ECE421 – Introduction to Machine Learning

Assignment 2

Neural Networks

Hard Copy Due: Friday, March 6 @ 3:00 PM, at BA3128
Code Submission Due: Friday, March 6 @ 5:00 PM, on Quercus

General Notes:

- Attach this cover page to your hard copy submission
- Please post assignment related questions on Piazza.

Please check section to which you would like the assignment returned.

Tutorial Sections:

☐ Tutorial 1: Thursdays 3-5pm (SF2202)
☐ Tutorial 2: Thursdays 3-5pm (GB304)
☐ Tutorial 3: Tuesdays 10-12 (SF2202)
☐ Tutorial 4: Fridays 9-11 (BA1230)

Group Members	
Name	Student ID

0 Setup

[1]:
```
# ignore all future warnings
from warnings import simplefilter
simplefilter(action='ignore', category=FutureWarning)
```

[2]:
```
# importing tensorflow
try:
    import google.colab
    import tensorflow as tf
    %tensorflow_version 1.13
except:
    import tensorflow as tf
    assert tf.__version__ == "1.13.1"

    # ignore tensorflow depreciation warnings
    import tensorflow.python.util.deprecation as deprecation
    deprecation._PRINT_DEPRECATION_WARNINGS = False
```

[3]:
```
# imports
import numpy as np
import matplotlib.pyplot as plt
from tensorflow.keras import layers, models
```

0.1 Visualizing the Dataset

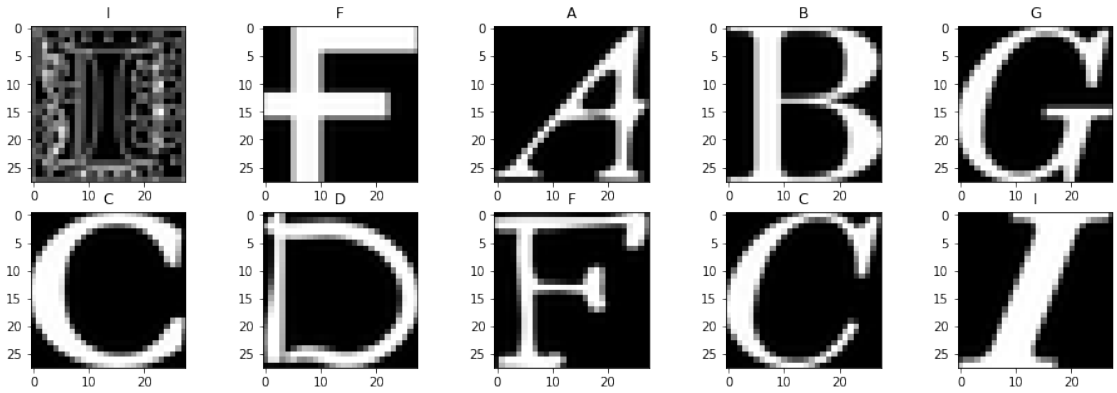
[4]:
```
# given by the assignment
def loadData():
    with np.load("notMNIST.npz") as data:
        Data, Target = data ["images"], data["labels"]
        np.random.seed(521)
        randIndx = np.arange(len(Data))
        np.random.shuffle(randIndx)
        Data = Data[randIndx]/255.
        Target = Target[randIndx]
        trainData, trainTarget = Data[:15000], Target[:15000]
        validData, validTarget = Data[15000:16000], Target[15000:16000]
        testData, testTarget = Data[16000:], Target[16000:]
    return trainData, validData, testData, trainTarget, validTarget, testTarget
```

[5]:
```
trainData, validData, testData, trainTarget, validTarget, testTarget =␣
↪loadData()
print(f"Training Data: {trainData.shape}\tTraining tagets: {trainTarget.shape}")
print(f"Validation Data: {validData.shape}\tValidation tagets: {validTarget.
↪shape}")
print(f"Testing Data: {testData.shape}\tTesting tagets:{testTarget.shape}")
```

```
Training Data: (15000, 28, 28)  Training tagets: (15000,)
Validation Data: (1000, 28, 28) Validation tagets: (1000,)
Testing Data: (2724, 28, 28)    Testing tagets:(2724,)
```

[6]:
```
def plot(image, target, ax=None):
    ax = plt.gca() if ax == None else ax
    ax.imshow(image, cmap=plt.cm.gray)
    target_names = ['A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J']
    ax.set_title(target_names[target])
    # targets interger encoded from 0 to 9 corresponding to 'A' to 'J',␣
↪respectively
```

[7]:
```
fig, axis = plt.subplots(2, 5, figsize=(16, 5))
for ax in axis.reshape(-1):
    r = np.random.randint(trainData.shape[0])
    plot(trainData[r], trainTarget[r], ax=ax)
plt.show()
```



0.2 Useful Functions

Some useful functions that will be used throughout the assignment such as getting random weights, getting the accuracy of a batch, making the loss and accuracy plots look nice, and global variables used throughout the code

[8]:
```
# given by the assignment
def convertOneHot(trainTarget, validTarget, testTarget):
    newtrain = np.zeros((trainTarget.shape[0], 10))
    newvalid = np.zeros((validTarget.shape[0], 10))
    newtest = np.zeros((testTarget.shape[0], 10))
    for item in range(0, trainTarget.shape[0]):
        newtrain[item][trainTarget[item]] = 1
    for item in range(0, validTarget.shape[0]):
        newvalid[item][validTarget[item]] = 1
    for item in range(0, testTarget.shape[0]):
        newtest[item][testTarget[item]] = 1
    return newtrain, newvalid, newtest
```

