Chapter 9 Sinusoids and Phasors

<table>
<tr><td>교과목명
(Course Title)</td><td>화로이론(2)</td><td>교과목번호
(Course Code)</td><td colspan="2">BC3000571</td><td>분반
(Section)</td><td>130</td></tr>
<tr><td>개설학과
(Department)</td><td>IT응용공학과</td><td>개설학년
(Level)</td><td colspan="2">2</td><td>학점-이론-실습</td><td>3-3-0</td></tr>
<tr><td>강의시간 및 강의실
(Class Hours & Classroom)</td><td colspan="6">화,목: 10:30~11:45/ 3503호실 / 강의자료 http://plato.pusan.ac.kr</td></tr>
<tr><td rowspan="2">담당교수
(Lecturer)</td><td rowspan="2">권혁숭</td><td colspan="2">연구실
(Office)</td><td>3563</td><td>상담가능시간
(Office Hours)</td><td>화,목: 14:00~16:30</td></tr>
<tr><td colspan="2">연락처
(Telephone)</td><td>010-9384-1372</td><td>이메일
(E-mail)</td><td>hskwon@pusan.ac.kr</td></tr>
<tr><td>수업방식
(Methodology of Instruction)</td><td colspan="6">☑ 강의식 □ PBL □ TBL □ 온라인콘텐츠활용 □ 기타 (학생간 멘토-멘티 학습)</td></tr>
<tr><td>평가방법
(Evaluation and Grading)</td><td colspan="6">출석: 5% 퀴즈:25% 과제:5% 중간고사:30% 기말고사:35%</td></tr>
<tr><td>선수과목 및 지식
(Prerequisites)</td><td colspan="6">공업수학, 회로이론(1)</td></tr>
<tr><td>교수목표
(Course Objectives)</td><td colspan="6">RLC소자에 대한 DC특성과 기본 법칙 등이 AC에서 어떻게 적용되는지 확인하고, 교류를 인가함으로 인하여 발생하는 여러 가지 특징과 주파수 특성에 대해 학습한다. 또한 회로를 해석하기 위한 여러 가지 변환 이론, Laplace Transformation, Fourier Series, Fourier Transformation을 회로를 해석하는 방법을 학습한다.</td></tr>
<tr><td>강의개요
(Course Description)</td><td colspan="6">- 전기소자 R, L, C에 대한 이해
- AC에 적용되는 주요 법칙 및 정리 회로망 해석
- 주파수 응답과 과도응답 해석
- Laplace transformation, Fourier Series, Fourier Transformation
- P-Spice 해석법</td></tr>
</table>

교과목과 핵심역량과의 관계(Relationship between Courses and Core Competencies)								
부산대학교 8대 핵심역량 (8 Core Competencies of PNU)	글로벌문화 역량 (Global-Cultural Competency)	소통역량 (Communication Competency)	융복합역량 (Convergence Competency)	응용역량 (Application Competency)	봉사역량 (Community Service Competency)	인성역량 (Human Character Competency)	기초지식역량 (Foundation Knowledge Competency)	고등사고역량 (High-order Thinking Competency)
				○				

교과목에 따른 핵심역량(Core Competencies Based on Courses and Educational Methods)

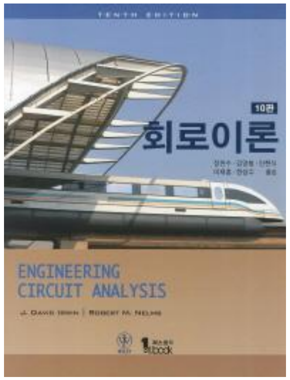
학과 핵심역량(Core Competencies of Department)		교육방법(Educational Methods)
	수학적 모델링을 기반으로 한 IT기술을 문제해결에 적용할 수 있다	R,L,C 수동소자의 교류입력에 대한 회로의 페이저 해석방법을 학습하고 주파수 응답에 대한 고찰을 한다.

교재 및 참고자료(Textbooks and References)

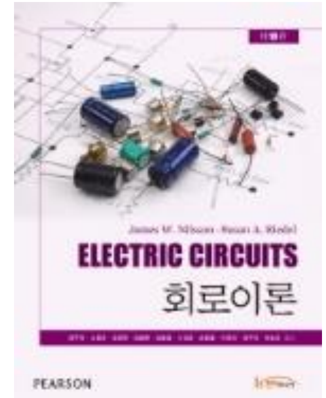
주교재 (Required Textbooks)	회로이론(제7판) 저자: Charles K Alexander and Matthew N. Sadiku	출판사: Mcgrow-Hill(교보문고)
참고자료 (References)	회로이론(9판) Irwin & Nelms	퍼스트북

주별 강의계획(Weekly Schedule of Classes)		
주차 (Week No.)	강의 및 실험 실기 내용 (Course Material)	과제 및 기타 참고사항 (Assignments and Other Notes)
제1주 (Week 1)	회로이론(1) 전반적인 개념 / [표절 등 학술적 부정행위 예방교육 실시]	
제2주 (Week 2)	제9장 정현파와 페이저 / [표절 등 학술적 부정행위 예방교육 실시]	
제3주 (Week 3)	제9장 정현파와 페이저	
제4주 (Week 4)	제10장 정현파의 정상상태 해석	
제5주 (Week 5)	제10장 정현파의 정상상태 해석/ 퀴즈	Quiz
제6주 (Week 6)	제11장 교류전력해석	
제7주 (Week 7)	제11장 교류전력해석	
제8주 (Week 8)	제13장 자기결합회로/ 중간고사	**Mid Exam.**
제9주 (Week 9)	제13장 자기결합회로	
제10주 (Week 10)	제13장 자기결합회로	
제11주 (Week 11)	제14장 주파수 응답/ 퀴즈	
제12주 (Week 12)	제14장 주파수 응답	
제13주 (Week 13)	제14장 주파수 응답	
제14주 (Week 14)	제14장 주파수 응답 / 제15장 라플라스변환	
제15주 (Week 15)	제16장 라플라스변환 응용 / 제17장 퓨리에급수	
제16주 (Week 16)	제18장 퓨리에변환 / 기말고사	**Final Exam.**
첨부파일 (attachment)		

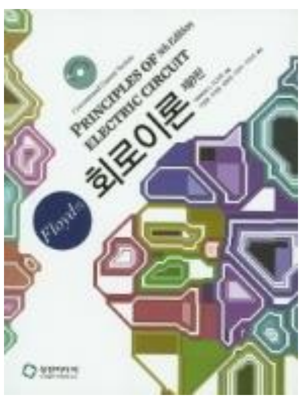
Reference(Text)



회로이론10th(Irwin)

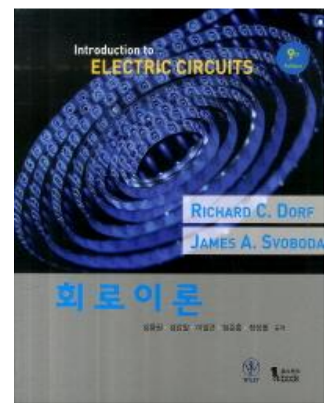


회로이론10th(Nilsson)



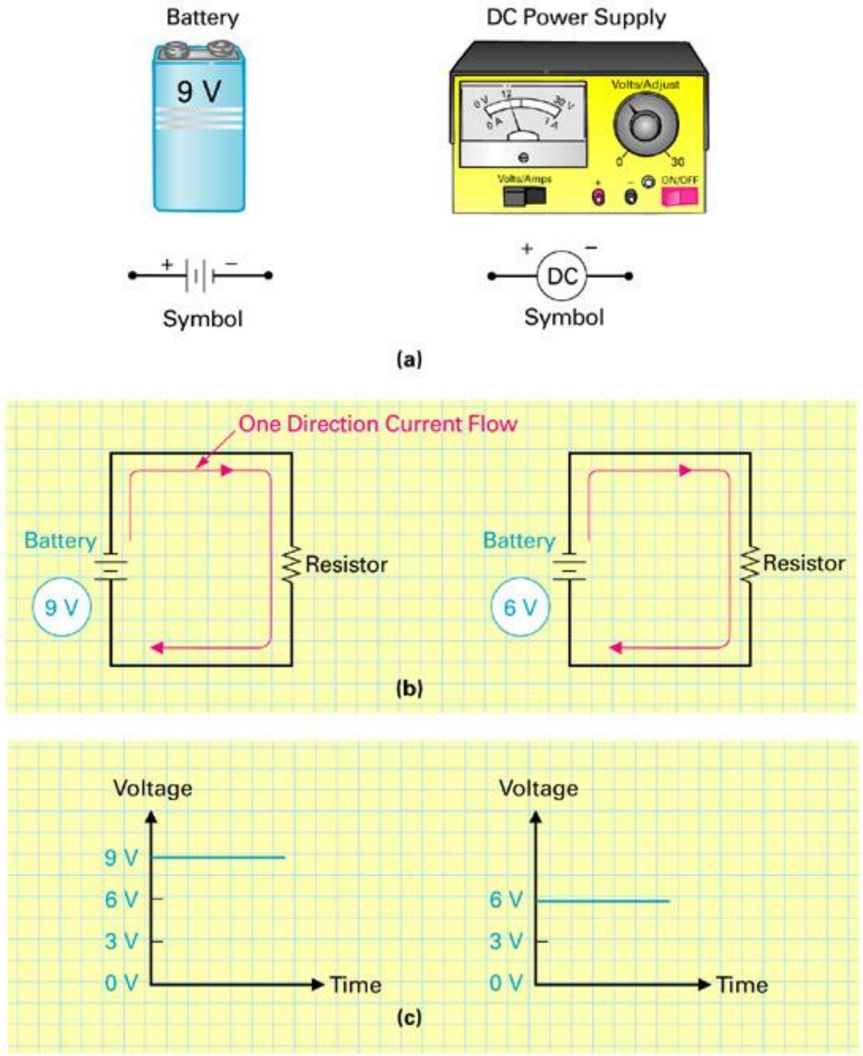
회로이론8th(Floyd)

http://www.bbc.com/news/magazine-38208814



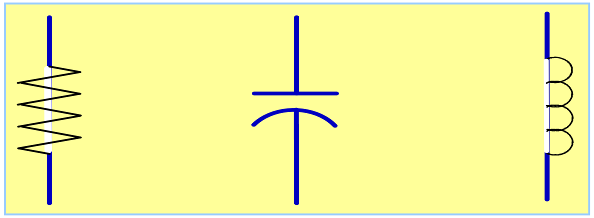
회로이론9th(Drof)

