ECE421 – Introduction to Machine Learning

Assignment 1

Linear and Logistic Regression

Hard Copy Due: Friday, Feb. 7 @ 2:00 PM, at BA3128
Code Submission Due: Friday, Feb. 7 @ 2:00 PM, on Quercus

General Notes:

- Attach this cover page to your hard copy submission
- Please post assignment related questions on Piazza.

Please check section to which you would like the assignment returned.

Tutorial Sections:

☐ Tutorial 1: Thursdays 3-5pm (SF2202)
☐ Tutorial 2: Thursdays 3-5pm (GB304)
☐ Tutorial 3: Tuesdays 10-12 (SF2202)
☐ Tutorial 4: Fridays 9-11 (BA1230)

Group Members	
Name	Student ID

0 Set Up

[1]:
```python
# ignore all future warnings
from warnings import simplefilter
simplefilter(action='ignore', category=FutureWarning)
```

[2]:
```python
# imports
import tensorflow as tf
import numpy as np
import matplotlib.pyplot as plt
```

[3]:
```python
# ignore tensorflow depreciation warnings
import tensorflow.python.util.deprecation as deprecation
deprecation._PRINT_DEPRECATION_WARNINGS = False
```

0.1 Visualizing Dataset

It's always important to visualize the dataset to gain and understanding of what the model is trying to accomplish. This can help in the debugging phase. The shape of each data set is printed out as well as a random sample of the training data is plotted.

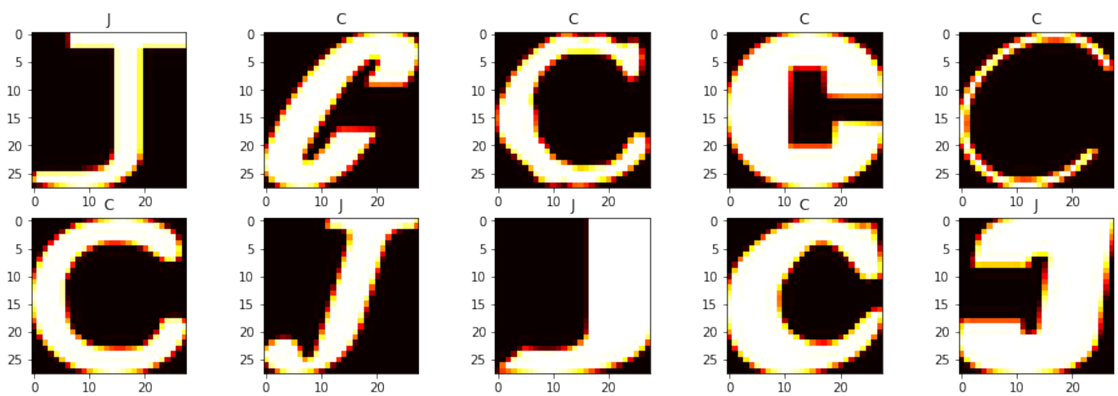
[4]:
```python
# given by the assignment
def loadData():
    with np.load('notMNIST.npz') as data :
        Data, Target = data['images'], data['labels']
        posClass = 2
        negClass = 9
        dataIndx = (Target==posClass) + (Target==negClass)
        Data = Data[dataIndx]/255.
        Target = Target[dataIndx].reshape(-1, 1)
        Target[Target==posClass] = 1
        Target[Target==negClass] = 0
        np.random.seed(421)
        randIndx = np.arange(len(Data))
        np.random.shuffle(randIndx)
        Data, Target = Data[randIndx], Target[randIndx]
        trainData, trainTarget = Data[:3500], Target[:3500]
        validData, validTarget = Data[3500:3600], Target[3500:3600]
        testData, testTarget = Data[3600:], Target[3600:]
    return trainData, validData, testData, trainTarget, validTarget, testTarget
```

[5]:
```
trainData, validData, testData, trainTarget, validTarget, testTarget =␣
 ↪loadData()
print(f"Training Data: {trainData.shape}\tTraining tagets: {trainTarget.shape}")
print(f"Validation Data: {validData.shape}\tValidation tagets: {validTarget.
 ↪shape}")
print(f"Testing Data: {testData.shape}\tTesting tagets:{testTarget.shape}")
```

```
Training Data: (3500, 28, 28)   Training tagets: (3500, 1)
Validation Data: (100, 28, 28)  Validation tagets: (100, 1)
Testing Data: (145, 28, 28)     Testing tagets:(145, 1)
```

[6]:
```
def plot(image, target, ax=None):
    ax = plt.gca() if ax == None else ax
    ax.imshow(image, cmap="hot")
    ax.set_title('J' if target == 0 else 'C')
    # targets are binary encoded 0 == 'J' and 1 == 'C'
```

[7]:
```
fig, axis = plt.subplots(2, 5, figsize=(16, 5))
for ax in axis.reshape(-1):
    r = np.random.randint(trainData.shape[0])
    plot(trainData[r], trainTarget[r], ax=ax)
plt.show()
```



0.2 Useful Functions

Some useful functions that will be used throughout the assignment such as getting random weights, getting the accuracy of a batch, making the loss and accuracy plots look nice, and global variables used throughout the code

[8]:
```
def augment(X, w, b):
    # flatten X
    if len(X.shape) == 3:
        X = X.reshape(X.shape[0], -1)

    # insert 1's at position 0 along the columns
    X = np.insert(X, 0, 1, axis=1)

    # insert b at the front of W
    w = np.insert(w, 0, b, axis=0)

    return X, w

def get_zero_parameters():
    w = np.zeros(d)
    b = np.zeros(1)
    return w, b

def get_random_parameters():
    w = np.random.uniform(low=-1.0, high=1.0, size=(d,))
    b = np.random.uniform(low=-1.0, high=1.0, size=(1,))
    return w, b
```

[9]:
```
def predict(w, b, X):
    X = X.reshape(X.shape[0], -1)
    return X.dot(w) + b

def accuracy(w, b, X, y):
    y = y.reshape(-1)
    y_pred = predict(w, b, X)
    y_pred = np.vectorize(lambda z: 1 if z > 0 else 0)(y_pred)
    return np.sum(y_pred == y) / y.shape[0]

def accuracy_with_predictions(y_pred, y):
    if y_pred.shape != y.shape:
        raise ValueError(f"prediction dimension {y_pred.shape} and label␣
↪dimensions {y.shape} don't match")
    y_pred = np.vectorize(lambda z: 1 if z > 0 else 0)(y_pred)
    return np.sum(y_pred == y) / y.shape[0]
```

```
[10]: def plot_loss(x, train_loss=None, valid_loss=None, test_loss=None, title=None,␣
↪ax=None):
    ax = plt.gca() if ax == None else ax
    if train_loss != None:
        ax.plot(x, train_loss, label="Training Loss")
    if valid_loss != None:
        ax.plot(x, valid_loss, label="Validation Loss")
    if test_loss != None:
        ax.plot(x, test_loss, label="Testing Loss")

    ax.set_title("Loss" if title == None else title)

    ax.set_xlabel("Iterations")
    ax.set_xlim(left=0)
    ax.set_ylabel("Loss")
    ax.set_ylim(bottom=0)
    ax.legend(loc="upper right")

def plot_accuracy(x, train_accuracy=None, valid_accuracy=None,␣
↪test_accuracy=None, title=None, ax=None):
    ax = plt.gca() if ax == None else ax
    if train_accuracy != None:
        ax.plot(x, train_accuracy, label="Training Accuracy")
    if valid_accuracy != None:
        ax.plot(x, valid_accuracy, label="Validation Accuracy")
    if test_accuracy != None:
        ax.plot(x, test_accuracy, label="Testing Accuracy")

    ax.set_title("Accuracy" if title == None else title)

    ax.set_xlabel("Iterations")
    ax.set_xlim(left=0)
    ax.set_ylabel("Accuracy")
    ax.set_yticks(np.arange(0, 1.1, step=0.1))
    ax.grid(linestyle='-', axis='y')
    ax.legend(loc="lower right")

def display_statistics(train_loss, train_acc, valid_loss, valid_acc, test_loss,␣
↪test_acc):
    print(f"Training loss: {train_loss[-1]:.4f}{'':.20s}\t\tTraining acc:␣
↪{train_acc[-1]*100:.2f}%")
    print(f"Validation loss: {valid_loss[-1]:.4f}\t\tValidation acc:␣
↪{valid_acc[-1]*100:.2f}%")
    print(f"Testing loss: {test_loss[-1]:.4f}\t\tTesting acc: {test_acc[-1]*100:
↪.2f}%")

    fig, ax = plt.subplots(1, 2, figsize=(18, 6))
```

```python
    plot_loss(np.arange(0, len(train_loss), 1), train_loss, valid_loss,␣
↪test_loss, ax=ax[0])
    plot_accuracy(np.arange(0, len(train_loss), 1), train_acc, valid_acc,␣
↪test_acc, ax=ax[1])
    plt.show()
    plt.close()
```

[11]:
```python
VTDatasets = {"validData" : validData, "validTarget" : validTarget,
              "testData" : testData, "testTarget" : testTarget}

N = trainData.shape[0]
d = trainData.shape[1] * trainData.shape[2]
```

1 Linear Regression

1. Loss Function and Gradient

$$\hat{y}^{(n)} = W^T \mathbf{x}^{(n)} + b$$

$$\mathcal{L}_{MSE} = \frac{1}{N} \sum_{n=1}^{N} (\hat{y}^{(n)} - y^{(n)})^2 + \lambda \|W\|_2^2$$

$$\frac{\partial \mathcal{L}_{MSE}}{\partial b} = \frac{2}{N} \sum_{n=1}^{N} (\hat{y}^{(n)} - y^{(n)})$$

$$\frac{\partial \mathcal{L}_{MSE}}{\partial W} = \frac{2}{N} X^T (\hat{\mathbf{y}} - \mathbf{y}) + \lambda W$$

