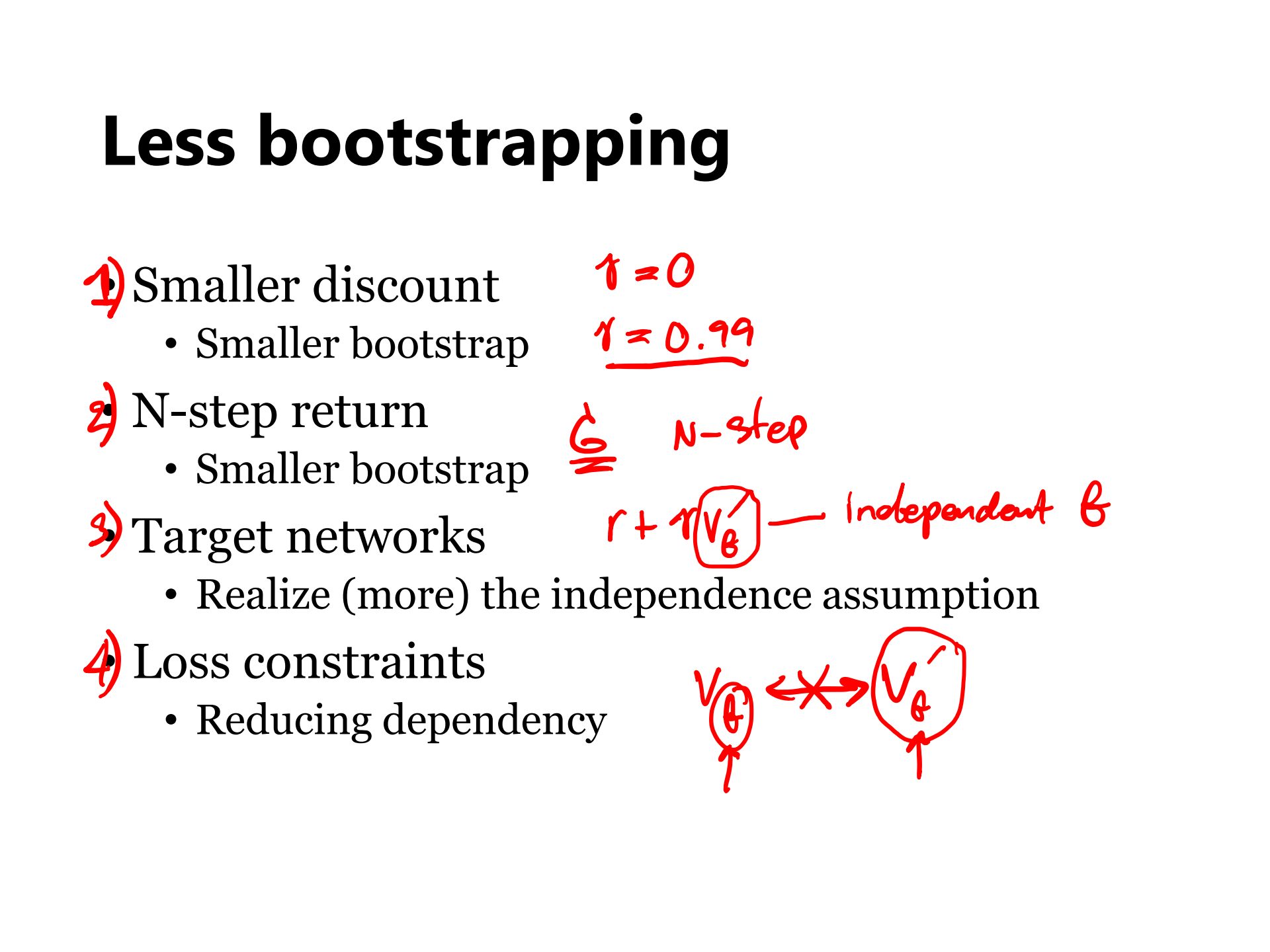
Less bootstrapping

1) Smaller discount
   - Smaller bootstrap

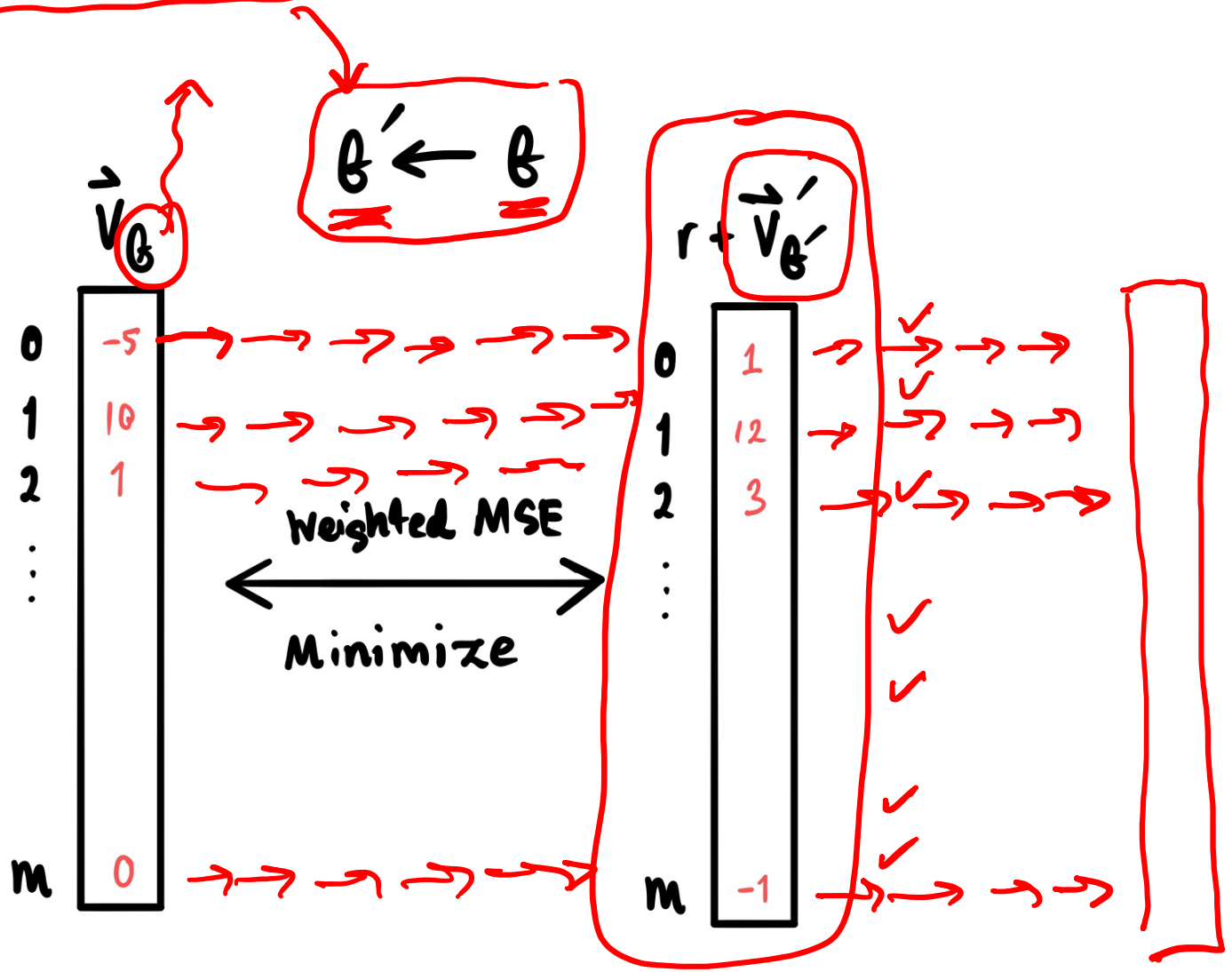
2) N-step return
   - Smaller bootstrap

3) Target networks
   - Realize (more) the independence assumption

4) Loss constraints
   - Reducing dependency



Target network visualized



Target networks (Q-learning)

