A1 WriteUp

February 5, 2020

0 Set Up

[1]:
```
# ignore all future warnings
from warnings import simplefilter
simplefilter(action='ignore', category=FutureWarning)
```

[2]:
```
# imports
import tensorflow as tf
import numpy as np
import matplotlib.pyplot as plt
```

[3]:
```
# ignore tensorflow depreciation warnings
import tensorflow.python.util.deprecation as deprecation
deprecation._PRINT_DEPRECATION_WARNINGS = False
```

0.1 Visualizing Dataset

It's always important to visualize the dataset to gain and understanding of what the model is trying to accomplish. This can help in the debugging phase. The shape of each data set is printed out as well as a random sample of the training data is plotted.

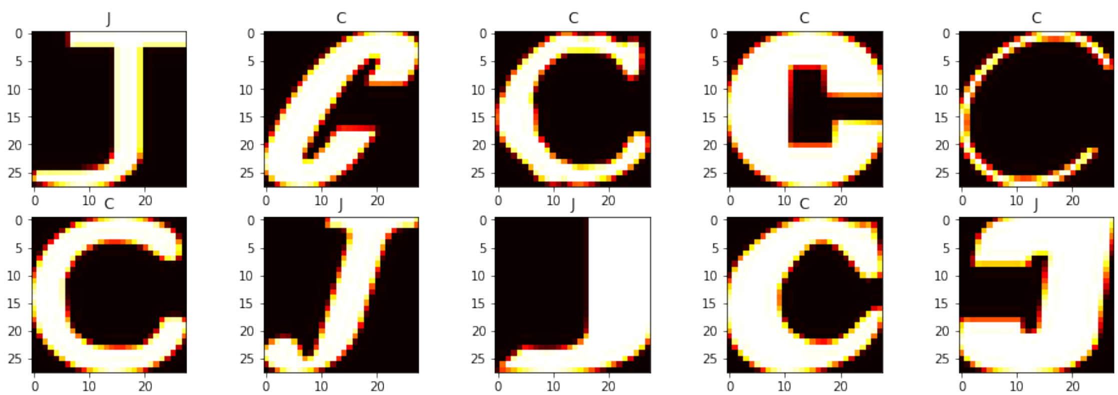
[4]:
```
# given by the assignment
def loadData():
    with np.load('notMNIST.npz') as data :
        Data, Target = data['images'], data['labels']
        posClass = 2
        negClass = 9
        dataIndx = (Target==posClass) + (Target==negClass)
        Data = Data[dataIndx]/255.
        Target = Target[dataIndx].reshape(-1, 1)
        Target[Target==posClass] = 1
        Target[Target==negClass] = 0
        np.random.seed(421)
        randIndx = np.arange(len(Data))
        np.random.shuffle(randIndx)
        Data, Target = Data[randIndx], Target[randIndx]
        trainData, trainTarget = Data[:3500], Target[:3500]
        validData, validTarget = Data[3500:3600], Target[3500:3600]
        testData, testTarget = Data[3600:], Target[3600:]
    return trainData, validData, testData, trainTarget, validTarget, testTarget
```

```
[5]: trainData, validData, testData, trainTarget, validTarget, testTarget =␣
      ↪loadData()
     print(f"Training Data: {trainData.shape}\tTraining tagets: {trainTarget.shape}")
     print(f"Validation Data: {validData.shape}\tValidation tagets: {validTarget.
      ↪shape}")
     print(f"Testing Data: {testData.shape}\tTesting tagets:{testTarget.shape}")
```

```
Training Data: (3500, 28, 28)   Training tagets: (3500, 1)
Validation Data: (100, 28, 28)  Validation tagets: (100, 1)
Testing Data: (145, 28, 28)     Testing tagets:(145, 1)
```

```
[6]: def plot(image, target, ax=None):
         ax = plt.gca() if ax == None else ax
         ax.imshow(image, cmap="hot")
         ax.set_title('J' if target == 0 else 'C')
         # targets are binary encoded 0 == 'J' and 1 == 'C'
```

```
[7]: fig, axis = plt.subplots(2, 5, figsize=(16, 5))
     for ax in axis.reshape(-1):
         r = np.random.randint(trainData.shape[0])
         plot(trainData[r], trainTarget[r], ax=ax)
     plt.show()
```



0.2 Useful Functions

Some useful functions that will be used throughout the assignment such as getting random weights, getting the accuracy of a batch, making the loss and accuracy plots look nice, and global variables used throughout the code

[8]:
```
def augment(X, w, b):
    # flatten X
    if len(X.shape) == 3:
        X = X.reshape(X.shape[0], -1)

    # insert 1's at position 0 along the columns
    X = np.insert(X, 0, 1, axis=1)

    # insert b at the front of W
    w = np.insert(w, 0, b, axis=0)

    return X, w

def get_zero_parameters():
    w = np.zeros(d)
    b = np.zeros(1)
    return w, b

def get_random_parameters():
    w = np.random.uniform(low=-1.0, high=1.0, size=(d,))
    b = np.random.uniform(low=-1.0, high=1.0, size=(1,))
    return w, b
```

[9]:
```
def predict(w, b, X):
    X = X.reshape(X.shape[0], -1)
    return X.dot(w) + b

def accuracy(w, b, X, y):
    y = y.reshape(-1)
    y_pred = predict(w, b, X)
    y_pred = np.vectorize(lambda z: 1 if z > 0 else 0)(y_pred)
    return np.sum(y_pred == y) / y.shape[0]

def accuracy_with_predictions(y_pred, y):
    if y_pred.shape != y.shape:
        raise ValueError(f"prediction dimension {y_pred.shape} and␣
↪dimensions {y.shape} don't match")
    y_pred = np.vectorize(lambda z: 1 if z > 0 else 0)(y_pred)
    return np.sum(y_pred == y) / y.shape[0]
```

```
[10]: def plot_loss(x, train_loss=None, valid_loss=None, test_loss=None, title=None,␣
      ↪ax=None):
          ax = plt.gca() if ax == None else ax
          if train_loss != None:
              ax.plot(x, train_loss, label="Training Loss")
          if valid_loss != None:
              ax.plot(x, valid_loss, label="Validation Loss")
          if test_loss != None:
              ax.plot(x, test_loss, label="Testing Loss")

          ax.set_title("Loss" if title == None else title)

          ax.set_xlabel("Iterations")
          ax.set_xlim(left=0)
          ax.set_ylabel("Loss")
          ax.set_ylim(bottom=0)
          ax.legend(loc="upper right")

      def plot_accuracy(x, train_accuracy=None, valid_accuracy=None,␣
      ↪test_accuracy=None, title=None, ax=None):
          ax = plt.gca() if ax == None else ax
          if train_accuracy != None:
              ax.plot(x, train_accuracy, label="Training Accuracy")
          if valid_accuracy != None:
              ax.plot(x, valid_accuracy, label="Validation Accuracy")
          if test_accuracy != None:
              ax.plot(x, test_accuracy, label="Testing Accuracy")

          ax.set_title("Accuracy" if title == None else title)

          ax.set_xlabel("Iterations")
          ax.set_xlim(left=0)
          ax.set_ylabel("Accuracy")
          ax.set_yticks(np.arange(0, 1.1, step=0.1))
          ax.grid(linestyle='-', axis='y')
          ax.legend(loc="lower right")

      def display_statistics(train_loss, train_acc, valid_loss, valid_acc, test_loss,␣
      ↪test_acc):
          print(f"Training loss: {train_loss[-1]:.4f}{'':.20s}\t\tTraining acc:␣
      ↪{train_acc[-1]*100:.2f}%")
          print(f"Validation loss: {valid_loss[-1]:.4f}\t\tValidation acc:␣
      ↪{valid_acc[-1]*100:.2f}%")
          print(f"Testing loss: {test_loss[-1]:.4f}\t\tTesting acc: {test_acc[-1]*100:
      ↪.2f}%")

          fig, ax = plt.subplots(1, 2, figsize=(18, 6))
```

```
    plot_loss(np.arange(0, len(train_loss), 1), train_loss, valid_loss,␣
↪test_loss, ax=ax[0])
    plot_accuracy(np.arange(0, len(train_loss), 1), train_acc, valid_acc,␣
↪test_acc, ax=ax[1])
    plt.show()
    plt.close()
```

[11]:
```
VTDatasets = {"validData" : validData, "validTarget" : validTarget,
              "testData" : testData, "testTarget" : testTarget}

N = trainData.shape[0]
d = trainData.shape[1] * trainData.shape[2]
```

1 Linear Regression

1. Loss Function and Gradient

$$\hat{y}^{(n)} = W^T \mathbf{x}^{(n)} + b$$

$$\mathcal{L}_{MSE} = \frac{1}{N} \sum_{n=1}^{N} (\hat{y}^{(n)} - y^{(n)})^2 + \lambda \|W\|_2^2$$

$$\frac{\partial \mathcal{L}_{MSE}}{\partial b} = \frac{2}{N} \sum_{n=1}^{N} (\hat{y}^{(n)} - y^{(n)})$$

$$\frac{\partial \mathcal{L}_{MSE}}{\partial W} = \frac{2}{N} X^T (\hat{\mathbf{y}} - \mathbf{y}) + \lambda W$$

