A2_WriteUp

February 22, 2020

0 Setup

[1]:
```
try:
    import google.colab
    IN_COLAB = True
except:
    IN_COLAB = False
    import tensorflow as tf

if IN_COLAB:
    %tensorflow_version 1.13
else:
    assert tf.__version__ == "1.13.1"

# ignore all future warnings
from warnings import simplefilter
simplefilter(action='ignore', category=FutureWarning)
```

```
`%tensorflow_version` only switches the major version: `1.x` or `2.x`.
You set: `1.13`. This will be interpreted as: `1.x`.


TensorFlow 1.x selected.
```

[0]:
```
# imports
import tensorflow as tf
import numpy as np
import matplotlib.pyplot as plt
from tensorflow.keras import layers, models
```

[3]:
```
print(tf.__version__)
```

```
1.15.0
```

[0]:
```
# ignore tensorflow depreciation warnings
import tensorflow.python.util.deprecation as deprecation
deprecation._PRINT_DEPRECATION_WARNINGS = False
```

0.1 Visualizing the Dataset

[0]:
```
# given by the assignment
def loadData():
    with np.load("notMNIST.npz") as data:
        Data, Target = data ["images"], data["labels"]
        np.random.seed(521)
        randIndx = np.arange(len(Data))
        np.random.shuffle(randIndx)
```

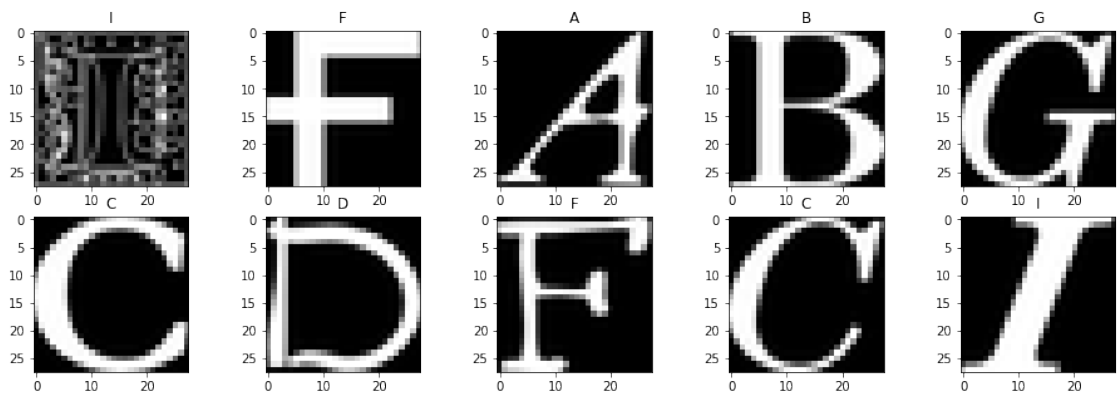
```
        Data = Data[randIndx]/255.
        Target = Target[randIndx]
        trainData, trainTarget = Data[:15000], Target[:15000]
        validData, validTarget = Data[15000:16000], Target[15000:16000]
        testData, testTarget = Data[16000:], Target[16000:]
    return trainData, validData, testData, trainTarget, validTarget, testTarget
```

[6]:
```
trainData, validData, testData, trainTarget, validTarget, testTarget = loadData()
print(f"Training Data: {trainData.shape}\tTraining tagets: {trainTarget.shape}")
print(f"Validation Data: {validData.shape}\tValidation tagets: {validTarget.
↪shape}")
print(f"Testing Data: {testData.shape}\tTesting tagets:{testTarget.shape}")
```

```
Training Data: (15000, 28, 28)  Training tagets: (15000,)
Validation Data: (1000, 28, 28) Validation tagets: (1000,)
Testing Data: (2724, 28, 28)    Testing tagets:(2724,)
```

[0]:
```
def plot(image, target, ax=None):
    ax = plt.gca() if ax == None else ax
    ax.imshow(image, cmap=plt.cm.gray)
    target_names = ['A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J']
    ax.set_title(target_names[target])
    # targets interger encoded from 0 to 9 corresponding to 'A' to 'J',␣
↪respectively
```

[8]:
```
fig, axis = plt.subplots(2, 5, figsize=(16, 5))
for ax in axis.reshape(-1):
    r = np.random.randint(trainData.shape[0])
    plot(trainData[r], trainTarget[r], ax=ax)
plt.show()
```



0.2 Useful Functions

Some useful functions that will be used throughout the assignment such as getting random weights, getting the accuracy of a batch, making the loss and accuracy plots look nice, and global variables used throughout the code

[0]:
```
# given by the assignment
def convertOneHot(trainTarget, validTarget, testTarget):
    newtrain = np.zeros((trainTarget.shape[0], 10))
    newvalid = np.zeros((validTarget.shape[0], 10))
    newtest = np.zeros((testTarget.shape[0], 10))
    for item in range(0, trainTarget.shape[0]):
        newtrain[item][trainTarget[item]] = 1
    for item in range(0, validTarget.shape[0]):
        newvalid[item][validTarget[item]] = 1
    for item in range(0, testTarget.shape[0]):
        newtest[item][testTarget[item]] = 1
    return newtrain, newvalid, newtest
```

[0]:
```
def accuracy(y_pred, y):
    if y_pred.shape != y.shape:
        raise ValueError(f"prediction dimension {y_pred.shape} and label␣
 ↪dimensions {y.shape} don't match")
    return np.sum(y_pred.argmax(axis=1) == y.argmax(axis=1)) / y.shape[0]
```