9.1 Introduction

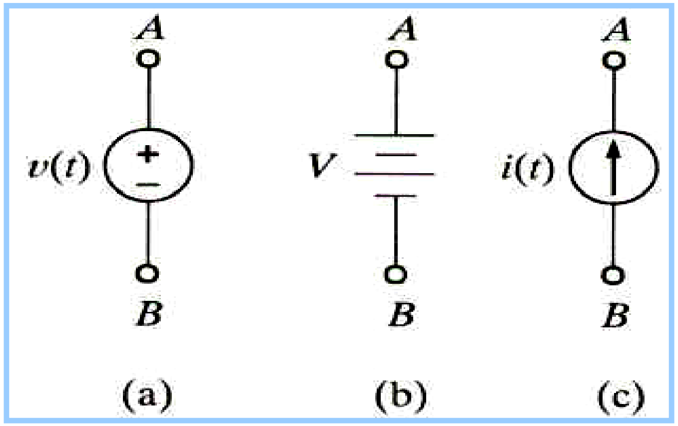


Direct Current. (a) DC Sources. (b) DC Flow. (c) Graphic Representation of DC

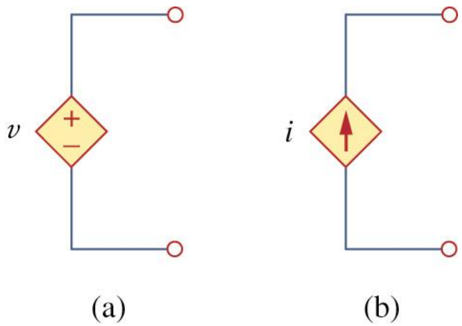
PASSIVE ELEMENTS



INDEPENDENT SOURCES

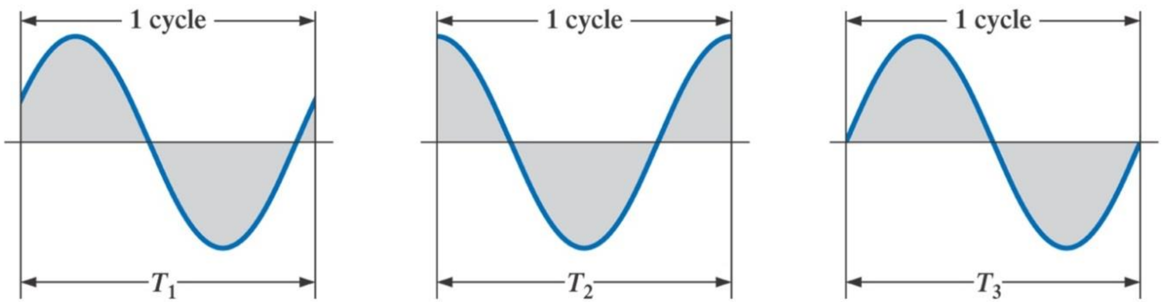


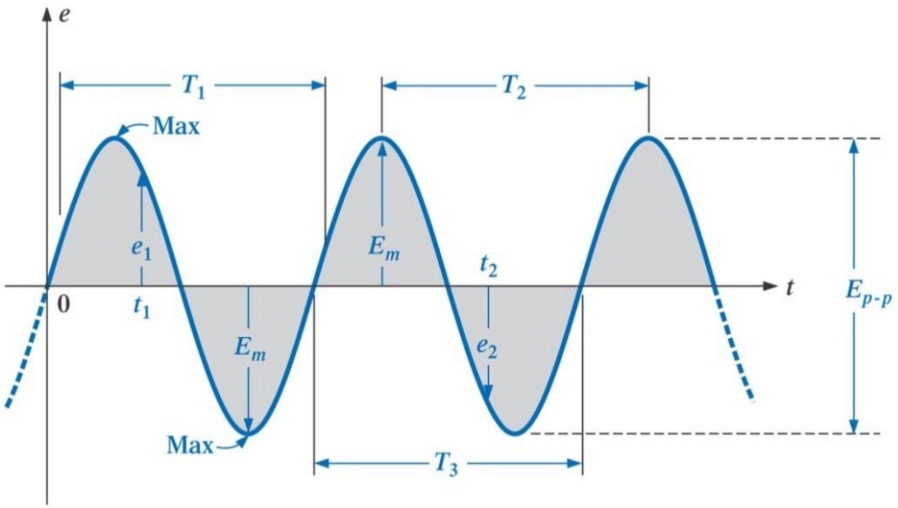
DEPENDENT SOURCES



- A Sinusoid is a signal that has the form of the sine or cosine function.

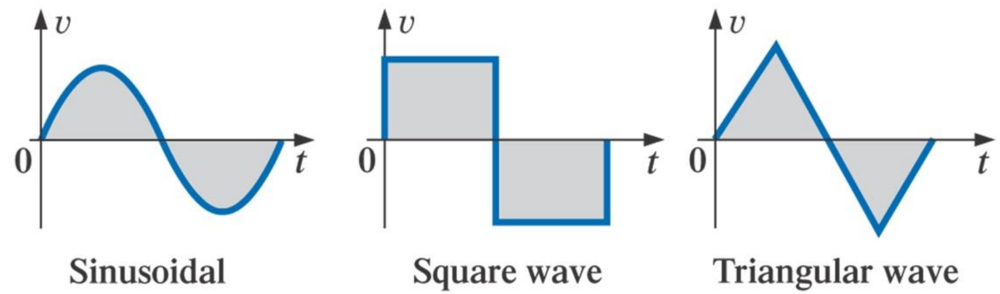
- Nature itself is characteristically sinusoidal. *(pendulum's variation, string's variation etc.)*
- A sinusoid signal is easy to generate and transmit.
- Fourier analysis*(periodic signal can be represented by a sum of sinusoid)*
- A sinusoid signal is easy to handle mathematically.*(derivative and integral of sin are themselves sin)*