$\theta' \leftarrow \theta$ target network θ'

for until q_θ is stable **do**

1) take action according to $q_\theta(s, a)$

2) collect (s, a, r, s')

3) $\delta = r + \gamma \max_{a'} q_{\theta'}(s', a') - q_\theta(s, a)$

4) $\theta \leftarrow \theta + \alpha \delta \nabla_\theta q_\theta(s, a)$

if every K steps **then**

$\theta' \leftarrow \theta$

end if

end for

DQN Atari

$K \approx 10{,}000$

how long do you need to get to $r + V_{\theta'}$

Loss constraints *

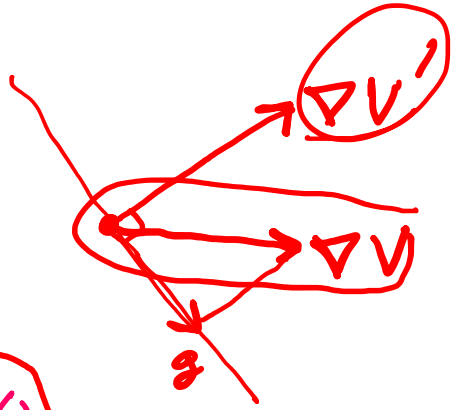
Reducing dependency between nearby states*

- Projected gradients

$V_\theta(s) \uparrow \quad V_\theta(s') \uparrow$

$V_\theta(s) \uparrow \quad V_\theta(s')$ unchanged

we use $g \approx \nabla V_\theta(s)$ s.t. $g \perp \nabla V_\theta(s')$



- Temporal consistency loss

$$TC(\theta) = \| V_t(s') - V_\theta(s') \|^2$$

↘ Pohlen, Tobias, Bilal Piot, Todd Hester, Mohammad Gheshlaghi Azar, Dan Horgan, David Budden, Gabriel Barth-Maron, et al. 2018. "Observe and Look Further: Achieving Consistent Performance on Atari." *arXiv [cs.LG]*. arXiv. http://arxiv.org/abs/1805.11593.

