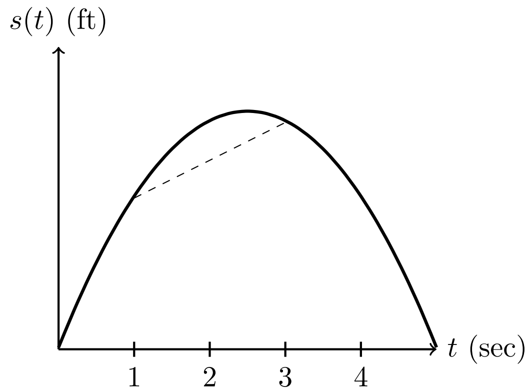
Name: ______________________________

1. The height $s(t)$ of a model rocket (in feet) after t seconds is given by the function with graph and table below.



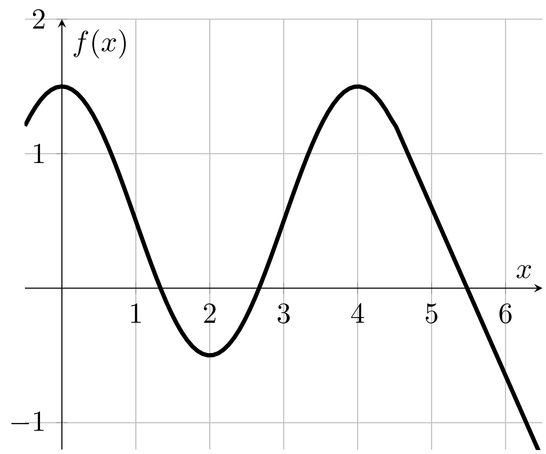
t (sec)	$s(t)$ (ft)
0	0
1	25
2	40
3	40
4	25

(a) Find the average velocity of the rocket between second 1 and second 3.

(b) Was the *average* velocity of the rocket between second 1 and second 3 more than or less than the *instantaneous* velocity at second 3? Explain using either the graph or the table above.

Name: ______________________________

1. The graph of the function f is given below.



(a) Use the graph to find $f'(5)$, the derivative of f at $a = 5$. Show your work or explain why your answer is correct.

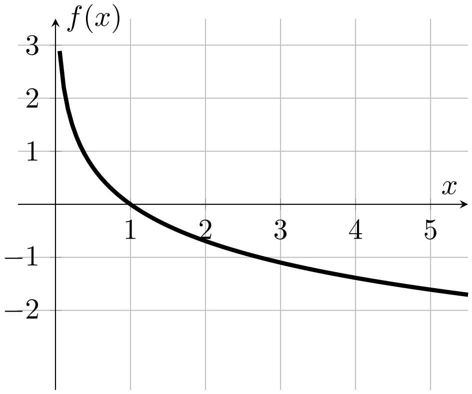
(b) Where is the derivative of f largest between $x = 0$ and $x = 6$? That is, find a value of a such that $f'(a)$ is as large as it is anywhere shown. Explain why your answer is correct.

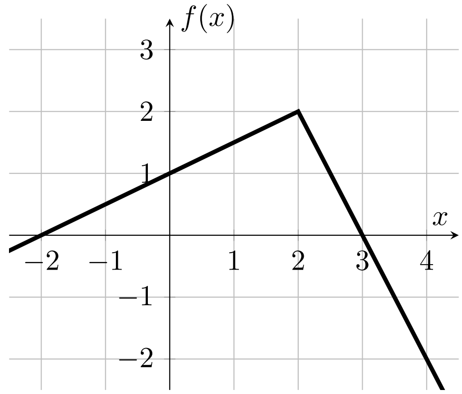
Name: ______________________________

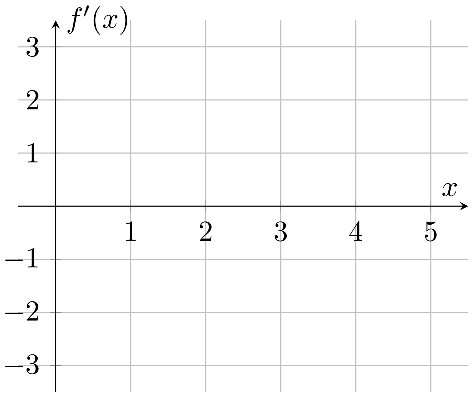
1. Let $f(x) = x^2 + 5x + 2$. Use the *limit definition of the derivative* to find $f'(x)$, the derivative of f. Show all your work.