[12]:
```
def MSE(w, b, X, y, reg):
    X = X.reshape(X.shape[0], -1)
    y = y.reshape(-1)
    return np.square(X.dot(w) + b - y).mean() + reg * np.square(w).sum()

def gradMSE(w, b, X, y, reg):
    X = X.reshape(X.shape[0], -1)
    y = y.reshape(-1)
    N = y.shape[0]

    w_grad = 2.0/N * X.T.dot(X.dot(w) + b - y) + reg * w
    b_grad = 2.0/N * np.sum(X.dot(w) + b - y)
    return w_grad, b_grad
```

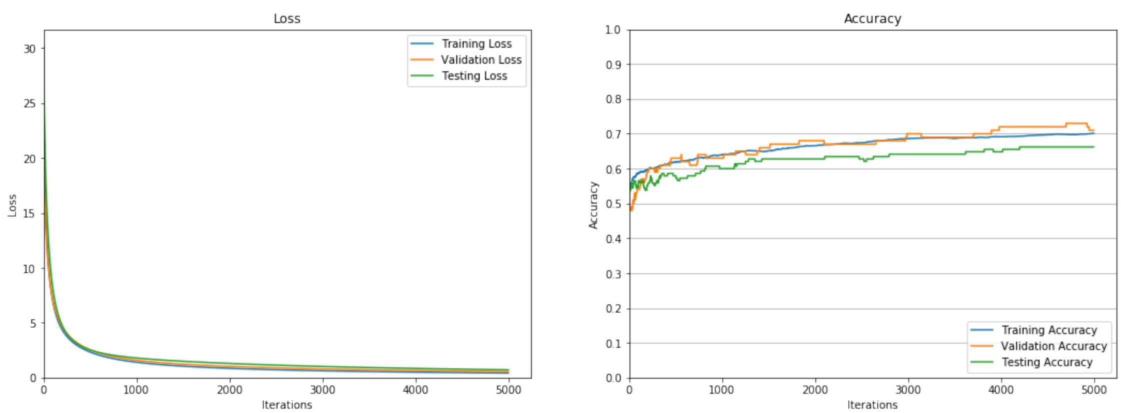
2. Gradient Descent Implementation

[13]:
```
def grad_descent_MSE(w, b, X, y, alpha, epochs, reg, error_tol=1e-7,
                     validData=None, validTarget=None, testData=None,␣
 ↪testTarget=None):
    train_loss, train_acc = [], []
    valid_loss, valid_acc = [], []
    test_loss, test_acc = [], []
    for i in range(epochs):
        grad_w, grad_b = gradMSE(w, b, X, y, reg)
        w -= alpha * grad_w
        b -= alpha * grad_b

        # Calculating Statistics
        train_loss.append( MSE(w, b, X, y, reg) )
        train_acc.append( accuracy(w, b, X, y) )

        if not validData is None and not validTarget is None:
            valid_loss.append( MSE(w, b, validData, validTarget, reg) )
            valid_acc.append( accuracy(w, b, validData, validTarget) )
        if not testData is None and not testTarget is None:
            test_loss.append( MSE(w, b, testData, testTarget, reg) )
            test_acc.append( accuracy(w, b, testData, testTarget) )

        # Check stopping condition
        if i > 1 and np.abs(train_loss[-2] - train_loss[-1]) <= error_tol:
            break

    statistics = (train_loss, train_acc)
    if not validData is None and not validTarget is None:
        statistics += (valid_loss, valid_acc, )
    if not testData is None and not testTarget is None:
        statistics += (test_loss, test_acc,)
    # Python 3.8 made this easier, but 3.7 you have to do this
    out = (w, b, *statistics)

    return out
```

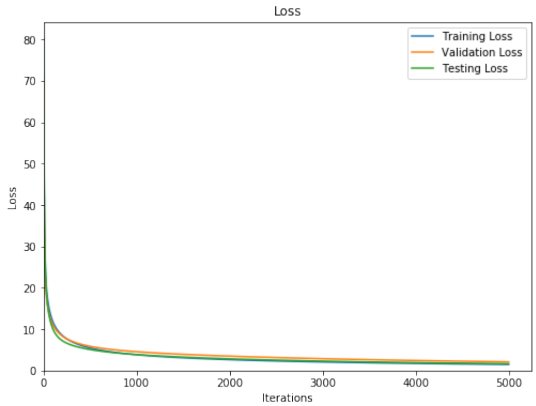
3. Tuning the Learning Rate

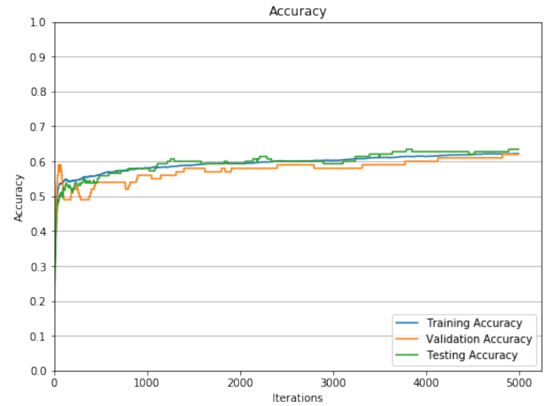
[14]:
```
for alpha in [0.005, 0.001, 0.0001]:

    print("alpha =", alpha)

    w, b = get_random_parameters()
    w, b, *statistics = grad_descent_MSE(w, b, trainData, trainTarget,
                                         alpha=alpha,
                                         epochs=5000,
                                         reg=0,
                                         **VTDatasets)
    display_statistics(*statistics)
```

```
alpha = 0.005
Training loss: 0.4199          Training acc: 70.11%
Validation loss: 0.5428        Validation acc: 71.00%
Testing loss: 0.7137           Testing acc: 66.21%
```

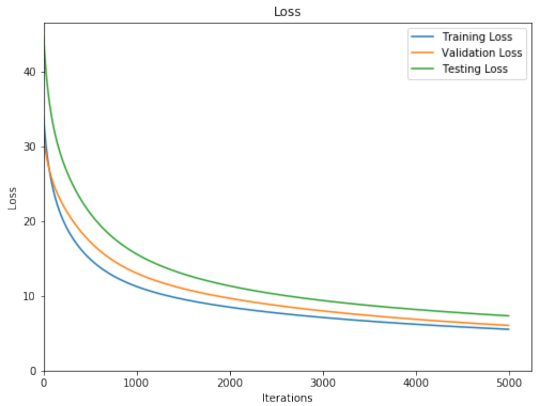


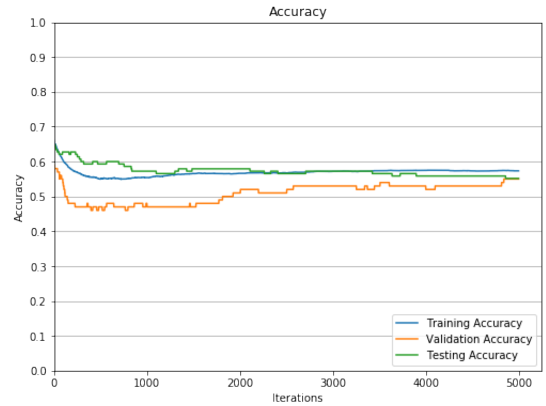

```
alpha = 0.001
Training loss: 1.4922          Training acc: 62.17%
Validation loss: 2.1204        Validation acc: 62.00%
Testing loss: 1.6711           Testing acc: 63.45%
```






```
alpha = 0.0001
Training loss: 5.5281          Training acc: 57.34%
Validation loss: 6.0496        Validation acc: 55.00%
Testing loss: 7.3279           Testing acc: 55.17%
```

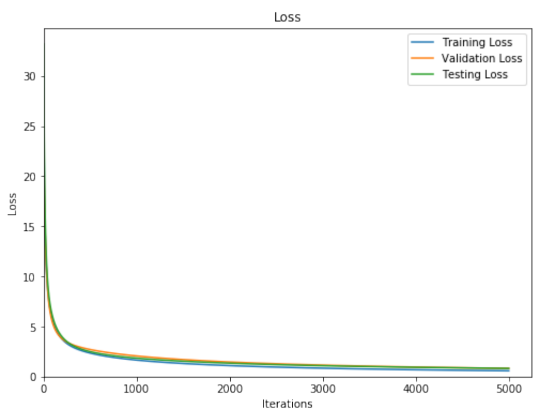




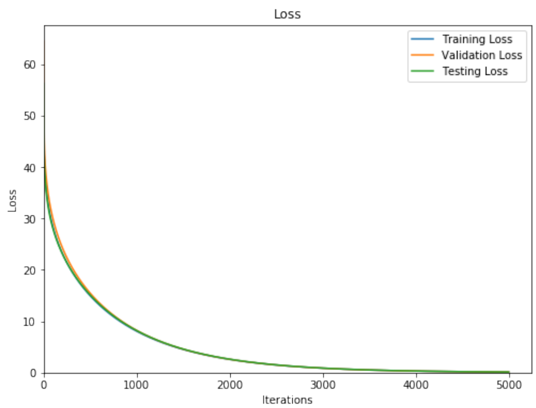
4. Generalization

[15]:
```
for reg in [0.001, 0.1, 0.5]:

    print("regularization =", reg)

    w, b = get_random_parameters()
    w, b, *statistics = grad_descent_MSE(w, b, trainData, trainTarget,
                                         alpha=0.005,
                                         epochs=5000,
                                         reg=reg,
                                         **VTDatasets)
    display_statistics(*statistics)
```