[12]:
```
def MSE(w, b, X, y, reg):
    X = X.reshape(X.shape[0], -1)
    y = y.reshape(-1)
    return np.square(X.dot(w) + b - y).mean() + reg * np.square(w).sum()

def gradMSE(w, b, X, y, reg):
    X = X.reshape(X.shape[0], -1)
    y = y.reshape(-1)
    N = y.shape[0]

    w_grad = 2.0/N * X.T.dot(X.dot(w) + b - y) + reg * w
    b_grad = 2.0/N * np.sum(X.dot(w) + b - y)
    return w_grad, b_grad
```

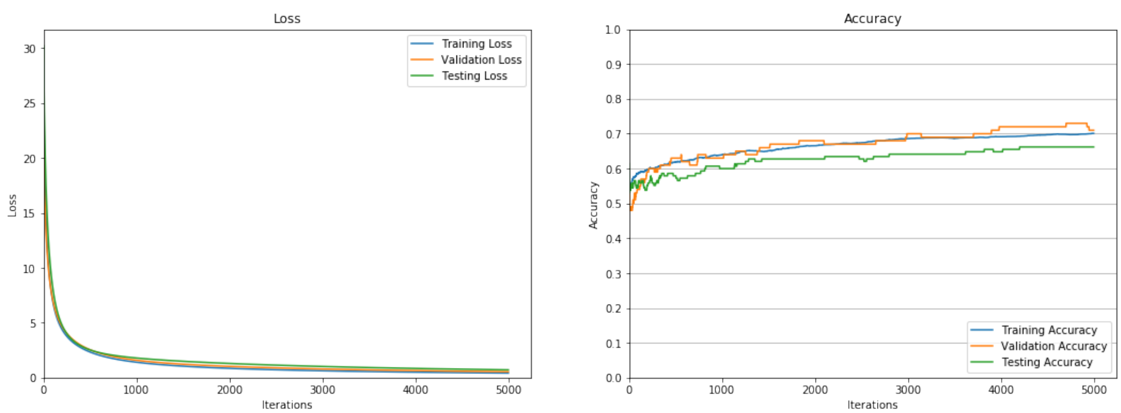
2. Gradient Descent Implementation

[13]:
```
def grad_descent_MSE(w, b, X, y, alpha, epochs, reg, error_tol=1e-7,
                     validData=None, validTarget=None, testData=None,␣
↪testTarget=None):
    train_loss, train_acc = [], []
    valid_loss, valid_acc = [], []
    test_loss, test_acc = [], []
    for i in range(epochs):
        grad_w, grad_b = gradMSE(w, b, X, y, reg)
        w -= alpha * grad_w
        b -= alpha * grad_b

        # Calculating Statistics
        train_loss.append( MSE(w, b, X, y, reg) )
        train_acc.append( accuracy(w, b, X, y) )

        if not validData is None and not validTarget is None:
            valid_loss.append( MSE(w, b, validData, validTarget, reg) )
            valid_acc.append( accuracy(w, b, validData, validTarget) )
        if not testData is None and not testTarget is None:
            test_loss.append( MSE(w, b, testData, testTarget, reg) )
            test_acc.append( accuracy(w, b, testData, testTarget) )

        # Check stopping condition
        if i > 1 and np.abs(train_loss[-2] - train_loss[-1]) <= error_tol:
            break

    statistics = (train_loss, train_acc)
    if not validData is None and not validTarget is None:
        statistics += (valid_loss, valid_acc, )
    if not testData is None and not testTarget is None:
        statistics += (test_loss, test_acc,)
    # Python 3.8 made this easier, but 3.7 you have to do this
    out = (w, b, *statistics)

    return out
```

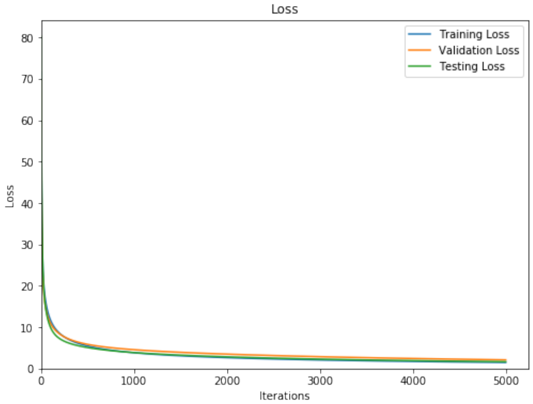
3. Tuning the Learning Rate

[14]:
```
for alpha in [0.005, 0.001, 0.0001]:

    print("alpha =", alpha)

    w, b = get_random_parameters()
    w, b, *statistics = grad_descent_MSE(w, b, trainData, trainTarget,
                                         alpha=alpha,
                                         epochs=5000,
                                         reg=0,
                                         **VTDatasets)
    display_statistics(*statistics)
```

```
alpha = 0.005
Training loss: 0.4199          Training acc: 70.11%
Validation loss: 0.5428        Validation acc: 71.00%
Testing loss: 0.7137           Testing acc: 66.21%
```




```
alpha = 0.001
Training loss: 1.4922          Training acc: 62.17%
Validation loss: 2.1204        Validation acc: 62.00%
Testing loss: 1.6711           Testing acc: 63.45%
```

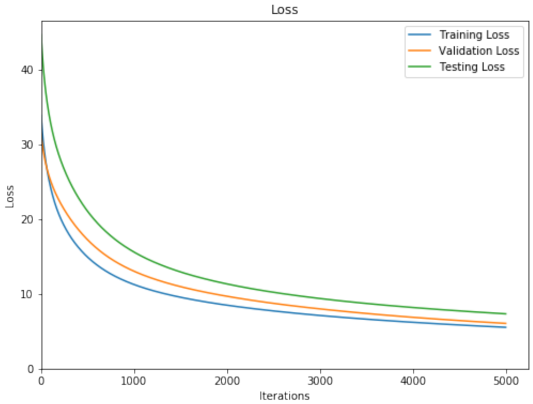


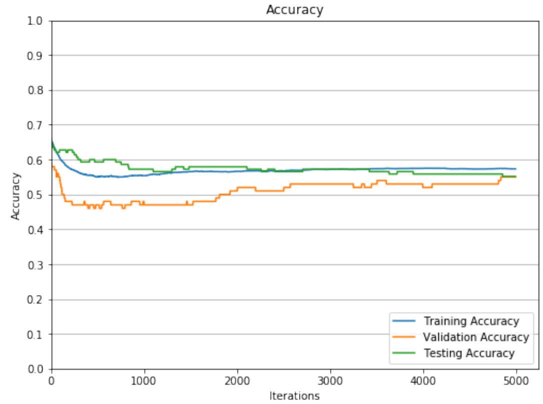
alpha = 0.0001

Training loss: 5.5281 Training acc: 57.34%

Validation loss: 6.0496 Validation acc: 55.00%

Testing loss: 7.3279 Testing acc: 55.17%

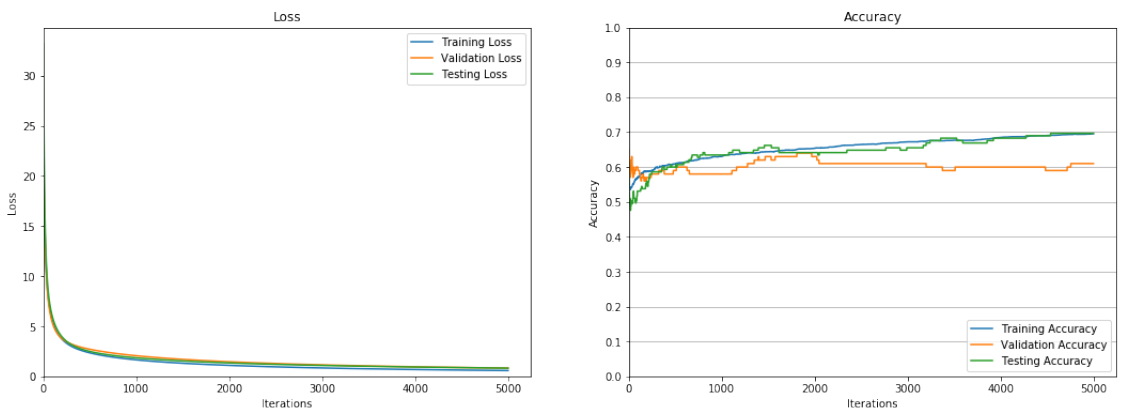




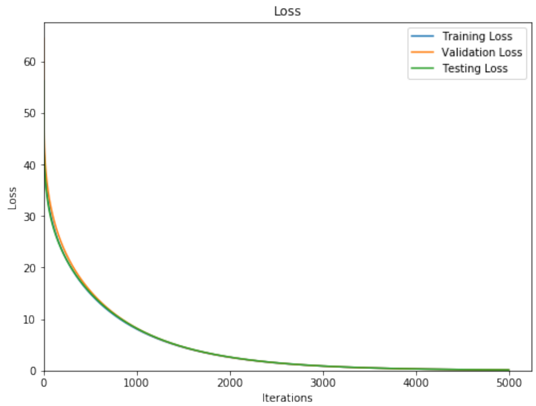
4. Generalization

```
[15]: for reg in [0.001, 0.1, 0.5]:

          print("regularization =", reg)

          w, b = get_random_parameters()
          w, b, *statistics = grad_descent_MSE(w, b, trainData, trainTarget,
                                               alpha=0.005,
                                               epochs=5000,
                                               reg=reg,
                                               **VTDatasets)
          display_statistics(*statistics)
```