[9]:
```
def accuracy(y_pred, y):
    if y_pred.shape != y.shape:
        raise ValueError(f"prediction dimension {y_pred.shape} and label␣
↪dimensions {y.shape} don't match")
    return np.sum(y_pred.argmax(axis=1) == y.argmax(axis=1)) / y.shape[0]
```

[10]:
```
def plot_loss(x, train_loss=None, valid_loss=None, test_loss=None, title=None,␣
↪ax=None):
    ax = plt.gca() if ax == None else ax
    if train_loss != None:
        ax.plot(x, train_loss, label="Training Loss")
    if valid_loss != None:
        ax.plot(x, valid_loss, label="Validation Loss")
    if test_loss != None:
        ax.plot(x, test_loss, label="Testing Loss")

    ax.set_title("Loss" if title == None else title)

    ax.set_xlabel("Iterations")
    ax.set_xlim(left=0)
    ax.set_ylabel("Loss")
    ax.set_ylim(bottom=0)
    ax.legend(loc="upper right")

def plot_accuracy(x, train_accuracy=None, valid_accuracy=None,␣
↪test_accuracy=None, title=None, ax=None):
    ax = plt.gca() if ax == None else ax
    if train_accuracy != None:
```

```
        ax.plot(x, train_accuracy, label="Training Accuracy")
    if valid_accuracy != None:
        ax.plot(x, valid_accuracy, label="Validation Accuracy")
    if test_accuracy != None:
        ax.plot(x, test_accuracy, label="Testing Accuracy")

    ax.set_title("Accuracy" if title == None else title)

    ax.set_xlabel("Iterations")
    ax.set_xlim(left=0)
    ax.set_ylabel("Accuracy")
    ax.set_yticks(np.arange(0, 1.1, step=0.1))
    ax.grid(linestyle='-', axis='y')
    ax.legend(loc="lower right")


def display_statistics(train_loss=None, train_acc=None, valid_loss=None,␣
↪valid_acc=None,
                       test_loss=None, test_acc=None, num=True, plot=True):

    tl = "-" if train_loss is None else round(train_loss[-1], 4)
    ta = "-" if train_acc is None else round(train_acc[-1]*100, 2)
    vl = "-\t" if valid_loss is None else round(valid_loss[-1], 4)
    va = "-" if valid_acc is None else round(valid_acc[-1]*100, 2)
    sl = "-\t\t" if test_loss is None else round(test_loss[-1], 4)
    sa = "-" if test_acc is None else round(test_acc[-1]*100, 2)

    if num:
        print(f"Training loss: {tl}{'':.20s}\t\tTraining acc: {ta}{'%' if ta !=␣
↪'-' else ''}")
        print(f"Validation loss: {vl}{'':.20s}\t\tValidation acc: {va}{'%' if␣
↪va != '-' else ''}")
        print(f"Testing loss: {sl}{'':.20s}\t\tTesting acc: {sa}{'%' if sa !=␣
↪'-' else ''}")

    if plot:
        fig, ax = plt.subplots(1, 2, figsize=(18, 6))
        plot_loss(np.arange(0, len(train_loss), 1), train_loss, valid_loss,␣
↪test_loss, ax=ax[0])
        plot_accuracy(np.arange(0, len(train_loss), 1), train_acc, valid_acc,␣
↪test_acc, ax=ax[1])
        plt.show()
        plt.close()
```

[11]:

```
TINY = 1e-20
newtrain, newvalid, newtest = convertOneHot(trainTarget, validTarget,␣
 ↪testTarget)
VTDatasets = {"validData" : validData.reshape(validData.shape[0], -1),␣
 ↪"validTarget" : newvalid,
              "testData" : testData.reshape(testData.shape[0], -1),␣
 ↪"testTarget" : newtest}

N = trainData.shape[0]
d = trainData.shape[1] * trainData.shape[2]
K = 10
```

1 Neural Networks using Numpy

1.1 Helper Functions

[12]:
```
def relu(x):
    return np.maximum(0, x)

# applies softmax to a single vector
def softmax(x):
    return np.exp(x) / np.exp(x).sum()

# applies softmax to a batch
# (more efficient than having a loop and calling the above function)
def softmax_batch(X):
    return np.exp(X) / np.exp(X).sum(axis=1, keepdims=True)
```

[13]:
```
def computeLayer(X, W, b):
    return X @ W.T + b
```

[14]:
```
# note: target is one-hot encoded
def averageCE(target, prediction):
    return -(target * np.log(prediction+TINY)).sum(axis=1).mean()

def gradCE(target, predication):
    return predication - target
```

1.2 Backpropagation Derivation

The code application of the following derivation is given in the next section.