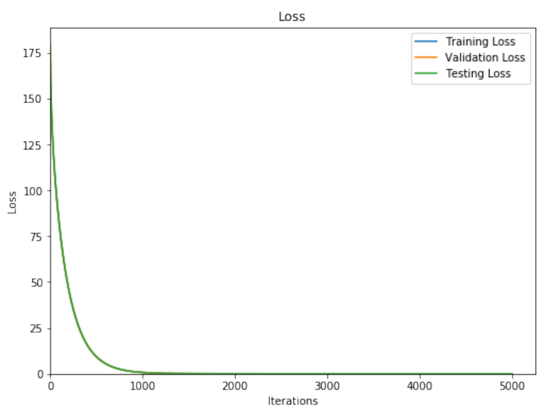
```
regularization = 0.001
Training loss: 0.5822           Training acc: 69.54%
Validation loss: 0.8218         Validation acc: 61.00%
Testing loss: 0.8229            Testing acc: 69.66%
```

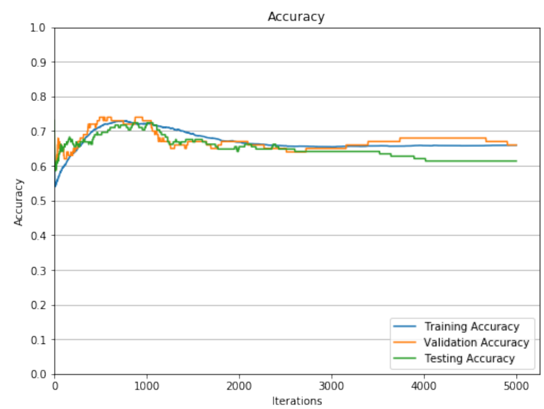



```
regularization = 0.1
Training loss: 0.1332           Training acc: 68.60%
Validation loss: 0.1355         Validation acc: 65.00%
Testing loss: 0.1393            Testing acc: 64.14%
```




```
regularization = 0.5
Training loss: 0.0416          Training acc: 65.94%
Validation loss: 0.0451        Validation acc: 66.00%
Testing loss: 0.0442           Testing acc: 61.38%
```





5. Comparing Batch GD with normal equation

[16]:
```
def least_squares(X, y):
    N = X.shape[0]
    d = X.shape[1] * X.shape[2]
    X = X.reshape(X.shape[0], -1)
    X = np.insert(X, 0, 1, axis=1)
    y = y.reshape(-1)

    # overparameterized (deep learning)
    if N < d:
        w_aug = X.T @ np.linalg.inv( X @ X.T ) @ y

    # underparameterized (typical case)
    else:
        w_aug = np.linalg.inv( X.T @ X ) @ X.T @ y

    return w_aug[1:], w_aug[0]
```

[17]:
```
w_LS, b_LS = least_squares(trainData, trainTarget)

loss = MSE(w_LS, b_LS, trainData, trainTarget, 0)
acc = accuracy(w_LS, b_LS, trainData, trainTarget)
print(f"Least Squares Training loss: {loss:.4f}\tLeast Squares Training acc:␣
 ↪{100*acc:.2f}%")
loss = MSE(w_LS, b_LS, validData, validTarget, 0)
acc = accuracy(w_LS, b_LS, validData, validTarget)
print(f"Least Squares Validation loss: {loss:.4f}\tLeast Squares Validation acc:
 ↪ {100*acc:.2f}%")
loss = MSE(w_LS, b_LS, testData, testTarget, 0)
acc = accuracy(w_LS, b_LS, testData, testTarget)
print(f"Least Squares Testing loss: {loss:.4f}\tLeast Squares Testing acc:␣
 ↪{100*acc:.2f}%")
```

```
Least Squares Training loss: 0.0187     Least Squares Training acc: 71.29%
Least Squares Validation loss: 0.0476   Least Squares Validation acc: 69.00%
Least Squares Testing loss: 0.0570      Least Squares Testing acc: 66.90%
```

The least squares solution achived a the training loss of 0.0187 and training accuracy of 71.29%. The training loss and accuracy for Batch Gradient Descent were 0.4199 and 70.11%, respectively. From the values, we see that the analytical solution performed marginally better. However, computing it grows increasingly difficult with the size of the problem. By constrast, batch gradient descent allows for faster convergence with less computation and comparable accuracies.

2 Logistic Regression

2.1 Binary cross-entropy loss

1. Loss Function and Gradient

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\hat{y}^{(n)} = \sigma(W^T \mathbf{x}^{(n)} + b)$$

$$\mathcal{L}_{CE} = \frac{1}{N} \sum_{n=1}^{N} \left[-y^{(n)} \log(\hat{y}^{(n)}) - (1-y^{(n)}) \log(1-\hat{y}^{(n)}) \right] + \frac{\lambda}{2} \|W\|_2^2$$

$$\frac{\partial \mathcal{L}_{CE}}{\partial b} = \frac{1}{N} \sum_{n=1}^{N} \left[\hat{y}^{(n)} - y^{(n)} \right]$$

$$\frac{\partial \mathcal{L}_{CE}}{\partial W} = \frac{1}{N} X^T \left(\hat{\mathbf{y}} - \mathbf{y} \right) + \lambda W$$

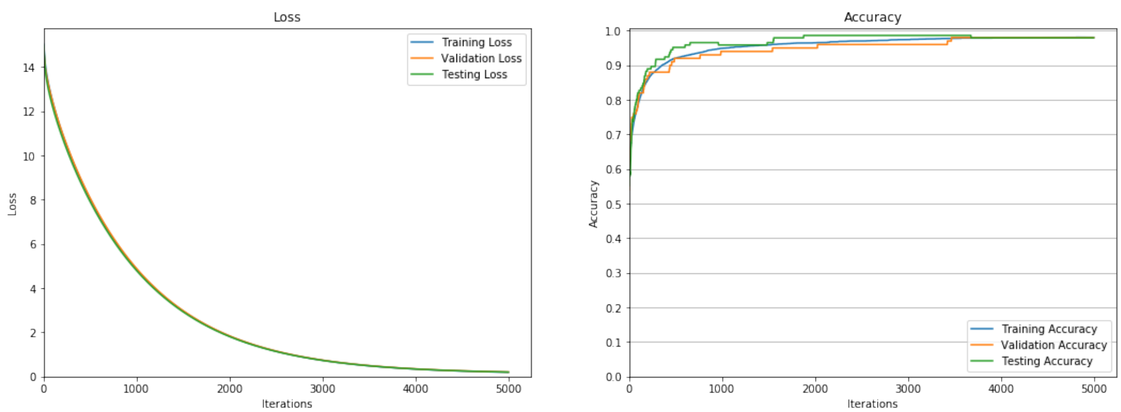
[18]:
```
def sigmoid(z):
    return 1.0 / (1 + np.exp(-z))

def crossEntropyLoss(w, b, X, y, reg):
    X, w = augment(X, w, b)
    y = y.reshape(-1)
    N = y.shape[0]

    y_hat = sigmoid(X.dot(w))

    return 1.0/N * (-y.dot(np.log(y_hat+1e-20)) - (1 - y).dot(np.log(1 -
 ↪y_hat+1e-20))) + reg/2.0 * np.square(w[1:]).sum()

def gradCE(w, b, X, y, reg):
    X, w = augment(X, w, b)
    y = y.reshape(-1)
    N = y.shape[0]

    y_hat = sigmoid(X.dot(w))

    grad = 1.0 /N * X.T.dot(y_hat - y) + reg * w

    return grad[1:], grad[0] - reg * w[0]
```

2. **Learning**

```
[19]: def grad_descent(w, b, X, y, alpha, epochs, reg, error_tol=1e-7, lossType="MSE",
                     validData=None, validTarget=None, testData=None,␣
      ↪testTarget=None):
          loss_func, grad_func = None, None
          if lossType == "MSE":
              loss_func, grad_func = MSE, gradMSE
          elif lossType == "CE":
              loss_func, grad_func = crossEntropyLoss, gradCE
          else:
              raise ValueError("Variable 'lossType' must be either 'MSE' or 'CE'.")

          train_loss, train_acc = [], []
          valid_loss, valid_acc = [], []
          test_loss, test_acc = [], []
          printing = False
          for i in range(epochs):
              grad_w, grad_b = grad_func(w, b, X, y, reg)
              w -= alpha * grad_w
              b -= alpha * grad_b

              # Calculating Statistics
              train_loss.append(loss_func(w, b, X, y, reg))
              train_acc.append(accuracy(w, b, X, y))

              if not validData is None and not validTarget is None:
                  valid_loss.append(loss_func(w, b, validData, validTarget, reg))
                  valid_acc.append(accuracy(w, b, validData, validTarget))
              if not testData is None and not testTarget is None:
                  test_loss.append(loss_func(w, b, testData, testTarget, reg))
                  test_acc.append(accuracy(w, b, testData, testTarget))

              # Check stopping condition
              if i > 1 and np.abs(train_loss[-2] - train_loss[-1]) <= error_tol:
                  break

          statistics = (train_loss, train_acc)
          if not validData is None and not validTarget is None:
              statistics += (valid_loss, valid_acc,)
          if not testData is None and not testTarget is None:
              statistics += (test_loss, test_acc,)
          out = (w, b, *statistics)

          return out
```

[20]:
```
w, b = get_random_parameters()
w, b, *statistics = grad_descent(w, b, trainData, trainTarget,
                                 alpha=0.005,
                                 epochs=5000,
                                 reg=0.1,
                                 lossType='CE',
                                 **VTDatasets)
display_statistics(*statistics)
```

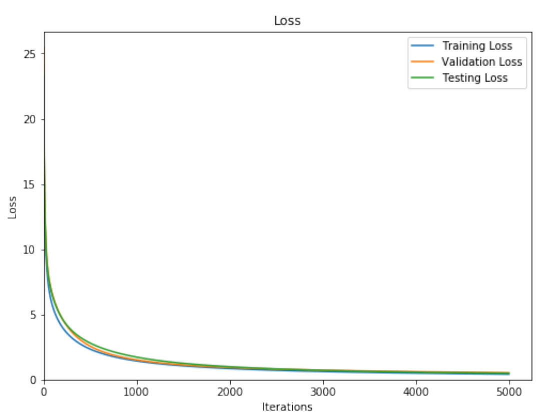
```
Training loss: 0.1992          Training acc: 98.06%
Validation loss: 0.2082        Validation acc: 98.00%
Testing loss: 0.2055           Testing acc: 97.93%
```