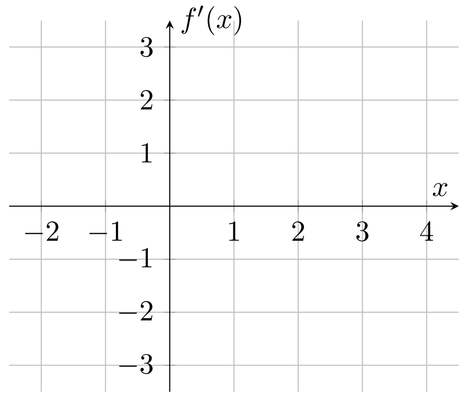
Name: ______________________________

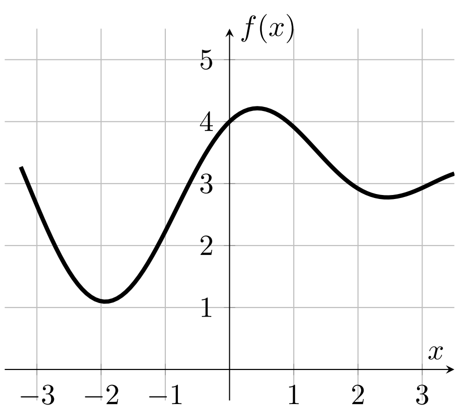
1. For each function graphed below, carefully sketch a graph of the functions derivative in the space provided below the original graph.

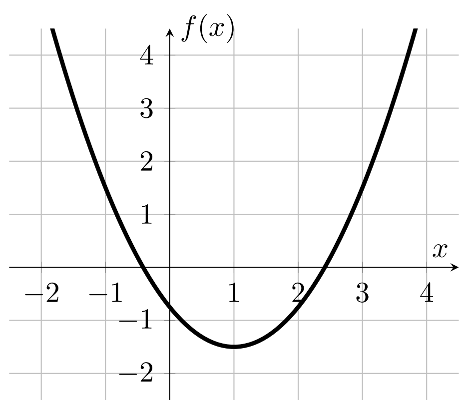


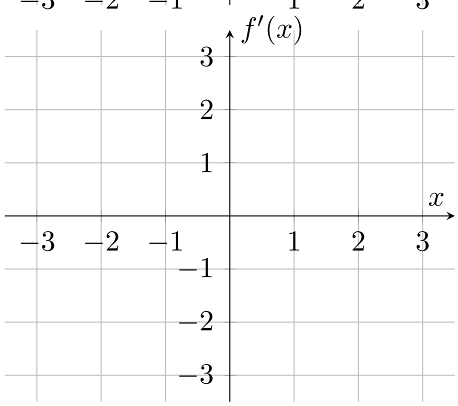


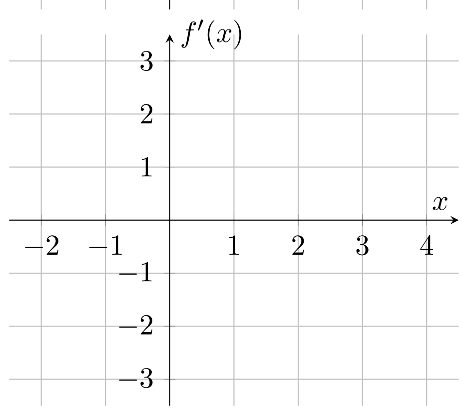












Name: ______________________________

1. The amount of food a carnivorous emu eats is a function of its age. That is, $m(t)$ is the number of ounces of meat the emu will eat per day, t weeks after it is born

   Write sentences accurately interpreting the two equations below, *including units*. Your sentences must use all numbers in the equation.

$$m(15) = 37 \qquad m'(15) = 3$$

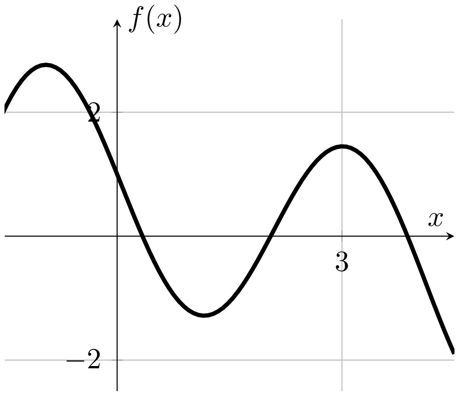
2. The number of eggs a carnivorous emu produces each week is a function of the amount it eats. That is, $g(m)$ is the number of eggs produced each week when an emu eats m ounces of food each day.

   Write a sentence accurately interpreting the two equations below, *including units*. Your sentence must use all numbers in the equation.

$$g(37) = 3 \qquad g'(37) = 0.25$$

Name: ______________________________

1. The function f is graphed below. Use the graph to determine whether the first or second derivatives of f are positive, negative, zero at the given points (circle the correct answer). Justify your answer by filling in the blanks.