Derivative of Softmax

$p_i = \texttt{softmax}(\mathbf{o})_i = \frac{e^{o_i}}{\sum_{k=1}^{K} e^{o_k}}$

if $i \neq j$

$$\frac{\partial p_j}{\partial o_i} = \frac{0 \cdot \sum_{k=1}^{K} e^{o_k} - e^{o_i} \cdot e^{o_j}}{\left(\sum_{k=1}^{K} e^{o_k}\right)^2} = \boxed{-p_i \cdot p_j}$$

if $i = j$

$$\frac{\partial p_j}{\partial o_i} = \frac{e^{o_i} \cdot \sum_{k=1}^{K} e^{o_k} - e^{o_i} \cdot e^{o_j}}{\left(\sum_{k=1}^{K} e^{o_k}\right)^2} = \boxed{(1 - p_j) \cdot p_i}$$

Derivative of Softmax + Cross Entropy Loss

$L_{CE}(\mathbf{y}, \mathbf{p}) = -\sum_{k=1}^{K} y_k \log p_k$

$$\frac{\partial L_{CE}}{\partial o_i} = -\sum_{k=1}^{K} \frac{y_k}{p_k} \cdot \frac{\partial p_k}{\partial o_i} = -y_i(1-p_i) - \sum_{k \neq i} \frac{y_k}{p_k} \cdot (-p_k p_i) = -y_i + y_i p_i + \sum_{k \neq i} y_k p_i = -y_i + p_i \cdot \sum_{k=1}^{K} y_k = p_i - y_i$$

In Vector Form: $\boxed{\frac{\partial L_{CE}}{\partial \mathbf{o}} = \mathbf{p} - \mathbf{y}}$

Remaining Backpropogation

$\mathbf{o} = W_o \mathbf{g} + \mathbf{b}_o$

$$\frac{\partial L}{\partial W_o} = \frac{\partial L}{\partial \mathbf{o}} \cdot \left(\frac{\partial \mathbf{o}}{\partial W_o}\right)^T = \frac{\partial L}{\partial \mathbf{o}} \cdot \mathbf{g}^T \qquad \frac{\partial L}{\partial \mathbf{b}_o} = \frac{\partial L}{\partial \mathbf{o}} \cdot \left(\frac{\partial \mathbf{o}}{\partial \mathbf{b}_o}\right)^T = \frac{\partial L}{\partial \mathbf{o}}$$

$g_i = \texttt{ReLU}(h_i) = \texttt{max}(h_i, 0)$

$$\frac{\partial L}{\partial h_i} = \frac{\partial L}{\partial g_i} \cdot \frac{\partial g_i}{\partial h_i} = \begin{cases} \frac{\partial L}{\partial g_i} & \text{if } h_i > 0 \\ 0 & \text{if } h_i < 0 \end{cases}$$

$\mathbf{h} = W_h \mathbf{x} + \mathbf{b}_h$

$$\frac{\partial L}{\partial W_h} = \frac{\partial L}{\partial \mathbf{h}} \cdot \left(\frac{\partial \mathbf{h}}{\partial W_h}\right)^T = \frac{\partial L}{\partial \mathbf{h}} \cdot \mathbf{x}^T \qquad \frac{\partial L}{\partial \mathbf{b}_h} = \frac{\partial L}{\partial \mathbf{h}} \cdot \left(\frac{\partial \mathbf{h}}{\partial \mathbf{b}_h}\right)^T = \frac{\partial L}{\partial \mathbf{h}}$$