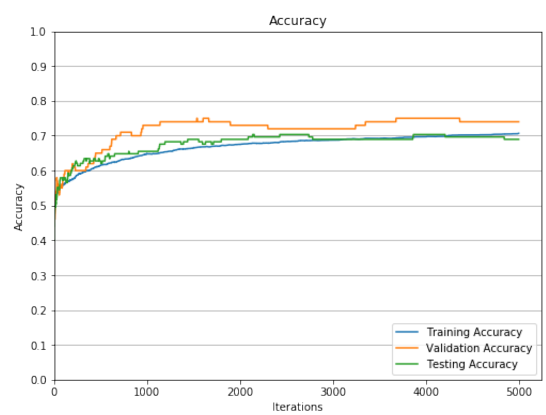


3. Comparision to Linear Regression

[21]:
```python
# same initialization for each to a more fair comparison
w, b = get_random_parameters()

# Linear Regresesion
print("Linear Regression")
w_lin, b_lin, *statistics = grad_descent(w, b, trainData, trainTarget,
                                          alpha=0.005,
                                          epochs=5000,
                                          reg=0,
                                          lossType='MSE',
                                          **VTDatasets)
display_statistics(*statistics)


# Logistic Regression
print("Logistic Regression")
w_log, b_log, *statistics = grad_descent(w, b, trainData, trainTarget,
                                          alpha=0.005,
                                          epochs=5000,
                                          reg=0,
                                          lossType='CE',
                                          **VTDatasets)
display_statistics(*statistics)
```

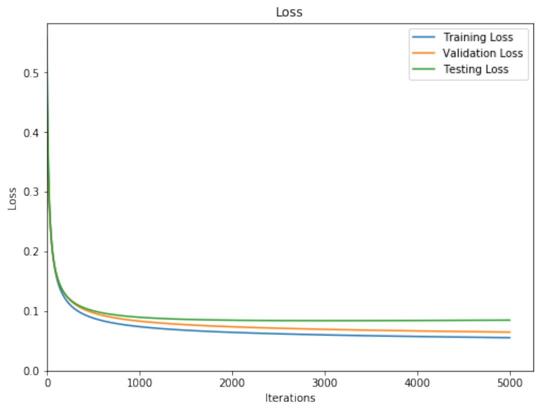
```
Linear Regression
Training loss: 0.4194           Training acc: 70.69%
Validation loss: 0.5375         Validation acc: 74.00%
Testing loss: 0.4956            Testing acc: 68.97%
```

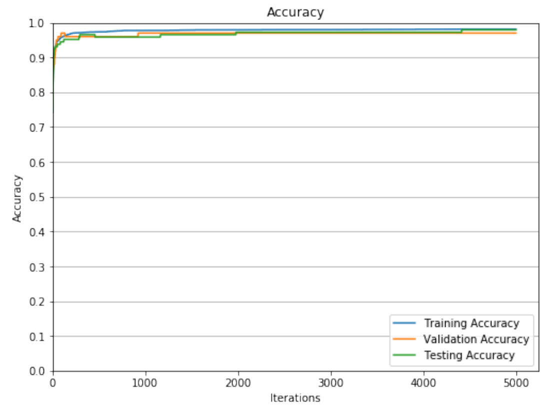





```
Logistic Regression
Training loss: 0.0553           Training acc: 98.14%
```

```
Validation loss: 0.0647          Validation acc: 97.00%
Testing loss: 0.0847             Testing acc: 97.93%
```





The cross-entropy loss function allows for much faster convergence to a more optimal minimum compared to the MSE loss function. This is due to the fact that cross-entropy punishes wrong predictions much more than MSE.

3 Batch Gradient Descent vs. SGD and Adam

3.1 SGD

1. Building the Computational Graph

[22]:
```
def buildGraph(alpha, reg=0, beta1=None, beta2=None, epsilon=None, loss="MSE",␣
↪seed=None):
    kwargs = {}
    if beta1 != None:
        kwargs["beta1"] = beta1
    if beta2 != None:
        kwargs["beta2"] = beta2
    if epsilon != None:
        kwargs["epsilon"] = epsilon

    # Initialize weight and bias tensors
    tf.set_random_seed(seed)
    tf.compat.v1.set_random_seed(seed)
    tf.compat.v1.disable_eager_execution()

    # Getting small random initial values for weights and bias
    w = tf.truncated_normal([d, 1], stddev=0.5)
    b = tf.truncated_normal([1, 1], stddev=0.5)
    # Converting into tensorflow Variable objects
    w = tf.Variable(w, name="weights")
    b = tf.Variable(b, name="bias")

    # tensorflow objects for data
    X = tf.placeholder(tf.float32, (None, d))
    y = tf.placeholder(tf.float32, (None, 1))
    y_pred = tf.matmul(X, w) + b

    cost = None
    if loss == "MSE":
        cost = tf.losses.mean_squared_error(y, y_pred) + reg * tf.norm(w)**2
    elif loss == "CE":
        cost = tf.losses.sigmoid_cross_entropy(y, y_pred)
    else:
        raise ValueError("Variable 'lossType' must be either 'MSE' or 'CE'.")

    opt = tf.train.AdamOptimizer(learning_rate=alpha, **kwargs).minimize(cost)

    return w, b, X, y, y_pred, cost, opt
```

2. Implementing Stochastic Gradient Descent

[23]:
```
class BatchLoader(object):

    """
    Custom robust batch loader class
    """

    def __init__(self, data, batch_size=None, randomize=True, drop_last=False,␣
↪seed=None):

        # error checking
        if len(data) > 1:
            for i in range(len(data)-1):
                if data[i].shape[0] != data[i+1].shape[0]:
                    raise ValueError("All inputs must have the same number of␣
↪elements")

        self.data = data if type(data) == tuple else (data, )
        self.N = data[0].shape[0]
        self.batch_size = batch_size if batch_size != None else self.N
        self.drop_last = drop_last

        # shuffling data
        if randomize:
            indices = np.arange(self.N)
            np.random.seed(seed)
            np.random.shuffle(indices)
            self.data = tuple([d[indices] for d in self.data])

        self.index = 0

    def __iter__(self):
        return self

    def __next__(self):

        # stop condition
        if self.index >= self.N:
            self.index = 0          # resetting index for next iteration
            raise StopIteration

        # iterating
        self.index += self.batch_size

        if self.index > self.N:
            if self.drop_last:
```

```
                self.index = 0      # resetting index for next iteration
                raise StopIteration
            else:
                return tuple([ d[self.index - self.batch_size: ] for d in self.
↪data ])
        else:
            return tuple([ d[self.index - self.batch_size: self.index] for d in␣
↪self.data ])
```

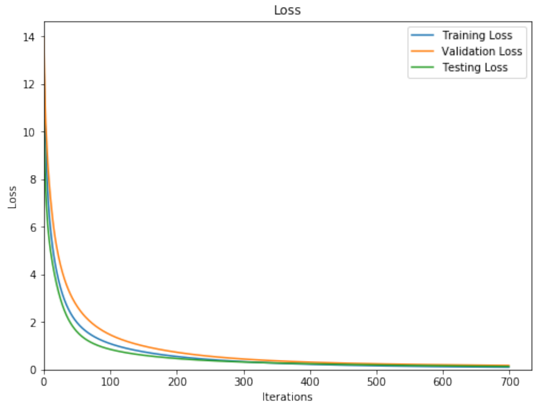
[24]:
```
def SGD(X, y, alpha, epochs, reg, error_tol=1e-7, lossType="MSE",
              batch_size=500, randomize=True, beta1=None, beta2=None,␣
↪epsilon=None,
              validData=None, validTarget=None, testData=None, testTarget=None):

    X = X.reshape(X.shape[0], -1)
    if not validData is None:
        validData = validData.reshape(validData.shape[0], -1)
    if not testData is None:
        testData = testData.reshape(testData.shape[0], -1)

    train_loss, train_acc = [], []
    valid_loss, valid_acc = [], []
    test_loss, test_acc = [], []
    printing = False

    running_loss = 0.0
    running_acc = 0.0

    w, b, X_tf, y_tf, y_pred_tf, cost, \
    optimizer = buildGraph(alpha=alpha, reg=reg, loss=lossType,
                           beta1=beta1, beta2=beta2, epsilon=epsilon)

    batch_iter = BatchLoader((X, y), batch_size=500, randomize=False)

    init = tf.initialize_all_variables()
    with tf.Session() as sess:
        sess.run(init)
        for e in range(epochs):
            for X_batch, y_batch in batch_iter:
                # updating the weights
                sess.run(optimizer, feed_dict={X_tf: X_batch, y_tf: y_batch})

                # getting the cost
                running_loss += sess.run(cost, feed_dict={X_tf: X_batch, y_tf:␣
↪y_batch}) * X_batch.shape[0]
                y_pred = sess.run(y_pred_tf, feed_dict={X_tf: X_batch, y_tf:␣
↪y_batch})
```

