NAME: RISHAV MONDAL ROLL NO: 19010172

Q1. What is a Gomory - Hu tree?

Ans). The Gomory - Hu tree is defined for a flow graph containing 'n' ~~edges~~ vertices and and 'm' edges as containing 'n' vertices and (n-1) edges. So ~~given~~ formally we can give the definition as :-

For any undirected graph $G(V, E)$ with edge weights, $w: E \to \mathbb{R}^+$, $\exists$ a Gomory - Hu tree, $T = (V, E_T)$ with an edge weight function $w': E \to \mathbb{R}^+$, such that the following hold true :-

1) $\forall$ u, v pairs in V, the value of the minimum u-v cut in T is the same as in G. This reinforces the fact that there are only (n-1) distinct min-cut values in G.

2) $\forall e = (a, b) \in T$, the cut induced by e in T is a min-cut in G between a & b. i.e $(w'(a,b) = c(a,b)$ where $c(a,b)$ is the capacity function b/w vertices a and b).

Q2. For a given graph G, how do we construct the Gomory - Hu tree?

Ans). We give a simple algorithm for constructing Gomory - Hu tree for graph $G(V, E)$:-

(1) Initialize the tree T to a single node, which is the entire vertex set V.

(2) Initialize the queue Q to the queue containing only one element which is again the entire vertex set V.

(3) while (! Q.empty) {
- delete the first element of Q; let's call it as S.
- call the minimum Steiner-cut algorithm with the set S as the Steiner set in new graph obtained by contracting the entire subtree rooted at each neighbour in T of S into a single node.
- let S_1 and S_2 be the two components that S is split into by the above cut and let c = size of this cut; now update T by splitting node S into S_1 & S_2 and

P.T.O

introduce an edge with weight = c between the two nodes S_1 and S_2.

— set the neighbours of S_1 and S_2 in the tree T appropriately as well.

— insert the node S_1 (also S_2) in the queue Q if S_1 (or S_2) contains more than 1 vertex.

end while

In this algorithm, we use the minimum Steiner-cut algorithm as a function call for efficient implementation of our algorithm.

Q3. Suggest some interesting research problems on Gomory-Hu tree

Ans). There are a lot of interesting research going on regarding the Gomory-Hu tree, and here are some of them:-

- Segmenting a webpage with Gomory-Hu tree based clustering.
- Trying to give more efficient algorithms for construction of the Gomory-Hu tree.
- Other interesting open problems include *derandomization, extension to the capacitated bases, and further improvement in speed and space.