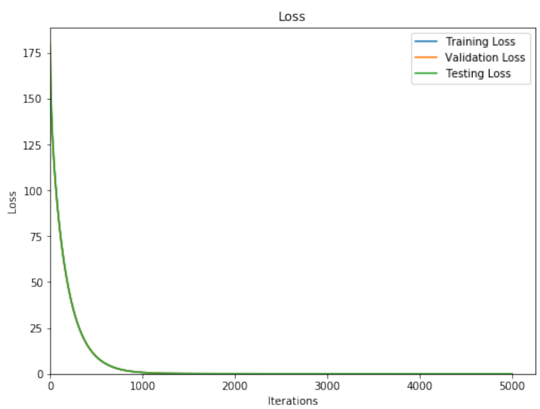
```
regularization = 0.001
Training loss: 0.5822          Training acc: 69.54%
Validation loss: 0.8218        Validation acc: 61.00%
Testing loss: 0.8229           Testing acc: 69.66%
```




```
regularization = 0.1
Training loss: 0.1332          Training acc: 68.60%
Validation loss: 0.1355        Validation acc: 65.00%
Testing loss: 0.1393           Testing acc: 64.14%
```




```
regularization = 0.5
Training loss: 0.0416          Training acc: 65.94%
Validation loss: 0.0451        Validation acc: 66.00%
Testing loss: 0.0442           Testing acc: 61.38%
```



5. Comparing Batch GD with normal equation

[16]:
```
def least_squares(X, y):
    N = X.shape[0]
    d = X.shape[1] * X.shape[2]
    X = X.reshape(X.shape[0], -1)
    X = np.insert(X, 0, 1, axis=1)
    y = y.reshape(-1)

    # overparameterized (deep learning)
    if N < d:
        w_aug = X.T @ np.linalg.inv( X @ X.T ) @ y

    # underparameterized (typical case)
    else:
        w_aug = np.linalg.inv( X.T @ X ) @ X.T @ y

    return w_aug[1:], w_aug[0]
```

[17]:
```
w_LS, b_LS = least_squares(trainData, trainTarget)

loss = MSE(w_LS, b_LS, trainData, trainTarget, 0)
acc = accuracy(w_LS, b_LS, trainData, trainTarget)
print(f"Least Squares Training loss: {loss:.4f}\tLeast Squares Training acc:␣
↪{100*acc:.2f}%")
loss = MSE(w_LS, b_LS, validData, validTarget, 0)
acc = accuracy(w_LS, b_LS, validData, validTarget)
print(f"Least Squares Validation loss: {loss:.4f}\tLeast Squares Validation acc:
↪ {100*acc:.2f}%")
loss = MSE(w_LS, b_LS, testData, testTarget, 0)
acc = accuracy(w_LS, b_LS, testData, testTarget)
print(f"Least Squares Testing loss: {loss:.4f}\tLeast Squares Testing acc:␣
↪{100*acc:.2f}%")
```

```
Least Squares Training loss: 0.0187      Least Squares Training acc: 71.29%
Least Squares Validation loss: 0.0476    Least Squares Validation acc: 69.00%
Least Squares Testing loss: 0.0570       Least Squares Testing acc: 66.90%
```

With the least squares solution achived a the training loss of 0.0187 and training accuracy of 71.29%. The training loss and accuracy for Batch Gradient Descent were 0.4199 and 70.11%, respectively. From the values, we see that the analytical solution performed marginally better. However, computing it grows increasingly difficult with the size of the problem. By constrast, batch gradient descent allows for faster convergence with less computation and comparable accuracies.

2 Logistic Regression

2.1 Binary cross-entropy loss

1. Loss Function and Gradient

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\hat{y}^{(n)} = \sigma(W^T \mathbf{x}^{(n)} + b)$$

$$\mathcal{L}_{CE} = \frac{1}{N}\sum_{n=1}^{N}\left[-y^{(n)}\log(\hat{y}^{(n)}) - (1-y^{(n)})\log(1-\hat{y}^{(n)})\right] + \frac{\lambda}{2}\|W\|_2^2$$

$$\frac{\partial \mathcal{L}_{CE}}{\partial b} = \frac{1}{N}\sum_{n=1}^{N}\left[\hat{y}^{(n)} - y^{(n)}\right]$$

$$\frac{\partial \mathcal{L}_{CE}}{\partial W} = \frac{1}{N}\sum_{n=1}^{N}\left[(\hat{y}^{(n)} - y^{(n)})\ \mathbf{x}^{(n)}\right] + \lambda W$$

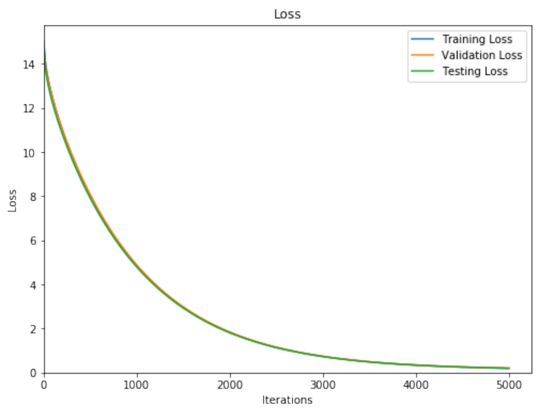
[18]:
```
def sigmoid(z):
    return 1.0 / (1 + np.exp(-z))

def crossEntropyLoss(w, b, X, y, reg):
    X, w = augment(X, w, b)
    y = y.reshape(-1)
    N = y.shape[0]

    y_hat = sigmoid(X.dot(w))

    return 1.0/N * (-y.dot(np.log(y_hat+1e-20)) - (1 - y).dot(np.log(1 -␣
 ↪y_hat+1e-20))) + reg/2.0 * np.square(w[1:]).sum()

def gradCE(w, b, X, y, reg):
    X, w = augment(X, w, b)
    y = y.reshape(-1)
    N = y.shape[0]

    y_hat = sigmoid(X.dot(w))

    grad = 1.0 /N * X.T.dot(y_hat - y) + reg * w

    return grad[1:], grad[0] - reg * w[0]
```

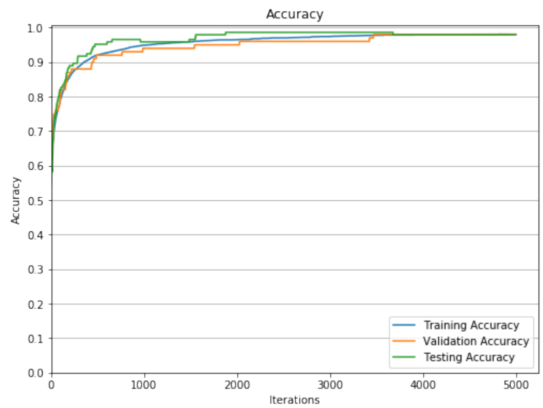
2. Learning

[19]:
```
def grad_descent(w, b, X, y, alpha, epochs, reg, error_tol=1e-7, lossType="MSE",
                 validData=None, validTarget=None, testData=None,␣
↪testTarget=None):
    loss_func, grad_func = None, None
    if lossType == "MSE":
        loss_func, grad_func = MSE, gradMSE
    elif lossType == "CE":
        loss_func, grad_func = crossEntropyLoss, gradCE
    else:
        raise ValueError("Variable 'lossType' must be either 'MSE' or 'CE'.")

    train_loss, train_acc = [], []
    valid_loss, valid_acc = [], []
    test_loss, test_acc = [], []
    printing = False
    for i in range(epochs):
        grad_w, grad_b = grad_func(w, b, X, y, reg)
        w -= alpha * grad_w
        b -= alpha * grad_b

        # Calculating Statistics
        train_loss.append(loss_func(w, b, X, y, reg))
        train_acc.append(accuracy(w, b, X, y))

        if not validData is None and not validTarget is None:
            valid_loss.append(loss_func(w, b, validData, validTarget, reg))
            valid_acc.append(accuracy(w, b, validData, validTarget))
        if not testData is None and not testTarget is None:
            test_loss.append(loss_func(w, b, testData, testTarget, reg))
            test_acc.append(accuracy(w, b, testData, testTarget))

        # Check stopping condition
        if i > 1 and np.abs(train_loss[-2] - train_loss[-1]) <= error_tol:
            break

    statistics = (train_loss, train_acc)
    if not validData is None and not validTarget is None:
        statistics += (valid_loss, valid_acc,)
    if not testData is None and not testTarget is None:
        statistics += (test_loss, test_acc,)
    out = (w, b, *statistics)

    return out
```