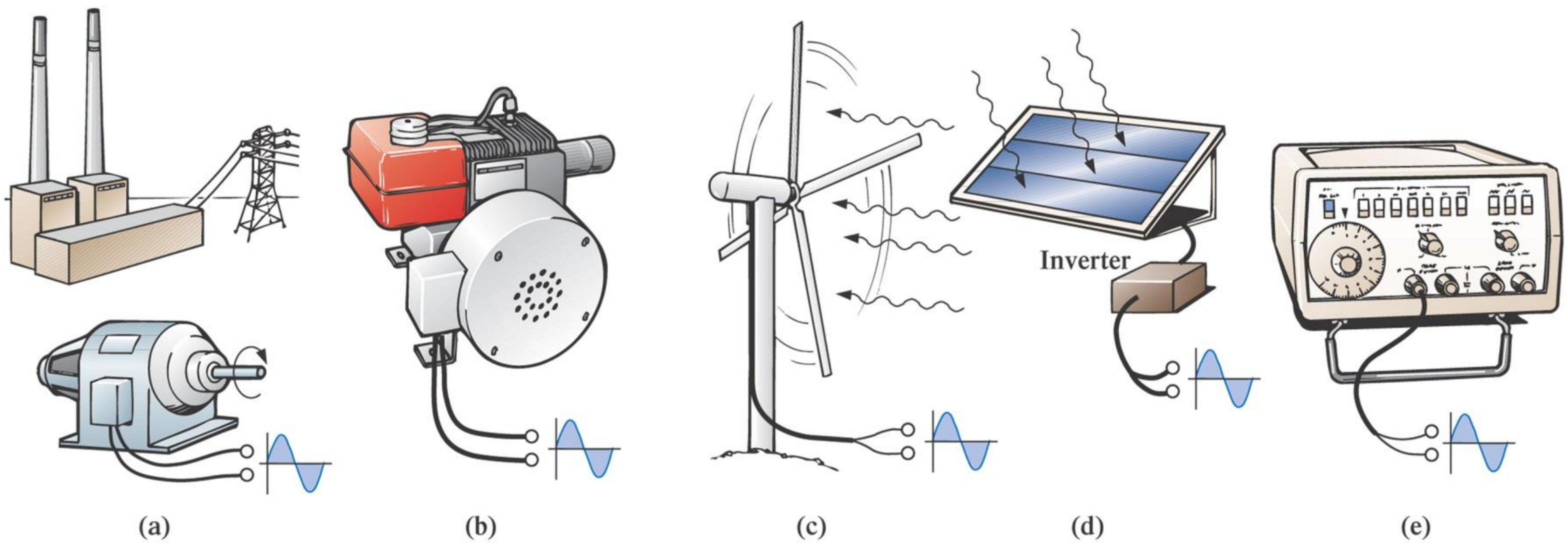




9.1 Introduction

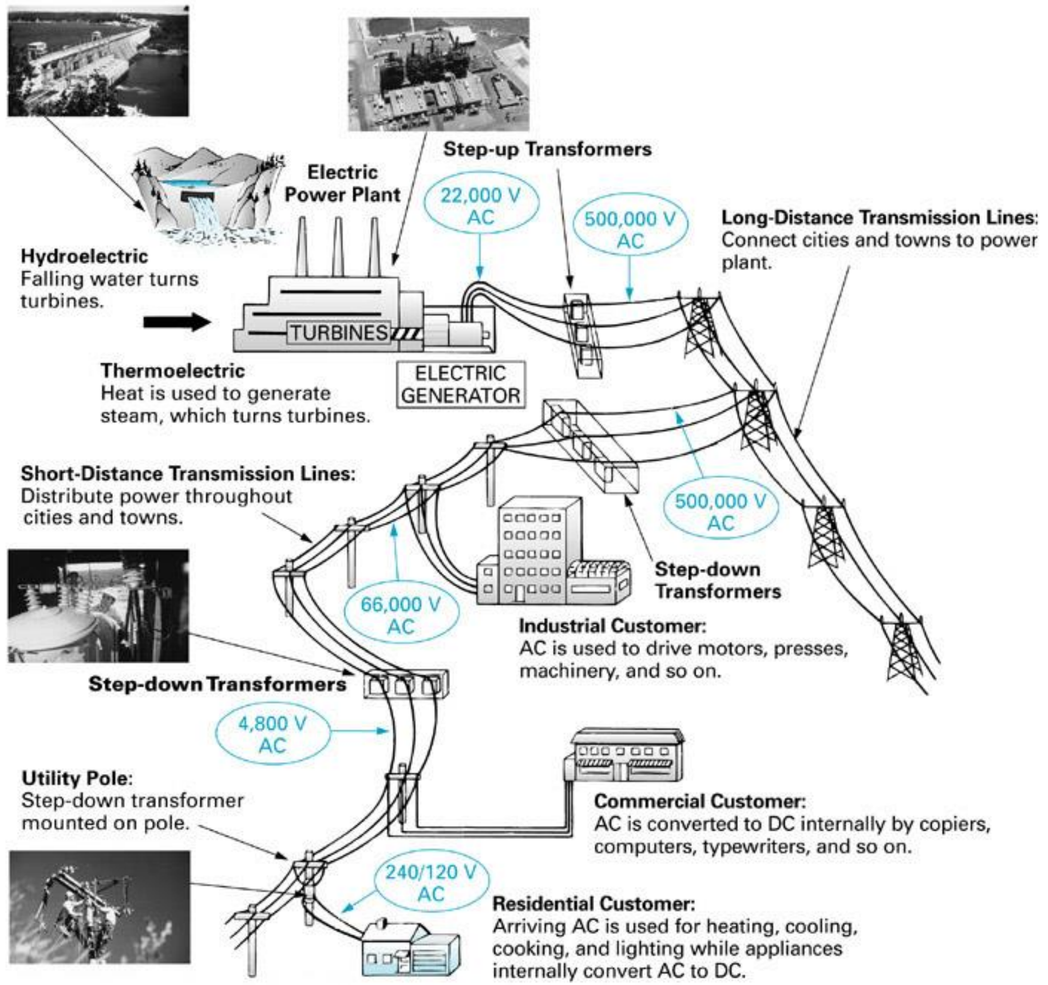
- Alternating waveforms



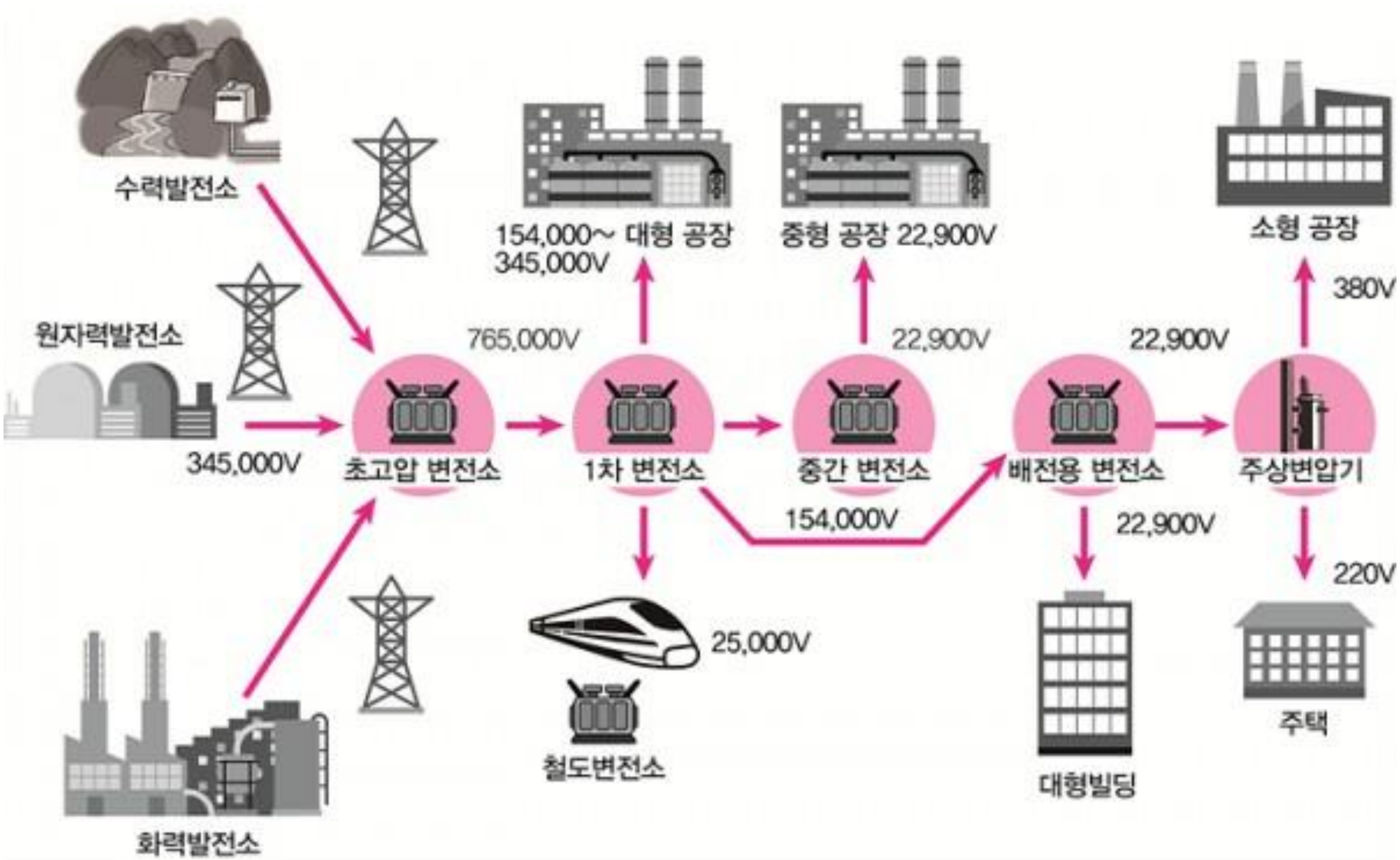


- Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

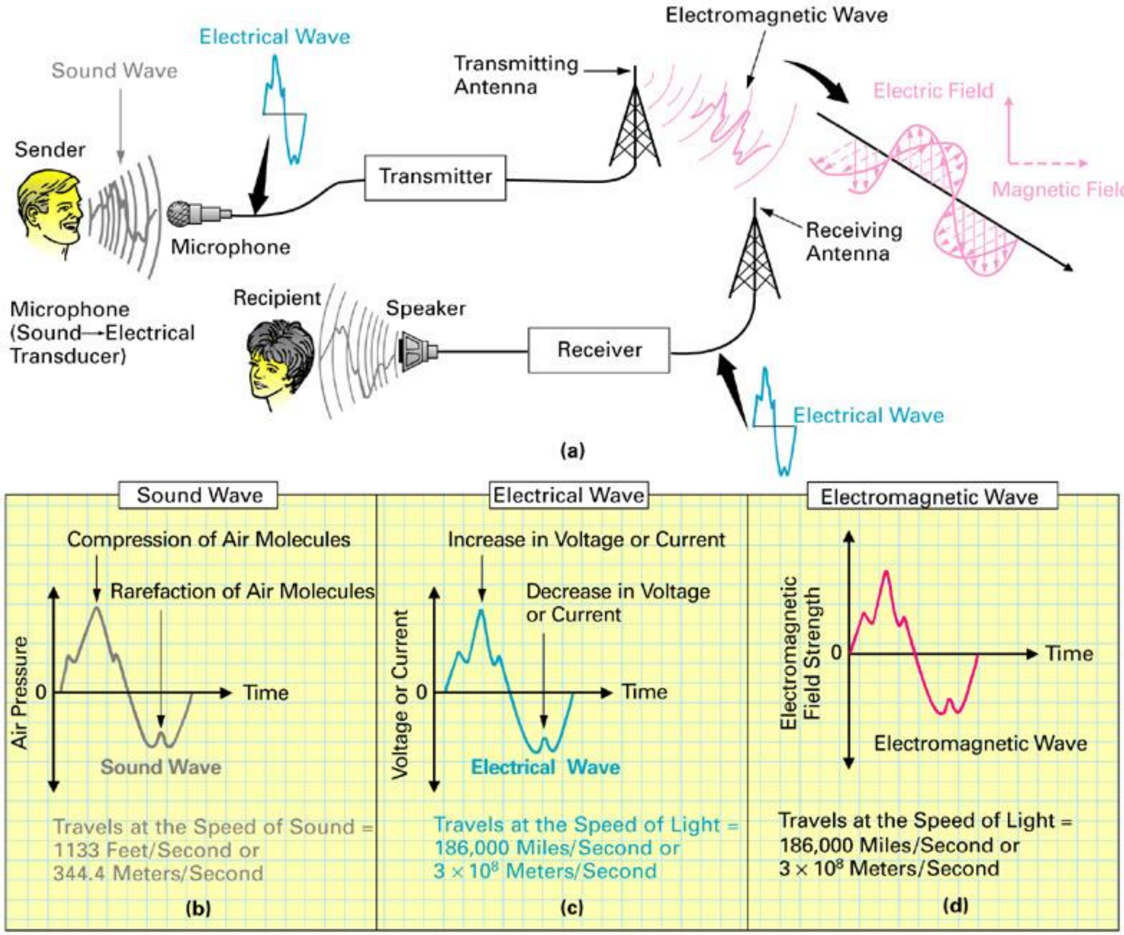
1. AC Power Distribution.

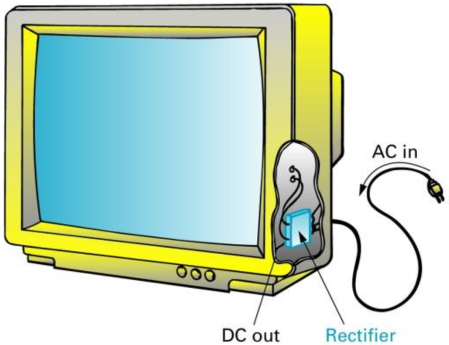


Electric generator(22.9kV)→step-up transformers (154kV/345kV/765kV)→long distance transmission line→ step-down transformers(22.9kV)→residential customer(380/220V)


수력발전소
원자력발전소
345,000V
화력발전소
765,000V
초고압 변전소
154,000~ 대형 공장
345,000V
1차 변전소
25,000V
철도변전소
154,000V
중형 공장 22,900V
22,900V
중간 변전소
22,900V
배전용 변전소
22,900V
대형빌딩
소형 공장
380V
주상변압기
220V
주택

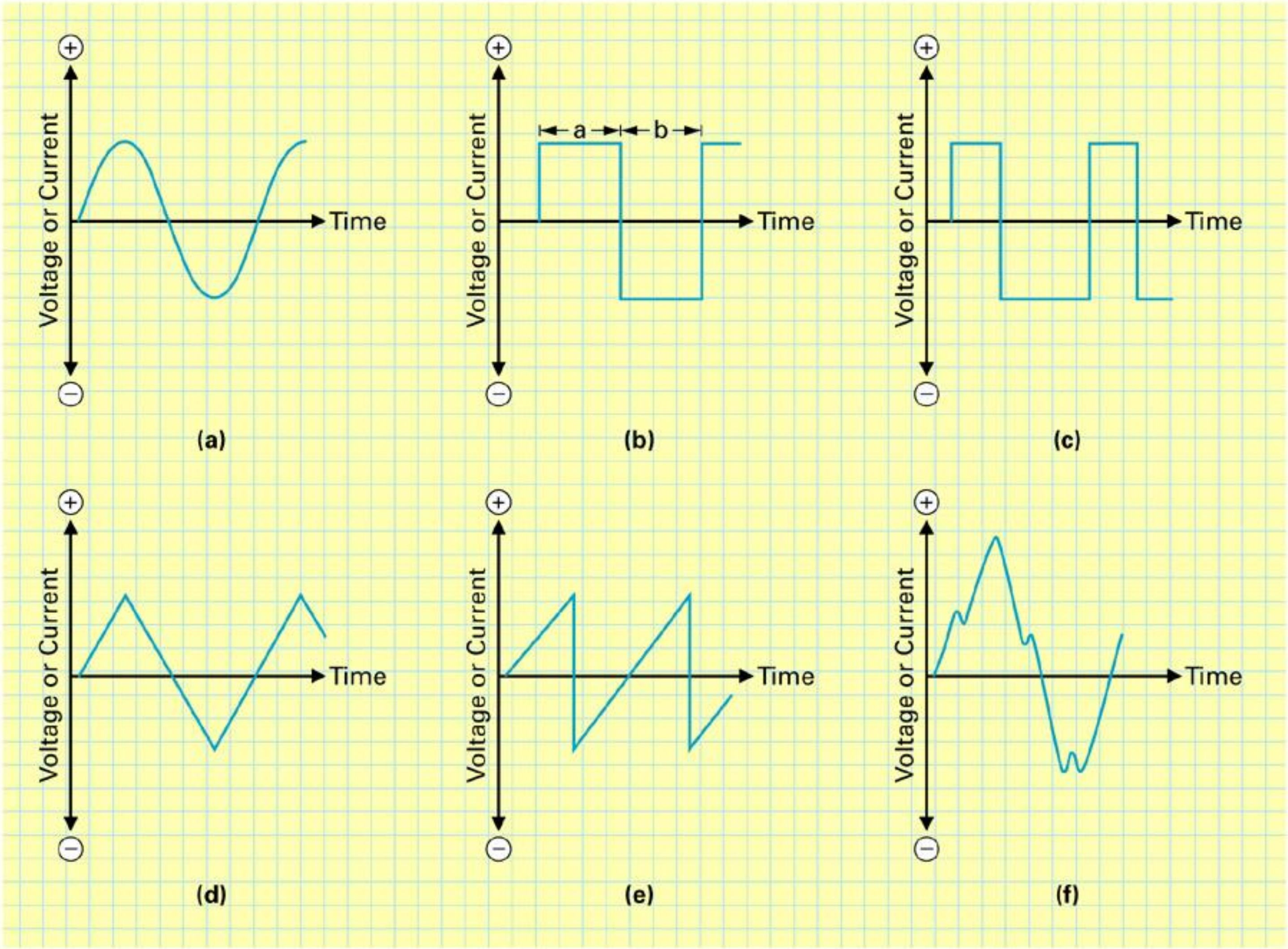
2. Information Transfer

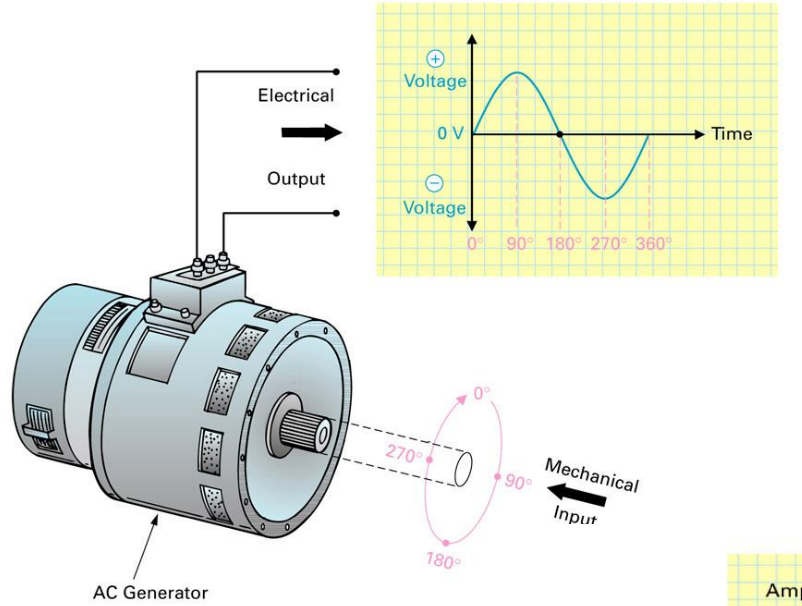




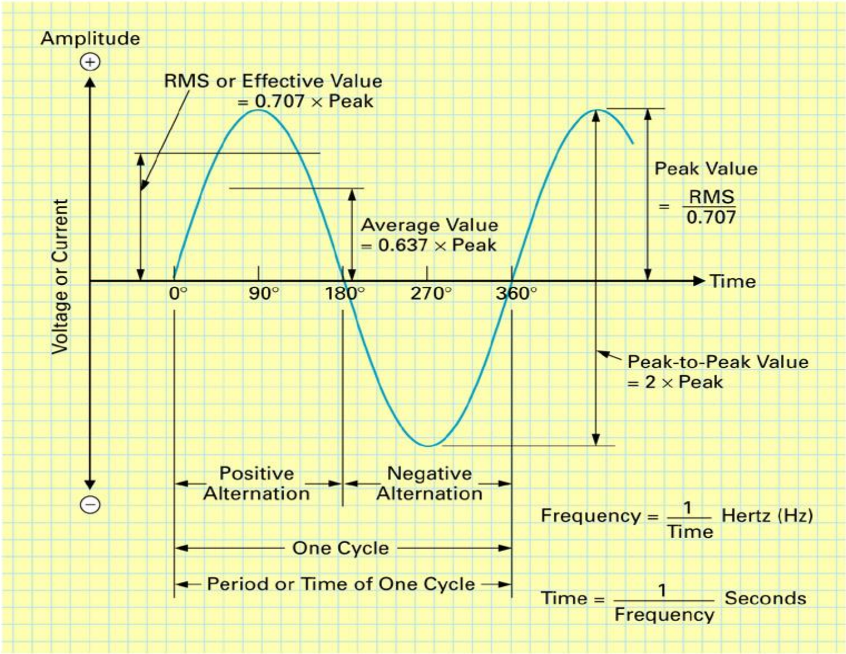
Rectification (Converting AC to DC) by Appliances.