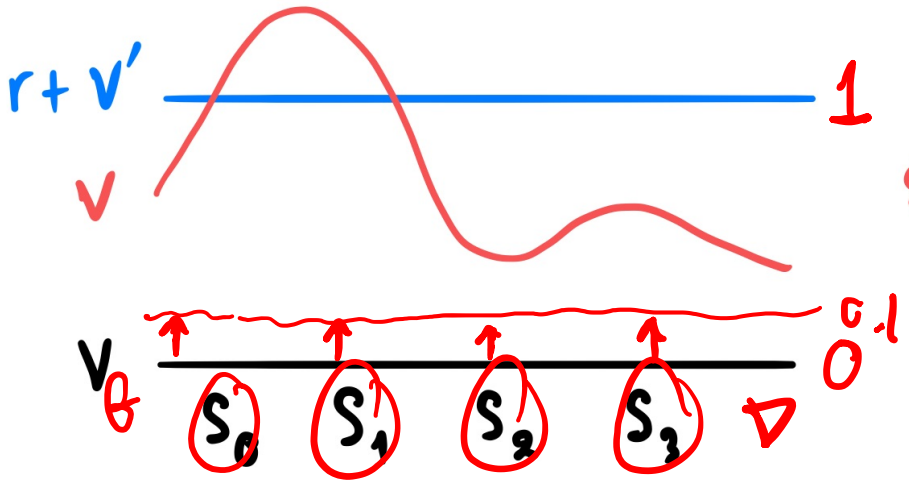
Less approximate

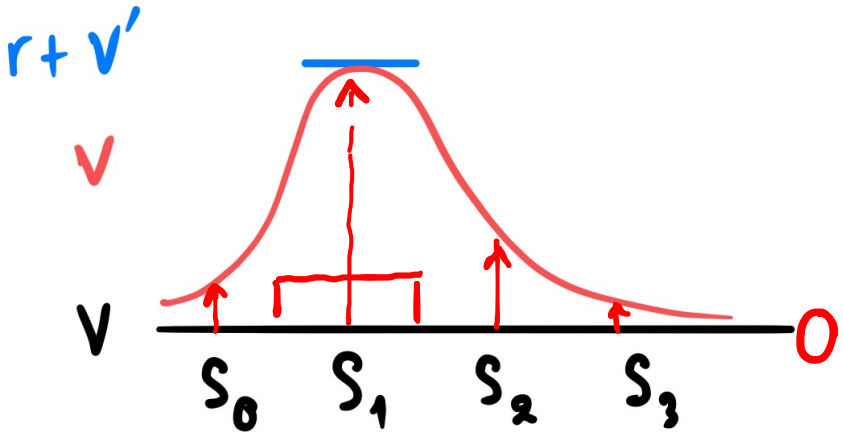
1) Increase the capacity of the approximator
   - This reduces the generalization effect

2) First-order tabular update approximation
   - Intuition, table is stable even with off-policy

First-order tabular update approximation

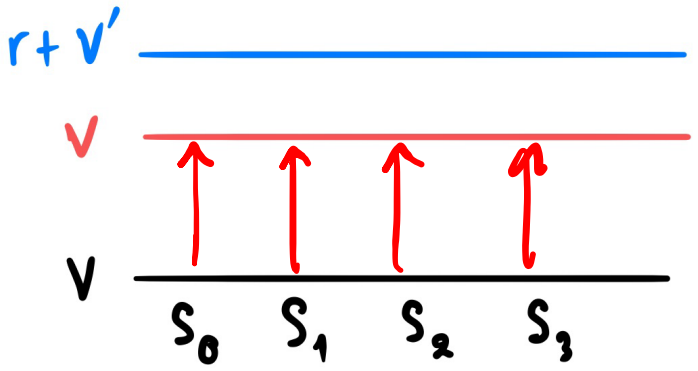
SGD (S) DU





First-order tabular update approximation

- Problems from “generalization”
- Problems from “slope”
  - SGD => steepest
  - Tabular => proportional
- If each update “knows” its “generalization”, it could correct itself!



Achiam, Joshua, Ethan Knight, and Pieter Abbeel. 2019. “Towards Characterizing Divergence in Deep Q-Learning.” *arXiv [cs.LG]*. arXiv. http://arxiv.org/abs/1903.08894.

Why do we love semi-gradient so much?

1. It is fast
2. It gives favorable fixed-point if trained successfully
   - TD fixed point
3. It is probably the one which is shown to work on large problems
   - Atari
   - Go

Moving to true gradient

Semi to true gradient

- **Semi-gradient is the culprit**
- True gradient guarantees convergence
  - Merit from SGD
  - Even in non-linear case
  - But to where?
- **Loss functions:**
  - TD error
  - Bellman error
  - Projected Bellman error

TD fixed point

TD Error (TDE)

$$\text{TDE}(\theta) = \mathbb{E}\left[(R_{t+1} + \gamma v_\theta(S_{t+1}) - v_\theta(S_t))^2 | A_t \sim \pi\right]$$

$$\text{TDE}(\theta) = \mathbb{E}\left[\delta^2 | A_t \sim \pi\right]$$

$S \sim p^\pi$

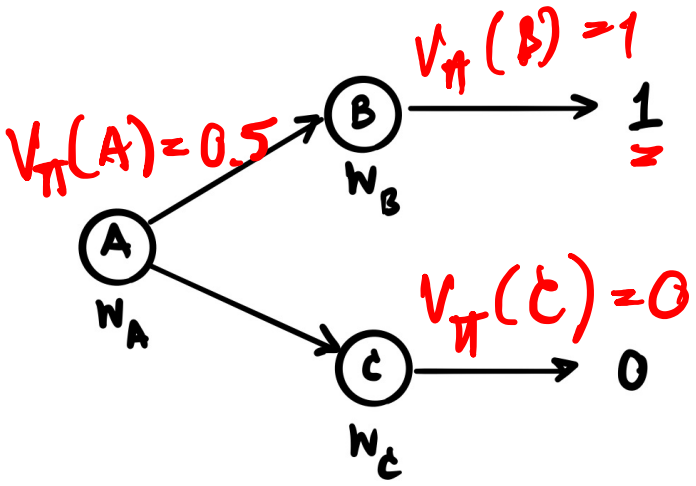
True gradient:

$$\theta \leftarrow \theta + \alpha\delta_t \left[\nabla_\theta v_\theta(s_t) - \gamma \nabla_\theta v_\theta(s_{t+1})\right]$$

