Prelim topic: Likelihood-free MCMC

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Paper

Darren Wilkinson's "Parameter inference for stochastic kinetic models of bacterial gene regulation," a book chapter in [1].

Likelihood Free MCMC

Cast of Characters

- r_j is a menu item in a restaurant.
- x_t is the amount of money in the cash register t.
- $\mathcal{D}_t$ incomplete observation of x_t with error.
- c is the popularity of menu items.
- τ governs measurement error.
- θ is τ and c together.

what is the statistical problem?

why is it important?

why is it hard?

what attempts were made to solve it in previous work?

introduce a running example

Likelihood Free MCMC

To produce a chain of samples from $P(\theta|D)$, using a proposal $q(\theta^*|\theta)$, accept with probability $p_{rej}(\theta^*|\theta) \equiv \min\{1, A\}$ if

$$A = \frac{q(\theta, x|\theta^*, x^*)}{q(\theta^*, x^*|\theta, x)} \times \frac{P(\theta^*, x^*|\mathcal{D})}{P(\theta, x|\mathcal{D})}$$

- *the method*
- *the theory behind it*

Likelihood Free MCMC

Cast of Characters (things DW can't evaluate are in red)

- $P(\theta)$
- $P(x|\theta)$
- $P(\mathcal{D}|x,\theta)$
- $P(x,\theta|\mathcal{D})$

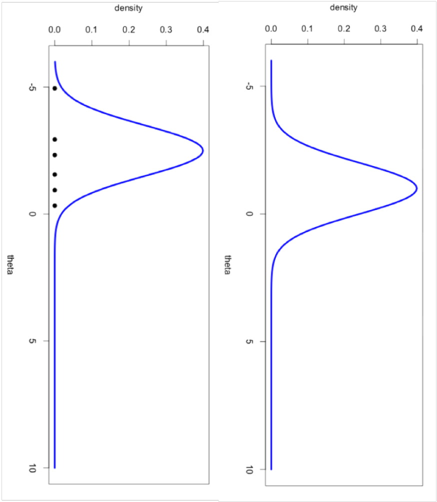
$$A = \frac{q(\theta, x|\theta^*, x^*)}{q(\theta^*, x^*|\theta, x)} \times \frac{P(\theta^*, x^*|\mathcal{D})}{P(\theta, x|\mathcal{D})}.$$

Quote: "Conditional on discrete-time observations, the Markov process breaks up into a collection of independent bridge processes that appear not to be analytically tractable."

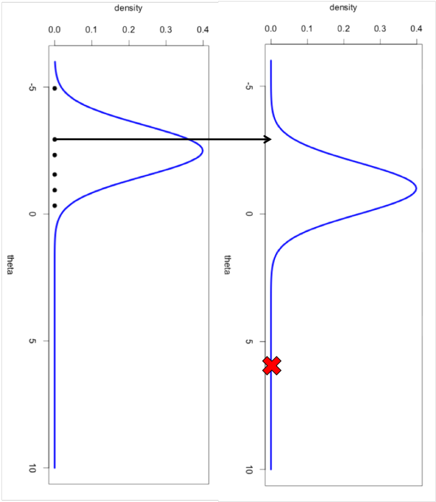
Likelihood Free MCMC

$$
\begin{aligned}
&\frac{q(\theta^*, x^*|\theta, x)}{q(\theta, x|\theta^*, x^*)} \times \frac{P(x|\theta)}{P(x^*|\theta^*)} \times \frac{P(\theta)}{P(\theta^*)} \times \frac{P(\mathcal{D}|x, \theta)}{P(\mathcal{D}|x^*, \theta^*)} \\
&= \frac{f(\theta^*|\theta)}{f(\theta|\theta^*)} \times \frac{P(x^*|\theta^*)}{P(x|\theta)} \times \frac{P(x|\theta)}{P(x^*|\theta^*)} \times \frac{P(\theta)}{P(\theta^*)} \times \frac{P(\mathcal{D}|x, \theta)}{P(\mathcal{D}|x^*, \theta^*)} \\
&= \frac{f(\theta^*|\theta)}{f(\theta|\theta^*)} \times \frac{P(\theta)}{P(\theta^*)} \times \frac{P(\mathcal{D}|x, \theta)}{P(\mathcal{D}|x^*, \theta^*)}.
\end{aligned}
$$