1.3 Learning

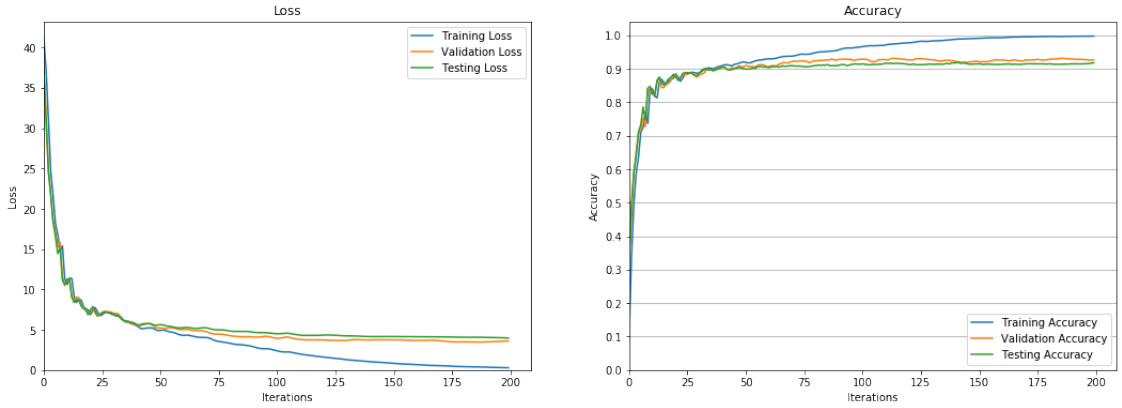
[16]:
```
class mini_NN(object):

    """
    Network Structure:
        input:  x
        hidden: h = W_h * x + b_h
                g = ReLU(h)
        output: o = W_o * g + b_o
                p = softmax(o)
    """

    def __init__(self, D, F, K):
        # D, F, and K are the number of neurons in the input, hidden, and␣
↪output layers
        self.D, self.F, self.K = D, F, K
        self.init_weights()

    def init_weights(self):
        # getting random parameters using Xaiver initialization scheme
        self.W_h = np.random.normal(0, np.sqrt(2.0/(self.D+self.F)), (self.F,␣
↪self.D))
        self.b_h = np.random.normal(0, np.sqrt(2.0/(self.D+self.F)), self.F)
        self.W_o = np.random.normal(0, np.sqrt(2.0/(self.F+self.K)), (self.K,␣
↪self.F))
        self.b_o = np.random.normal(0, np.sqrt(2.0/(self.F+self.K)), self.K)

    def feedforward(self, X):
        # python can dynamically create attributes
        self.H = computeLayer(X, self.W_h, self.b_h)
        self.G = relu(self.H)
        self.O = computeLayer(self.G, self.W_o, self.b_o)
        self.P = softmax_batch(self.O)
        return self.P

    def backpropagation(self, X, y):

        # This function assumes that feedforward was called before,
        # which instantiates the needed activations

        # output layer activations
        dL_do = gradCE(y, self.P)

        # output layer parameters
        dL_dWo = dL_do.T @ self.G
        dL_dbo = dL_do
```

```
        # hidden layer activations
        dL_dg = dL_do @ self.W_o
        dL_dh = dL_dg.copy()
        dL_dh[self.H <= 0] = 0

        # hidden layer parameters
        dL_dWh = dL_dh.T @ X
        dL_dbh = dL_dh

        return dL_dWo , dL_dbo.sum(axis=0), dL_dWh, dL_dbh.sum(axis=0)

    def train(self, X, y, epochs=200, gamma=0.99, alpha=1e-5, F=None,
              validData=None, validTarget=None, testData=None, testTarget=None):
        # initializations
        self.F = self.F if F is None else F
        self.init_weights()

        train_loss, train_acc = [], []
        valid_loss, valid_acc = [], []
        test_loss, test_acc = [], []

        v_Wo, v_Wh = 0, 0

        for e in range(epochs):
            # getting predictions
            p = self.feedforward(X)
            train_loss.append( averageCE(p, y) )
            train_acc.append( accuracy(p, y) )

            # getting gradients
            dL_dWo, dL_dbo, dL_dWh, dL_dbh = self.backpropagation(X, y)

            # updating parameters
            v_Wo = gamma * v_Wo + alpha * dL_dWo
            self.W_o -= v_Wo

            self.b_o -= alpha * dL_dbo

            v_Wh = gamma * v_Wh + alpha * dL_dWh
            self.W_h -= v_Wh

            self.b_h -= alpha * dL_dbh

            # calculating statistics
            if not validData is None and not validTarget is None:
                p = self.feedforward(validData)
```

```
                valid_loss.append(averageCE(p, validTarget))
                valid_acc.append(accuracy(p, validTarget))
            if not testData is None and not testTarget is None:
                p = self.feedforward(testData)
                test_loss.append(averageCE(p, testTarget))
                test_acc.append(accuracy(p, testTarget))

        statistics = (train_loss, train_acc)
        if not validData is None and not validTarget is None:
            statistics += (valid_loss, valid_acc, )
        if not testData is None and not testTarget is None:
            statistics += (test_loss, test_acc,)
        return statistics
```

[17]:
```
# For investigation, analyze how long each hyperparameter set takes to train
import time

X, y = trainData.reshape(N, d), newtrain
F = 1000  # hidden unit size
model = mini_NN(d, F, K)

start = time.time()
statistics = model.train(X, y, epochs=200, gamma=0.99, alpha=1e-5, **VTDatasets)
end = time.time()

display_statistics(*statistics)
print(f"Time is {end - start}.")
```

```
Training loss: 0.2957           Training acc: 99.73%
Validation loss: 3.6131 Validation acc: 92.6%
Testing loss: 3.9853    Testing acc: 91.74%
```




```
Time is 372.057715177536.
```

1.4 Hyperparameter Investigation

The following hyperparameter search was done on google colab using code similar to the snippet above, but with the corresponding hyperparameters changed.

1.4.1 Number of Hidden Units

Number of hidden units: 100

Statistic	Value	Statistic	Value
Training loss:	4.6824	Training acc:	92.31%
Validation loss:	5.5578	Validation acc:	89.8%
Testing loss:	5.7423	Testing acc:	89.46%
Training Time(s):	42.705		

Comments: Accuracy seems to have converged by 50 iterations. Training this network with 100 hidden units was the fastest of the three. This is likely because there were very few parameters to optimize.

Number of hidden units: 500

Statistic	Value	Statistic	Value
Training loss:	0.7009	Training acc:	99.36%
Validation loss:	3.894	Validation acc:	91.9%
Testing loss:	4.1808	Testing acc:	91.19%
Training Time(s):	176.361		

Comments: Accuracy seems to have converged by 50 iterations. Training this network with 500 hidden units was slower than with 100 units, but the validation/testing accuracies showed slight improvement. Further, Training loss and training accuracy near perfection. Since this trend is not matched by validation/testing accuracies, we see that the network is overfitting as it is learning the training examples too well.

Number of hidden units: 2000

Statistic	Value	Statistic	Value
Training loss:	0.2303	Training acc:	99.82%
Validation loss:	3.3766	Validation acc:	93.0%
Testing loss:	3.8814	Testing acc:	91.85%
Training Time(s):	645.494		

Comments: Accuracy seems to have convered by 50 iterations. Training this network with 2000 hidden units was considerably slower than both other networks, depite nearly identical validation/testing accuracies. Training loss and training accuracy are essentially perfect by the end of 200 epochs. Since this trend is not matched by validation/testing accuracies, we see that the

network is overfitting considerably. It has essentially memorized the training examples, but can not generalize as well (beyond the performance attained by smaller networks) to validation/testing data.