[20]:
```
w, b = get_random_parameters()
w, b, *statistics = grad_descent(w, b, trainData, trainTarget,
                                 alpha=0.005,
                                 epochs=5000,
                                 reg=0.1,
                                 lossType='CE',
                                 **VTDatasets)
display_statistics(*statistics)
```

```
Training loss: 0.1992          Training acc: 98.06%
Validation loss: 0.2082        Validation acc: 98.00%
Testing loss: 0.2055           Testing acc: 97.93%
```

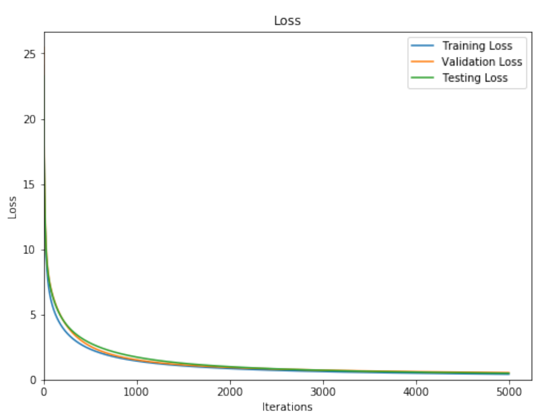




3. Comparision to Linear Regression

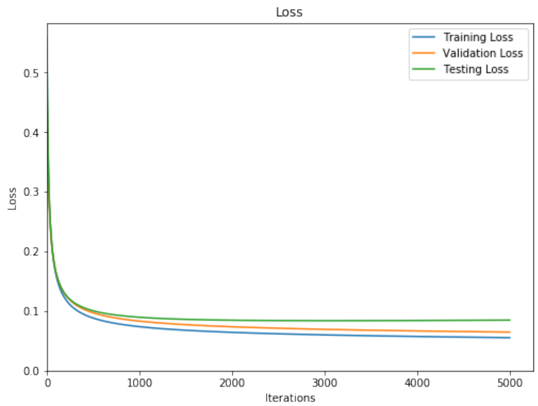
[21]:
```
# same initialization for each to a more fair comparison
w, b = get_random_parameters()

# Linear Regresesion
print("Linear Regression")
w_lin, b_lin, *statistics = grad_descent(w, b, trainData, trainTarget,
                                          alpha=0.005,
                                          epochs=5000,
                                          reg=0,
                                          lossType='MSE',
                                          **VTDatasets)
display_statistics(*statistics)


# Logistic Regression
print("Logistic Regression")
w_log, b_log, *statistics = grad_descent(w, b, trainData, trainTarget,
                                          alpha=0.005,
                                          epochs=5000,
                                          reg=0,
                                          lossType='CE',
                                          **VTDatasets)
display_statistics(*statistics)
```

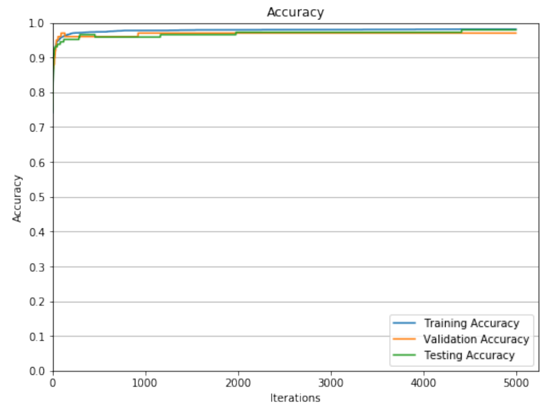
```
Linear Regression
Training loss: 0.4194          Training acc: 70.69%
Validation loss: 0.5375        Validation acc: 74.00%
Testing loss: 0.4956           Testing acc: 68.97%
```




```
Logistic Regression
Training loss: 0.0553          Training acc: 98.14%
```

```
Validation loss: 0.0647          Validation acc: 97.00%
Testing loss: 0.0847             Testing acc: 97.93%
```





The cross-entropy loss function allows for much faster convergence to a more optimal minimum compared to the MSE loss function. This is due to the fact that cross-entropy punishes wrong predictions much more than MSE.

3 Batch Gradient Descent vs. SGD and Adam

3.1 SGD

1. Building the Computational Graph

[22]:
```
def buildGraph(alpha, reg=0, beta1=None, beta2=None, epsilon=None, loss="MSE",␣
 ↪seed=None):
    kwargs = {}
    if beta1 != None:
        kwargs["beta1"] = beta1
    if beta2 != None:
        kwargs["beta2"] = beta2
    if epsilon != None:
        kwargs["epsilon"] = epsilon

    # Initialize weight and bias tensors
    tf.set_random_seed(seed)
    tf.compat.v1.set_random_seed(seed)
    tf.compat.v1.disable_eager_execution()

    # Getting small random initial values for weights and bias
    w = tf.truncated_normal([d, 1], stddev=0.5)
    b = tf.truncated_normal([1, 1], stddev=0.5)
    # Converting into tensorflow Variable objects
    w = tf.Variable(w, name="weights")
    b = tf.Variable(b, name="bias")

    # tensorflow objects for data
    X = tf.placeholder(tf.float32, (None, d))
    y = tf.placeholder(tf.float32, (None, 1))

    y_pred = None
    cost = None
    if loss == "MSE":
        y_pred = tf.matmul(X, w) + b
        cost = tf.losses.mean_squared_error(y, y_pred) + reg * tf.norm(w)**2
    elif loss == "CE":
        y_pred = tf.sigmoid(tf.matmul(X, w) + b)
        cost = tf.losses.sigmoid_cross_entropy(y, y_pred)
    else:
        raise ValueError("Variable 'lossType' must be either 'MSE' or 'CE'.")

    opt = tf.train.AdamOptimizer(learning_rate=alpha, **kwargs).minimize(cost)

    return w, b, X, y, y_pred, cost, opt
```

2. Implementing Stochastic Gradient Descent

[23]:
```
class BatchLoader(object):

    """
    Custom robust batch loader class
    """

    def __init__(self, data, batch_size=None, randomize=True, drop_last=False,␣
↪seed=None):

        # error checking
        if len(data) > 1:
            for i in range(len(data)-1):
                if data[i].shape[0] != data[i+1].shape[0]:
                    raise ValueError("All inputs must have the same number of␣
↪elements")

        self.data = data if type(data) == tuple else (data, )
        self.N = data[0].shape[0]
        self.batch_size = batch_size if batch_size != None else self.N
        self.drop_last = drop_last

        # shuffling data
        if randomize:
            indices = np.arange(self.N)
            np.random.seed(seed)
            np.random.shuffle(indices)
            self.data = tuple([d[indices] for d in self.data])

        self.index = 0

    def __iter__(self):
        return self

    def __next__(self):

        # stop condition
        if self.index >= self.N:
            self.index = 0          # resetting index for next iteration
            raise StopIteration

        # iterating
        self.index += self.batch_size

        if self.index > self.N:
            if self.drop_last:
```

```
                self.index = 0      # resetting index for next iteration
                raise StopIteration
            else:
                return tuple([ d[self.index - self.batch_size: ] for d in self.
↪data ])
        else:
            return tuple([ d[self.index - self.batch_size: self.index] for d in␣
↪self.data ])
```

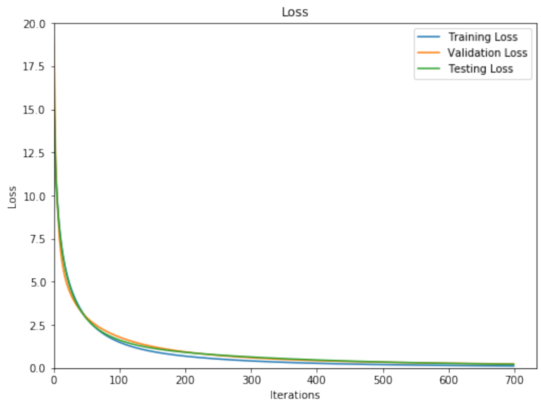
[24]:
```
def SGD(X, y, alpha, epochs, reg, error_tol=1e-7, lossType="MSE",
              batch_size=500, randomize=True, beta1=None, beta2=None,␣
↪epsilon=None,
              validData=None, validTarget=None, testData=None, testTarget=None):

    X = X.reshape(X.shape[0], -1)
    if not validData is None:
        validData = validData.reshape(validData.shape[0], -1)
    if not testData is None:
        testData = testData.reshape(testData.shape[0], -1)

    train_loss, train_acc = [], []
    valid_loss, valid_acc = [], []
    test_loss, test_acc = [], []
    printing = False

    running_loss = 0.0
    running_acc = 0.0

    w, b, X_tf, y_tf, y_pred_tf, cost, \
    optimizer = buildGraph(alpha=alpha, reg=reg, loss=lossType,
                           beta1=beta1, beta2=beta2, epsilon=epsilon)

    batch_iter = BatchLoader((X, y), batch_size=500, randomize=False)

    init = tf.initialize_all_variables()
    with tf.Session() as sess:
        sess.run(init)
        for e in range(epochs):
            for X_batch, y_batch in batch_iter:
                # updating the weights
                sess.run(optimizer, feed_dict={X_tf: X_batch, y_tf: y_batch})

                # getting the cost
                running_loss += sess.run(cost, feed_dict={X_tf: X_batch, y_tf:␣
↪y_batch}) * X_batch.shape[0]
                y_pred = sess.run(y_pred_tf, feed_dict={X_tf: X_batch, y_tf:␣
↪y_batch})
```