AC Wave Shapes. (a) Sine Wave. (b) Square Wave. (c) Pulse Wave.
(d) Triangular Wave. (e) Sawtooth Wave. (f) Irregular Wave





Degrees of a Sine Wave



9.2 Sinusoids

$$v(t) = V_m \sin \omega t$$

V_m = the amplitude of the sinusoid

ω = the angular frequency in radians/s

ωt = the argument of the sinusoid

$$T = \frac{2\pi}{\omega}$$

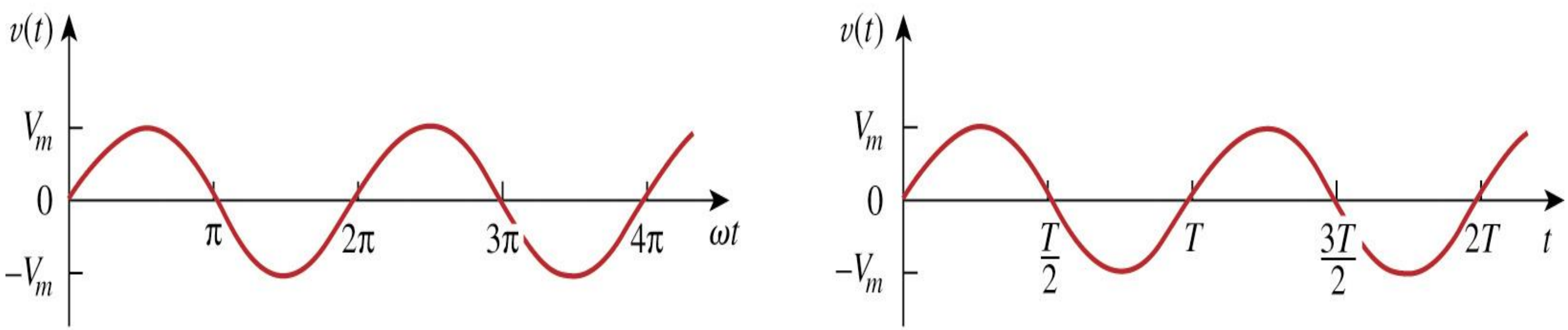
$$v(t+T) = V_m \sin \omega(t+T) = V_m \sin \omega\left(t + \frac{2\pi}{\omega}\right)$$

$$= V_m \sin(\omega t + 2\pi) = V_m \sin \omega t = v(t)$$

Hence,

$$v(t+T) = v(t)$$

- A periodic function is one that satisfies $f(t)=f(t+nT)$, for all t and for all integers n.



(a) $\boxed{\omega t}$

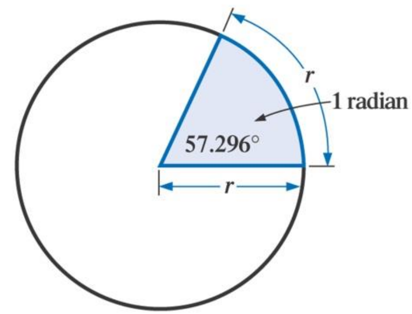
(b) $\boxed{T}$

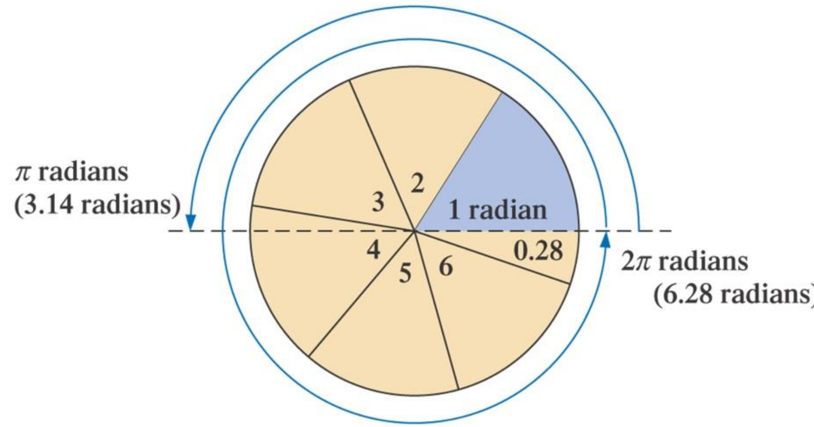
$$1\,\text{rad} = 57.296^{\circ} \cong 57.3^{\circ}$$

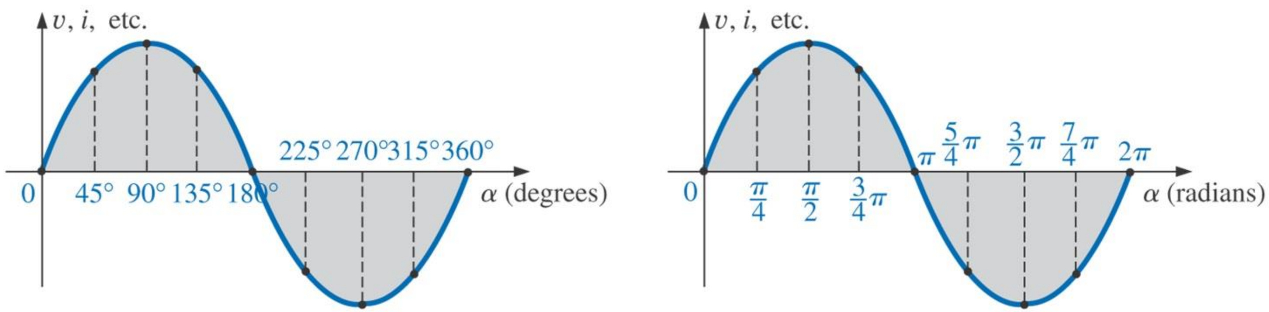
$$2\pi\ \text{rad} = 360^{\circ}$$

$$\text{radians} = \left(\frac{\pi}{180^{\circ}}\right) \times (\text{degrees})$$

$$\text{degrees} = \left(\frac{180^{\circ}}{\pi}\right) \times (\text{radians})$$







(a) (b)

- A periodic function is one that satisfies $f(t)=f(t+nT)$, for all t and for all integers n.

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T}$$

$$\omega = 2\pi f$$

$*$ General expression

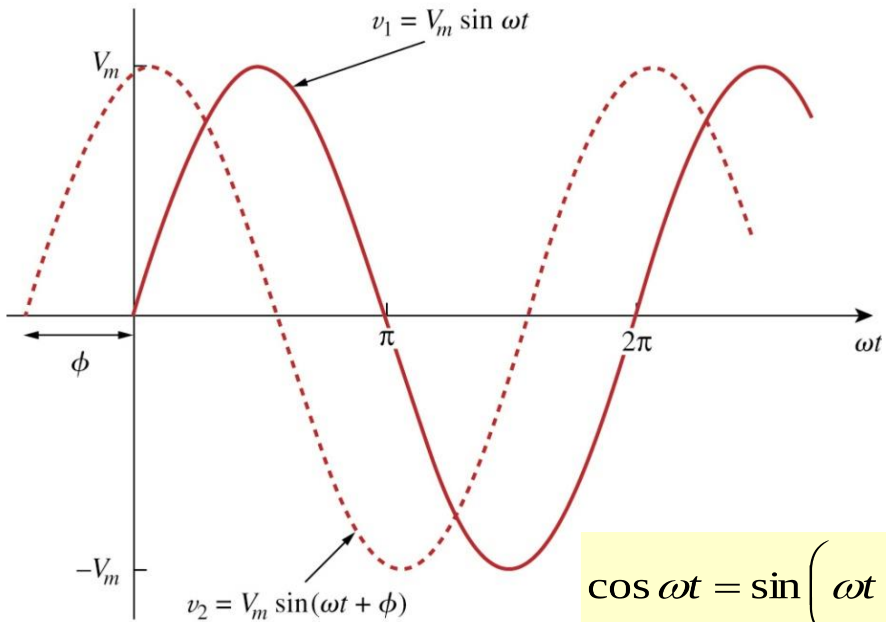
$$v(t) = V_m \sin(\omega t + \phi)$$

$$v_1(t) = V_m \sin \omega t \qquad and$$

$$v_2(t) = V_m \sin(\omega t + \boxed{\phi})$$

argument

Phase (lead)



$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\sin \omega t = \cos\left(\omega t - \frac{\pi}{2}\right)$$

- Only two sinusoidal values with the same frequency can be compared by their amplitude and phase difference.
- If phase difference is zero, they are **in phase**;
- If phase difference is not zero, they are **out of phase**.

- 정현파를 **sin** 과 **cosin** 형태로 표현 가능(삼각함수 적용)
  진폭과 위상을 파악하기 쉽다

(same frequency, positive amplitude, expression sin or cos wave)

$$\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\sin\omega t = \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

with these identities, it is easy to show that

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$

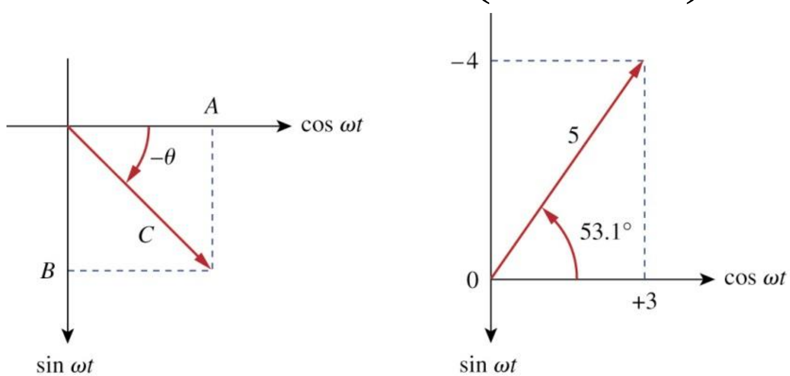
$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

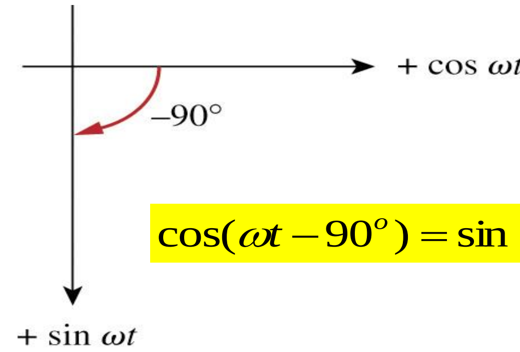
$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$

$$C = \sqrt{A^2 + B^2}, \qquad \theta = \tan^{-1}\frac{B}{A}$$

$$3\cos\omega t - 4\sin\omega t = 5\cos(\omega t + 53.1^\circ)$$

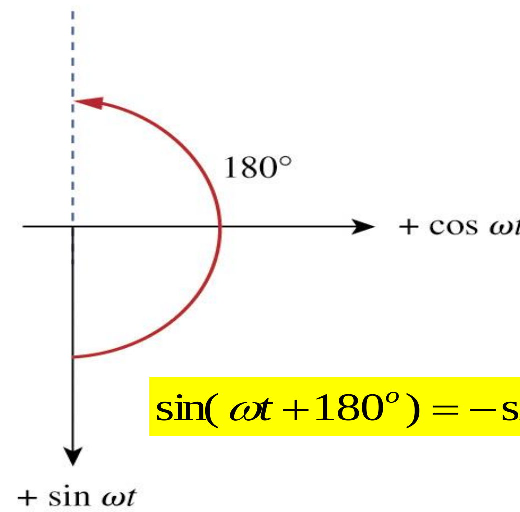


(a) (b)



$$\cos(\omega t - 90^\circ) = \sin\omega t$$

(a)



$$\sin(\omega t + 180^\circ) = -\sin\omega t$$

(b)

Ex. 9.1

Find the amplitude, phase, period and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

The amplitude is $V_m = 12V$

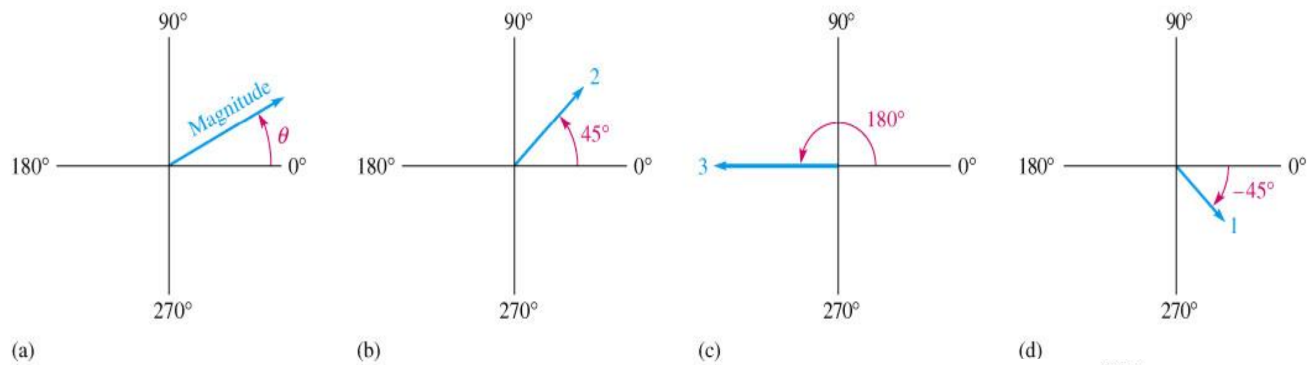
The phase is $\phi = 10^\circ$.