[0]:
```
def plot_loss(x, train_loss=None, valid_loss=None, test_loss=None, title=None,␣
 ↪ax=None):
    ax = plt.gca() if ax == None else ax
    if train_loss != None:
        ax.plot(x, train_loss, label="Training Loss")
    if valid_loss != None:
        ax.plot(x, valid_loss, label="Validation Loss")
    if test_loss != None:
        ax.plot(x, test_loss, label="Testing Loss")

    ax.set_title("Loss" if title == None else title)

    ax.set_xlabel("Iterations")
    ax.set_xlim(left=0)
    ax.set_ylabel("Loss")
    ax.set_ylim(bottom=0)
    ax.legend(loc="upper right")

def plot_accuracy(x, train_accuracy=None, valid_accuracy=None,␣
 ↪test_accuracy=None, title=None, ax=None):
    ax = plt.gca() if ax == None else ax
    if train_accuracy != None:
        ax.plot(x, train_accuracy, label="Training Accuracy")
```

```
    if valid_accuracy != None:
        ax.plot(x, valid_accuracy, label="Validation Accuracy")
    if test_accuracy != None:
        ax.plot(x, test_accuracy, label="Testing Accuracy")

    ax.set_title("Accuracy" if title == None else title)

    ax.set_xlabel("Iterations")
    ax.set_xlim(left=0)
    ax.set_ylabel("Accuracy")
    ax.set_yticks(np.arange(0, 1.1, step=0.1))
    ax.grid(linestyle='-', axis='y')
    ax.legend(loc="lower right")


def display_statistics(train_loss=None, train_acc=None, valid_loss=None,␣
 ↪valid_acc=None,
                       test_loss=None, test_acc=None, num=True, plot=True):

    tl = "-" if train_loss is None else round(train_loss[-1], 4)
    ta = "-" if train_acc is None else round(train_acc[-1]*100, 2)
    vl = "-\t" if valid_loss is None else round(valid_loss[-1], 4)
    va = "-" if valid_acc is None else round(valid_acc[-1]*100, 2)
    sl = "-\t\t" if test_loss is None else round(test_loss[-1], 4)
    sa = "-" if test_acc is None else round(test_acc[-1]*100, 2)

    if num:
        print(f"Training loss: {tl}{'':.20s}\t\tTraining acc: {ta}{'%' if ta !=␣
 ↪'-' else ''}")
        print(f"Validation loss: {vl}{'':.20s}\tValidation acc: {va}{'%' if va !
 ↪= '-' else ''}")
        print(f"Testing loss: {sl}{'':.20s}\tTesting acc: {sa}{'%' if sa != '-'␣
 ↪else ''}")

    if plot:
        fig, ax = plt.subplots(1, 2, figsize=(18, 6))
        plot_loss(np.arange(0, len(train_loss), 1), train_loss, valid_loss,␣
 ↪test_loss, ax=ax[0])
        plot_accuracy(np.arange(0, len(train_loss), 1), train_acc, valid_acc,␣
 ↪test_acc, ax=ax[1])
        plt.show()
        plt.close()
```

[0]:
```
TINY = 1e-20
newtrain, newvalid, newtest = convertOneHot(trainTarget, validTarget, testTarget)
```

```
VTDatasets = {"validData" : validData.reshape(validData.shape[0], -1),␣
 ↪"validTarget" : newvalid,
              "testData" : testData.reshape(testData.shape[0], -1), "testTarget"␣
 ↪: newtest}

N = trainData.shape[0]
d = trainData.shape[1] * trainData.shape[2]
K = 10
```

1 Neural Networks using Numpy

1.1 Helper Functions

[0]:
```
def relu(x):
    return np.maximum(0, x)

def softmax(x):
    return np.exp(x) / np.exp(x).sum()

def softmax_batch(X):
    return np.exp(X) / np.exp(X).sum(axis=1, keepdims=True)
```

[0]:
```
def computeLayer(X, W, b):
    return X @ W.T + b
```

[0]:
```
# target is one-hot encoded
def averageCE(target, prediction):
    return -(target * np.log(prediction+TINY)).sum(axis=1).mean()

# target is one-hot encoded
def gradCE(target, predication):
    return predication - target
```

1.2 Backpropagation Derivation

Derivative of Softmax $p_i = \texttt{softmax}(\mathbf{o})_i = \frac{e^{o_i}}{\sum_{k=1}^{K} e^{o_k}}$

if $i \neq j$

$$\frac{\partial p_j}{\partial o_i} = \frac{0 \cdot \sum_{k=1}^{K} e^{o_k} - e^{o_i} \cdot e^{o_j}}{\left(\sum_{k=1}^{K} e^{o_k}\right)^2} = \boxed{-p_i \cdot p_j}$$

if $i = j$

$$\frac{\partial p_j}{\partial o_i} = \frac{e^{o_i} \cdot \sum_{k=1}^{K} e^{o_k} - e^{o_i} \cdot e^{o_j}}{\left(\sum_{k=1}^{K} e^{o_k}\right)^2} = \boxed{(1 - p_j) \cdot p_i}$$

Derivative of Softmax + Cross Entropy Loss $L_{CE}(\mathbf{y}, \mathbf{p}) = -\sum_{k=1}^{K} y_k \log p_k$

$$\frac{\partial L_{CE}}{\partial o_i} = -\sum_{k=1}^{K} \frac{y_k}{p_k} \cdot \frac{\partial p_k}{\partial o_i} = -y_i(1 - p_i) - \sum_{k \neq i} \frac{y_k}{p_k} \cdot (-p_k p_i) = -y_i + y_i p_i + \sum_{k \neq i} y_k p_i = -y_i + p_i \cdot \sum_{k=1}^{K} y_k = p_i - y_i$$

In Vector Form: $\boxed{\frac{\partial L_{CE}}{\partial \mathbf{o}} = \mathbf{p} - \mathbf{y}}$

Remaining Backpropogation $\mathbf{o} = W_o\mathbf{g} + \mathbf{b}_o$

$$\frac{\partial L}{\partial W_o} = \frac{\partial L}{\partial \mathbf{o}} \cdot \left(\frac{\partial \mathbf{o}}{\partial W_o}\right)^T = \frac{\partial L}{\partial \mathbf{o}} \cdot \mathbf{g}^T$$

$$\frac{\partial L}{\partial \mathbf{b}_o} = \frac{\partial L}{\partial \mathbf{o}} \cdot \left(\frac{\partial \mathbf{o}}{\partial \mathbf{b}_o}\right)^T = \frac{\partial L}{\partial \mathbf{o}}$$

$g_i = \mathtt{ReLU}(h_i) = \mathtt{max}(h_i, 0)$

$$\frac{\partial L}{\partial h_i} = \frac{\partial L}{\partial g_i} \cdot \frac{\partial g_i}{\partial h_i} = \begin{cases} \frac{\partial L}{\partial g_i} & \text{if } h_i > 0 \\ 0 & \text{if } h_i < 0 \end{cases}$$

$\mathbf{h} = W_h\mathbf{x} + \mathbf{b}_h$

$$\frac{\partial L}{\partial W_h} = \frac{\partial L}{\partial \mathbf{h}} \cdot \left(\frac{\partial \mathbf{h}}{\partial W_h}\right)^T = \frac{\partial L}{\partial \mathbf{h}} \cdot \mathbf{x}^T$$

$$\frac{\partial L}{\partial \mathbf{b}_h} = \frac{\partial L}{\partial \mathbf{h}} \cdot \left(\frac{\partial \mathbf{h}}{\partial \mathbf{b}_h}\right)^T = \frac{\partial L}{\partial \mathbf{h}}$$