$f'(0)$ is: positive / negative / zero

because near $x = 0$, $f(x)$ is ____________

$f''(0)$ is: positive / negative / zero

because near $x = 0$, $f(x)$ is ____________

$f'(3)$ is: positive / negative / zero

because near $x = 3$, $f(x)$ is ____________

$f''(3)$ is: positive / negative / zero

because near $x = 3$, $f(x)$ is ____________

2. The function g has a **derivative** g' with values given in the following table. Circle the characteristics of g, g', and g'' that you can conclude from the table at the specified point.

x	0	1	2	3	4
$g'(x)$	-1	1	3	6	10

(a) What can you conclude about $g'(2)$?

$g'(2)$ is: positive / negative / zero / sign can't be determined.

$g'(2)$ is: increasing / decreasing / flat / direction can't be determined.

(b) What can you say about $g''(2)$?

$g''(2)$ is: positive / negative / zero / sign can't be determined.

(c) What can you say about $g(2)$, the original function whose derivative is given in the table?

$g(2)$ is: concave up / concave down / linear / shape can't be determined.

$g(2)$ is: increasing / decreasing / flat / direction can't be determined.

$g(2)$ is: positive / negative / zero / sign can't be determined.

Name: ______________________________

1. Suppose you know that the function f passes through the point $(1, 5)$ and has first derivative

$$f'(x) = 3^x.$$

   (a) Find the equation of the tangent line to the the function $f(x)$ at the point $(1, 5)$.

   (b) Use the tangent line (or the equivalent *local linearization*) to approximate $f(0.9)$. Show your work.

   (c) Suppose you found out that $f''(1) = 3.3$. What does this tell you about the shape of f near $x = 1$? Does this mean your approximation for $f(0.9)$ is an over estimate or under estimate? Briefly explain.

Name: ______________________________

For each function given below, find its derivative (using basic derivative rules).

1. $f(x) = 2 + 3x + 4x^5$

2. $f(x) = 3^x - e^x + 3^e$

3. $f(x) = 2\cos(x) - 3\sin(x)$

4. $f(x) = \dfrac{3}{x^2} - \sqrt[5]{x}$

Name: ______________________________

Find the derivatives of the following functions, using the product and quotient rules as appropriate.

1. $f(x) = \cos(x)x^{-2}$

2. $f(x) = \dfrac{e^x + 5}{x^2 + 1}$

3. $f(x) = x^3 \sin(x)(x^5 - 2)$

Name: ______________________________

Find the derivatives of the following functions, using the chain rule.

1. $f(x) = \cos(5^x)$

2. $f(x) = \sqrt{x^3 + 2x}$

3. $f(x) = e^{\sin(\ln(x))+2}$

Name: ______________________________

Find the derivatives of the following functions. You will need to use a combination of multiple rules.

1. $f(x) = \cos\left(\dfrac{x^4+2}{x^7+1}\right)$

2. $f(x) = 3^{x^2+1}\ln(x^3+4)$

Name: ______________________________

1. Consider the curve defined implicitly by

$$x^2 - xy = 5y - y^2.$$

   (a) Use implicit differentiation to find $\dfrac{dy}{dx}$. Show your work.

   (b) Find the slope of the line tangent to the curve at the point $(0, 5)$.

Name: ______________________________

1. A mommy emu is out foraging due north of her next when her baby takes off running away from the nest due east. By the time the baby emu is 30 feet away from the nest, it is running at 10 feet per second. At this time, the mom is 40 feet from the nest, running due south at 15 feet per second (directly towards the nest, because emu's don't understand diagonal shortcuts).

   At what rate is the (diagonal) distance between the baby and momma decreasing at this instant?

Name: ______________________________

You want to build a new habitat for your emus so they don't run away. You must enclose a rectangular fields with fence, plus also add a fence separating the field into two equal sized rectangles. If you have 500 feet of fence, what is the largest habitat you can construct? Show all your work.

Name: ______________________________

1. The function $f(x)$ (which you don't know) has *first and second derivatives*

$$f'(x) = x(x+3)(x-2)$$

$$f''(x) = 3x^2 + 2x - 6$$

Using these provided derivatives, find all <u>critical numbers</u> of the original function $f(x)$, and then use the first or second derivative tests to classify them as local maximums, local minimums, or neither. Then give the intervals on which f is increasing or decreasing.

Use the middle of the page to show your work and record your answers at the bottom of the page.