The angular frequency is $\omega = 50\text{rad/s}$.

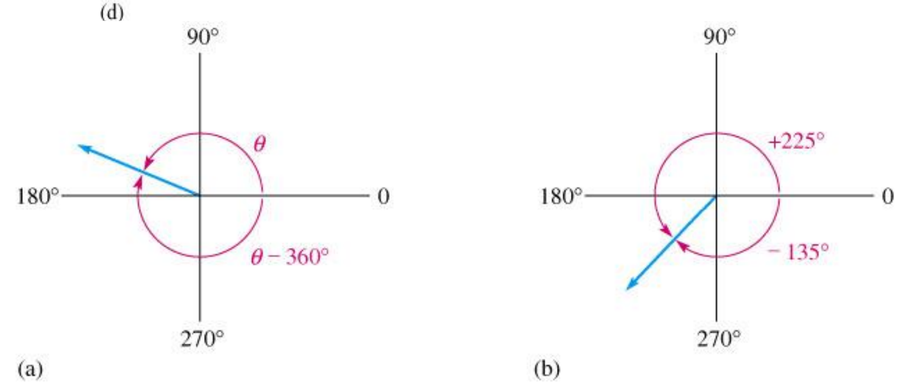
The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257s$.

The frequency is $f = \frac{1}{T} = 7.958Hz$

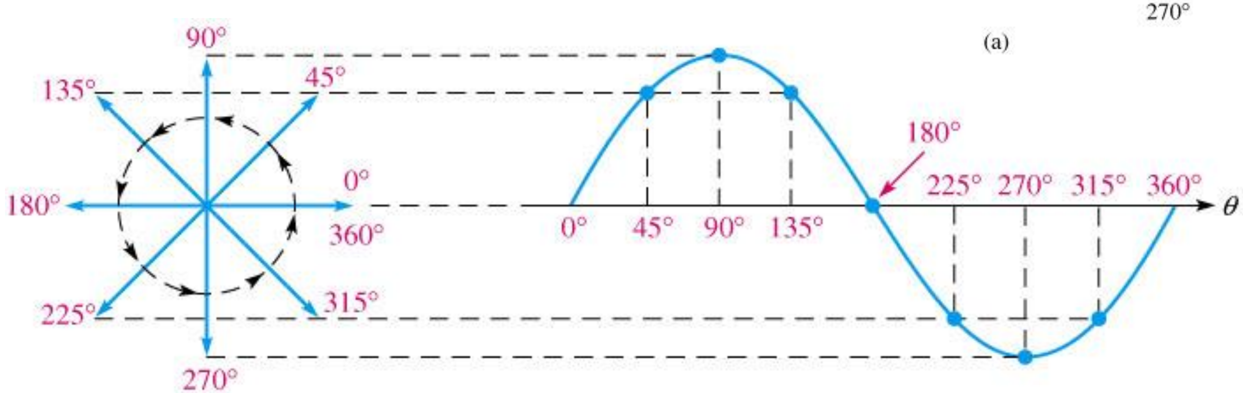
9.3 Phasors



Example of the Phasor
(magnitude, phase)

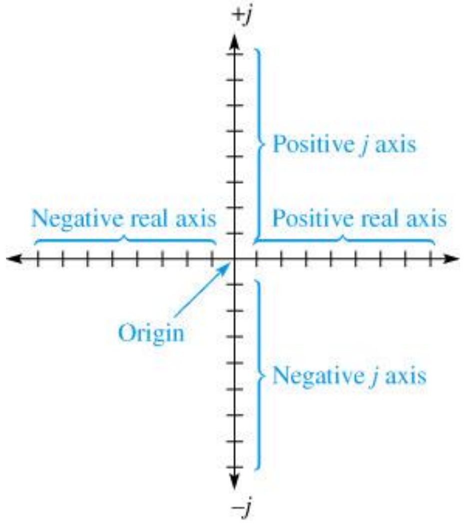


Positive and negative phasor angles

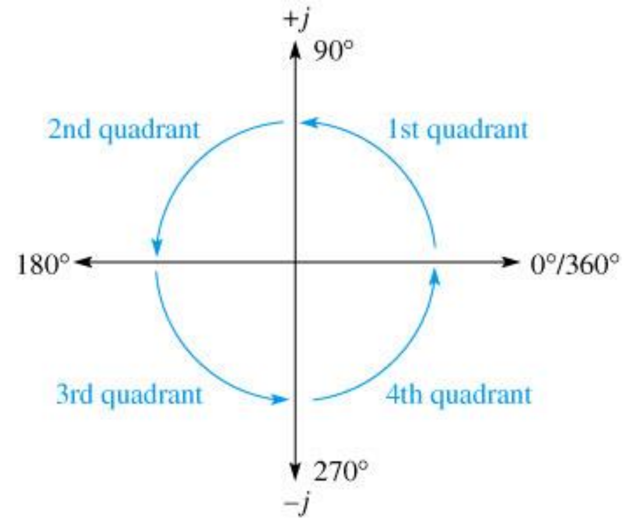


Sine wave represented by rotational phasor motion

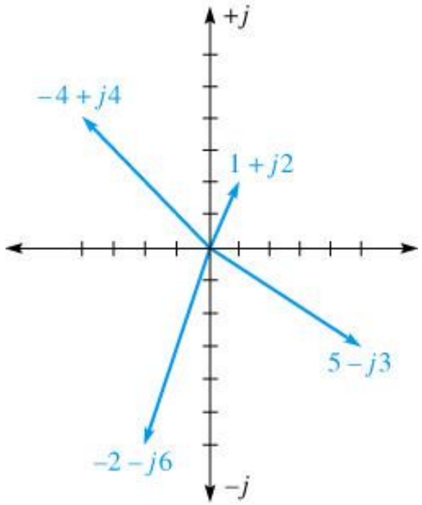
Complex plane



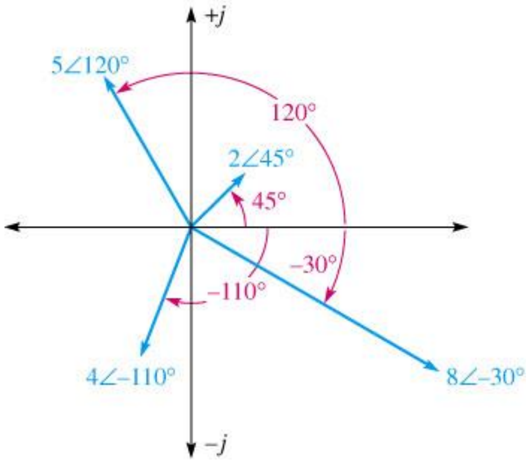
Angle on complex plane



complex specified by rectangular coordinate



complex specified by polar values



Complex plane

- A complex number defines a point in a two-dimensional plane established by two axes at 90° to one another
  - –the horizontal axis is the real axis
  - –the vertical axis is the imaginary axis
- Three forms are used to represent a complex number: **rectangular** and **polar and exponential**
  - –each can define the vector drawn from the origin to a point in the two-dimensional plane

$$z = x + jy$$

$$z = x + jy = r\angle\phi = re^{j\phi}$$

$$z = x + jy \quad \Rightarrow \quad \text{Rectangular form}$$

$$z = r\angle\phi \quad \Rightarrow \quad \text{Polar form}$$

$$z = re^{j\phi} \quad \Rightarrow \quad \text{Exponential form}$$

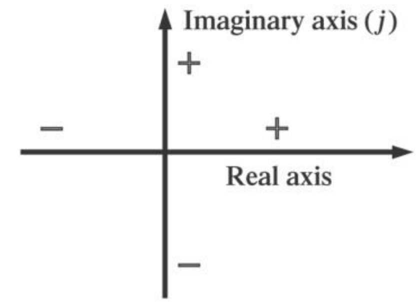
- Rectangular ⇒ Polar

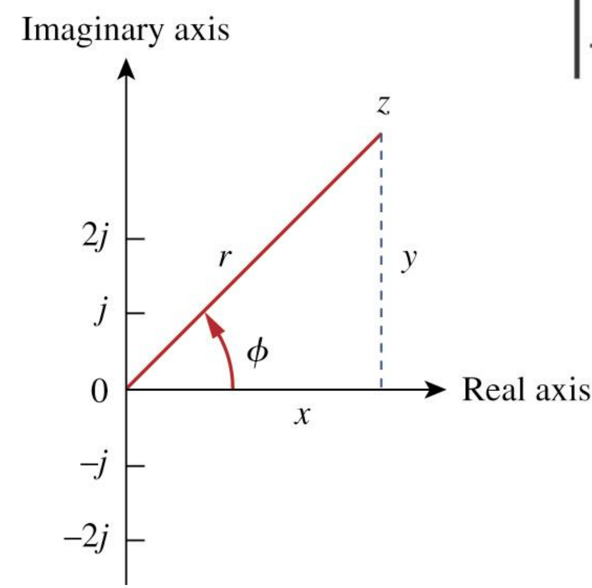
$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

- Polar ⇒ Rectangular

$$x = r\cos\phi, \qquad y = r\sin\phi$$

$$\therefore z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$



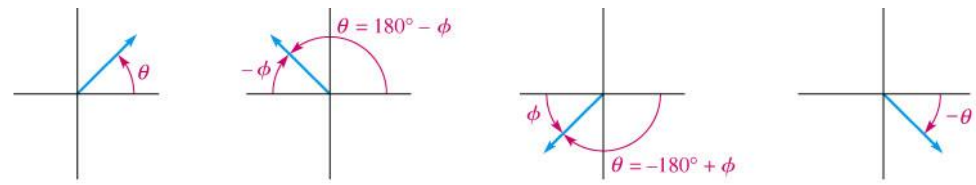


- Rectangular $\Rightarrow$ Polar

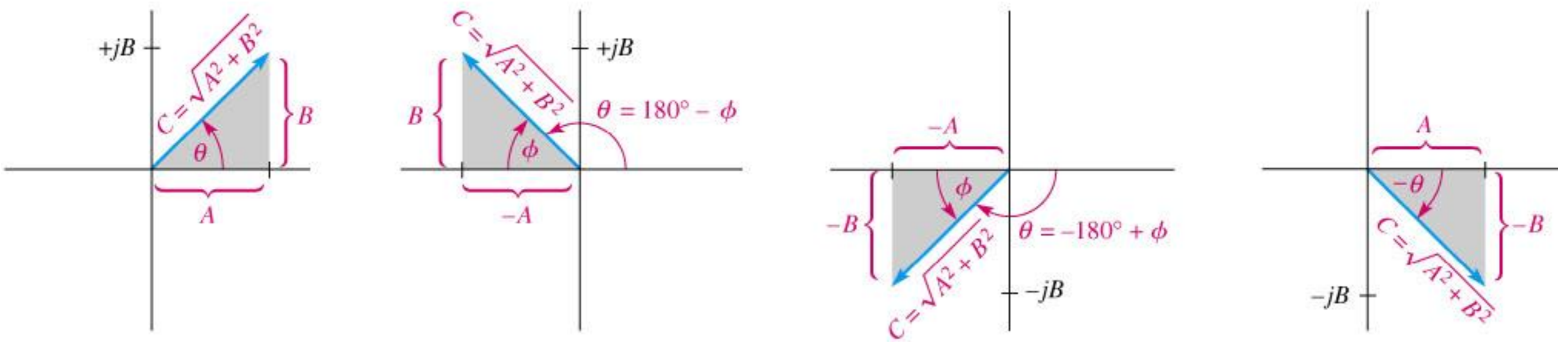
$$C=\sqrt{A^2+B^2},\qquad \theta=\tan^{-1}\left(\frac{\pm B}{A}\right)\Rightarrow \text{(a), (d)}$$

$$\text{or } \theta = 360-\tan^{-1}\left(\frac{B}{A}\right)\ \text{(d) 4th quadrant}$$

$$\theta=\pm 180^{\circ}\mp\tan^{-1}\left(\frac{B}{A}\right)\Rightarrow \text{(b), (c)}$$



