Prelim topic: Likelihood-free MCMC

Eric Kernfeld[1]

[1]University of Washington Department of Statistics

Paper

Darren Wilkinson's "Parameter inference for stochastic kinetic models of bacterial gene regulation," a book chapter in [1].

Likelihood Free MCMC

Cast of Characters

- r_j is a menu item in a restaurant.
- x_t is the amount of money in the cash register t.
- $\mathcal{D}_t$ incomplete observation of x_t with error.
- c is the popularity of menu items.
- τ governs measurement error.
- θ is τ and c together.

Likelihood Free MCMC

To produce a chain of samples from $P(\theta|D)$, using a proposal $q(\theta^*|\theta)$, accept with probability $p_{rej}(\theta^*|\theta) \equiv \min\{1, A\}$ if

$$A = \frac{q(\theta, x|\theta^*, x^*)}{q(\theta^*, x^*|\theta, x)} \times \frac{P(\theta^*, x^*|D)}{P(\theta, x|D)}$$

Likelihood Free MCMC

Cast of Characters (things DW can't evaluate are in red)

- $P(\theta)$
- $P(x|\theta)$
- $P(\mathcal{D}|x,\theta)$
- $P(x,\theta|\mathcal{D})$

$$A = \frac{q(\theta, x|\theta^*, x^*)}{q(\theta^*, x^*|\theta, x)} \times \frac{P(\theta^*, x^*|D)}{P(\theta, x|D)}.$$

Quote: "Conditional on discrete-time observations, the Markov process breaks up into a collection of independent bridge processes that appear not to be analytically tractable."

Likelihood Free MCMC

$$\frac{q(\theta^*, x^*|\theta, x)}{q(\theta, x|\theta^*, x^*)} \times \frac{P(x|\theta)}{P(x^*|\theta^*)} \times \frac{P(\theta)}{P(\theta^*)} \times \frac{P(\mathcal{D}|x, \theta)}{P(\mathcal{D}|x^*, \theta^*)}$$

$$= \frac{f(\theta^*|\theta)}{f(\theta|\theta^*)} \times \frac{P(x^*|\theta^*)}{P(x|\theta)} \times \frac{P(x|\theta)}{P(x^*|\theta^*)} \times \frac{P(\theta)}{P(\theta^*)} \times \frac{P(\mathcal{D}|x, \theta)}{P(\mathcal{D}|x^*, \theta^*)}$$

$$= \frac{f(\theta^*|\theta)}{f(\theta|\theta^*)} \times \frac{P(\theta)}{P(\theta^*)} \times \frac{P(\mathcal{D}|x, \theta)}{P(\mathcal{D}|x^*, \theta^*)}.$$

Questions?

Wilkinson's paper is a chapter from this book:

Bernardo, J.M., Bayarri, M.J., Berger, J.O., Dawid, A.P., Heckerman, D., Smith, A.F.M., West, M.:
Ninth Valencia international meeting on Bayesian statistics, Benidorm, Spain, 03-08.06.2010.
Oxford U.P., Oxford (2012)