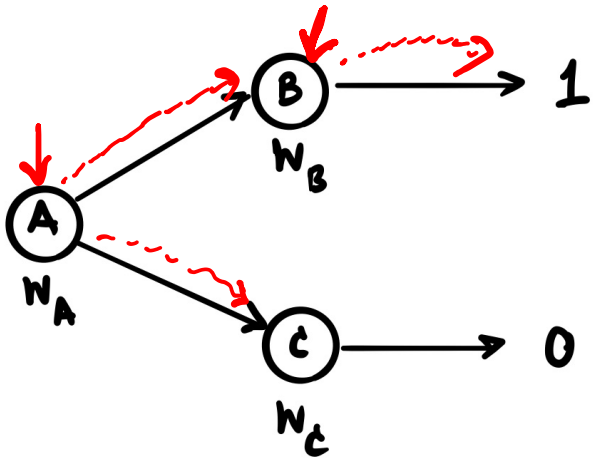
$$\theta \leftarrow \theta + \alpha \delta_t \nabla_\theta v_\theta(s_t)$$

How good is the fixed point?

A-split problem



Semi-grad TD on A-split



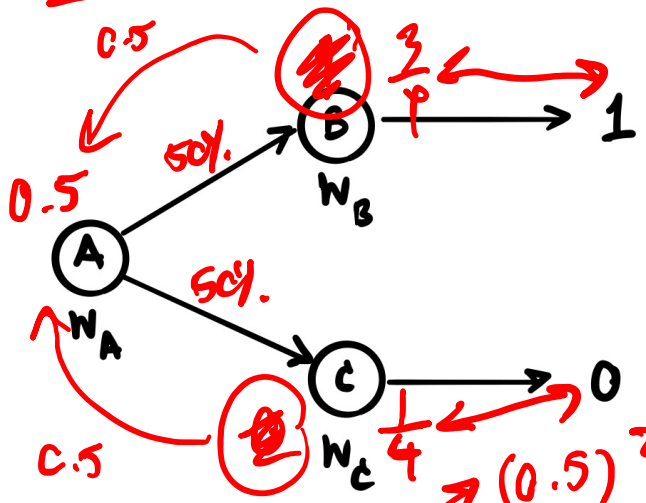
$$\min_{v_\theta} \mathbb{E}_\pi[\delta^2]$$

$w_A = 0.5$

$w_B = 1$

$w_C = 0$

TDE on A-split



TDE fixed point

$$\min_{\theta} \mathbb{E}_{\pi}[\delta^2]$$ Tends to be large

① $W_A = 0.5$

$W_B = 1 \leftarrow 0$

$W_C = 0 \leftarrow 0$

TD fixed point

$(0.5)^2 \Rightarrow \frac{1}{4}$

$(0.5)^2 \Rightarrow \frac{1}{4}$

$\frac{1}{4}$

② $W_A = 0.5$

$W_B = \frac{3}{4}$

$W_E = \frac{1}{4}$

$\frac{1}{16} = \frac{1}{2}\left(\frac{1}{2} - \frac{3}{4}\right)^2 + \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)^2$