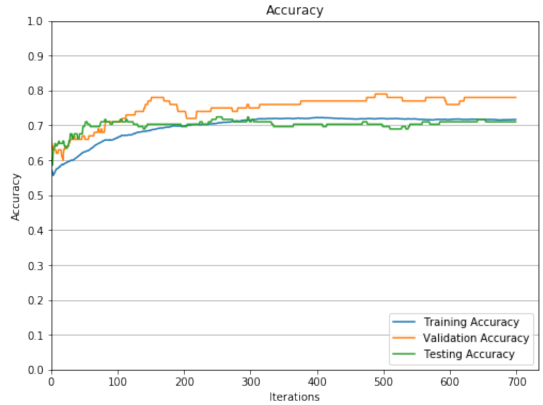
```
                running_acc += accuracy_with_predictions(y_pred.flatten(),␣
↪y_batch.flatten()) * y_batch.shape[0]

            else:
                # Calculating Statistics
                train_loss.append(running_loss / X.shape[0])
                train_acc.append(running_acc / X.shape[0])
                running_loss = 0.0
                running_acc = 0.0

                if not validData is None and not validTarget is None:
                    valid_loss.append(sess.run(cost, feed_dict={X_tf:␣
↪validData, y_tf: validTarget}))
                    y_pred = sess.run(y_pred_tf, feed_dict={X_tf: validData,␣
↪y_tf: validTarget})
                    valid_acc.append(accuracy_with_predictions(y_pred.
↪flatten(), validTarget.flatten()))
                if not testData is None and not testTarget is None:
                    test_loss.append(sess.run(cost, feed_dict={X_tf: testData,␣
↪y_tf: testTarget}))
                    y_pred = sess.run(y_pred_tf, feed_dict={X_tf: testData,␣
↪y_tf: testTarget})
                    test_acc.append(accuracy_with_predictions(y_pred.flatten(),␣
↪testTarget.flatten()))

                continue
            break

    statistics = (train_loss, train_acc)
    if not validData is None and not validTarget is None:
        statistics += (valid_loss, valid_acc,)
    if not testData is None and not testTarget is None:
        statistics += (test_loss, test_acc,)
    out = (w, b, *statistics)

    return out
```

[25]:
```
w, b, *statistics = SGD(trainData, trainTarget,
                        alpha=0.001,
                        epochs=700,
                        reg=0,
                        lossType='MSE',
                        batch_size=500,
                        **VTDatasets)
display_statistics(*statistics)
```

```
Training loss: 0.1018           Training acc: 71.69%
```

```
Validation loss: 0.1753          Validation acc: 78.00%
Testing loss: 0.1355             Testing acc: 71.03%
```

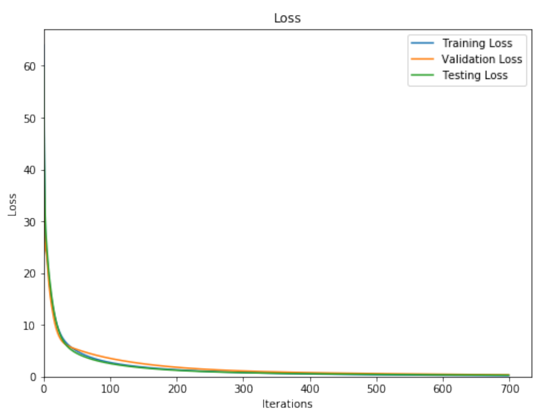


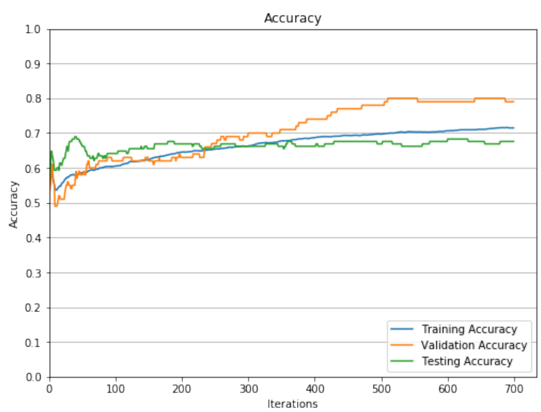


3. Batch Size Investigation

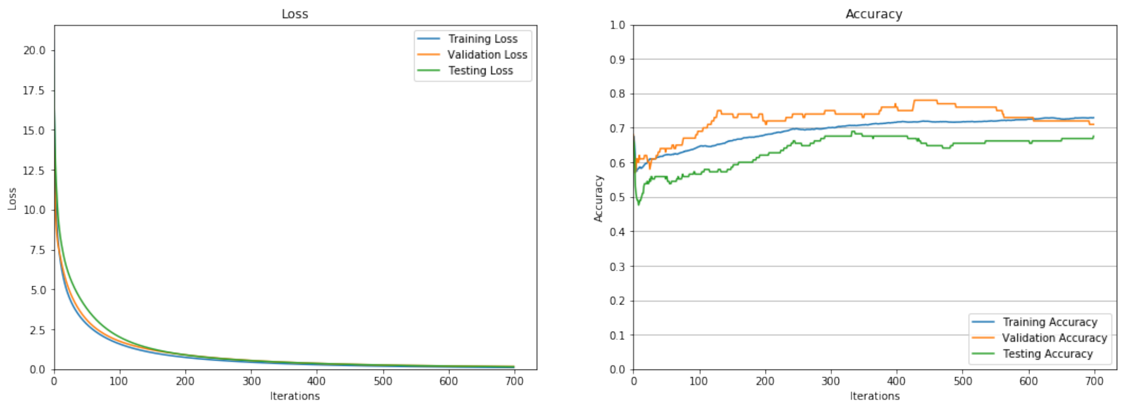
```
[26]: for batch_size in [100, 700, 1750]:

          print("batch size =", batch_size)

          w, b, *statistics = SGD(trainData, trainTarget,
                                  alpha=0.001,
                                  epochs=700,
                                  reg=0,
                                  lossType='MSE',
                                  batch_size=batch_size,
                                  **VTDatasets)
          display_statistics(*statistics)
```

```
batch size = 100
Training loss: 0.2088           Training acc: 71.49%
Validation loss: 0.3629         Validation acc: 79.00%
Testing loss: 0.2983            Testing acc: 67.59%
```

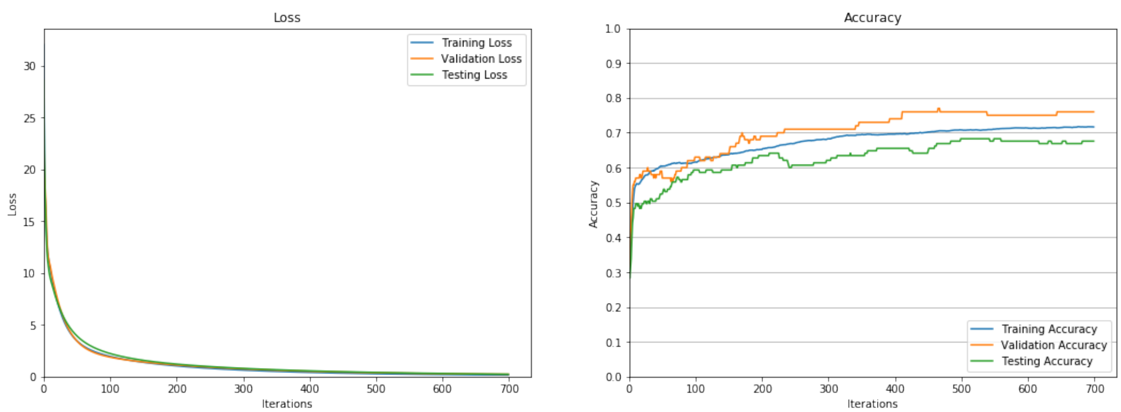





```
batch size = 700
Training loss: 0.0998           Training acc: 72.94%
Validation loss: 0.1584         Validation acc: 71.00%
Testing loss: 0.1527            Testing acc: 67.59%
```




```
batch size = 1750
Training loss: 0.1533          Training acc: 71.66%
Validation loss: 0.2316        Validation acc: 76.00%
Testing loss: 0.2374           Testing acc: 67.59%
```

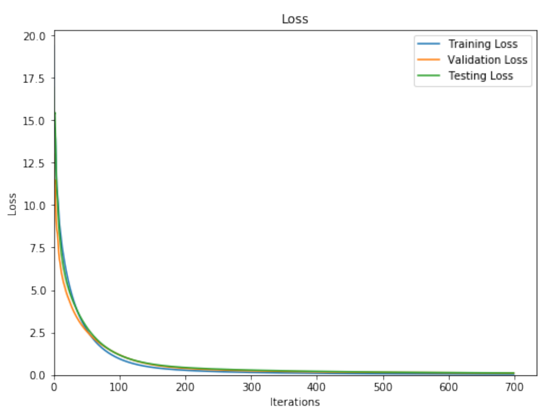


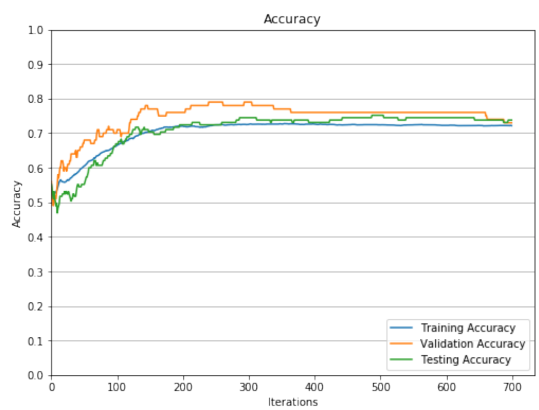
A smaller batch size converges better initially as seen in the graphs. A batch size of 100 jumps immediately to 60% accuracy where as a batch size of 1750 actually goes down initially. However, the batch size that was too small could not reach as optimal performance as the larger batch size. This is likely due to additional noise from the small sample size.