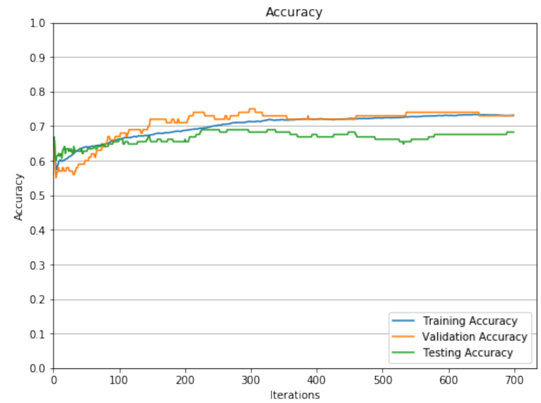
```
                running_acc += accuracy_with_predictions(y_pred.flatten(),␣
↪y_batch.flatten()) * y_batch.shape[0]

            else:
                # Calculating Statistics
                train_loss.append(running_loss / X.shape[0])
                train_acc.append(running_acc / X.shape[0])
                running_loss = 0.0
                running_acc = 0.0

                if not validData is None and not validTarget is None:
                    valid_loss.append(sess.run(cost, feed_dict={X_tf:␣
↪validData, y_tf: validTarget}))
                    y_pred = sess.run(y_pred_tf, feed_dict={X_tf: validData,␣
↪y_tf: validTarget})
                    valid_acc.append(accuracy_with_predictions(y_pred.
↪flatten(), validTarget.flatten()))
                if not testData is None and not testTarget is None:
                    test_loss.append(sess.run(cost, feed_dict={X_tf: testData,␣
↪y_tf: testTarget}))
                    y_pred = sess.run(y_pred_tf, feed_dict={X_tf: testData,␣
↪y_tf: testTarget})
                    test_acc.append(accuracy_with_predictions(y_pred.flatten(),␣
↪testTarget.flatten()))

                continue
            break

    statistics = (train_loss, train_acc)
    if not validData is None and not validTarget is None:
        statistics += (valid_loss, valid_acc,)
    if not testData is None and not testTarget is None:
        statistics += (test_loss, test_acc,)
    out = (w, b, *statistics)

    return out
```

[25]:
```
w, b, *statistics = SGD(trainData, trainTarget,
                        alpha=0.001,
                        epochs=700,
                        reg=0,
                        lossType='MSE',
                        batch_size=500,
                        **VTDatasets)
display_statistics(*statistics)
```

```
Training loss: 0.1192          Training acc: 73.17%
```

```
Validation loss: 0.2329          Validation acc: 73.00%
Testing loss: 0.2164             Testing acc: 68.28%
```



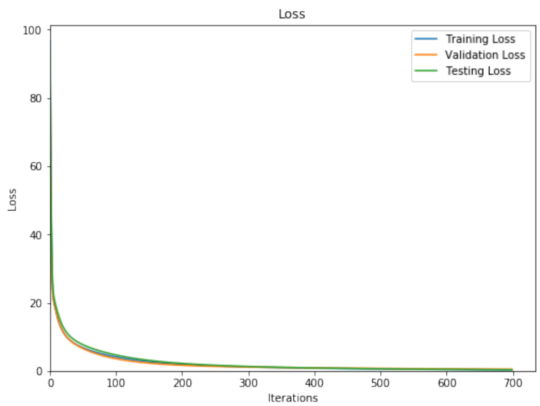


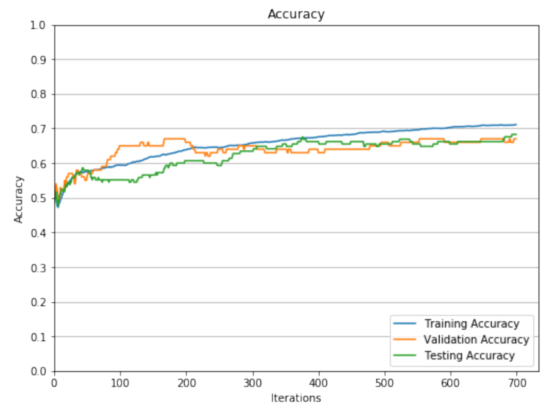
3. Batch Size Investigation

[29]:
```
for batch_size in [100, 700, 1750]:

    print("batch size =", batch_size)

    w, b, *statistics = SGD(trainData, trainTarget,
                            alpha=0.001,
                            epochs=700,
                            reg=0,
                            lossType='MSE',
                            batch_size=batch_size,
                            **VTDatasets)
    display_statistics(*statistics)
```

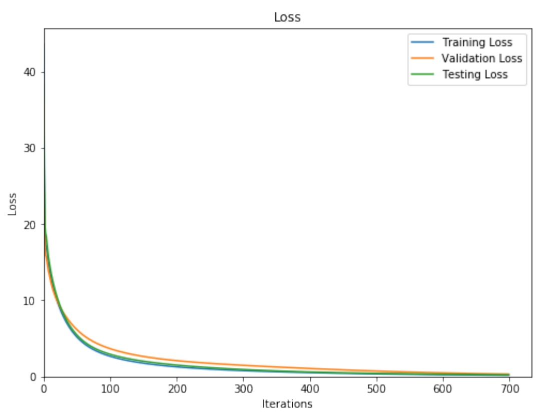
```
batch size = 100
Training loss: 0.3860          Training acc: 69.80%
Validation loss: 0.4299        Validation acc: 69.00%
Testing loss: 0.4305           Testing acc: 68.97%
```


```
batch size = 700
Training loss: 0.3032          Training acc: 71.11%
Validation loss: 0.4878        Validation acc: 67.00%
Testing loss: 0.3485           Testing acc: 68.28%
```






```
batch size = 1750
Training loss: 0.1794          Training acc: 72.20%
Validation loss: 0.3305        Validation acc: 72.00%
Testing loss: 0.2431           Testing acc: 71.72%
```

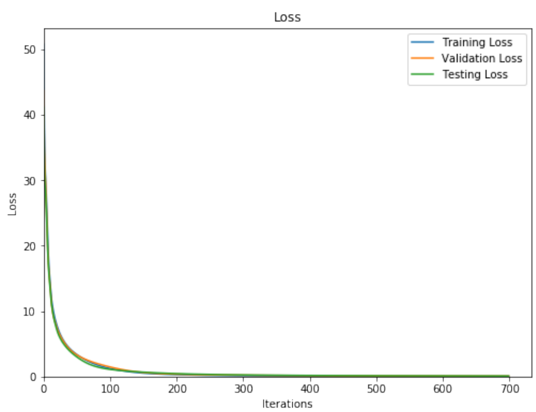


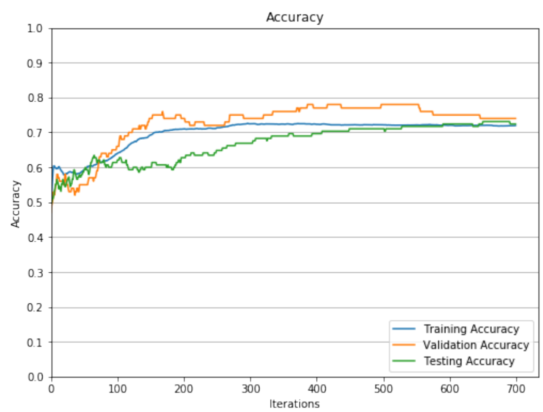
A smaller batch size converges much at first. This can be seen in the graphs. A batch size of 100 jumps immediately to 60% accuracy where as a batch size of 1750 actually goes down initially. However, a batch size that was too small could not reach as optimium of performance as the larger batch size. This is due to the noise from the small sample size.