1.3 Learning

[16]:
```
class mini_NN(object):

    """
    Network Structure:
        input:  x
        hidden: h = W_h * x + b_h
                g = ReLU(h)
        output: o = W_o * g + b_o
                p = softmax(o)
    """

    def __init__(self, D, F, K):
        # D, F, and K are the number of neurons in the input, hidden, and output␣
 ↪layers
        self.D = D
        self.F = F
        self.K = K
        self.init_weights()

    def init_weights(self):
        # getting random parameters using Xaiver initialization scheme
        self.W_h = np.random.normal(0, np.sqrt(2.0/(self.D+self.F)), (self.F,␣
 ↪self.D))
```

```
        self.b_h = np.random.normal(0, np.sqrt(2.0/(self.D+self.F)), self.F)
        self.W_o = np.random.normal(0, np.sqrt(2.0/(self.F+self.K)), (self.K,␣
↪self.F))
        self.b_o = np.random.normal(0, np.sqrt(2.0/(self.F+self.K)), self.K)

    def feedforward(self, X):
        # python can dynamically create attributes
        self.H = computeLayer(X, self.W_h, self.b_h)
        self.G = relu(self.H)
        self.O = computeLayer(self.G, self.W_o, self.b_o)
        self.P = softmax_batch(self.O)
        return self.P

    def backpropagation(self, X, y):

        # This function assumes that feedforward was called before,
        # which instantiates the needed activations

        # output layer activations
        dL_do = gradCE(y, self.P)

        # output layer parameters
        dL_dWo = dL_do.T @ self.G
        dL_dbo = dL_do

        # hidden layer activations
        dL_dg = dL_do @ self.W_o
        dL_dh = dL_dg.copy()
        dL_dh[self.H <= 0] = 0

        # hidden layer parameters
        dL_dWh = dL_dh.T @ X
        dL_dbh = dL_dh

        return dL_dWo , dL_dbo.sum(axis=0), dL_dWh, dL_dbh.sum(axis=0)

    def train(self, X, y, epochs=200, gamma=0.99, alpha=1e-5, F=None,
              validData=None, validTarget=None, testData=None, testTarget=None):
        # initializations
        self.F = self.F if F is None else F
        self.init_weights()

        train_loss, train_acc = [], []
        valid_loss, valid_acc = [], []
        test_loss, test_acc = [], []

        v_Wo, v_Wh = 0, 0
```

```
        for e in range(epochs):
            if e > 0:
                print(f"epoch: {e+1}\tloss: {train_loss[-1]:.4f}\tacc:␣
 ↪{train_acc[-1]:.4f}")
            else:
                print("epoch:", e+1)

            # getting predictions
            p = self.feedforward(X)
            train_loss.append( averageCE(p, y) )
            train_acc.append( accuracy(p, y) )

            # getting gradients
            dL_dWo, dL_dbo, dL_dWh, dL_dbh = self.backpropagation(X, y)

            # updating parameters
            v_Wo = gamma * v_Wo + alpha * dL_dWo
            self.W_o -= v_Wo

            self.b_o -= alpha * dL_dbo

            v_Wh = gamma * v_Wh + alpha * dL_dWh
            self.W_h -= v_Wh

            self.b_h -= alpha * dL_dbh

            # calculating statistics
            if not validData is None and not validTarget is None:
                p = self.feedforward(validData)
                valid_loss.append(averageCE(p, validTarget))
                valid_acc.append(accuracy(p, validTarget))
            if not testData is None and not testTarget is None:
                p = self.feedforward(testData)
                test_loss.append(averageCE(p, testTarget))
                test_acc.append(accuracy(p, testTarget))

        statistics = (train_loss, train_acc)
        if not validData is None and not validTarget is None:
            statistics += (valid_loss, valid_acc, )
        if not testData is None and not testTarget is None:
            statistics += (test_loss, test_acc,)
        return statistics

X = trainData.reshape(N, d)

y = newtrain
```

```python
# For investigation, analyze how long each hyperparameter set takes to train
import time
start = time.time()

model = mini_NN(d, 1000, K)
statistics = model.train(X, y, epochs=200, gamma=0.99, alpha=1e-5, **VTDatasets)
display_statistics(*statistics)
print(f"Time is {time.time() - start}.")
```

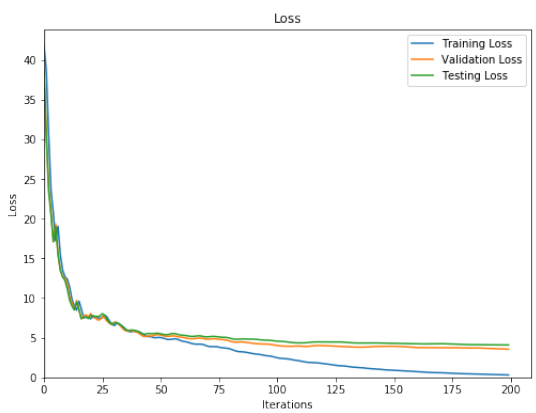
```
epoch: 1
epoch: 2        loss: 41.8202   acc: 0.0623
epoch: 3        loss: 38.6344   acc: 0.1826
epoch: 4        loss: 30.5366   acc: 0.5814
epoch: 5        loss: 23.6037   acc: 0.6173
epoch: 6        loss: 20.7250   acc: 0.7061
epoch: 7        loss: 17.2101   acc: 0.7915
epoch: 8        loss: 19.0484   acc: 0.6860
epoch: 9        loss: 15.4555   acc: 0.7875
epoch: 10       loss: 13.4947   acc: 0.7948
epoch: 11       loss: 12.6952   acc: 0.7845
epoch: 12       loss: 12.3658   acc: 0.8017
epoch: 13       loss: 11.3749   acc: 0.8021
epoch: 14       loss: 9.8147    acc: 0.8311
epoch: 15       loss: 9.0415    acc: 0.8425
epoch: 16       loss: 8.4522    acc: 0.8576
epoch: 17       loss: 9.6073    acc: 0.8306
epoch: 18       loss: 8.6273    acc: 0.8579
epoch: 19       loss: 7.4822    acc: 0.8685
epoch: 20       loss: 7.6507    acc: 0.8607
epoch: 21       loss: 7.6104    acc: 0.8610
epoch: 22       loss: 7.3691    acc: 0.8727
epoch: 23       loss: 7.8118    acc: 0.8719
epoch: 24       loss: 7.4743    acc: 0.8791
epoch: 25       loss: 7.5346    acc: 0.8783
epoch: 26       loss: 7.4533    acc: 0.8836
epoch: 27       loss: 7.5341    acc: 0.8862
epoch: 28       loss: 7.8440    acc: 0.8823
epoch: 29       loss: 7.6133    acc: 0.8800
epoch: 30       loss: 7.1122    acc: 0.8833
epoch: 31       loss: 6.7606    acc: 0.8868
epoch: 32       loss: 6.5217    acc: 0.8953
epoch: 33       loss: 6.7583    acc: 0.8958
epoch: 34       loss: 6.7986    acc: 0.8963
epoch: 35       loss: 6.5820    acc: 0.8950
epoch: 36       loss: 6.3596    acc: 0.8940
epoch: 37       loss: 6.0831    acc: 0.8988
```