Critical numbers:

Local maximum(s) at $x =$

Local minimum(s) at $x =$

f is increasing on the interval(s):

f is decreasing on the interval(s):

Name: ______________________________

1. Find the limit, using L'Hôpital's rule if appropriate:

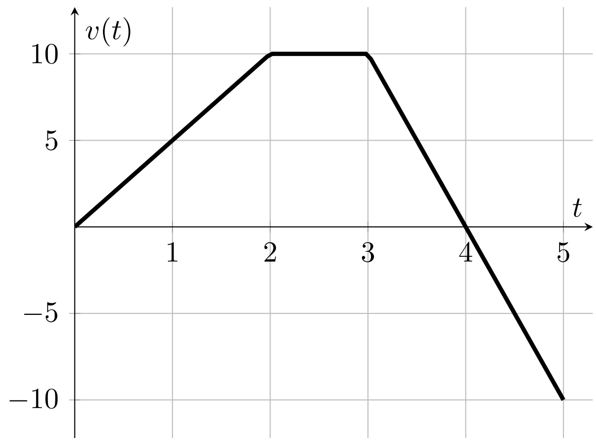
$$\lim_{x \to 1} \frac{x^2 + x - 2}{7x^2 - 4x - 3}$$

2. Evaluate the limit at infinity, using L'Hôpital's rule if appropriate:

$$\lim_{x \to \infty} \frac{15x + 12}{3e^x - 100}$$

Name: ______________________________

1. The *velocity* $v(t)$, in feet per second, of a model rocket, t seconds after launch, has graph shown below.

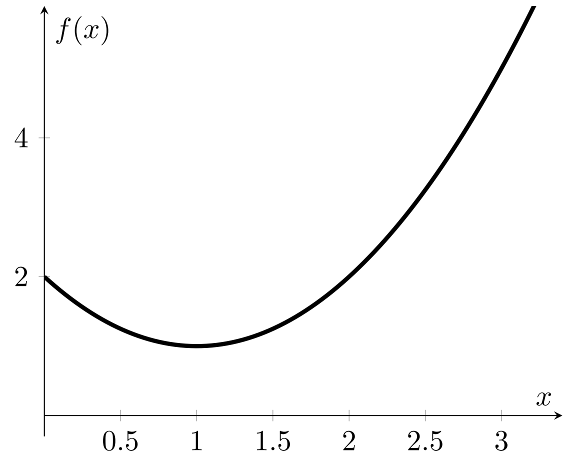


(a) Find the total distance traveled by the rocket in the first four seconds (between seconds 0 and 4). Briefly explain how you found your answer.

(b) Find the total *change in position* of the rocket in the first *five* seconds (between seconds 0 and 5). Explain why this is less than the distance traveled in the first four seconds.

Name: ______________________________

1. Consider the function $f(x) = (x-1)^2 + 1$. Compute the left and right Riemann sums for $f(x)$ with $n = 4$ sub-intervals on the interval $[1, 3]$. Sketch the rectangles whose area represent the *left* Riemann sum on the graph of f below.



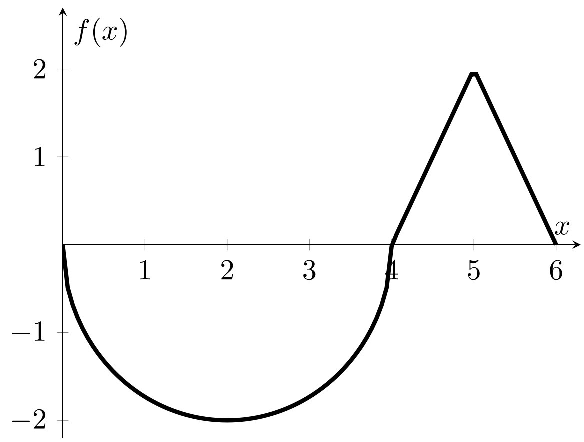
Left Riemann sum:

Right Riemann sum:

2. If the function $f(x)$ above represents the *rate of change* in height of your pet emu, in inches per week, x weeks after you adopted him, what do the sums you computed above represent?

Name: ______________________________

1. Consider the function $f(x)$ graphed below. Assume each portion of the graph is either part of a line or part of a circle.



   (a) Give the exact value of $\int_0^6 f(x)\,dx$. Briefly explain your answer.

   (b) Find the total area between the curve and the x-axis (from 0 to 6). What sum of integrals describes this (positive) area?

Name: ______________________________

1. Consider the function
$$f(x) = x^2 + 2x + 5.$$
Find the antiderivative $F(x)$ of $f(x)$. Then check you work by computing $F'(x)$.

2. Use your answer to the previous question to find the exact value of the definite integral:
$$\int_1^3 x^2 + 2x + 5\,dx.$$