(a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant

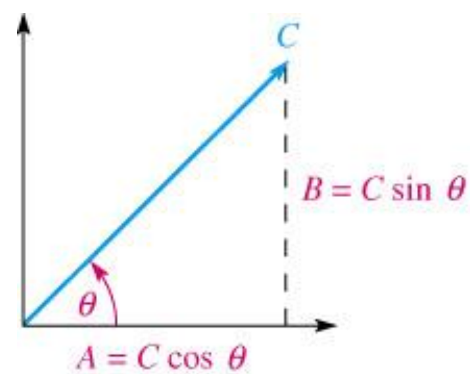


(a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant

- Polar $\Rightarrow$ Rectangular

$$A=C\cos\theta,\qquad B=C\sin\theta$$

$$\therefore z=A+jB=C\angle\theta=C(\cos\theta+j\sin\theta)$$



복소수의 계산

-. 덧셈과 뺄셈: 직각좌표형식

-. 곱셈과 나눗셈: 극좌표형식

$z = x + jy,$	$\phi = \tan^{-1}\dfrac{y}{x}$	(1st quadrant)
$z = -x + jy,$	$\phi = 180^0 - \tan^{-1}\dfrac{y}{x}$	(2nd quadrant)
$z = -x - jy,$	$\phi = 180^0 + \tan^{-1}\dfrac{y}{x}$	(3rd quadrant)
$z = x - jy,$	$\phi = 360^0 - \tan^{-1}\dfrac{y}{x}$	(4th quadrant)

$$z_1 = x_1 + jy_1 = r_1\angle\phi_1 \qquad z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

Addition :

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtrancti on :

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplica tion :

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

Division :

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1 - \phi_2$$

Square Root :

$$\sqrt{z} = \sqrt{r}\angle\phi/2$$

Reciprocal :

$$\frac{1}{z} = \frac{1}{r}\angle -\phi$$

Complex Conjugate :

$$z^* = x - jy = r\angle -\phi = re^{-j\phi}$$

$$\frac{1}{j} = -j$$

9.3 Phasors

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

$$\cos\phi = \mathrm{Re}(e^{j\phi})$$

$$\sin\phi = \mathrm{Im}(e^{j\phi})$$

$$v(t) = V_m\cos(\omega t+\phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)})$$

$$v(t) = \mathrm{Re}(V_m e^{j\phi}e^{j\omega t})$$

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})$$

where

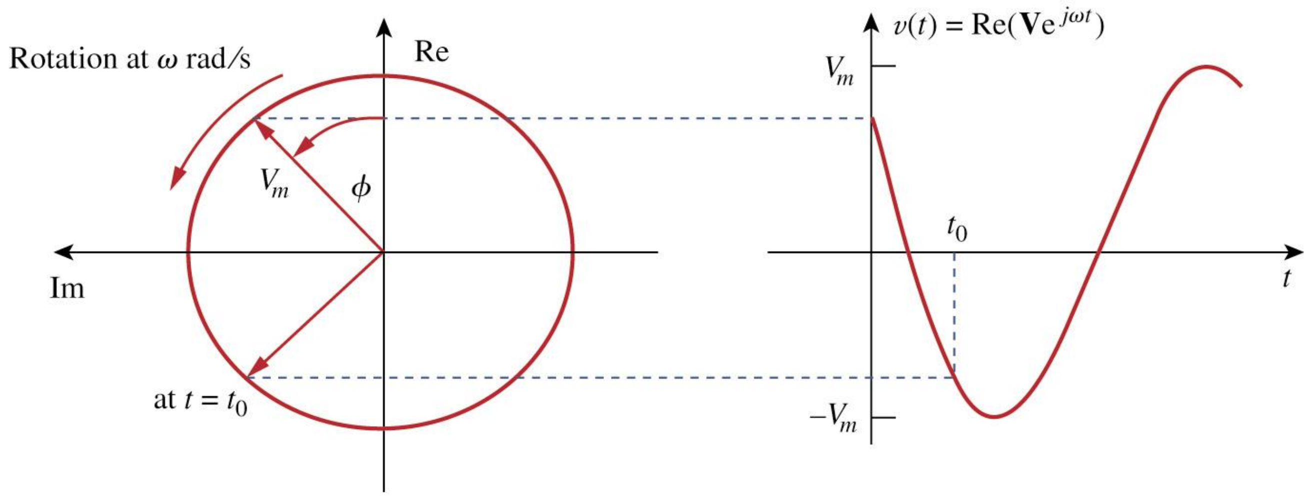
$$\mathbf{V} = V_m e^{j\phi} = V_m\angle\phi$$

→ V:정현파 $v(t)$의 페이저 표현

$$v(t) = V_m\sin(\omega t+\phi) = \mathrm{Im}(V_m e^{j(\omega t+\phi)})$$

$$v(t) = \mathrm{Im}(V_m e^{j\phi}e^{j\omega t})$$

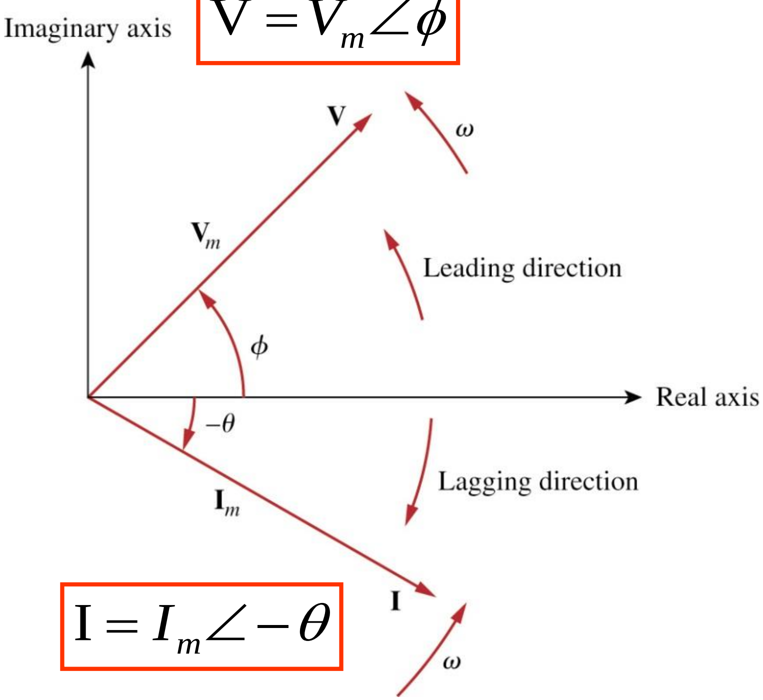
$$v(t) = \mathrm{Im}(\mathbf{V}e^{j\omega t})$$



(a) (b)

$$v(t) = \underset{(Time-domain\,representation)}{V_m \cos(\omega t + \phi)} \Leftrightarrow \underset{(Phasor-domain\,representation)}{\mathbf{V} = V_m \angle \phi}$$

$$\mathbf{V} = V_m \angle \phi$$



$$\mathbf{I} = I_m \angle -\theta$$

TABLE 9.1 Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

$$\frac{dv}{dt} = \frac{d\left(V_m \cos(\omega t + \phi)\right)}{dt}$$

$$= -\omega V_m \sin(\omega t + \phi) = \omega V_m \cos(\omega t + \phi + 90^\circ)$$

$$= \mathrm{Re}(\omega V_m e^{j\omega t} e^{j\phi} e^{j90^\circ}) = \mathrm{Re}(j\omega \mathbf{V} e^{j\omega t})$$

$$\therefore e^{j90^\circ} = j$$

$$\underset{Time\ domain}{\frac{dv}{dt}} \Leftrightarrow \underset{Phasor\ domain}{j\omega \mathbf{V}}$$

$$\underset{Time\ domain}{\int v\, dt} \Leftrightarrow \underset{Phasor\ domain}{\frac{\mathbf{V}}{j\omega}}$$

- The differences between *v(t)* and **V** should be emphasized:

1. *v(t)* is the instantaneous or time domain representation, while **V** is the frequency or phasor domain representation.
2. *v(t)* is time dependent, while **V** is not. (This fact is often forgotten by students.)
3. *v(t)* is always real with no complex term, while **V** is generally complex.

Relationship between differential, integral operation in phasor listed as follow:

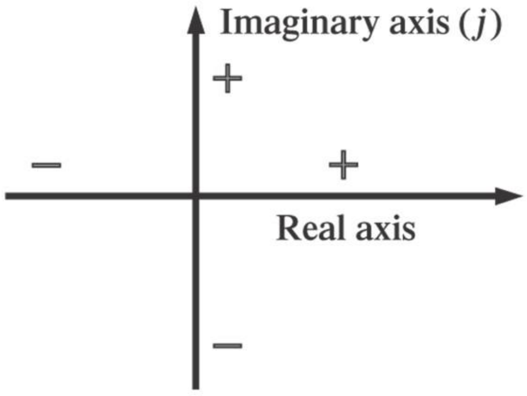
$v(t)$	$\longleftrightarrow$	$\mathrm{V} = V\angle\phi$
$\dfrac{dv}{dt}$	$\longleftrightarrow$	$j\omega\mathrm{V}$
$\int v\,dt$	$\longleftrightarrow$	$\dfrac{\mathrm{V}}{j\omega}$

- Adding and Subtracting Sinusoidal Waveforms

- Finding the sum or difference of two sinusoidal waveforms requires putting each in the phasor format and performing the required vector algebra
  - the angle associated with each vector is the phase angle associated with the standard format for a sinusoidal waveform

$$v_1 = V_m \cos(\omega t + \phi) \Rightarrow \mathrm{V}_1 = V_m \angle \phi$$
$$i_1 = I_m \cos(\omega t + \phi) \Rightarrow \mathrm{I}_1 = I_m \angle \phi$$

$$v_1 = V_m \sin(\omega t + \phi) \Rightarrow \mathrm{V}_1 = V_m \angle \phi - 90^\circ$$
$$i_1 = I_m \sin(\omega t + \phi) \Rightarrow \mathrm{I}_1 = I_m \angle \phi - 90^\circ$$



Example 9.4

Transform these sinusoids to phasors:

$$(a)\ \ i = 6\cos(50t - 40°)A$$

$$(b)\ \ v = -4\sin(30t + 50°)V$$

$$(a) i = 6\cos(50t - 40°)$$

$$\mathrm{I} = 6\angle\text{-}40° A$$

$$(b) v = -4\sin(30t + 50°)V$$

$$-\sin A = \cos(A + 90°)$$

$$v = -4\sin(30t + 50°) = 4\cos(30t + 50° + 90°)$$

$$= 4\cos(30t + 140°)V$$

The phasor form of v is

$$\mathrm{V} = 4\angle 140° V$$

Example 9.7

Use phasor approach, determine the current *i(t)* in a circuit described by the integro-differential equation

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75°)$$

transfer : time domain $\Rightarrow$ phasor domain

$$4\mathrm{I} + \frac{8\mathrm{I}}{j\omega} - 3j\omega\mathrm{I} = 50\angle 75^{\circ}$$

$$\omega = 2$$

$$\mathrm{I}(4 - j4 - j6) = 50\angle 75^{\circ}$$

$$\mathrm{I} = \frac{50\angle 75^{\circ}}{4 - j10} = \frac{50\angle 75^{\circ}}{10.77\angle -68.2^{\circ}} = 4.642\angle 143.2^{\circ}[A]$$

transfer time domain

$$i(t) = 4.642\cos(2t + 143.2^{\circ})[A]$$

$$r = \sqrt{x^2 + y^2}, \qquad \varphi = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\phi, \qquad y = r\sin\phi$$

$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$

$$v(t) \longleftrightarrow \mathbf{V} = V\angle\phi$$

$$\frac{dv}{dt} \longleftrightarrow j\omega V$$

$$\int v\,dt \longleftrightarrow \frac{V}{j\omega}$$

9.4 Phasor Relationships for Circuit Elements

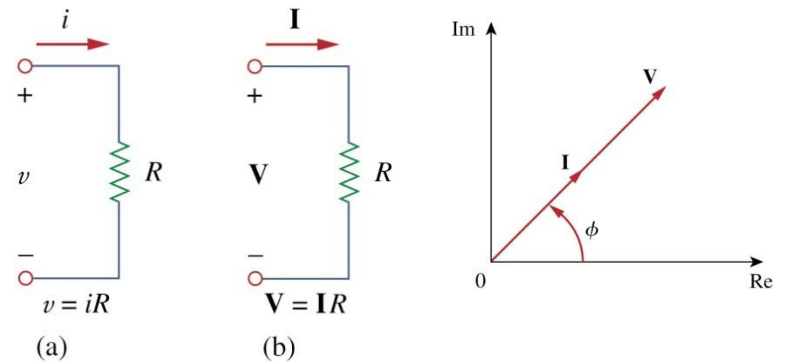
$i = I_m \cos(\omega t + \phi)$

[1] $v = iR = RI_m \cos(\omega t + \phi)$

$\mathrm{V} = RI_m \cos(\omega t + \phi)$

$\mathrm{V} = RI_m \angle \phi \quad \Rightarrow \mathrm{I} = I_m \angle \phi$

$\mathrm{V} = R\mathrm{I}$



[2] $v = L\dfrac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$

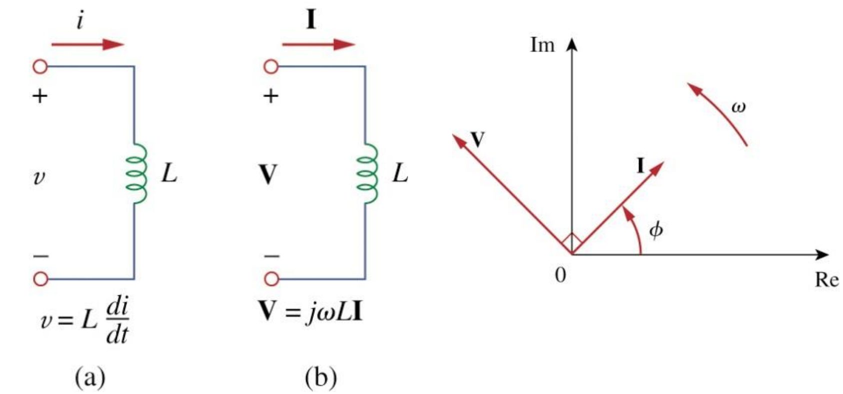
$v = \omega L I_m \cos(\omega t + \phi + 90°)$