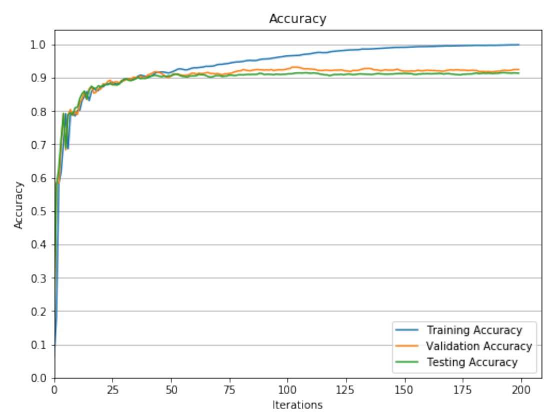
```
epoch: 38       loss: 5.8347    acc: 0.9041
epoch: 39       loss: 5.7389    acc: 0.9072
epoch: 40       loss: 5.7982    acc: 0.9054
epoch: 41       loss: 5.8156    acc: 0.9033
epoch: 42       loss: 5.7793    acc: 0.9019
epoch: 43       loss: 5.7104    acc: 0.9035
epoch: 44       loss: 5.5470    acc: 0.9058
epoch: 45       loss: 5.3071    acc: 0.9114
epoch: 46       loss: 5.1573    acc: 0.9159
epoch: 47       loss: 5.1544    acc: 0.9161
epoch: 48       loss: 5.1194    acc: 0.9167
epoch: 49       loss: 5.0175    acc: 0.9161
epoch: 50       loss: 5.0072    acc: 0.9149
epoch: 51       loss: 5.0473    acc: 0.9134
epoch: 52       loss: 5.0269    acc: 0.9153
epoch: 53       loss: 4.9502    acc: 0.9182
epoch: 54       loss: 4.8594    acc: 0.9213
epoch: 55       loss: 4.7863    acc: 0.9250
epoch: 56       loss: 4.7844    acc: 0.9261
epoch: 57       loss: 4.8328    acc: 0.9245
epoch: 58       loss: 4.8698    acc: 0.9230
epoch: 59       loss: 4.8308    acc: 0.9230
epoch: 60       loss: 4.7091    acc: 0.9255
epoch: 61       loss: 4.5888    acc: 0.9278
epoch: 62       loss: 4.5251    acc: 0.9293
epoch: 63       loss: 4.4795    acc: 0.9290
epoch: 64       loss: 4.4006    acc: 0.9298
epoch: 65       loss: 4.3017    acc: 0.9309
epoch: 66       loss: 4.2276    acc: 0.9315
epoch: 67       loss: 4.1908    acc: 0.9337
epoch: 68       loss: 4.1885    acc: 0.9333
epoch: 69       loss: 4.1939    acc: 0.9337
epoch: 70       loss: 4.1573    acc: 0.9341
epoch: 71       loss: 4.0575    acc: 0.9361
epoch: 72       loss: 3.9474    acc: 0.9387
epoch: 73       loss: 3.8890    acc: 0.9396
epoch: 74       loss: 3.8850    acc: 0.9396
epoch: 75       loss: 3.8930    acc: 0.9409
epoch: 76       loss: 3.8705    acc: 0.9416
epoch: 77       loss: 3.8124    acc: 0.9427
epoch: 78       loss: 3.7506    acc: 0.9445
epoch: 79       loss: 3.7120    acc: 0.9455
epoch: 80       loss: 3.6882    acc: 0.9465
epoch: 81       loss: 3.6495    acc: 0.9467
epoch: 82       loss: 3.5668    acc: 0.9474
epoch: 83       loss: 3.4519    acc: 0.9483
epoch: 84       loss: 3.3431    acc: 0.9499
epoch: 85       loss: 3.2696    acc: 0.9510
```

```
epoch: 86       loss: 3.2365    acc: 0.9515
epoch: 87       loss: 3.2230    acc: 0.9508
epoch: 88       loss: 3.1943    acc: 0.9508
epoch: 89       loss: 3.1376    acc: 0.9517
epoch: 90       loss: 3.0705    acc: 0.9536
epoch: 91       loss: 3.0146    acc: 0.9551
epoch: 92       loss: 2.9787    acc: 0.9553
epoch: 93       loss: 2.9516    acc: 0.9559
epoch: 94       loss: 2.9135    acc: 0.9561
epoch: 95       loss: 2.8565    acc: 0.9574
epoch: 96       loss: 2.7934    acc: 0.9586
epoch: 97       loss: 2.7478    acc: 0.9593
epoch: 98       loss: 2.7193    acc: 0.9608
epoch: 99       loss: 2.6844    acc: 0.9615
epoch: 100      loss: 2.6219    acc: 0.9625
epoch: 101      loss: 2.5404    acc: 0.9639
epoch: 102      loss: 2.4663    acc: 0.9646
epoch: 103      loss: 2.4184    acc: 0.9649
epoch: 104      loss: 2.3951    acc: 0.9649
epoch: 105      loss: 2.3812    acc: 0.9657
epoch: 106      loss: 2.3583    acc: 0.9661
epoch: 107      loss: 2.3147    acc: 0.9663
epoch: 108      loss: 2.2548    acc: 0.9680
epoch: 109      loss: 2.1970    acc: 0.9692
epoch: 110      loss: 2.1519    acc: 0.9697
epoch: 111      loss: 2.1104    acc: 0.9705
epoch: 112      loss: 2.0592    acc: 0.9714
epoch: 113      loss: 1.9977    acc: 0.9730
epoch: 114      loss: 1.9409    acc: 0.9739
epoch: 115      loss: 1.9038    acc: 0.9751
epoch: 116      loss: 1.8881    acc: 0.9751
epoch: 117      loss: 1.8827    acc: 0.9745
epoch: 118      loss: 1.8700    acc: 0.9745
epoch: 119      loss: 1.8424    acc: 0.9747
epoch: 120      loss: 1.8024    acc: 0.9766
epoch: 121      loss: 1.7576    acc: 0.9779
epoch: 122      loss: 1.7173    acc: 0.9786
epoch: 123      loss: 1.6858    acc: 0.9793
epoch: 124      loss: 1.6565    acc: 0.9799
epoch: 125      loss: 1.6186    acc: 0.9804
epoch: 126      loss: 1.5700    acc: 0.9811
epoch: 127      loss: 1.5217    acc: 0.9816
epoch: 128      loss: 1.4884    acc: 0.9823
epoch: 129      loss: 1.4716    acc: 0.9827
epoch: 130      loss: 1.4576    acc: 0.9827
epoch: 131      loss: 1.4301    acc: 0.9832
epoch: 132      loss: 1.3875    acc: 0.9833
epoch: 133      loss: 1.3418    acc: 0.9840
```

```
epoch: 134      loss: 1.3063    acc: 0.9853
epoch: 135      loss: 1.2854    acc: 0.9853
epoch: 136      loss: 1.2719    acc: 0.9855
epoch: 137      loss: 1.2554    acc: 0.9853
epoch: 138      loss: 1.2300    acc: 0.9858
epoch: 139      loss: 1.1959    acc: 0.9864
epoch: 140      loss: 1.1591    acc: 0.9870
epoch: 141      loss: 1.1282    acc: 0.9873
epoch: 142      loss: 1.1075    acc: 0.9877
epoch: 143      loss: 1.0932    acc: 0.9881
epoch: 144      loss: 1.0777    acc: 0.9885
epoch: 145      loss: 1.0536    acc: 0.9891
epoch: 146      loss: 1.0210    acc: 0.9899
epoch: 147      loss: 0.9879    acc: 0.9897
epoch: 148      loss: 0.9628    acc: 0.9903
epoch: 149      loss: 0.9474    acc: 0.9903
epoch: 150      loss: 0.9357    acc: 0.9902
epoch: 151      loss: 0.9207    acc: 0.9904
epoch: 152      loss: 0.9005    acc: 0.9904
epoch: 153      loss: 0.8784    acc: 0.9908
epoch: 154      loss: 0.8589    acc: 0.9912
epoch: 155      loss: 0.8433    acc: 0.9913
epoch: 156      loss: 0.8296    acc: 0.9920
epoch: 157      loss: 0.8149    acc: 0.9926
epoch: 158      loss: 0.7980    acc: 0.9926
epoch: 159      loss: 0.7787    acc: 0.9927
epoch: 160      loss: 0.7587    acc: 0.9927
epoch: 161      loss: 0.7393    acc: 0.9929
epoch: 162      loss: 0.7216    acc: 0.9932
epoch: 163      loss: 0.7062    acc: 0.9931
epoch: 164      loss: 0.6915    acc: 0.9931
epoch: 165      loss: 0.6746    acc: 0.9933
epoch: 166      loss: 0.6547    acc: 0.9939
epoch: 167      loss: 0.6337    acc: 0.9939
epoch: 168      loss: 0.6162    acc: 0.9941
epoch: 169      loss: 0.6050    acc: 0.9945
epoch: 170      loss: 0.6002    acc: 0.9948
epoch: 171      loss: 0.5981    acc: 0.9946
epoch: 172      loss: 0.5938    acc: 0.9946
epoch: 173      loss: 0.5833    acc: 0.9948
epoch: 174      loss: 0.5662    acc: 0.9949
epoch: 175      loss: 0.5455    acc: 0.9952
epoch: 176      loss: 0.5258    acc: 0.9955
epoch: 177      loss: 0.5108    acc: 0.9956
epoch: 178      loss: 0.5011    acc: 0.9958
epoch: 179      loss: 0.4948    acc: 0.9961
epoch: 180      loss: 0.4887    acc: 0.9961
epoch: 181      loss: 0.4800    acc: 0.9960
```

```
epoch: 182      loss: 0.4688    acc: 0.9965
epoch: 183      loss: 0.4572    acc: 0.9965
epoch: 184      loss: 0.4473    acc: 0.9965
epoch: 185      loss: 0.4399    acc: 0.9965
epoch: 186      loss: 0.4343    acc: 0.9963
epoch: 187      loss: 0.4288    acc: 0.9963
epoch: 188      loss: 0.4220    acc: 0.9967
epoch: 189      loss: 0.4134    acc: 0.9965
epoch: 190      loss: 0.4034    acc: 0.9965
epoch: 191      loss: 0.3930    acc: 0.9965
epoch: 192      loss: 0.3827    acc: 0.9967
epoch: 193      loss: 0.3728    acc: 0.9969
epoch: 194      loss: 0.3631    acc: 0.9971
epoch: 195      loss: 0.3540    acc: 0.9973
epoch: 196      loss: 0.3460    acc: 0.9974
epoch: 197      loss: 0.3394    acc: 0.9978
epoch: 198      loss: 0.3334    acc: 0.9978
epoch: 199      loss: 0.3273    acc: 0.9977
epoch: 200      loss: 0.3204    acc: 0.9981
Training loss: 0.3129           Training acc: 99.81%
Validation loss: 3.558  Validation acc: 92.4%
Testing loss: 4.0922    Testing acc: 91.3%
```






```
Time is 342.4068307876587.
```