4. Hyperparameter Investigation

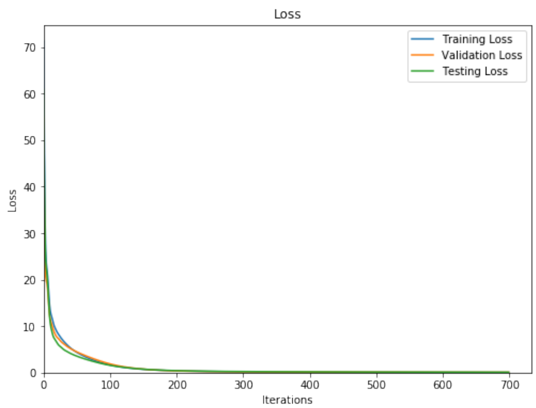
[30]:
```
for beta1 in [0.95, 0.99]:
    for beta2 in [0.99, 0.9999]:
        for epsilon in [1e-9, 1e-4]:

            print(f"beta1: {beta1}\tbeta2: {beta2}\tepsilon: {epsilon}")

            w, b, *statistics = SGD(trainData, trainTarget,
                                    alpha=0.001,
                                    epochs=700,
                                    reg=0,
                                    lossType='MSE',
                                    batch_size=500,
                                    beta1=beta1, beta2=beta2, epsilon=epsilon,
                                    **VTDatasets)
            display_statistics(*statistics)
```

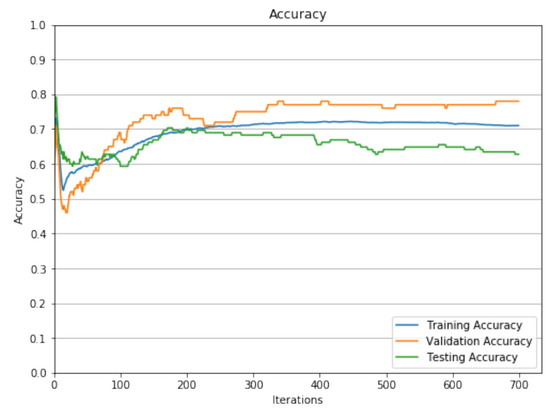
```
beta1: 0.95     beta2: 0.99     epsilon: 1e-09
Training loss: 0.0457           Training acc: 72.00%
Validation loss: 0.0839         Validation acc: 74.00%
Testing loss: 0.0960            Testing acc: 72.41%
```



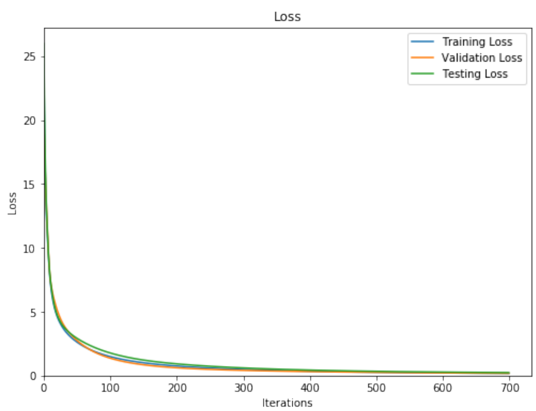


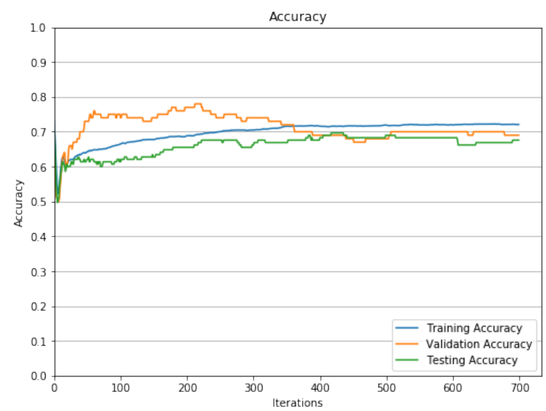

```
beta1: 0.95     beta2: 0.99     epsilon: 0.0001
Training loss: 0.0538           Training acc: 71.03%
Validation loss: 0.0890         Validation acc: 78.00%
Testing loss: 0.1007            Testing acc: 62.76%
```





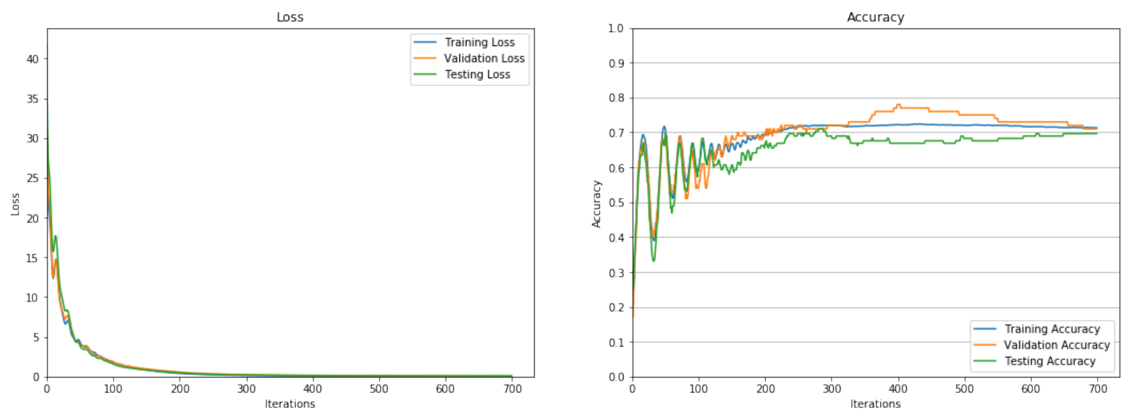

```
beta1: 0.95     beta2: 0.9999   epsilon: 1e-09
Training loss: 0.1798           Training acc: 72.09%
Validation loss: 0.1843         Validation acc: 69.00%
Testing loss: 0.2388            Testing acc: 67.59%
```



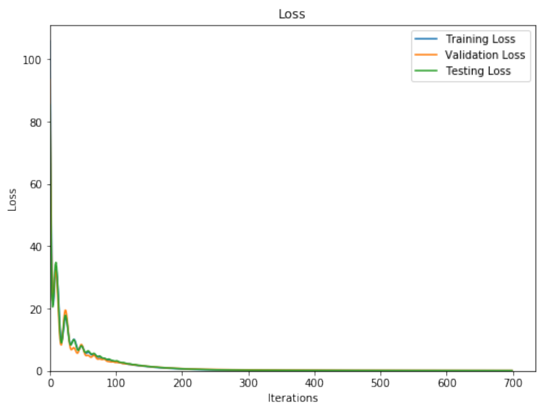



```
beta1: 0.95     beta2: 0.9999   epsilon: 0.0001
Training loss: 0.6275           Training acc: 69.06%
Validation loss: 0.7622         Validation acc: 68.00%
Testing loss: 0.8242            Testing acc: 73.10%
```

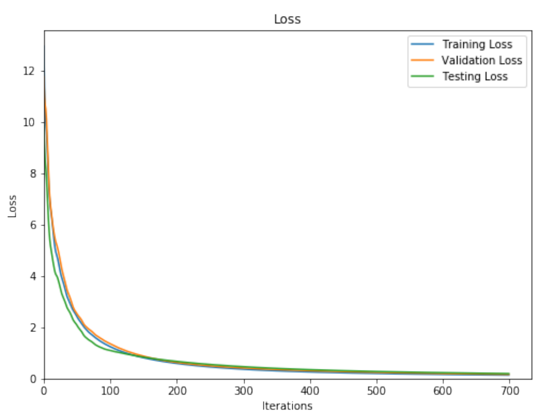
beta1: 0.99 beta2: 0.99 epsilon: 1e-09
Training loss: 0.0331 Training acc: 71.34%
Validation loss: 0.0707 Validation acc: 71.00%
Testing loss: 0.0816 Testing acc: 69.66%

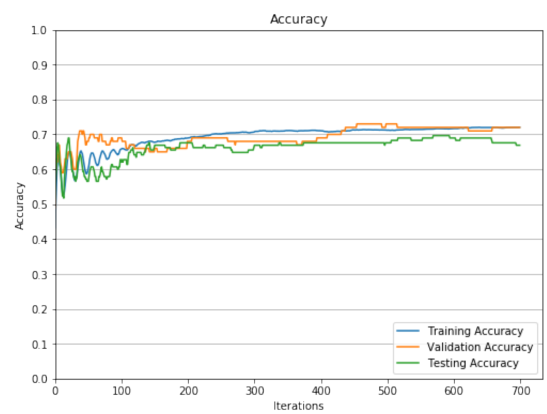


beta1: 0.99 beta2: 0.99 epsilon: 0.0001
Training loss: 0.0401 Training acc: 71.63%
Validation loss: 0.1119 Validation acc: 71.00%
Testing loss: 0.0986 Testing acc: 63.45%