$\mathrm{V} = \omega L I_m e^{j(\phi + 90°)} = \omega L I_m e^{j\phi} e^{j90°}$

$= \omega L I_m \angle \phi + 90°$

$\boxed{e^{j90°} = j}$

$\mathrm{V} = j\omega L \mathrm{I}$



$v = V_m \cos(\omega t + \phi)$

[3] $i = C\dfrac{dv}{dt} \quad \Rightarrow \quad v = V_m \cos(\omega t + \phi)$

$\mathrm{I} = j\omega C \mathrm{V} \quad \Rightarrow \quad \mathrm{V} = \dfrac{\mathrm{I}}{j\omega C}$

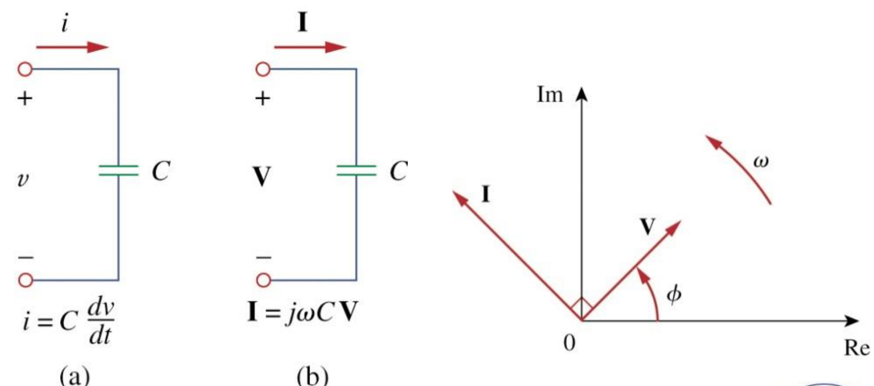


TABLE 9.2 Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Example 9.8 The voltage v=12 cos(60t+45˚) is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

For the inductor, $\mathrm{V} = j\omega L\mathrm{I}$, where $\omega = 60$ rad/s and $\mathrm{V} = 12\angle 45°\mathrm{V}$.

Hence,

$$\mathrm{I} = \frac{\mathrm{V}}{j\omega L} = \frac{12\angle 45°}{j60\times 0.1} = \frac{12\angle 45°}{6\angle 90°} = 2\angle -45° A$$

$$i(t) = 2\cos(60t - 45°) A$$

9.5 Impedance and Admittance

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I} \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

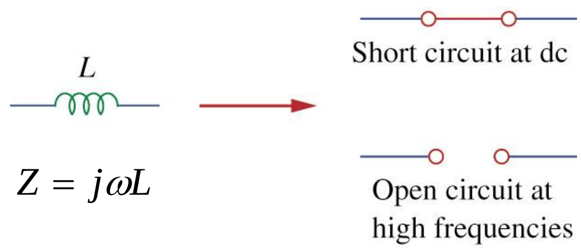
$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}$$

TABLE 9.3

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

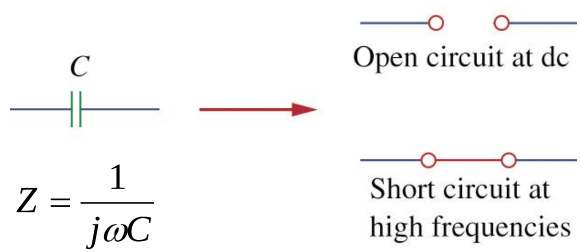


$Z = j\omega L$

(a)

$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$



$Z = \frac{1}{j\omega C}$

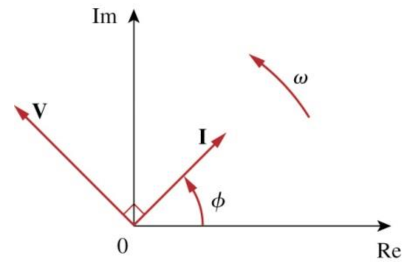
(b)

$$\omega = 0; Z \to \infty$$

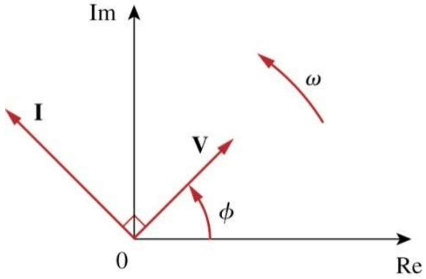
$$\omega \to \infty; Z = 0$$

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms (Ω)

$$Z = R + jX$$
$$\Rightarrow \text{inductive reactance (I lags V : lagging)}$$



$$Z = R - jX$$
$$\Rightarrow \text{capactive reactance (I leads V : leading)}$$



$$Z = R + jX$$
$$Z = |Z| \angle \theta$$
$$Z = R + jX = |Z| \angle \theta$$
$$where$$
$$|Z| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$
$$and$$
$$R = |Z|\cos\theta, \qquad X = |Z|\sin\theta$$

TABLE 9.3

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

- The admittance **Y** is the reciprocal of impedance, measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

$$Y = G + jB \Rightarrow G : \text{conductance}, \quad B : \text{susceptance}$$

$$G + jB = \frac{1}{R + jX}$$

By rationalization

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

equating the real and imaginary parts gives

$$G = \frac{R}{R^2 + X^2} \quad , \qquad B = -\frac{X}{R^2 + X^2}$$

showing that $G \neq 1/R$ as it is in resistive circuits.

$X = 0, G = 1/R.$

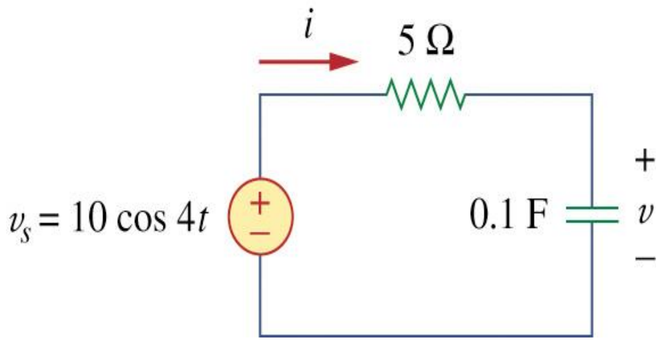
TABLE 9.3

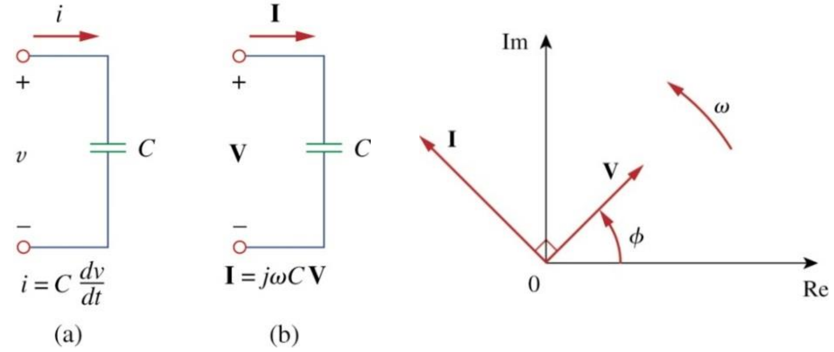
Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

Example 9.9

Find $v(t)$ and $i(t)$ the circuit shown in Fig. 9.16





voltage source $v_s = 10\ \cos 4t, \quad \omega = 4,$

$V_s = 10\angle 0° V$

The impedance is

$$Z = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\Omega$$

the current

$$I = \frac{V_s}{Z} = \frac{10\angle 0°}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$

$$= 1.6 + j0.8 = 1.789\angle 26.57° A$$

voltage across the capacitor is

$$V = IZ_C = \frac{I}{j\omega C} = \frac{1.789\angle 26.57°}{j4 \times 0.1}$$

$$= \frac{1.789\angle 26.57°}{0.4\angle 90°} = 4.47\angle -63.43° V$$

Converting I and V into the time domain,

$i(t) = 1.789\cos(4t + 26.57°)A$

$v(t) = 4.47\cos(4t - 63.43°)V$

$i(t)$ leads $v(t)$ by 90° as expected.

9.6 Kirchhoff's Laws in the Frequency Domain

- Both KVL and KCL are hold in the **phasor domain** or more commonly called **frequency domain.**
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

$$\text{KVL} \quad \Rightarrow \quad v_1 + v_2 + \cdots + v_n = 0$$

$$V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2) + \cdots + V_{mn}\cos(\omega t + \theta_n) = 0$$

$$\text{Re}\left(V_{m1}e^{j\theta_1}e^{j\omega t}\right) + \text{Re}\left(V_{m2}e^{j\theta_2}e^{j\omega t}\right) + \cdots + \text{Re}\left(V_{mn}e^{j\theta_n}e^{j\omega t}\right) = 0$$

$$\text{Re}\left[\left(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \cdots + V_{mn}e^{j\theta_n}\right)e^{j\omega t}\right] = 0$$

$$\Rightarrow \ \mathrm{V_k} = V_{mk}e^{j\theta_k},$$

$$\text{Re}[(\mathrm{V_1} + \mathrm{V_2} + \cdots + \mathrm{V_n})e^{j\omega t}] = 0$$

$$\Rightarrow e^{j\omega t} \neq 0,$$

$$\mathrm{V_1} + \mathrm{V_2} + \cdots + \mathrm{V_n} = 0$$

$$i_1 + i_2 + \cdots + i_n = 0$$

If $\mathrm{I_1}, \mathrm{I_2} \cdots \mathrm{I_n}$ are the phasor forms of the sinusoids $i_1, i_2 \cdots i_n$, then

$$\mathrm{I_1} + \mathrm{I_2} + \cdots + \mathrm{I_n} = 0$$

which is Kirchhoff' s current law in the frequency domain.