General comments: Also, the small bump and incresae in loss before once again decreasing suggest that the parameters went through a local minimum before converging. Momentum likely helped it converge faster as it doesn't appear to have gotten stuck in the local min. The smaller networks trained considerably faster, but had lower accuracy (even though the difference was very small). The larger networks clearly overfit the training data; this indicates that they are too complex for their classification tasks and/or the input data should be improved.

1.4.2 Early Stopping

From the plots, we observe that training should have stopped at an early stopping point of 50 iterations. Beyond this point, there is very little improvement in either validation/testing losses or accuracies.

2 Neural Networks in Tensorflow

2.1 Model implementation

[0]:
```
# load + reshape data
trainData, validData, testData, trainTarget, validTarget, testTarget =␣
↪loadData()
trainData = trainData.reshape(15000,28,28,1)
validData = validData.reshape(1000,28,28,1)
testData = testData.reshape(2724,28,28,1)

# one-hot encode
train_labels, valid_labels, test_labels = convertOneHot(trainTarget,␣
↪validTarget, testTarget)
```

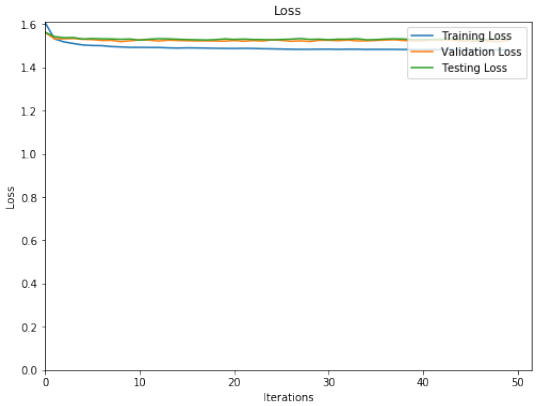
[0]:
```
# training params
learning_rate = 0.0001
epochs = 50
batch_size = 32

# create model
model = models.Sequential()
model.add(layers.InputLayer(input_shape=(28, 28,1)))    # input layer
model.add(layers.Conv2D(                                # conv layer
        filters=32,
        strides=(1,1),
        kernel_size=[3, 3],
        padding="same",
        activation='relu',
        kernel_initializer=tf.contrib.layers.xavier_initializer(uniform=False)))
model.add(layers.BatchNormalization())                  # batch norm
model.add(layers.MaxPooling2D((2, 2)))                  # max pooling
model.add(layers.Flatten())                             # flatten
model.add(layers.Dense(784, activation='relu'))         # fully-connected 784
model.add(layers.Dense(10))                             # fully-connected 10
model.add(layers.Softmax())                             # softmax output

# compile model w/ Adam optimizer + cross entropy loss
model.compile(optimizer=tf.keras.optimizers.Adam(lr=learning_rate),
              loss=tf.keras.losses.CategoricalCrossentropy(from_logits=True),
              metrics=['accuracy'])
```

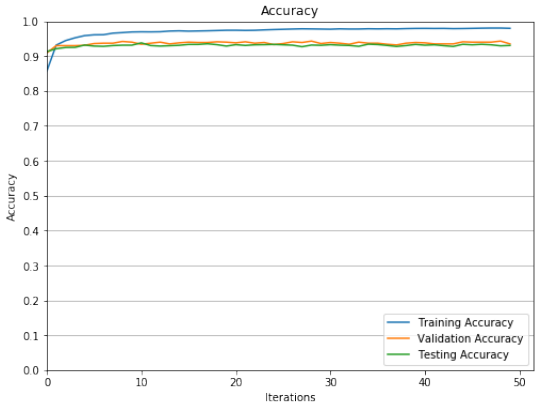
2.2 Model Training

[ ]:
```
# callback to test after each epoch
class TestCallback(tf.keras.callbacks.Callback):
    def __init__(self, test_data):
        self.test_data = test_data
        self.test_acc = []
        self.test_loss = []

    def on_epoch_end(self, epoch, logs=None):
        # perform a test per epoch
        x, y = self.test_data
        loss, acc = self.model.evaluate(x, y, verbose=0, batch_size=32)
        self.test_loss.append(loss)
        self.test_acc.append(acc)
        # append to returned dictionary
        logs["test_loss"] = self.test_loss
        logs["test_acc"] = self.test_acc
```

[0]:
```
# training
history = model.fit(trainData, train_labels,
                    validation_data = (validData, valid_labels),
                    epochs=epochs,
                    batch_size=batch_size,
                    callbacks=[TestCallback((testData, test_labels))],
                    verbose=0,  # 0 = silent, 1 = per epoch
                    shuffle=True)

# display statistics
train_acc, train_loss = history.history["acc"], history.history["loss"]
val_acc, val_loss = history.history["val_acc"], history.history["val_loss"]
test_acc, test_loss = history.history["test_acc"][0], history.
 ↪history["test_loss"][0]

display_statistics(train_loss=train_loss, train_acc=train_acc,
                   valid_loss=val_loss, valid_acc=val_acc,
                   test_loss=test_loss, test_acc=test_acc)
```

```
Training loss: 1.4814           Training acc: 97.97%
Validation loss: 1.5248 Validation acc: 93.5%
Testing loss: 1.5293    Testing acc: 93.06%
```


Loss
Training Loss
Validation Loss
Testing Loss
Loss
Iterations
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0
10
20
30
40
50