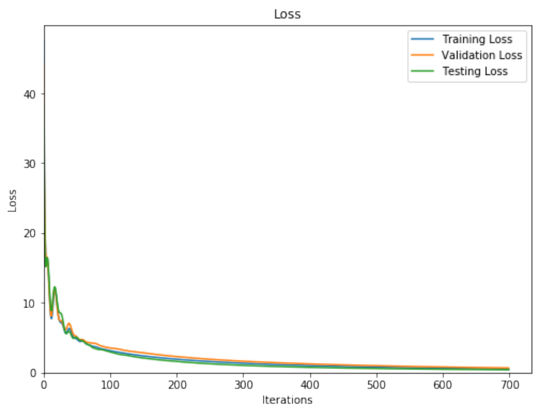


beta1: 0.99 beta2: 0.9999 epsilon: 1e-09
Training loss: 0.1370 Training acc: 72.03%
Validation loss: 0.1667 Validation acc: 72.00%
Testing loss: 0.1961 Testing acc: 66.90%





beta1: 0.99 beta2: 0.9999 epsilon: 0.0001
Training loss: 0.5262 Training acc: 68.83%
Validation loss: 0.6822 Validation acc: 65.00%
Testing loss: 0.4243 Testing acc: 71.03%



The equations for the Adam optimizer given by TensorFlow are the following

$$lr_t = \alpha \cdot \frac{\sqrt{1-\beta_2^t}}{1-\beta_1^t}$$

$$m_t = \beta_1 \cdot m_{t-1} + (1-\beta_1) \cdot g$$

$$v_t = \beta_2 \cdot v_{t-1} + (1-\beta_2) \cdot g^2$$

$$var = var - lr_t \cdot \frac{m_t}{\sqrt{v_t} + \epsilon}$$

epsilon is a small constant that ensures we don't divide by zero in the optimization step. Epsilon of 10^{-4} is far too large and causes the optimization to make wrong corrections. This can be seen in the every second graph where initially all accuracies dip and the final accuracies of the datasets do not necessarily converge like they do when epsilon is 10^{-9}.

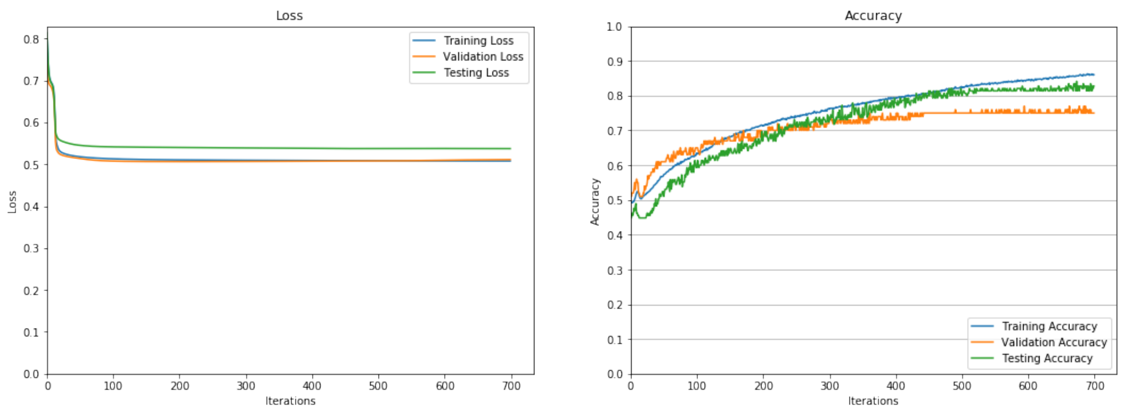
beta1 is the exponential decay rate for the first momentum estimate. Graphs with beta1 of 0.99 oscillate a lot in early iterations compared to a value of 0.95. Looking at the equations, this is most likely due to the adaptive step size: lr_t. The term $1 - \beta_1^t$ for beta1's close to 1 in early iterations will be very small, causing the step size to be very large and the parameters to oscillate. In later iterations, since beta1 is take to the power of t, this effect smooths out.

beta2 is the exponential decay rate for the second momentum estimate. Graphs with beta2 of 0.99 seem to oscillate less at the beginning, but converge faster at the end compared to graphs with beta2 of 0.9999. The oscillation is due to the opposite reason as beta1, since the term $1 - \beta_2$ is in the numerate of lr_t. The convergence at the end is most likely due to the $(1 - \beta_2) \cdot g^2$ term. If beta2 is too small, then $(1 - \beta_2)$ will be too large and the squared gradient will begin to dominate, causing the momentum to become too large.

5. Cross Entropy Loss Investigation

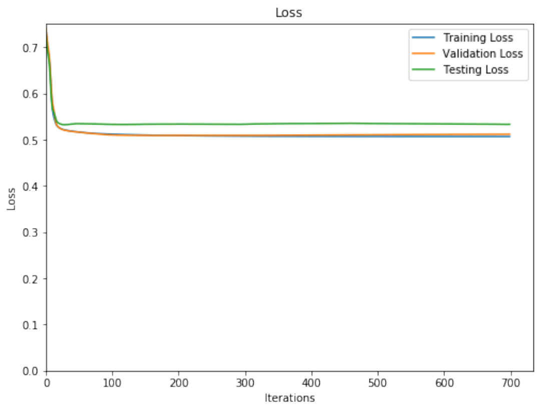
[31]:
```
# 3.1.2 with Cross Entropy Loss
w, b, *statistics = SGD(trainData, trainTarget,
                        alpha=0.001,
                        epochs=700,
                        reg=0,
                        lossType='CE',
                        batch_size=500,
                        **VTDatasets)
display_statistics(*statistics)
```

```
Training loss: 0.5077          Training acc: 86.03%
Validation loss: 0.5110        Validation acc: 75.00%
Testing loss: 0.5372           Testing acc: 82.76%
```

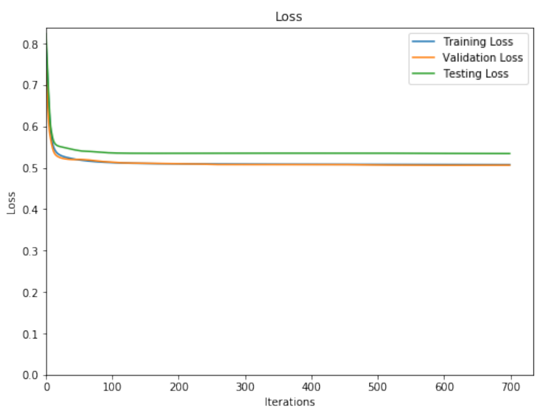


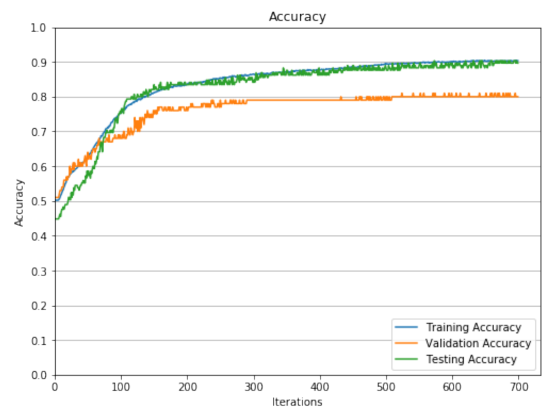
[32]:
```
# 3.1.4 with Cross Entropy Loss
for beta1 in [0.95, 0.99]:
    for beta2 in [0.99, 0.9999]:
        for epsilon in [1e-9, 1e-4]:

            print(f"beta1: {beta1}\tbeta2: {beta2}\tepsilon: {epsilon}")

            w, b, *statistics = SGD(trainData, trainTarget,
                                    alpha=0.001,
                                    epochs=700,
                                    reg=0,
                                    lossType='CE',
                                    batch_size=500,
                                    beta1=beta1, beta2=beta2, epsilon=epsilon,
                                    **VTDatasets)
            display_statistics(*statistics)
```

beta1: 0.95 beta2: 0.99 epsilon: 1e-09
Training loss: 0.5071 Training acc: 93.54%
Validation loss: 0.5122 Validation acc: 89.00%
Testing loss: 0.5338 Testing acc: 93.79%

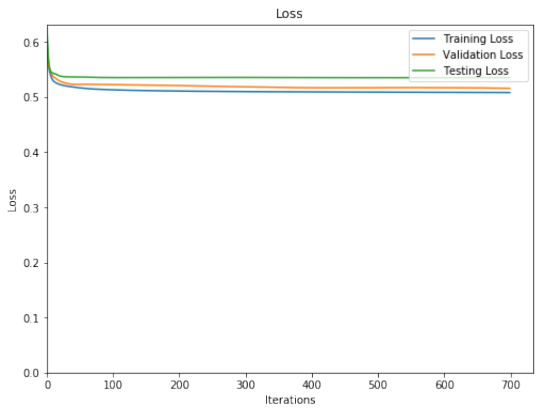


beta1: 0.95 beta2: 0.99 epsilon: 0.0001
Training loss: 0.5077 Training acc: 90.34%
Validation loss: 0.5063 Validation acc: 80.00%
Testing loss: 0.5344 Testing acc: 89.66%