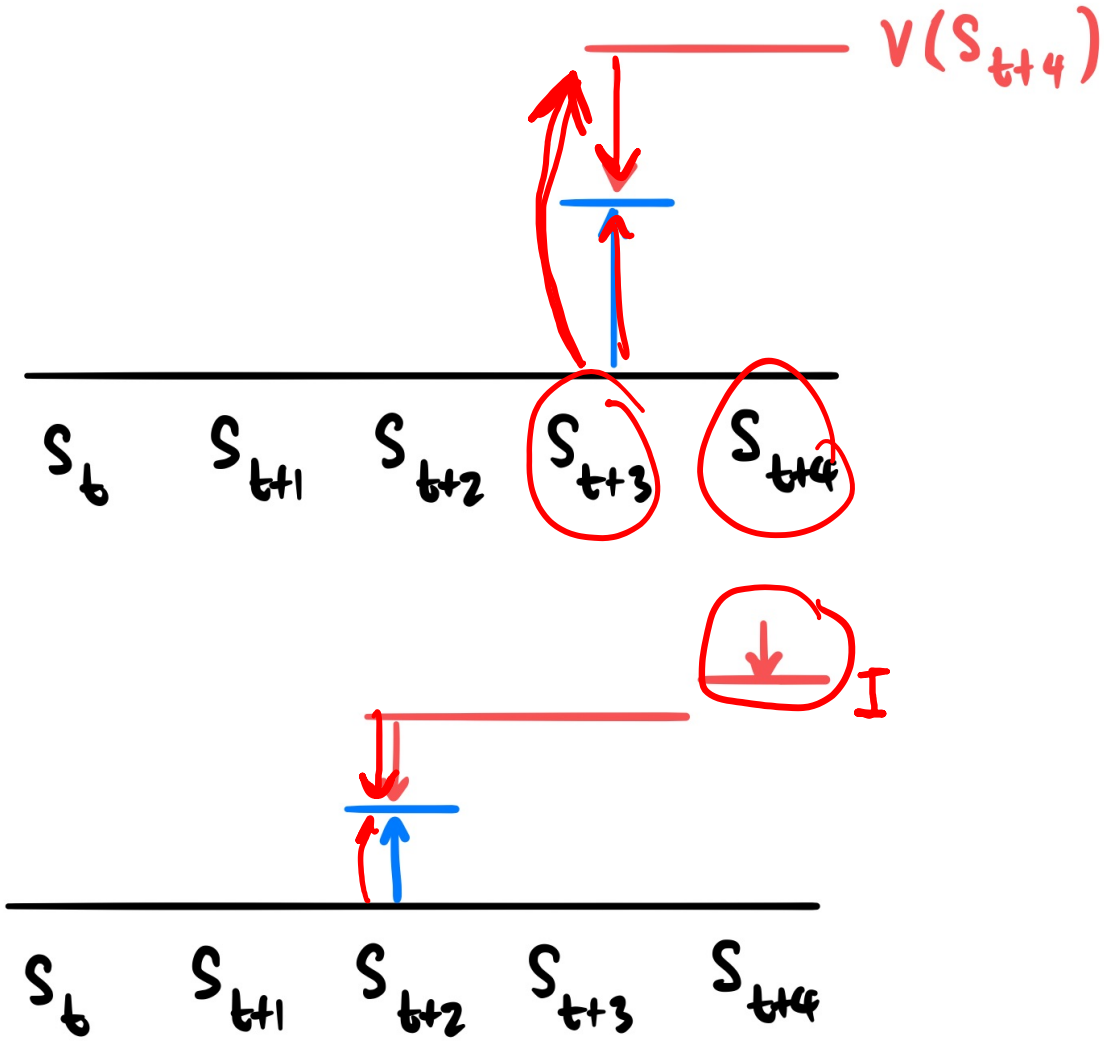
$\left(\frac{3}{4} - 1\right)^2 = \frac{1}{16}$