1.4 Hyperparameter Investigation

0.1 Number of Hidden Units

0.1.1 Number of hidden units: 100

Statistic	Value	Statistic	Value
Training loss:	4.6824	Training acc:	92.31%
Validation loss:	5.5578	Validation acc:	89.8%

Statistic	Value	Statistic	Value
Testing loss:	5.7423	Testing acc:	89.46%
Training Time(s):	42.705		

Comments: Accuracy seems to have converged by 50 iterations. Training this network with 100 hidden units was the fastest of the three. This is likely because there were very few parameters to optimize.

0.1.2 Number of hidden units: 500

Statistic	Value	Statistic	Value
Training loss:	0.7009	Training acc:	99.36%
Validation loss:	3.894	Validation acc:	91.9%
Testing loss:	4.1808	Testing acc:	91.19%
Training Time(s):	176.361		

Comments: Accuracy seems to have converged by 50 iterations. Training this network with 500 hidden units was slower than with 100 units, but the validation/testing accuracies showed slight improvement. Further, Training loss and training accuracy near perfection. Since this trend is not matched by validation/testing accuracies, we see that the network is overfitting as it is learning the training examples too well.

0.1.3 Number of hidden units: 2000

Statistic	Value	Statistic	Value
Training loss:	0.2303	Training acc:	99.82%
Validation loss:	3.3766	Validation acc:	93.0%
Testing loss:	3.8814	Testing acc:	91.85%
Training Time(s):	645.494		

Comments: Accuracy seems to have convered by 50 iterations. Training this network with 2000 hidden units was considerably slower than both other networks, depite nearly identical validation/testing accuracies. Training loss and training accuracy are essentially perfect by the end of 200 epochs. Since this trend is not matched by validation/testing accuracies, we see that the network is overfitting considerably. It has essentially memorized the training examples, but can not generalize as well (beyond the performance attained by smaller networks) to validation/testing data.

General comments: Also, the small bump and incresae in loss before once again decreasing suggest that the parameters went through a local minimum before converging. Momentum likely helped it converge faster as it doesn't appear to have gotten stuck in the local min. The smaller networks trained considerably faster, but had lower accuracy (even though the difference was very small). The larger networks clearly overfit the training data; this indicates that they are too complex for their classification tasks and/or the input data should be improved.

0.2 Early Stopping

From the plots, we observe that training should have stopped at an early stopping point of 50 iterations. Beyond this point, there is very little improvement in either validation/testing losses or accuracies.