Accuracy
Training Accuracy
Validation Accuracy
Testing Accuracy
Accuracy
Iterations
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
10
20
30
40
50

2.3 Hyperparameter Investigation

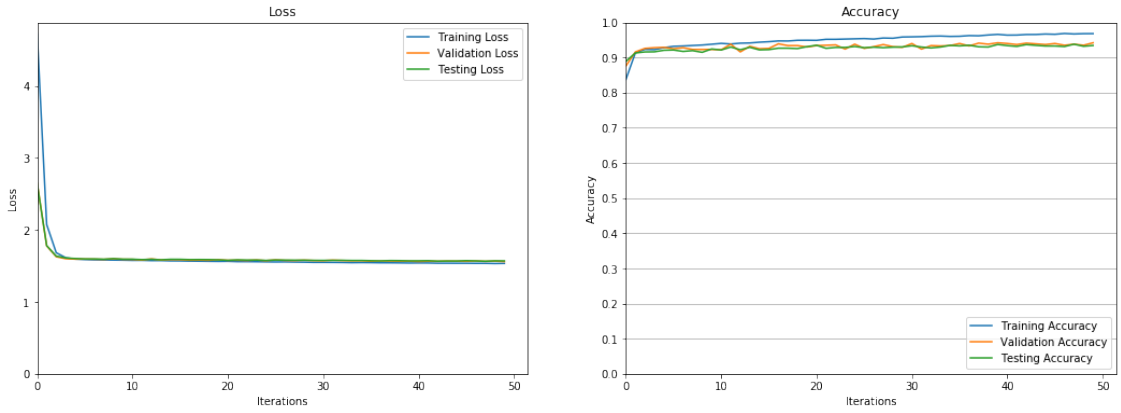
2.3.1 L2 Regularization

[0]:
```
# training params
learning_rate = 0.0001
epochs = 50
batch_size = 32

# test all weight decays [0.01, 0.1, 0.5]
for scale in [0.01, 0.1, 0.5]:
    print("\nL2 Normalization with {}\n".format(scale))

    # create model
    model = models.Sequential()
    model.add(layers.InputLayer(input_shape=(28, 28,1)))    # input layer
    model.add(layers.Conv2D(                                # conv layer
          filters=32,
          strides=(1,1),
          kernel_size=[3, 3],
          padding="same",
          activation='relu',
          kernel_initializer=tf.contrib.layers.
↪xavier_initializer(uniform=False)))
    model.add(layers.BatchNormalization())                  # batch norm
    model.add(layers.MaxPooling2D((2, 2)))                  # max pooling
    model.add(layers.Flatten())                             # flatten
    model.add(layers.Dense(784,
                        activation='relu',
                        kernel_regularizer=tf.contrib.layers.
↪l2_regularizer(scale=scale)))                                  #␣
↪fully-connected 784 w/ ReLu
    model.add(layers.Dense(10))                             # fully-connected 10
    model.add(layers.Softmax())                             # softmax output

    # compile model w/ Adam optimizer + cross entropy loss
    model.compile(optimizer=tf.keras.optimizers.Adam(lr=learning_rate),
                loss=tf.keras.losses.CategoricalCrossentropy(from_logits=True),
                metrics=['accuracy'])

    # train
    history_1 = model.fit(trainData, train_labels,
                        validation_data = (validData, valid_labels),
                        epochs=epochs,
                        batch_size=batch_size,
                        callbacks=[TestCallback((testData, test_labels))],
                        verbose=0,  # 0 = silent, 1 = per epoch
```

```
                    shuffle=True)
    # display statistics
    train_acc, train_loss = history.history["acc"], history.history["loss"]
    val_acc, val_loss = history.history["val_acc"], history.history["val_loss"]
    test_acc, test_loss = history.history["test_acc"][0], history.
↪history["test_loss"][0]

    display_statistics(train_loss=train_loss, train_acc=train_acc,
                       valid_loss=val_loss, valid_acc=val_acc,
                       test_loss=test_loss, test_acc=test_acc)
```

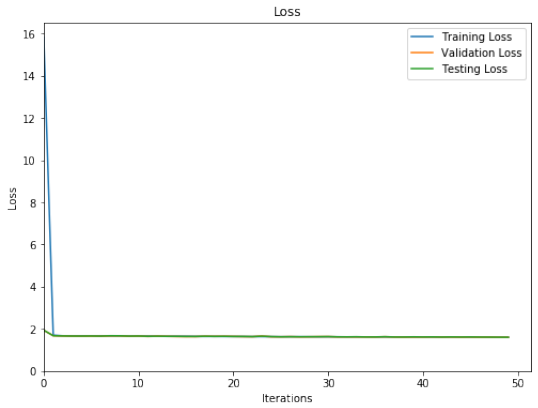
L2 Normalization with 0.01

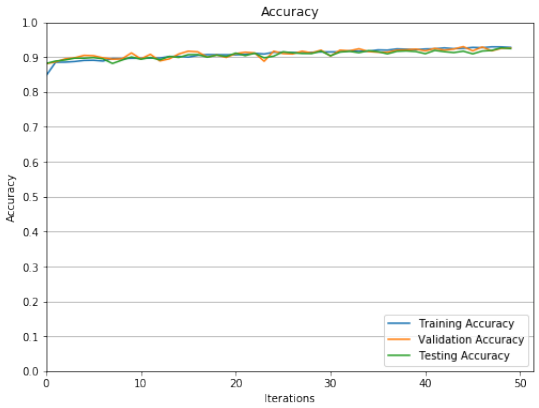
Training loss: 1.5349 Training acc: 96.81%
Validation loss: 1.5601 Validation acc: 94.2%
Testing loss: 1.5696 Testing acc: 93.43%