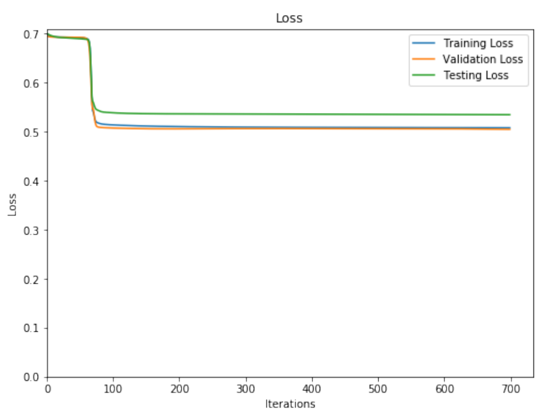




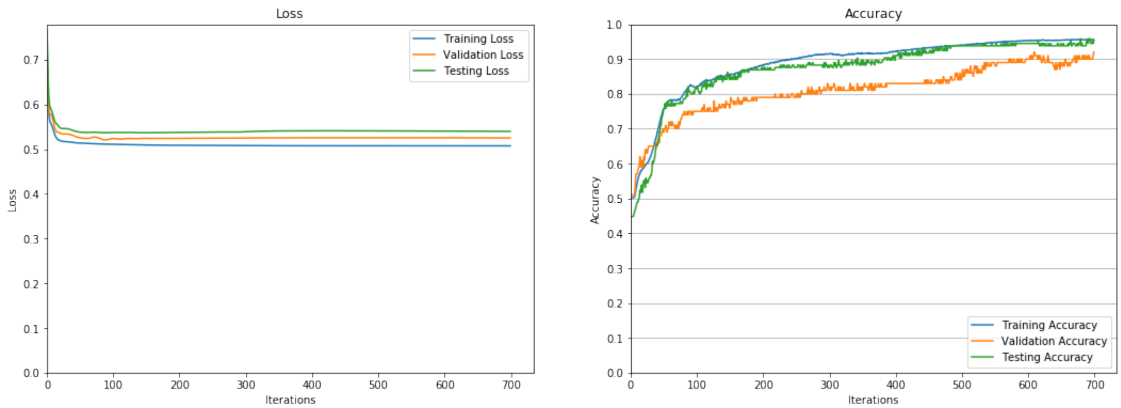
beta1: 0.95 beta2: 0.9999 epsilon: 1e-09
Training loss: 0.5084 Training acc: 83.34%
Validation loss: 0.5160 Validation acc: 77.00%
Testing loss: 0.5356 Testing acc: 80.00%



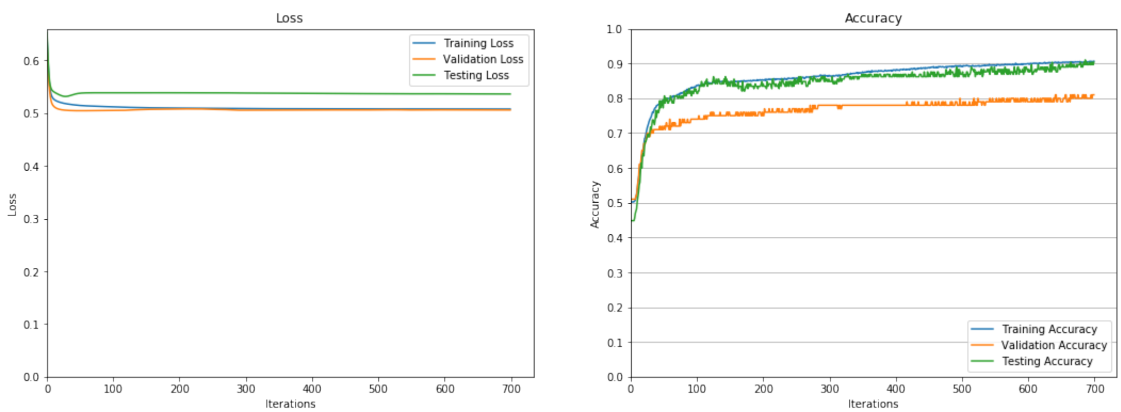

```
beta1: 0.95      beta2: 0.9999    epsilon: 0.0001
Training loss: 0.5082             Training acc: 83.06%
Validation loss: 0.5052           Validation acc: 76.00%
Testing loss: 0.5349              Testing acc: 83.45%
```



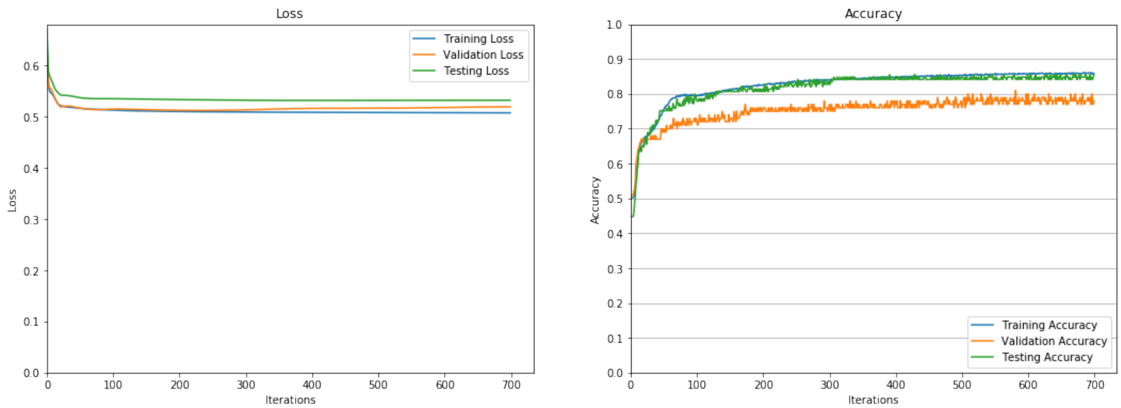

```
beta1: 0.99      beta2: 0.99      epsilon: 1e-09
Training loss: 0.5073             Training acc: 95.54%
Validation loss: 0.5249           Validation acc: 92.00%
Testing loss: 0.5397              Testing acc: 95.17%
```



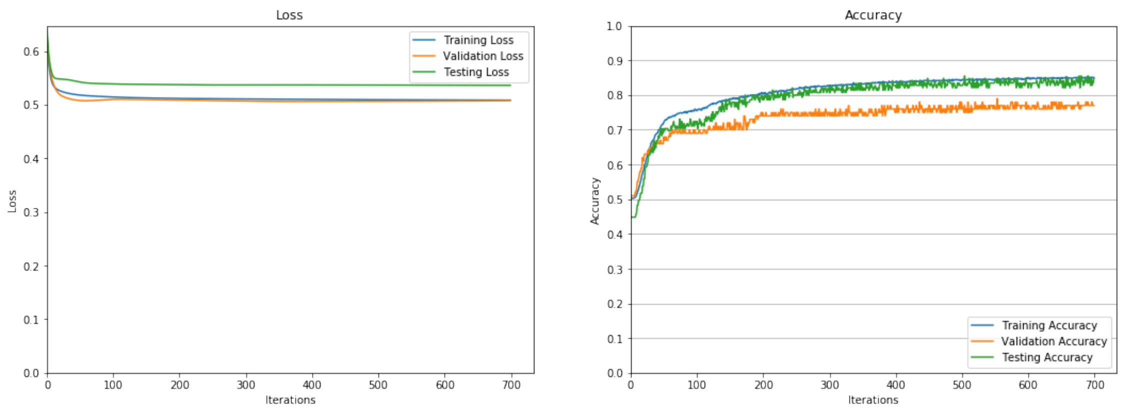

```
beta1: 0.99     beta2: 0.99     epsilon: 0.0001
Training loss: 0.5077           Training acc: 90.66%
Validation loss: 0.5059         Validation acc: 81.00%
Testing loss: 0.5362            Testing acc: 90.34%
```




```
beta1: 0.99     beta2: 0.9999   epsilon: 1e-09
Training loss: 0.5078           Training acc: 85.80%
Validation loss: 0.5195         Validation acc: 78.00%
Testing loss: 0.5325            Testing acc: 85.52%
```




```
beta1: 0.99     beta2: 0.9999   epsilon: 0.0001
Training loss: 0.5085           Training acc: 85.06%
Validation loss: 0.5077         Validation acc: 77.00%
Testing loss: 0.5361            Testing acc: 84.83%
```



The binary cross entrop loss does much better than the MSE loss function. In all tests, the finall accuracy is larger then the MSE. The best final test accuracy for cross entropy was 95.17% where as the best for MSE was 73.10%, which is a dramatic difference.

6. Comparison against Batch GD

One of the big differences between the SGD with Adam and batch gradient descent is how much quicker SGD is able to achieve the same results as BGD. BGD usually required 1000 epochs at minimum to stabilize the curves. SGD, on the other hand, was able to do so in less than 200 epochs.

Concerning the accuracy plots, SGD with Adam creates creates an oscillating curve in most cases, before smoothing out. BGD, on the other hand, maintains a smooth curve throughout the training. This is most likely due to the nature of SGD working in random mini batches. As the algorithm finishes training on one mini batch, the accuracy is high. With the next random batch, however, the initial results are poor and the model needs to “retrain”. This produces the oscillation in the curve. The loss graphs mirror this behaviour as well, with some of the loss graphs oscillating with the accuracy.