2 Neural Networks in Tensorflow

2.1 Model implementation

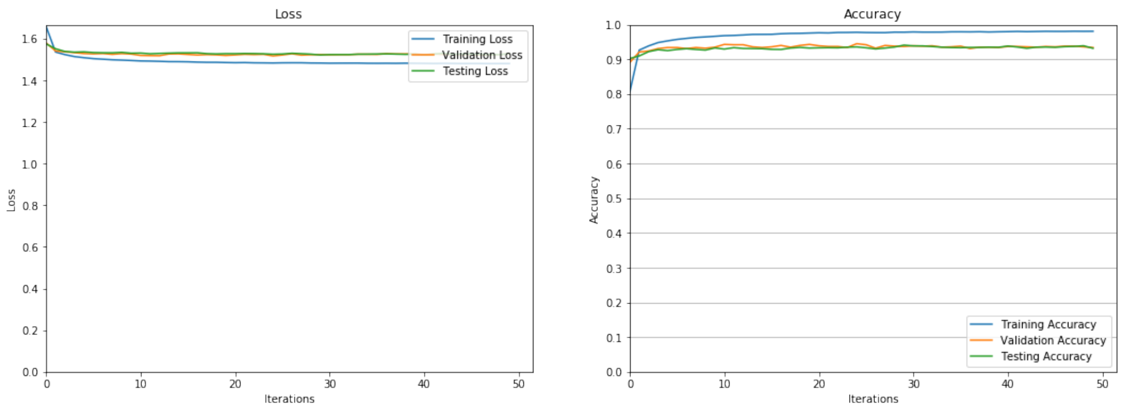
[0]:
```
# load + reshape data
trainData, validData, testData, trainTarget, validTarget, testTarget = loadData()
trainData = trainData.reshape(15000,28,28,1)
validData = validData.reshape(1000,28,28,1)
testData = testData.reshape(2724,28,28,1)

# one-hot encode
train_labels, valid_labels, test_labels = convertOneHot(trainTarget, validTarget, testTarget)

# training params
learning_rate = 0.0001
epochs = 50
batch_size = 32

# create model
model = models.Sequential()
model.add(layers.InputLayer(input_shape=(28, 28,1)))                  # input layer
model.add(layers.Conv2D(                                                # conv layer
        filters=32,
        strides=(1,1),
        kernel_size=[3, 3],
        padding="same",
        activation='relu',
        kernel_initializer=tf.contrib.layers.xavier_initializer(uniform=False)))
model.add(layers.BatchNormalization())                                  # batch norm
model.add(layers.MaxPooling2D((2, 2)))                                  # max pooling
model.add(layers.Flatten())                                             # flatten
model.add(layers.Dense(784, activation='relu'))                         # fully-connected 784 w/ ReLu
model.add(layers.Dense(10))                                             # fully-connected 10
model.add(layers.Softmax())                                             # softmax output

# compile model w/ Adam optimizer + cross entropy loss
model.compile(optimizer=tf.keras.optimizers.Adam(lr=learning_rate),
              loss=tf.keras.losses.CategoricalCrossentropy(from_logits=True),
```

```
                metrics=['accuracy'])
```

2.2 Model Training

[18]:
```
# callback to test after each epoch
class TestCallback(tf.keras.callbacks.Callback):
    def __init__(self, test_data):
        self.test_data = test_data
        self.test_acc = []
        self.test_loss = []

    def on_epoch_end(self, epoch, logs=None):
        # perform a test per epoch
        x, y = self.test_data
        loss, acc = self.model.evaluate(x, y, verbose=0, batch_size=32)
        self.test_loss.append(loss)
        self.test_acc.append(acc)
        # append to returned dictionary
        logs["test_loss"] = self.test_loss
        logs["test_acc"] = self.test_acc


# train
history = model.fit(trainData, train_labels,
                    validation_data=(validData, valid_labels),
                    epochs=epochs,
                    batch_size=batch_size,
                    callbacks=[TestCallback((testData, test_labels))],
                    verbose=0,  # 0 = silent, 1 = per epoch
                    shuffle=True)

# print(history.history)

# plot accuracy + loss
train_acc = history.history["acc"]
val_acc = history.history["val_acc"]
test_acc = history.history["test_acc"][0]
train_loss = history.history["loss"]
val_loss = history.history["val_loss"]
test_loss = history.history["test_loss"][0]

statistics = (train_loss, train_acc, val_loss, val_acc, test_loss, test_acc)
display_statistics(*statistics)
```

```
Training loss: 1.4806           Training acc: 98.08%
Validation loss: 1.5268 Validation acc: 93.4%
Testing loss: 1.5286    Testing acc: 93.17%
```

