Problems 1 & 2

Problem 1: Big-O Characterization

a) $2 \log n + 100{,}000$
Answer: $O(\log n)$
Explanation: Constants are ignored in Big-O notation. The dominating term is $\log n$.

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b) $n^2 + 2n$
Answer: $O(n^2)$
Explanation: n^2 grows faster than $2n$, so n^2 dominates for large n.

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c) $(2n+1) + (2n-1) + \ldots + 3 + 1$
Answer: $O(n^2)$
Explanation: This is the sum of the first n odd numbers, which equals n^2.

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d) $2^{20} + 3^{10}$
Answer: $O(1)$
Explanation: Both terms are constants, so the runtime is constant.

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e) $1 + n^2 + 2n + n!$
Answer: $O(n!)$
Explanation: $n!$ grows faster than all other terms, so it dominates.

Problem 2: Function Classifications

Functions:

- $3n$
- $2n + 3$
- $n^2 + n$
- $\log(n^2)$
- $\sqrt[3]{n}$
- $\log(2n)$

$O(n)$:

- $3n$
- $2n + 3$
- $\log(n^2)$
- $\sqrt[3]{n}$
- $\log(2n)$

$\Theta(n)$:

- $3n$
- $2n + 3$

$\Omega(n)$:

- $3n$
- $2n + 3$
- $n^2 + n$

Explanations:

- $3n$ and $2n + 3$ are linear functions $\Rightarrow O(n), \Theta(n), \Omega(n)$
- $\log(n^2) = 2 \log(n) \Rightarrow O(\log n)$
- $\sqrt[3]{n}$ grows slower than n $\Rightarrow O(n)$
- $\log(2n) = \log(2) + \log(n) \Rightarrow O(\log n)$
- $n^2 + n$ grows faster than n $\Rightarrow \Omega(n)$, but not $O(n)$