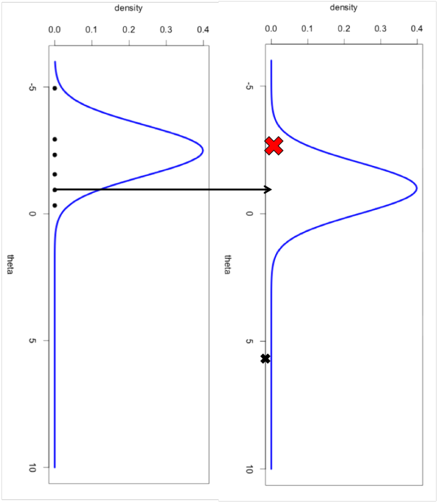
Likelihood Free Particle MCMC



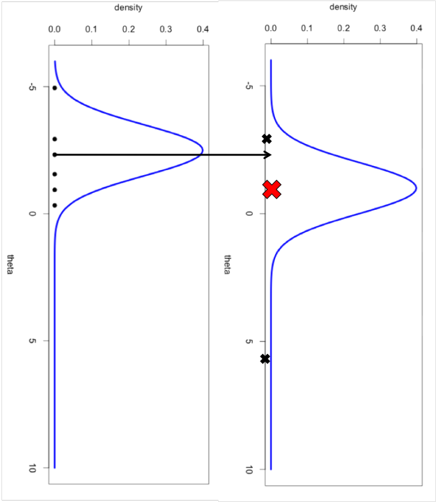
Likelihood Free Particle MCMC



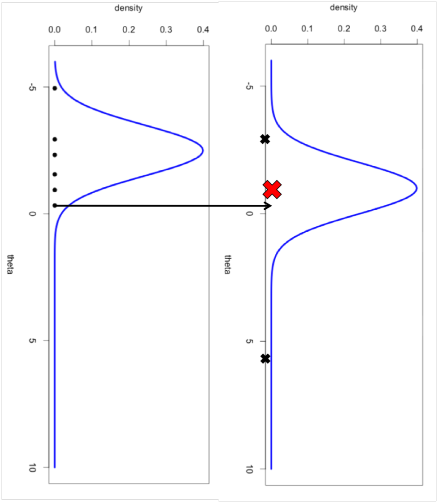
Likelihood Free Particle MCMC



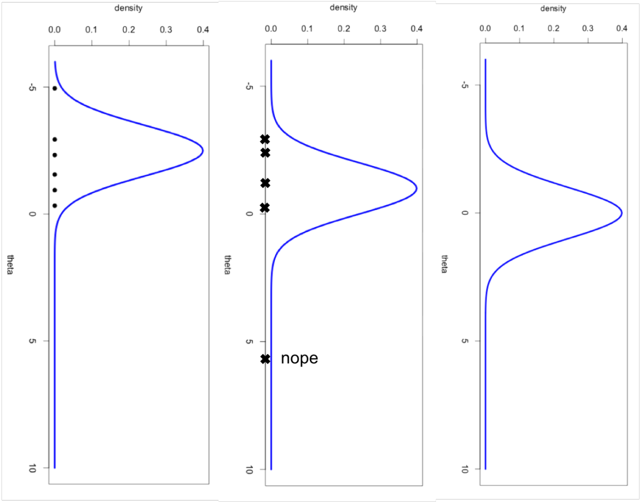
Likelihood Free Particle MCMC



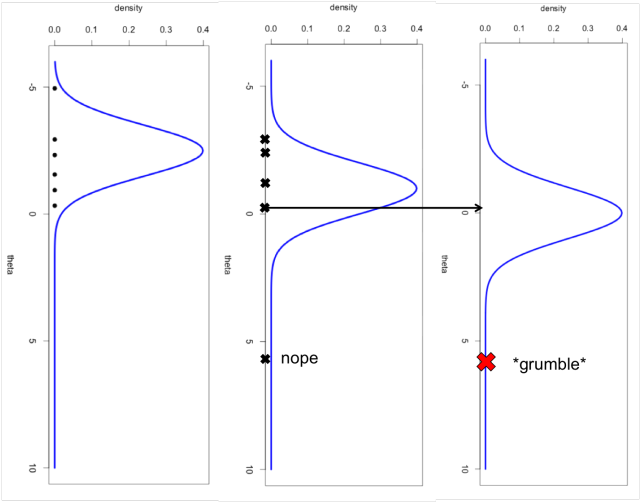
Likelihood Free Particle MCMC



Likelihood Free Particle MCMC



Likelihood Free Particle MCMC



Likelihood Free Particle MCMC

Given a hidden continuous-time Markov process $\{x_t\}_{t=0}^T$ with:

Unknown parameters θ and known initial state x_0

Data points $\mathcal{D}_{t_i}$ at times t_i, $i \in \{1, ...I\}$

A simple, tractable error model $P(\mathcal{D}_{t_i} | x_{t_i}, \theta)$

A simulator for paths of x given θ

An array B_0 of 1,000,000 samples from a prior on θ, x_0

Empty arrays B_i of the same length

For each time point (for $i \in \{1, ...I\}$):

Until B_i is full:

Draw $(\theta^*, x^*_{t_{i-1}})$ from B_{i-1} or a KDE of its contents

Using $(\theta^*, x^*_{t_{i-1}})$, simulate up to $x^*_{t_i}$, the state at time t_i

Set $A = \min(1, \frac{P(\mathcal{D}_{t_i} | x^*_{t_i}, \theta^*)}{P(\mathcal{D}_{t_i} | x_{t_i}, \theta)})$