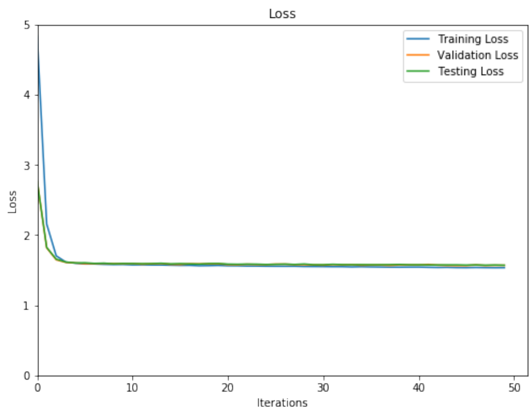


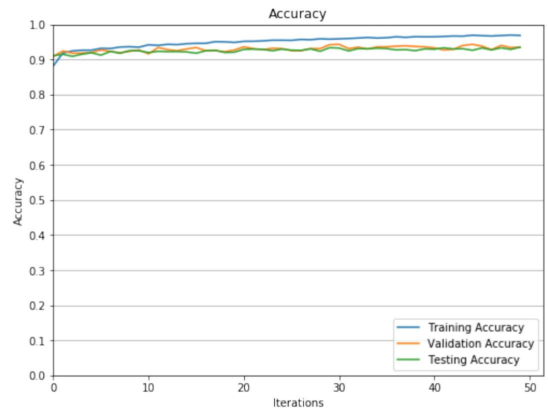
2.3 Hyperparameter Investigation

[20]:
```
# 2.3.1: L2 Regularization

# training params
learning_rate = 0.0001
epochs = 50
batch_size = 32

# test all weight decays [0.01, 0.1, 0.5]
for scale in [0.01, 0.1, 0.5]:
    print("\nL2 Normalization with {}\n".format(scale))

    # create model
    model = models.Sequential()
    model.add(layers.InputLayer(input_shape=(28, 28,1)))     # input layer
    model.add(layers.Conv2D(                                  # conv layer
            filters=32,
            strides=(1,1),
            kernel_size=[3, 3],
            padding="same",
            activation='relu',
            kernel_initializer=tf.contrib.layers.xavier_initializer(uniform=False)))
    model.add(layers.BatchNormalization())                    # batch norm
    model.add(layers.MaxPooling2D((2, 2)))                    # max pooling
    model.add(layers.Flatten())                               # flatten
    model.add(layers.Dense(784,
```

```
                         activation='relu',
                         kernel_regularizer=tf.contrib.layers.
↪l2_regularizer(scale=scale)))  # fully-connected 784 w/ ReLu
    model.add(layers.Dense(10))                                                      ␣
↪    # fully-connected 10
    model.add(layers.Softmax())                                                      ␣
↪    # softmax output

    # compile model w/ Adam optimizer + cross entropy loss
    model.compile(optimizer=tf.keras.optimizers.Adam(lr=learning_rate),
                  loss=tf.keras.losses.CategoricalCrossentropy(from_logits=True),
                  metrics=['accuracy'])

    # train
    history_1 = model.fit(trainData, train_labels,
                        validation_data = (validData, valid_labels),
                        epochs=epochs,
                        batch_size=batch_size,
                        callbacks=[TestCallback((testData, test_labels))],
                        verbose=0,  # 0 = silent, 1 = per epoch
                        shuffle=True)
    # display stats
    train_acc = history_1.history["acc"]
    val_acc = history_1.history["val_acc"]
    test_acc = history_1.history["test_acc"][0]
    train_loss = history_1.history["loss"]
    val_loss = history_1.history["val_loss"]
    test_loss = history_1.history["test_loss"][0]

    statistics = (train_loss, train_acc, val_loss, val_acc, test_loss, test_acc)
    display_statistics(*statistics)
```

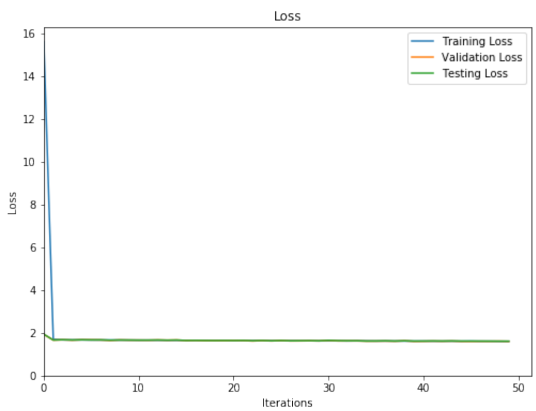
```
L2 Normalization with 0.01

Training loss: 1.5345           Training acc: 96.86%
Validation loss: 1.5678 Validation acc: 93.5%
Testing loss: 1.569     Testing acc: 93.5%
```

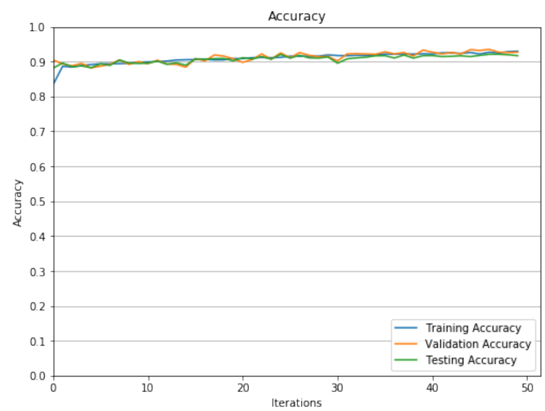




L2 Normalization with 0.1

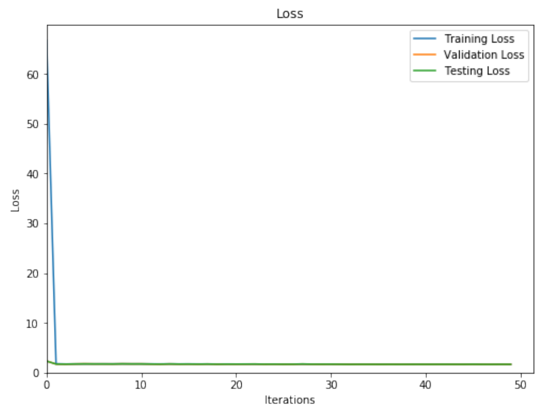
Training loss: 1.6037 Training acc: 92.97%
Validation loss: 1.6036 Validation acc: 92.7%
Testing loss: 1.6122 Testing acc: 91.7%

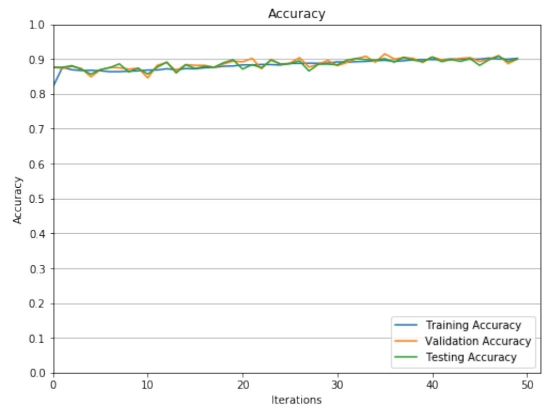




L2 Normalization with 0.5

Training loss: 1.6725 Training acc: 90.21%
Validation loss: 1.6759 Validation acc: 90.0%
Testing loss: 1.6763 Testing acc: 90.01%





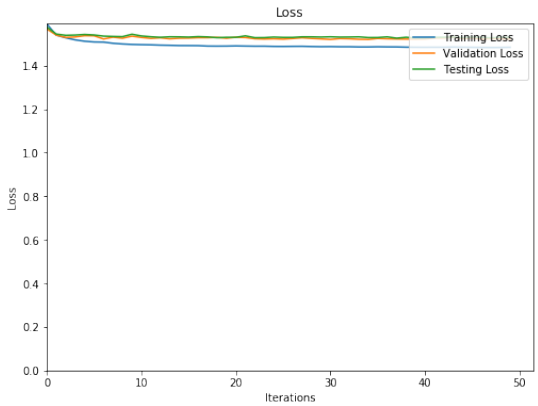
General Comments: L2 regularization is a technique to reduce overfitting. For small values of λ, we expect a slight improvement in model performance and a reduced discrepancy between training and validation/test accuracies. This is exactly what we observe here. For $\lambda = 0.01$, we observe validation and testing accuracy both improve slightly while training accuracy decreases by $\sim 1.5\%$. So, the model performance improves slightly and the model overfits less as expected. For values of λ that are too large, we expect the model to start underfitting as we are harshly penalizing parameter vectors which are large in magnitude. As λ increases, we see the discrepancy between training and validation/test accuracies decrease dramatically (the model is no longer overfitting). However, this change is accompanied by a reduction in overall accuracy (by $\sim 2\%$) as the model is now underfitting.