4. Hyperparameter Investigation

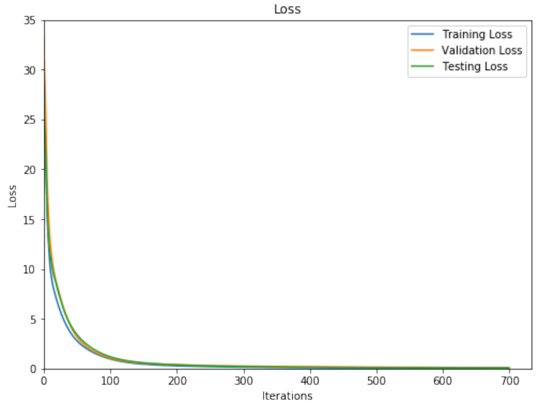
[27]:
```
for beta1 in [0.95, 0.99]:
    for beta2 in [0.99, 0.9999]:
        for epsilon in [1e-9, 1e-4]:

            print(f"beta1: {beta1}\tbeta2: {beta2}\tepsilon: {epsilon}")

            w, b, *statistics = SGD(trainData, trainTarget,
                                    alpha=0.001,
                                    epochs=700,
                                    reg=0,
                                    lossType='MSE',
                                    batch_size=500,
                                    beta1=beta1, beta2=beta2, epsilon=epsilon,
                                    **VTDatasets)
            display_statistics(*statistics)
```

```
beta1: 0.95     beta2: 0.99     epsilon: 1e-09
Training loss: 0.0506           Training acc: 72.17%
Validation loss: 0.1181         Validation acc: 73.00%
Testing loss: 0.1208            Testing acc: 73.79%
```

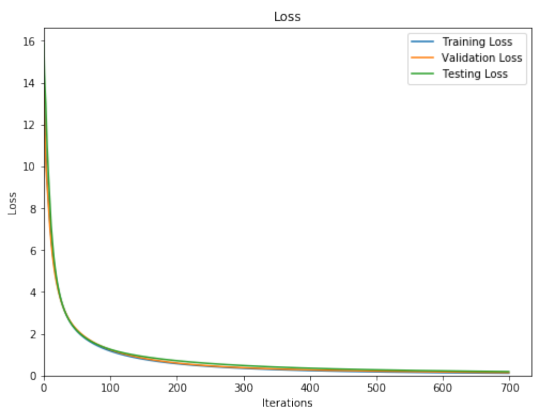


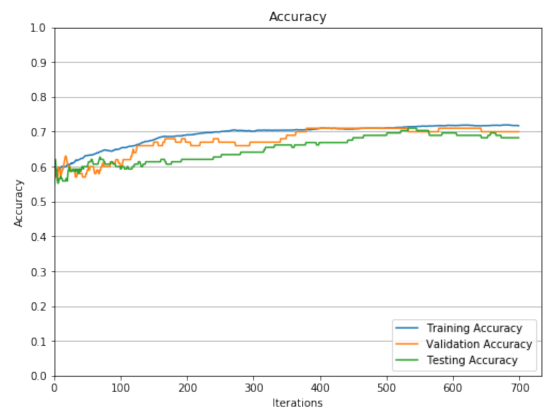



```
beta1: 0.95     beta2: 0.99     epsilon: 0.0001
Training loss: 0.0503           Training acc: 71.66%
Validation loss: 0.0994         Validation acc: 67.00%
Testing loss: 0.0815            Testing acc: 71.72%
```

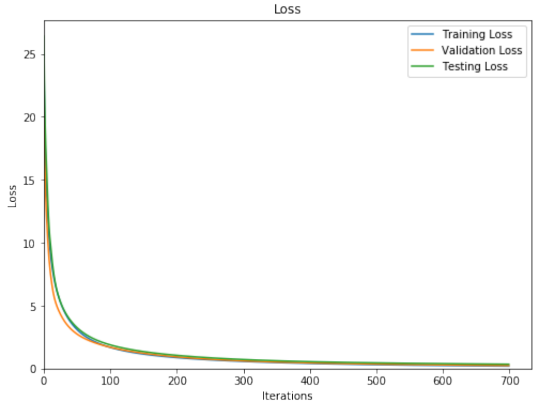



```
beta1: 0.95     beta2: 0.9999   epsilon: 1e-09
Training loss: 0.1179           Training acc: 71.71%
Validation loss: 0.1509         Validation acc: 70.00%
Testing loss: 0.1953            Testing acc: 68.28%
```

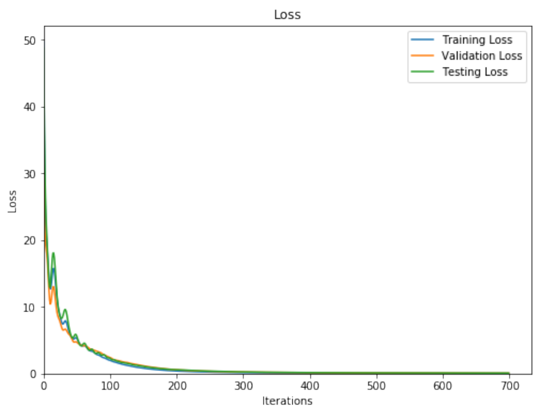




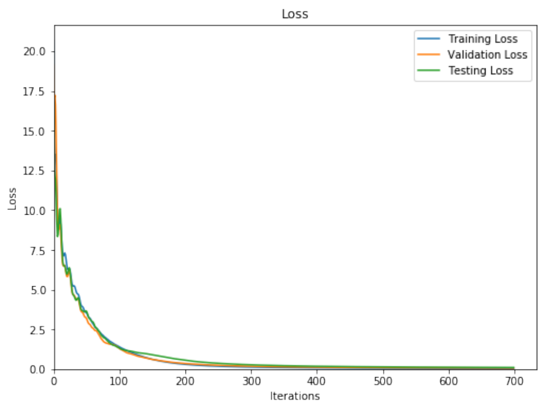

```
beta1: 0.95     beta2: 0.9999   epsilon: 0.0001
Training loss: 0.2036           Training acc: 72.06%
Validation loss: 0.2494         Validation acc: 69.00%
Testing loss: 0.3512            Testing acc: 71.03%
```

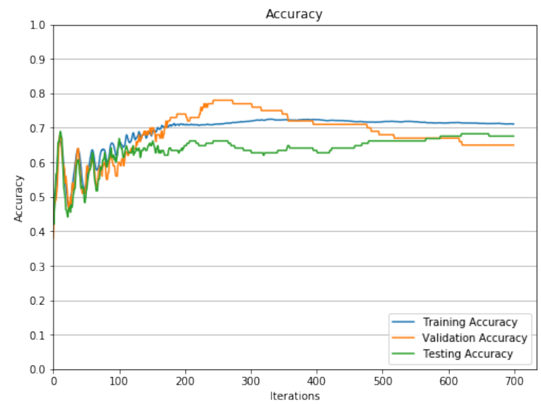



```
beta1: 0.99      beta2: 0.99       epsilon: 1e-09
Training loss: 0.0384              Training acc: 72.03%
Validation loss: 0.0759            Validation acc: 68.00%
Testing loss: 0.0870               Testing acc: 68.28%
```

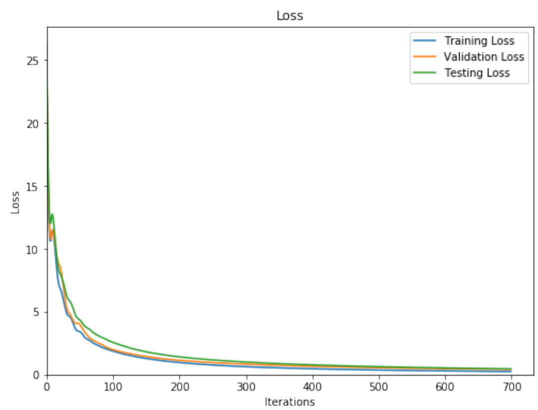


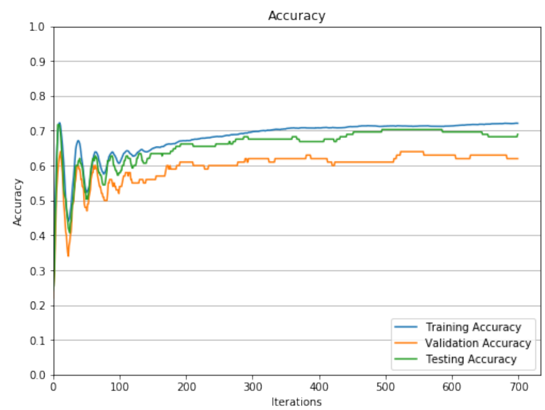

```
beta1: 0.99      beta2: 0.99       epsilon: 0.0001
Training loss: 0.0347              Training acc: 71.11%
Validation loss: 0.0649            Validation acc: 65.00%
Testing loss: 0.1013               Testing acc: 67.59%
```



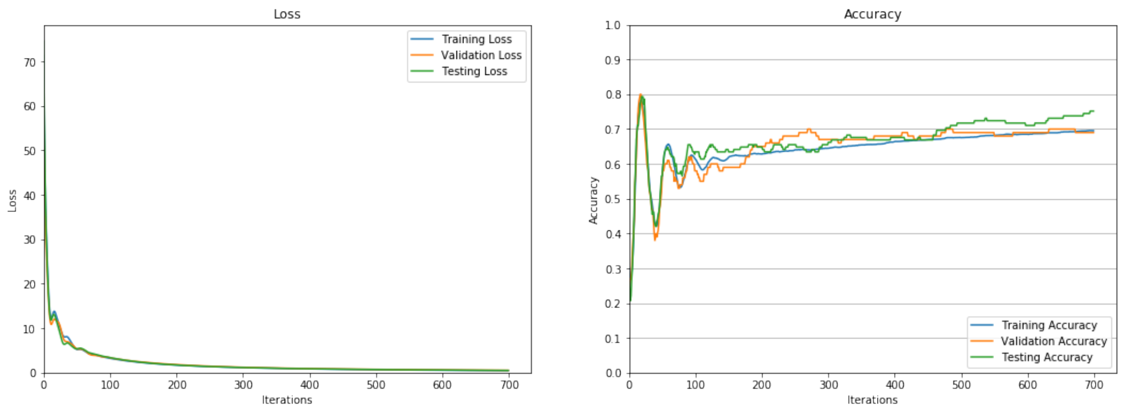



```
beta1: 0.99      beta2: 0.9999    epsilon: 1e-09
Training loss: 0.2517             Training acc: 72.14%
Validation loss: 0.4157           Validation acc: 62.00%
Testing loss: 0.4632              Testing acc: 68.97%
```






```
beta1: 0.99      beta2: 0.9999    epsilon: 0.0001
Training loss: 0.4685             Training acc: 69.54%
Validation loss: 0.5341           Validation acc: 69.00%
Testing loss: 0.5164              Testing acc: 75.17%
```



The equations for the Adam optimizer given by TensorFlow are the following:

$$lr_t = \alpha \cdot \frac{\sqrt{1 - \beta_2^t}}{1 - \beta_1^t}$$

$$m_t = \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g$$

$$v_t = \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g^2$$

$$var = var - lr_t \cdot \frac{m_t}{\sqrt{v_t} + \epsilon}$$

Epsilon is a small constant which helps with numerical stability. It ensures we don't divide by zero in the optimization step (in the case where $v_t = 0$). Epsilon of 10^{-4} is far too large and causes the optimization to make wrong corrections. This can be seen in the every second graph where initially all accuracies dip and the final accuracies of the datasets do not necessarily converge like they do when epsilon is 10^{-9}.