$\left(\frac{1}{4} - 0\right)^2 = \frac{1}{16}$

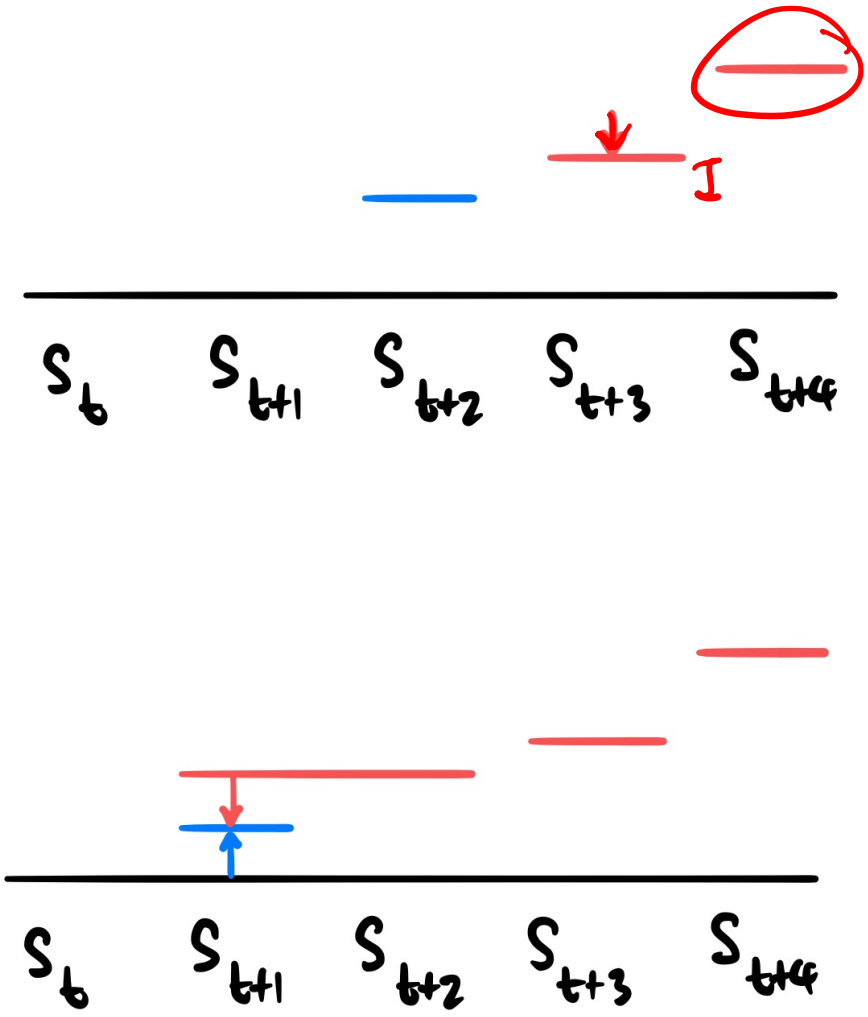
$\frac{3}{16}$

Temporal smoothing

Update visualized true SGD TD



Update visualized true SGD TD



TDE seems bad, Alternatives?

Bellman error

- Bellman equation

$$v(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) \left[r + \gamma v(s')\right]$$

- Formulation for single state

$$\begin{aligned}\overline{\delta_\theta}(s) &= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) \left[r + \gamma v_\theta(s')\right] - v_\theta(s) \\ &= \mathbb{E}_\pi \left[\delta_s\right]\end{aligned}$$

Bellman error

$$\overline{\delta_\theta}(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) \left[r + \gamma v_\theta(s')\right] - v_\theta(s)$$

$$= \mathbb{E}_\pi \left[\delta_s\right]$$

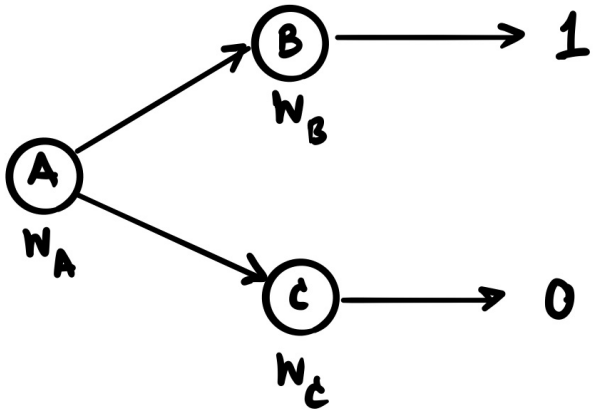
- Weighted squared error for all states

$$\|\overline{\delta_\theta}\|_\pi^2 = \mathrm{BE}(\theta) = \sum_s \mathbb{P}^\pi(s) \left[\mathbb{E}_\pi[\delta_s]^2\right]$$