L2 Normalization with 0.1

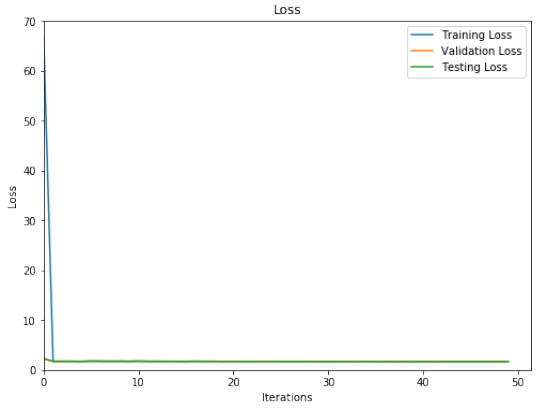
Training loss: 1.6037 Training acc: 92.77%
Validation loss: 1.6033 Validation acc: 92.6%
Testing loss: 1.6058 Testing acc: 92.44%

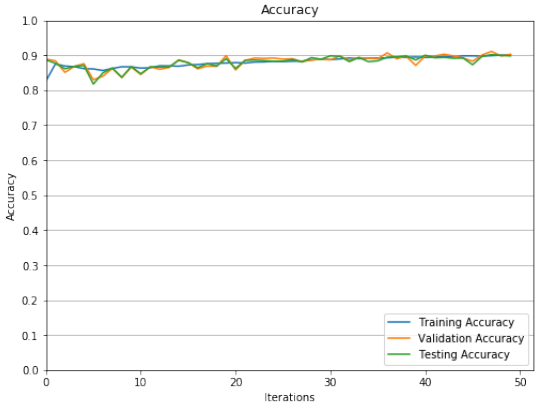




`L2 Normalization with 0.5`

```
Training loss: 1.6729          Training acc: 90.07%
Validation loss: 1.6705 Validation acc: 90.3%
Testing loss: 1.6746    Testing acc: 89.83%
```





General Comments: L2 regularization is a technique to reduce overfitting. For small values of λ, we expect a slight improvement in model performance and a reduced discrepancy between training and validation/test accuracies. This is exactly what we observe here. For $\lambda = 0.01$, we observe validation and testing accuracy both improve slightly while training accuracy decreases by $\sim 1.5\%$. So, the model performance improves slightly and the model overfits less as expected. For values of λ that are too large, we expect the model to start underfitting as we are harshly penalizing parameter vectors which are large in magnitude. As λ increases, we see the discrepancy between training and validation/test accuracies decrease dramatically (the model is no longer overfitting). However, this change is accompanied by a reduction in overall accuracy (by $\sim 2\%$) as the model is now underfitting.

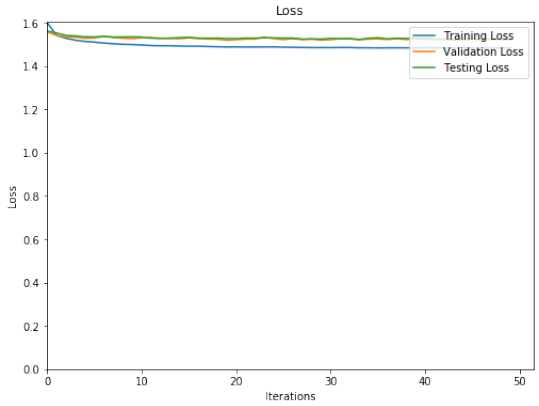
2.3.2 Dropout

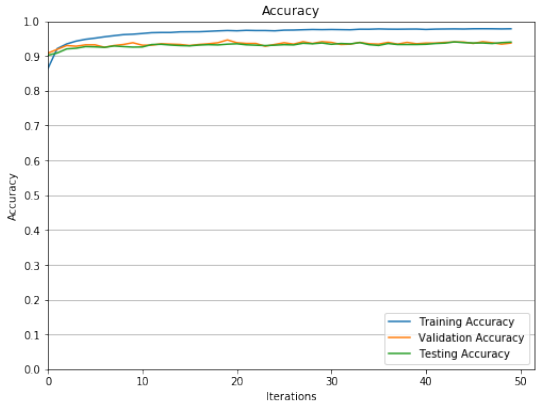
[0]:
```
# training params
learning_rate = 0.0001
epochs = 50
batch_size = 32

# for rate in [0.9, 0.75, 0.5]:
for rate in [0.1, 0.25, 0.5]:
    print("\nDropout with probability {}\n".format(rate))

    # create model
    model = models.Sequential()
    model.add(layers.InputLayer(input_shape=(28, 28,1)))    # input layer
    model.add(layers.Conv2D(                                # conv layer
          filters=32,
          strides=(1,1),
          kernel_size=[3, 3],
          padding="same",
          activation='relu',
          kernel_initializer=tf.contrib.layers.
 ↪xavier_initializer(uniform=False)))
    model.add(layers.BatchNormalization())                  # batch norm
    model.add(layers.MaxPooling2D((2, 2)))                  # max pooling
    model.add(layers.Flatten())                             # flatten
    model.add(layers.Dense(784))                            # fully-connected 784
    model.add(layers.Dropout(rate=rate))                    # dropout
    model.add(layers.ReLU())                                # Relu activation
    model.add(layers.Dense(10))                             # fully-connected 10
    model.add(layers.Softmax())                             # softmax output

    # compile model w/ Adam optimizer + cross entropy loss
    model.compile(optimizer=tf.keras.optimizers.Adam(lr=learning_rate),
                loss=tf.keras.losses.CategoricalCrossentropy(from_logits=True),
                metrics=['accuracy'])

    # train
    history_2 = model.fit(trainData, train_labels,
                      validation_data = (validData, valid_labels),
                      epochs=epochs,
                      batch_size=batch_size,
                      callbacks=[TestCallback((testData, test_labels))],
                      verbose=0,  # 0 = silent, 1 = per epoch
                      shuffle=True)
    # display stats
    train_acc, train_loss = history.history["acc"], history.history["loss"]
    val_acc, val_loss = history.history["val_acc"], history.history["val_loss"]
```

```
   test_acc, test_loss = history.history["test_acc"][0], history.
↪history["test_loss"][0]

   display_statistics(train_loss=train_loss, train_acc=train_acc,
                      valid_loss=val_loss, valid_acc=val_acc,
                      test_loss=test_loss, test_acc=test_acc)
```