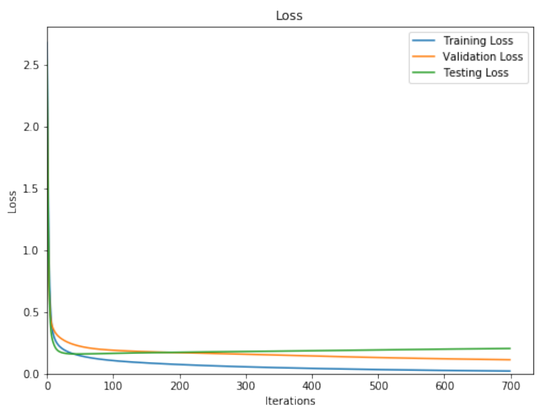
beta1 is the exponential decay rate for the first momentum estimate. Graphs with beta1 of 0.99 oscillate a lot in early iterations compared to a value of 0.95. Looking at the equations, this is most likely due to the adaptive step size: lr_t. The term $1 - \beta_1^t$ for beta1's close to 1 in early iterations will be very small, causing the step size to be very large and the parameters to oscillate. In later iterations, since beta1 is take to the power of t, this effect smooths out.

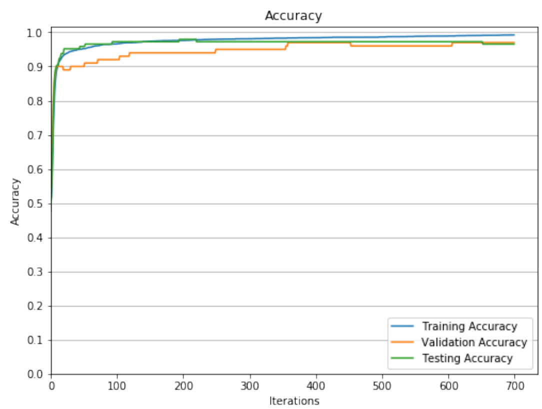
beta2 is the exponential decay rate for the second momentum estimate. Graphs with beta2 of 0.99 seem to oscillate less at the beginning, but converge faster at the end compared to graphs with beta2 of 0.9999. The oscillation is due to the opposite reason as beta1, since the term $1 - \beta_2$ is in the numerate of lr_t. The convergence at the end is most likely due to the $(1 - \beta_2) \cdot g^2$ term. If beta2 is too small, then $(1 - \beta_2)$ will be too large and the squared gradient will begin to dominate, causing the momentum to become too large.

5. Cross Entropy Loss Investigation

[28]:
```
# 3.1.2 with Cross Entropy Loss
w, b, *statistics = SGD(trainData, trainTarget,
                        alpha=0.001,
                        epochs=700,
                        reg=0,
                        lossType='CE',
                        batch_size=500,
                        **VTDatasets)
display_statistics(*statistics)
```

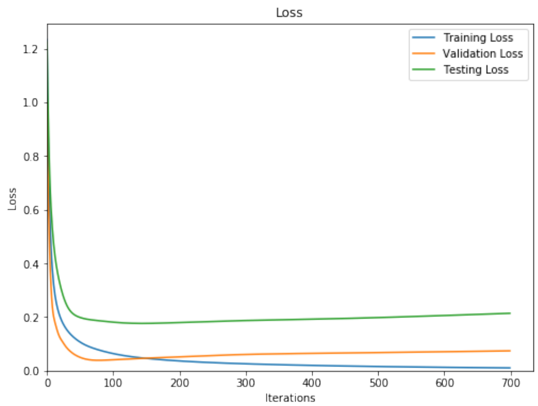
```
Training loss: 0.0220          Training acc: 99.23%
Validation loss: 0.1136        Validation acc: 97.00%
Testing loss: 0.2045           Testing acc: 96.55%
```

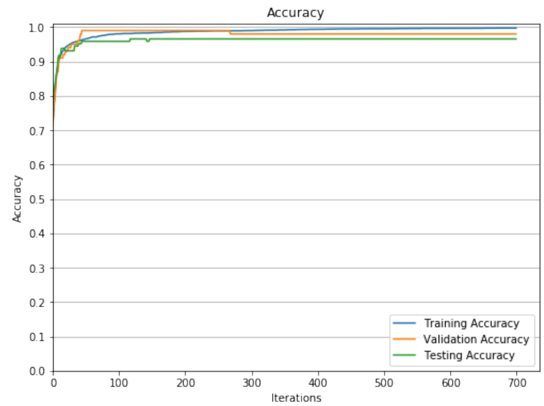




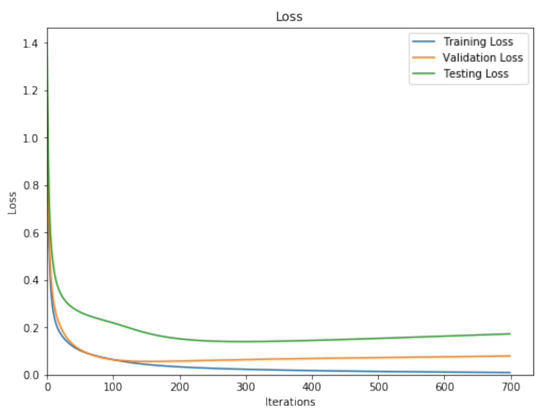
[29]:
```
# 3.1.4 with Cross Entropy Loss
for beta1 in [0.95, 0.99]:
    for beta2 in [0.99, 0.9999]:
        for epsilon in [1e-9, 1e-4]:

            print(f"beta1: {beta1}\tbeta2: {beta2}\tepsilon: {epsilon}")

            w, b, *statistics = SGD(trainData, trainTarget,
                                    alpha=0.001,
                                    epochs=700,
                                    reg=0,
                                    lossType='CE',
                                    batch_size=500,
                                    beta1=beta1, beta2=beta2, epsilon=epsilon,
                                    **VTDatasets)
            display_statistics(*statistics)
```

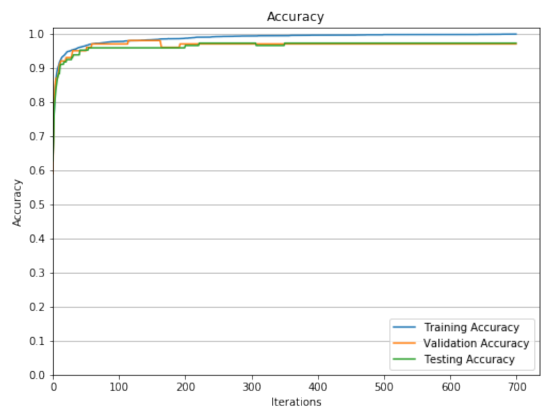
```
beta1: 0.95      beta2: 0.99      epsilon: 1e-09
Training loss: 0.0107             Training acc: 99.71%
Validation loss: 0.0742           Validation acc: 98.00%
Testing loss: 0.2142              Testing acc: 96.55%
```



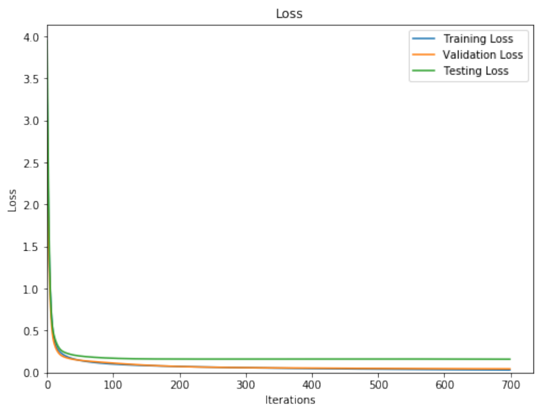


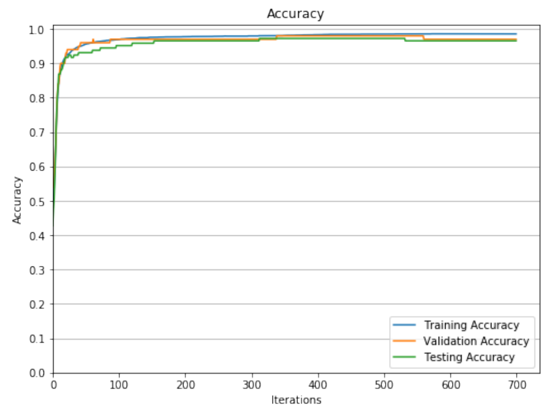

```
beta1: 0.95      beta2: 0.99      epsilon: 0.0001
Training loss: 0.0095             Training acc: 99.89%
Validation loss: 0.0794           Validation acc: 97.00%
Testing loss: 0.1724              Testing acc: 97.24%
```