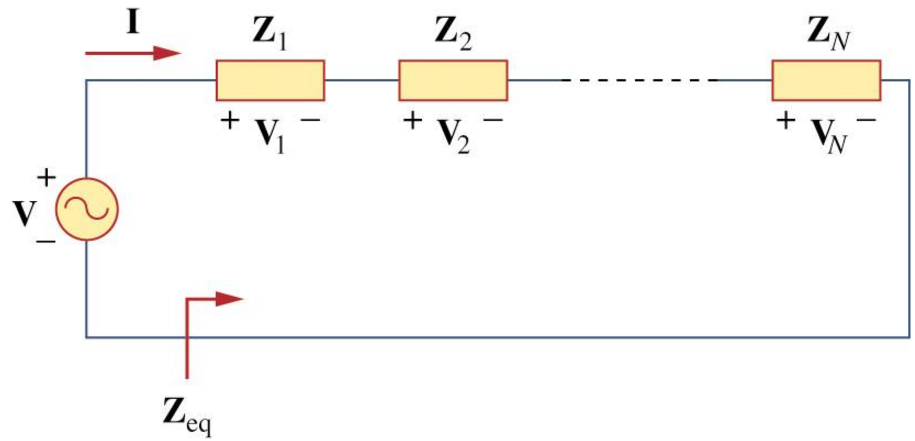
9.7 Impedance Combinations

$$V = V_1 + V_2 + \cdots + V_N = I(Z_1 + Z_2 + \cdots + Z_N)$$

$$Z_{eq} = \frac{V}{I} = Z_1 + Z_2 + \Lambda\ Z_N$$

or

$$Z_{eq} = Z_1 + Z_2 + \Lambda\ Z_N$$

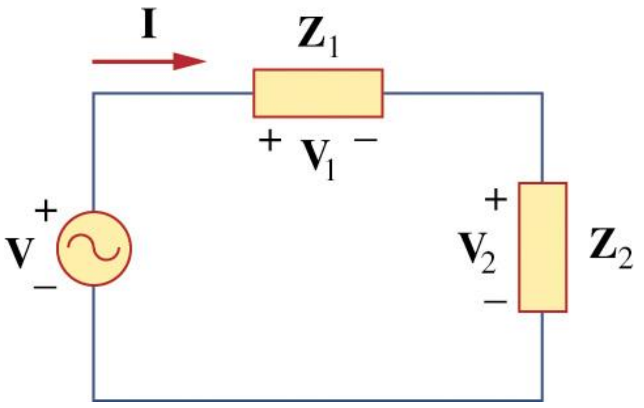


Voltage division

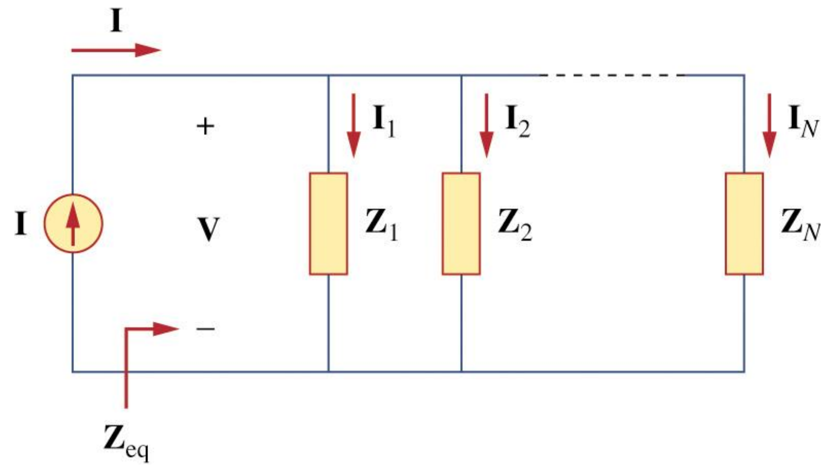
$$I = \frac{V}{Z_1 + Z_2}$$

Since $V_1 = Z_1 I$ and $V_2 = Z_2 I$, then

$$V_1 = \frac{Z_1}{Z_1 + Z_2} V, \qquad V_2 = \frac{Z_2}{Z_1 + Z_2} V$$



$$I = I_1 + I_2 + \cdots + I_N = V\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}\right)$$



$$\frac{1}{Z_{eq}} = \frac{I}{V} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_N}$$

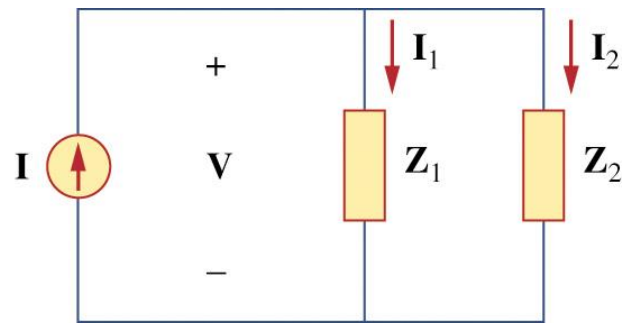
$$Y_{eq} = Y_1 + Y_2 + \cdots + Y_N$$

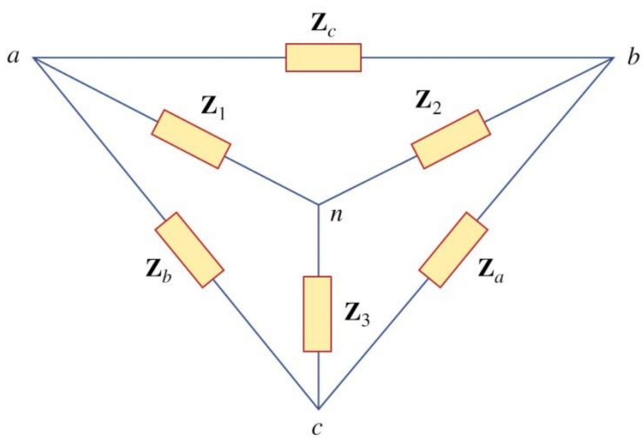
$$Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{Y_1} + \frac{1}{Y_2} = \frac{1}{1/Z_1 + 1/Z_2} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

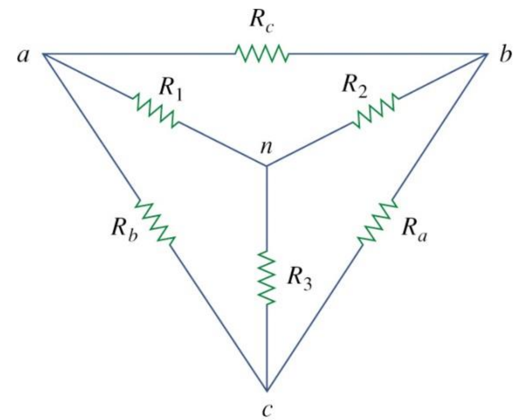
current division

$$V = IZ_{eq} = I_1 Z_1 = I_2 Z_2$$

$$I_1 = \frac{Z_2}{Z_1 + Z_2} I, \qquad I_2 = \frac{Z_1}{Z_1 + Z_2} I$$







$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$\mathrm{Y}-\Delta$ Conversion :

$$\mathrm{Z_a} = \frac{\mathrm{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}}{\mathrm{Z_1}}$$

$$\mathrm{Z_b} = \frac{\mathrm{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}}{\mathrm{Z_2}}$$

$$\mathrm{Z_c} = \frac{\mathrm{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}}{\mathrm{Z_3}}$$

$\Delta-\mathrm{Y}$ Conversion :

$$\mathrm{Z_1} = \frac{\mathrm{Z_bZ_c}}{\mathrm{Z_a + Z_b + Z_c}}$$

$$\mathrm{Z_2} = \frac{\mathrm{Z_cZ_a}}{\mathrm{Z_a + Z_b + Z_c}}$$

$$\mathrm{Z_3} = \frac{\mathrm{Z_aZ_b}}{\mathrm{Z_a + Z_b + Z_c}}$$

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_cR_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

$$\mathrm{Z_\Delta} = 3\mathrm{Y_Y} \quad \text{or} \quad \mathrm{Z_Y} = \frac{1}{3}\mathrm{Z_\Delta}$$

where $\mathrm{Z_Y = Z_1 = Z_2 = Z_3}$ and $\mathrm{Z_\Delta = Z_a = Z_b = Z_c}$

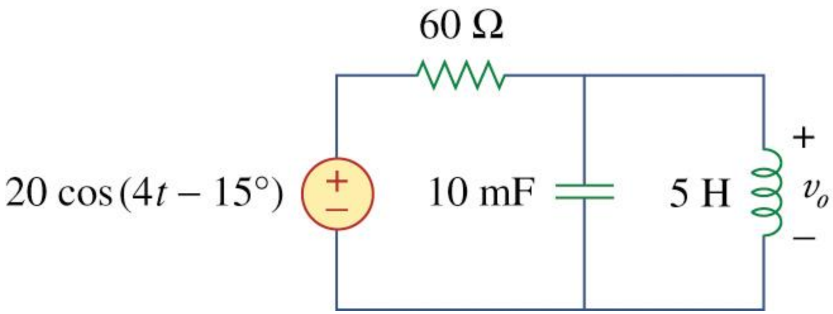
- A delta or wye circuit is said to be balanced if it has equal impedances in all three branches.

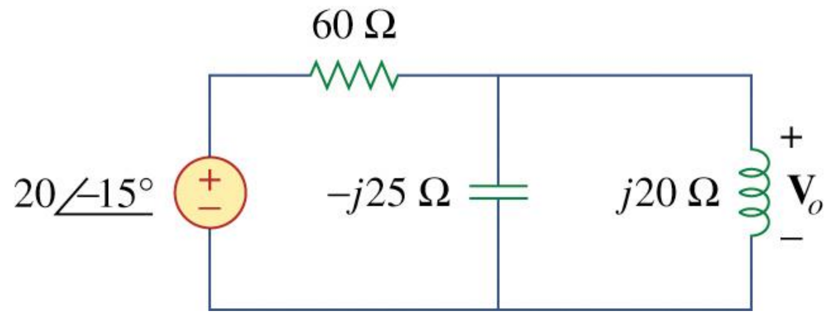
$$Z_\Delta = 3Y_Y \qquad \text{or} \qquad Z_Y = \frac{1}{3} Z_\Delta$$

$$\text{where } Z_Y = Z_1 = Z_2 = Z_3 \text{ and } Z_\Delta = Z_a = Z_b = Z_c$$

Example 9.11

Find current $v_o(t)$ in the circuit of Fig. 9.25





$$v_s = 20\cos(4t - 15^{\circ}) \quad \Rightarrow \quad \mathrm{V}_s = 20\angle -15^{\circ} \;,\; \omega = 4$$

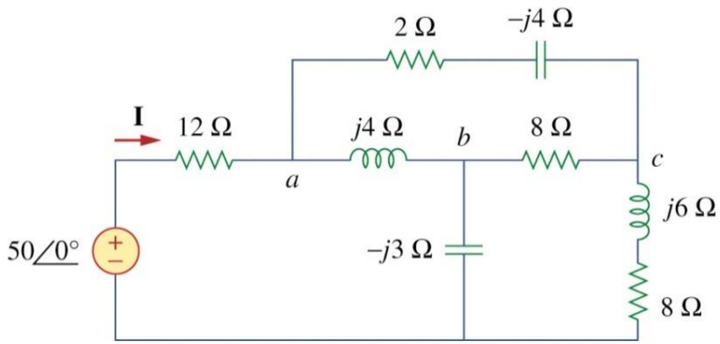
$$10mF \quad \Rightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4\times 10\times 10^{-3}} = -j25\Omega$$

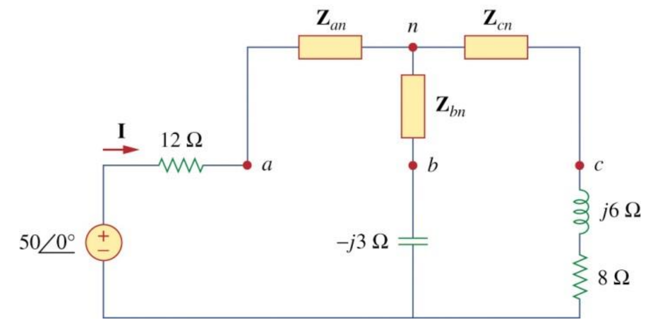
$$5H \quad \Rightarrow \quad j\omega L = j4\times 5 = j20\Omega$$

$$Z_1 = 60\Omega \quad , \quad \mathrm{Z}_2 = -j25\Omega \| j20\Omega = \frac{-j25\times j20}{-j25 + j20} = j100\Omega$$

$$\mathrm{V}_0 = \frac{\mathrm{Z}_2}{\mathrm{Z}_1 + \mathrm{Z}_2}\mathrm{V}_s = \frac{j100}{60 + j100}(20\angle -15^{\circ}) = 17.15\angle 15.96^{\circ}[V]$$

$$\therefore v_0(t) = 17.15\cos(4t + 15.96^{\circ})[V]$$



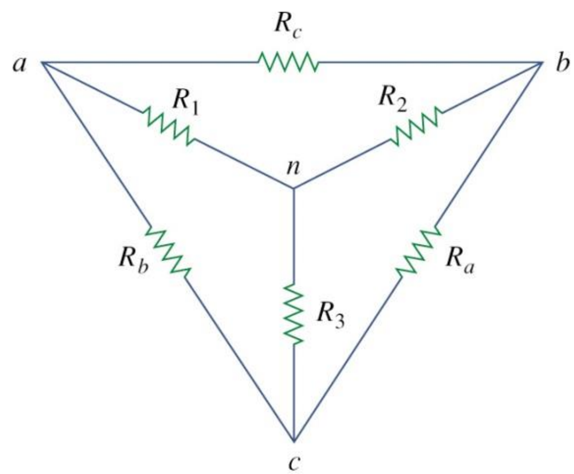


$\Delta - \mathrm{Y}$ Conversion :

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$



$\Delta - \mathrm{Y}$ Conversion :

$$Z_{an} = \frac{j4(2 - J4)}{j4 + 2 - j4 + 8} = (1.6 + j0.8)\Omega$$

$$Z_{bn} = \frac{j4(8)}{j4 + 2 - j4 + 8} = j3.2\Omega$$

$$Z_{cn} = \frac{8(2 - J4)}{j4 + 2 - j4 + 8} = (1.6 - j3.2)\Omega$$

The total impedance at the source terminals is

$$Z = 12 + Z_{an} + (Z_{bn} - j3) \parallel (Z_{cn} + j6 + 8)$$

$$= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)$$

$$= 13.6 + j0.8 + \frac{j0.29(9.6 + j2.8)}{9.6 + j3}$$

$$= 13.6 + j1 = 13.64\angle 4.204°\Omega$$

The desired current is

$$I = \frac{V}{Z} = \frac{50\angle 0°}{13.64\angle 4.204°} = 3.666\angle -4.204° A$$