↖ vs. $\mathbb{E}_\pi[\delta_s^2]$ TDE

- Minimize the above, using SGD

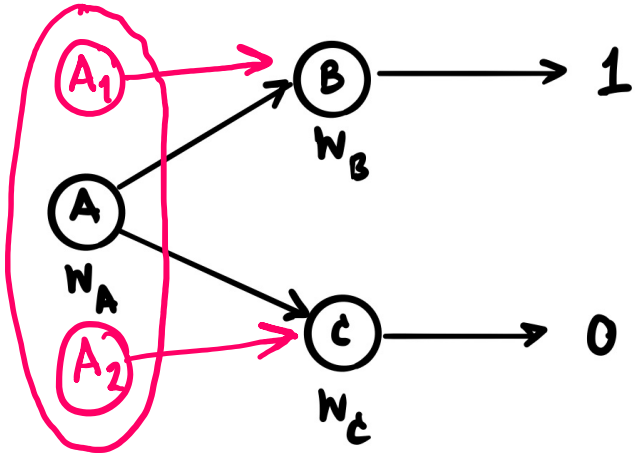
BE on A-split



$$\min_{\theta} \mathbb{E}_{\pi}[\delta]^2$$

Tends to be smaller

BE on A-split (v2)



$$\min_{\theta} \mathbb{E}_{\pi}[\delta]^2$$

Tends to be smaller

Doesn't help anymore

Indistinguishable state features

Bellman error's gradient

$$\overline{\delta_\theta}(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) \left[r + \gamma v_\theta(s')\right] - v_\theta(s)$$

$$\|\overline{\delta_\theta}\|_\pi^2 = \mathrm{BE}(\theta) = \sum_s \mathbb{P}^\pi(s) \left[\mathbb{E}_\pi[\delta_s]^2\right]$$

- Gradient

$$\nabla_\theta \mathrm{BE}(\theta) = \sum_s \mathbb{P}^\pi(s) \left[\mathbb{E}_\pi[\delta_s] \mathbb{E}_\pi[\gamma \nabla_\theta v_\theta(s') - \nabla_\theta v_\theta(s)]\right]$$

Double sampling problem

$$\|\overline{\delta_\theta}\|_\pi^2 = \mathrm{BE}(\theta) = \sum_s \mathbb{P}^\pi(s) \left[\mathbb{E}_\pi[\delta_s]^2\right]$$

$$\nabla_\theta \mathrm{BE}(\theta) = \sum_s \mathbb{P}^\pi(s) \left[\mathbb{E}_\pi[\delta_s]\mathbb{E}_\pi[\gamma \nabla_\theta v_\theta(s') - \nabla_\theta v_\theta(s)]\right]$$

Bellman error summary

- Impractical
- Might converge to undesirable fixed point
- A better loss function needed
  - Projected Bellman error
  - How to get its gradients (GTD2)

Read further

- Sutton 2018, Chapter 11
- Maei, Hamid Reza. 2011. “Gradient Temporal-Difference Learning Algorithms.” University of Alberta.
- Topics:
  - Projected Bellman error
  - GTD2

Levels of guarantees

- **Stability vs convergence**
  - Stability is easier to guarantee
- **On-policy vs off-policy**
  - On-policy is easier to guarantee
- **Linear vs non-linear**
  - Linear is easier to guarantee

Convergence of off-policy non-linear function approximation:

- Doesn’t imply a “good” fixed point

Summary of gradients

1. **Semi gradients** = converge only on linear with on-policy
   - Semi-gradient TD
   - You need luck and tricks
2. **True gradients** = converge even on non-linear, both on-policy and off-policy
   - Value error
   - TD error
   - Bellman error
   - ~~~~~

Assignment

Off policy + bootstrapping + approximate

- Q-learning with function approximation notebook (Github)