Dropout with probability 0.1

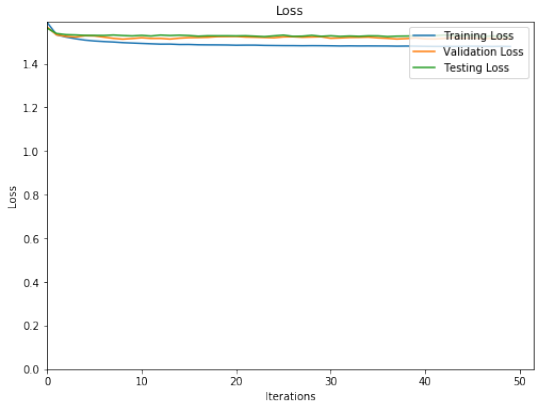
Training loss: 1.4831 Training acc: 97.81%
Validation loss: 1.5229 Validation acc: 93.7%
Testing loss: 1.5219 Testing acc: 93.98%

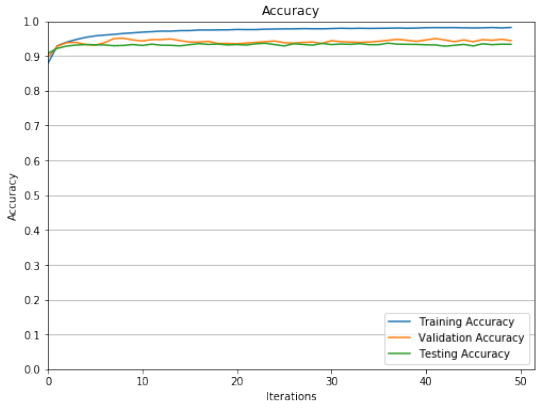




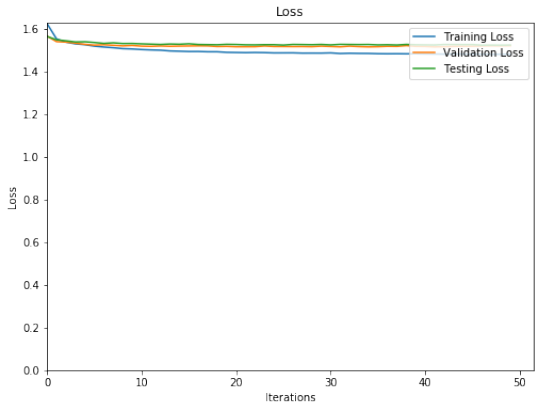
Dropout with probability 0.25

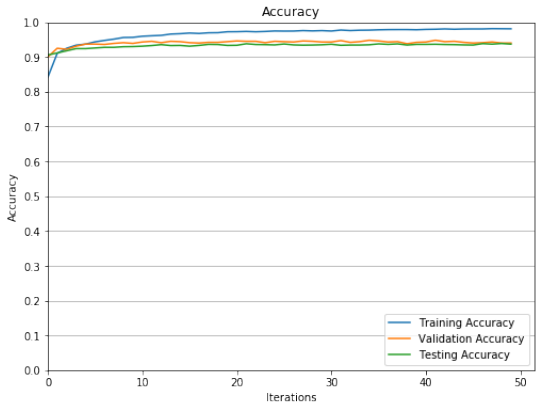
Training loss: 1.4794 Training acc: 98.19%
Validation loss: 1.5156 Validation acc: 94.4%
Testing loss: 1.527 Testing acc: 93.36%





```
Dropout with probability 0.5

Training loss: 1.4803           Training acc: 98.09%
Validation loss: 1.5206 Validation acc: 94.0%
Testing loss: 1.5238    Testing acc: 93.69%
```





General Comments: Dropout is a technique which aims to decrease overfitting in our model. We expect see this in the form of improved accuracy and loss statistics. Indeed, this is exactly observed. As the amount of dropout increases (keeping the rate within reason), the validation and testing accuracies both improve. Note that we also see training accuracy improve. So, it seems that the model as an increased capacity to learn overall. With dropout, it is able to learn the training data better, but it is also able to generalize better.