With probability A, overwrite (θ, x_{t_i}) with $(\theta^*, x^*_{t_i})$

After burn-in and thinning, add (θ, x_{t_i}) to B_i

Likelihood Free Particle MCMC

Likelihood Free Particle MCMC

$$\frac{P(x^*_{t_i}|x^*_{t_{i-1}},\theta^*,\mathcal{D}_{t_{i-1}})}{P(x_{t_i}|x_{t_{i-1}},\theta,\mathcal{D}_{t_{i-1}})}\frac{P(x^*_{t_{i-1}},\theta^*|\mathcal{D}_{t_{i-1}})}{P(x_{t_{i-1}},\theta|\mathcal{D}_{t_{i-1}})}\times$$

$$\frac{P(x^*_{t_i}|x^*_{t_{i-1}},\theta^*,\mathcal{D}_{t_{i-1}})}{P(x_{t_i}|x_{t_{i-1}},\theta,\mathcal{D}_{t_{i-1}})}\frac{P(x_{t_{i-1}},\theta|\mathcal{D}_{t_{i-1}})}{P(x^*_{t_{i-1}},\theta^*|\mathcal{D}_{t_{i-1}})}\frac{P(\mathcal{D}_{t_i}|x_{t_i},\theta)}{P(\mathcal{D}_{t_i}|x^*_{t_i},\theta^*)}$$

- plan for experiments
(this part will be replaced later;
if you are Eric or Alec this part is optional)

Questions?

Wilkinson's paper is a chapter from this book:

Bernardo, J.M., Bayarri, M.J., Berger, J.O., Dawid, A.P., Heckerman, D., Smith, A.F.M., West, M.:
Ninth Valencia international meeting on Bayesian statistics, Benidorm, Spain, 03-08.06.2010.
Oxford U.P., Oxford (2012)