Question 1

The decision version of the problem can be stated as follows:

Inputs:

- A set of projects **T**, where each project t is associated with positive integers $\boldsymbol{e_t}$ and $\boldsymbol{p_t}$.
- A positive integer **c** denoting the maximum amount of effort that can be spent on the projects.
- A positive integer **r** denoting the amount of profit to be exceeded (lower bound profit).

Outputs:

- **YES** if there exists a subset of projects in T such that the total amount of effort spent to complete the projects is at most c and the profit gained is greater than r.
- **NO** otherwise.

Decision Problem: "Given a set of projects T, where each project t is associated with positive integers e_t and p_t, and a positive integer c denoting the maximum amount of effort that can be spent on the projects, determine if it is possible to select a subset of projects from T such that the total effort spent on those projects does not exceed c and the profit gained is greater than r. The output should be YES if such a subset exists, and NO if it does not."

In this problem, the goal is to determine if it is possible to select a subset of projects from T such that the total effort spent on those projects does not exceed c and the profit gained is greater than r. The inputs to the problem are the set of projects T, the maximum effort c and the lower bound profit r, and the output is a YES or NO answer indicating whether it is possible to find a subset of projects that meets the given constraints.

Question 2

To prove that the decision problem is in NP (nondeterministic polynomial-time), we need to show that given a proposed solution (a subset of projects) or a guess, we can verify its correctness in polynomial time.

In this problem, a solution is considered correct if the total effort spent on the selected projects does not exceed c and the profit gained is greater than r. To verify that a proposed solution is correct, we can simply check that the total effort spent on the selected projects is at most c and that the profit gained by completing the projects is greater than r. To check that the total effort spent on the selected projects is at most c, we can simply sum the efforts required for each of the selected projects and compare the result to c. This can be done in $O(n)$ time, where n is the number of projects in the set T. To check that the profit gained by completing the projects is greater than r, we can simply sum the profits for each of the selected projects and compare the result to r. This can also be done in $O(n)$ time. Therefore, both checks can be done in $O(n)$ time, and the solution can be verified in polynomial time. The decision version is in NP.

To verify the solution, we can simply perform both checks. Return YES if both checks are successful and NO if either check fails. This can be done in $O(n)$ time, as both checks can be performed in $O(n)$ time.

Additionally, there are 2^n possible subsets of T which takes $O(2^n)$ time to generate all solutions (brute–force algorithm to guess the solution). Therefore, the problem can be solved in $O(2^n)$ time (exponential solveability), and can be verified in $O(n)$ time (polynomial verifiability) which makes the decision problem an element of NP.