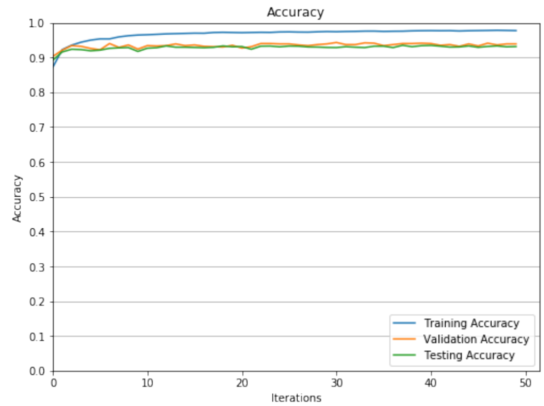
[21]:
```
# 2.3.2: Dropout

# training params
learning_rate = 0.0001
epochs = 50
batch_size = 32

# for rate in [0.9, 0.75, 0.5]:
for rate in [0.1, 0.25, 0.5]:
    print("\nDropout with probability {printed: .2f}\n".format(printed=1-rate))

    # create model
    model = models.Sequential()
    model.add(layers.InputLayer(input_shape=(28, 28,1)))    # input layer
    model.add(layers.Conv2D(
 ↪    # conv layer
            filters=32,
            strides=(1,1),
            kernel_size=[3, 3],
            padding="same",
            activation='relu',
```

```
        kernel_initializer=tf.contrib.layers.
↪xavier_initializer(uniform=False)))
    model.add(layers.BatchNormalization())                                                      ␣
↪     # batch norm
    model.add(layers.MaxPooling2D((2, 2)))                                                      ␣
↪     # max pooling
    model.add(layers.Flatten())                                                                 ␣
↪     # flatten
    model.add(layers.Dense(784))                                                                ␣
↪     # fully-connected 784 w/ ReLu
    model.add(layers.Dropout(rate=rate))                                                        ␣
↪     # dropout
    model.add(layers.ReLU())                                                                    ␣
↪      # Relu activation
    model.add(layers.Dense(10))                                                                 ␣
↪     # fully-connected 10
    model.add(layers.Softmax())                                                                 ␣
↪     # softmax output

    # compile model w/ Adam optimizer + cross entropy loss
    model.compile(optimizer=tf.keras.optimizers.Adam(lr=learning_rate),
                  loss=tf.keras.losses.CategoricalCrossentropy(from_logits=True),
                  metrics=['accuracy'])

    # train
    history_2 = model.fit(trainData, train_labels,
                          validation_data = (validData, valid_labels),
                          epochs=epochs,
                          batch_size=batch_size,
                          callbacks=[TestCallback((testData, test_labels))],
                          verbose=0,  # 0 = silent, 1 = per epoch
                          shuffle=True)
    # display stats
    train_acc = history_2.history["acc"]
    val_acc = history_2.history["val_acc"]
    test_acc = history_2.history["test_acc"][0]
    train_loss = history_2.history["loss"]
    val_loss = history_2.history["val_loss"]
    test_loss = history_2.history["test_loss"][0]

    statistics = (train_loss, train_acc, val_loss, val_acc, test_loss, test_acc)
    display_statistics(*statistics)
```

```
Dropout with probability  0.90

Training loss: 1.484           Training acc: 97.71%
```

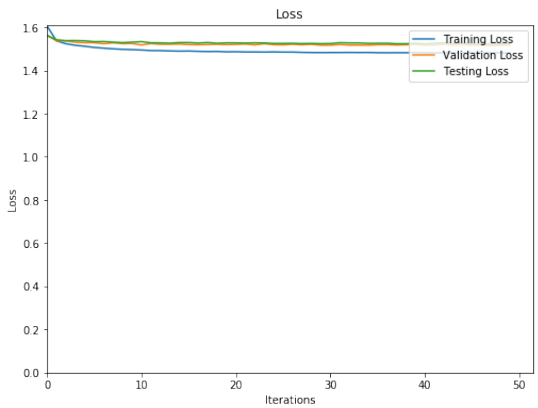
Validation loss: 1.5219 Validation acc: 93.9%
Testing loss: 1.5291   Testing acc: 93.14%

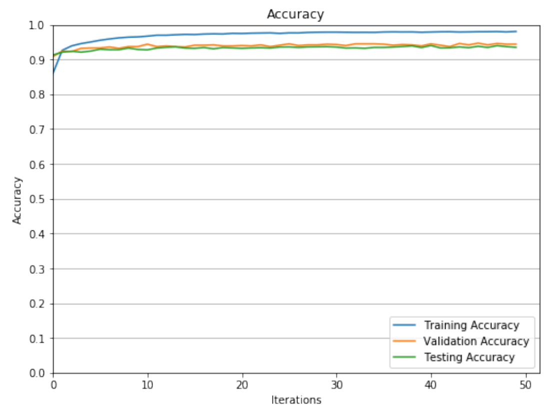




Dropout with probability 0.75

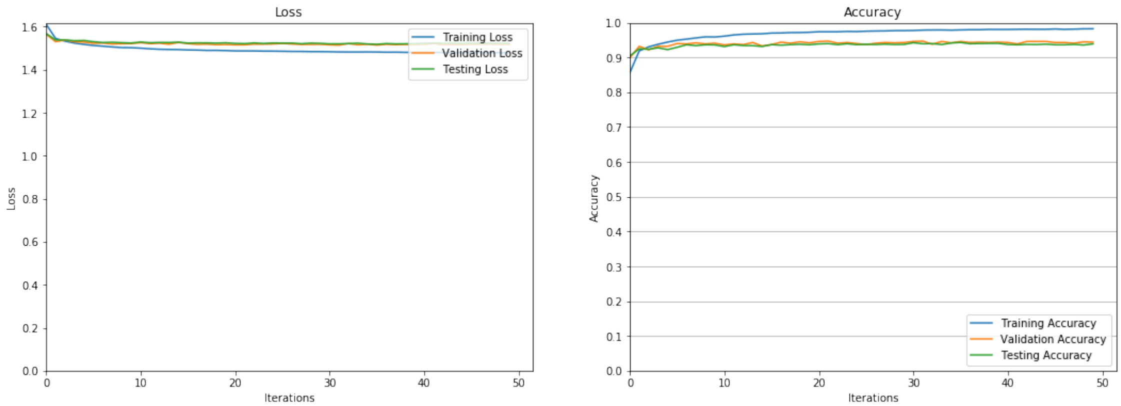
Training loss: 1.4808   Training acc: 98.04%
Validation loss: 1.5174 Validation acc: 94.4%
Testing loss: 1.5249   Testing acc: 93.5%





Dropout with probability 0.50

Training loss: 1.4788   Training acc: 98.25%
Validation loss: 1.5178 Validation acc: 94.4%
Testing loss: 1.523   Testing acc: 93.91%



General Comments: Dropout is a technique which aims to decrease overfitting in our model. We expect see this in the form of improved accuracy and loss statistics. Indeed, this is exactly observed. As the amount of dropout increases (keeping the rate within reason), the validation and testing accuracies both improve. Note that we also see training accuracy improve. So, it seems that the model as an increased capacity to learn overall. With dropout, it is able to learn the training data better, but it is also able to generalize better.