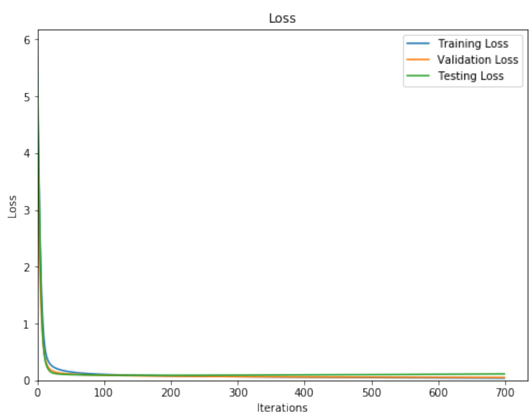


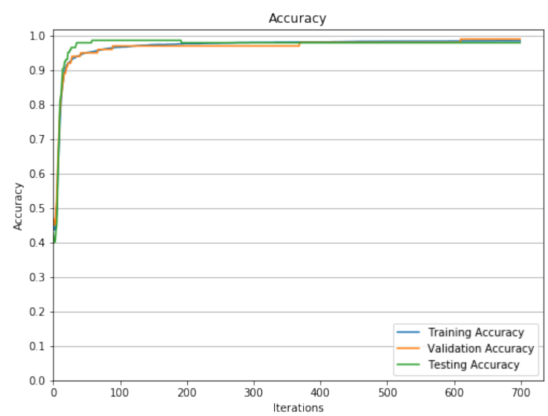

```
beta1: 0.95      beta2: 0.9999    epsilon: 1e-09
Training loss: 0.0322             Training acc: 98.57%
Validation loss: 0.0453           Validation acc: 97.00%
Testing loss: 0.1603              Testing acc: 96.55%
```



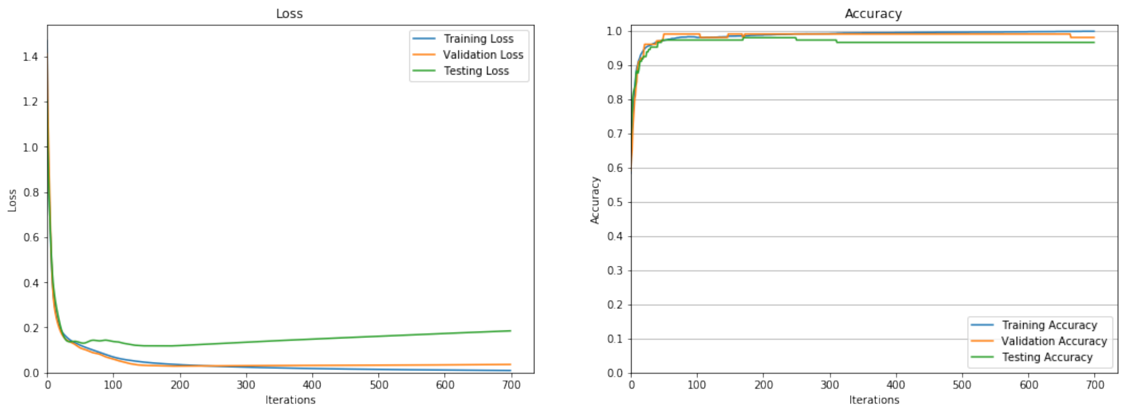



```
beta1: 0.95      beta2: 0.9999    epsilon: 0.0001
Training loss: 0.0387             Training acc: 98.46%
Validation loss: 0.0514           Validation acc: 99.00%
Testing loss: 0.1136              Testing acc: 97.93%
```

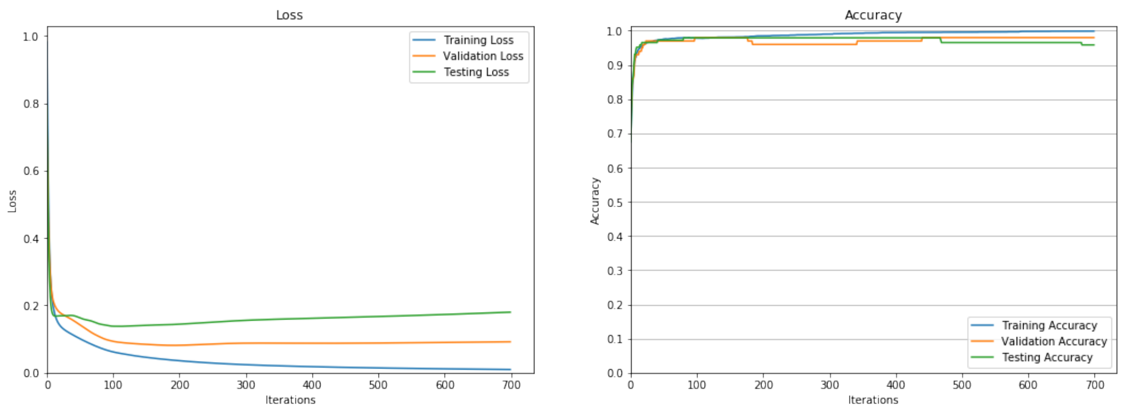




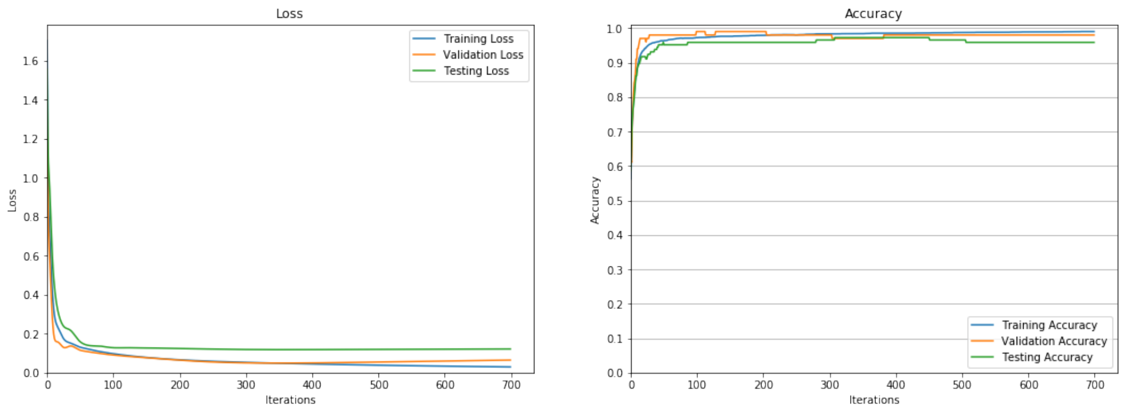

```
beta1: 0.99      beta2: 0.99      epsilon: 1e-09
Training loss: 0.0096             Training acc: 99.80%
Validation loss: 0.0364           Validation acc: 98.00%
Testing loss: 0.1850              Testing acc: 96.55%
```



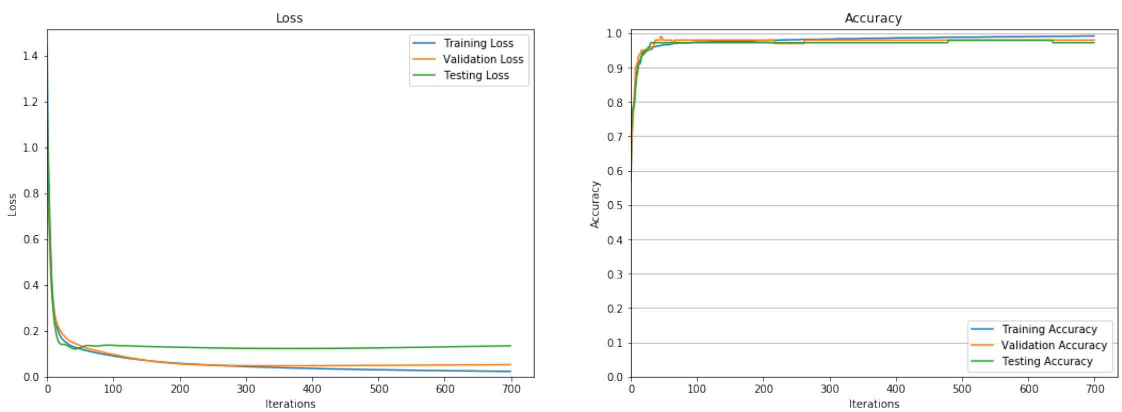

```
beta1: 0.99     beta2: 0.99     epsilon: 0.0001
Training loss: 0.0099           Training acc: 99.89%
Validation loss: 0.0922         Validation acc: 98.00%
Testing loss: 0.1800            Testing acc: 95.86%
```




```
beta1: 0.99     beta2: 0.9999   epsilon: 1e-09
Training loss: 0.0295           Training acc: 99.00%
Validation loss: 0.0646         Validation acc: 98.00%
Testing loss: 0.1215            Testing acc: 95.86%
```




```
beta1: 0.99      beta2: 0.9999    epsilon: 0.0001
Training loss: 0.0233             Training acc: 99.20%
Validation loss: 0.0528           Validation acc: 98.00%
Testing loss: 0.1349              Testing acc: 97.24%
```



The binary cross entrop loss does much better than the MSE loss function. In all tests, the finall accuracy is larger than the MSE. The best final test accuracy for cross entropy was 99.89% compared to the best for MSE at 72.16%. It's clear that the difference is dramatic. However, in some of the loss plots the training and testing lines sometimes diverge signifying overfitting. Overfitting did not occur as often with MSE loss.

6. Comparison against Batch GD

One of the big differences between the SGD with Adam and batch gradient descent is how much quicker SGD is able to achieve the same results as BGD. BGD usually required 1000 epochs at minimum to stabilize the curves. SGD, on the other hand, was able to do so in fewer than 200 epochs.

Concerning the accuracy plots, SGD with Adam creates creates an oscillating curve in most cases, before smoothing out. BGD, on the other hand, maintains a smooth curve throughout the training. This is most likely due to the nature of SGD working in random mini batches. As the algorithm finishes training on one mini batch, the accuracy is high. With the next random batch, however, the initial results are poor and the model needs to "retrain". This produces the oscillation in the curve. The loss graphs mirror this behaviour as well, with some of the loss graphs oscillating with the accuracy.