BEE 4530 Assignment 7

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0.1 Programming the finite difference equation in a spreadsheet program

What is the maximum Δt you can choose?
You will want a Δt small enough such that heat does not diffuse more than one element width in a single timestep. Following the Nyquist limit, we should have at least two points per interval. For a 5cm slab with 6 nodes, each element must have width 1cm. Therefore, the minimum timestep must be:

$$\Delta t = \frac{\Delta x^2}{2\alpha} \tag{1}$$

$$\Delta t = \frac{(.01m)^2}{20^{-7}\frac{m^2}{s}} \tag{2}$$

And thus $\Delta t_{max} = 500$s.

Repeat computations with Δt higher than this value and comment on the results.
At values of $\Delta t > \Delta t_{max}$, internal temperatures begin to show non-physical final states in which the internal temperature falls below the boundary temperature.

0.2 Error in finite element computation and its reduction

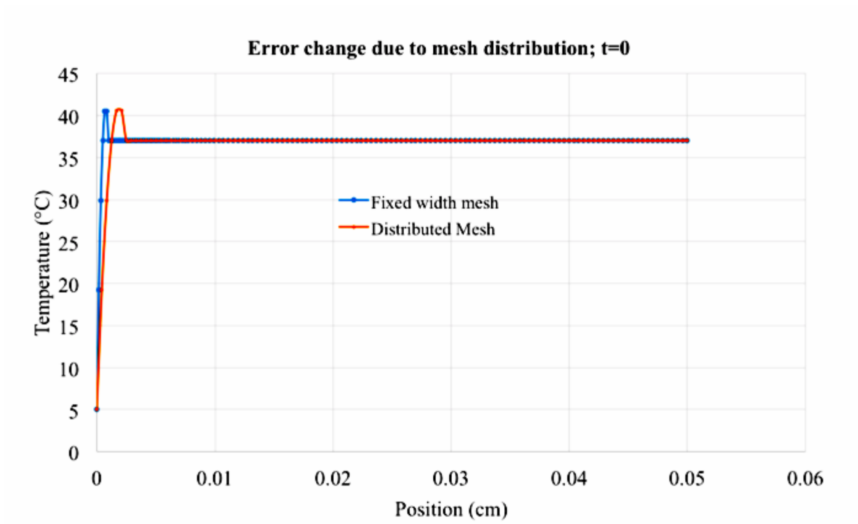
Is there a region where the computed values are unphysical?
Yes. Near time 0, the region closest to the uninsulated boundary increases to over 37°C. Since no heat energy is added to the system, this should not be the case.

Do these unphysical values occur at an early or later time, within the duration of 5 minutes?
They occur early, from 0 to 5 seconds.

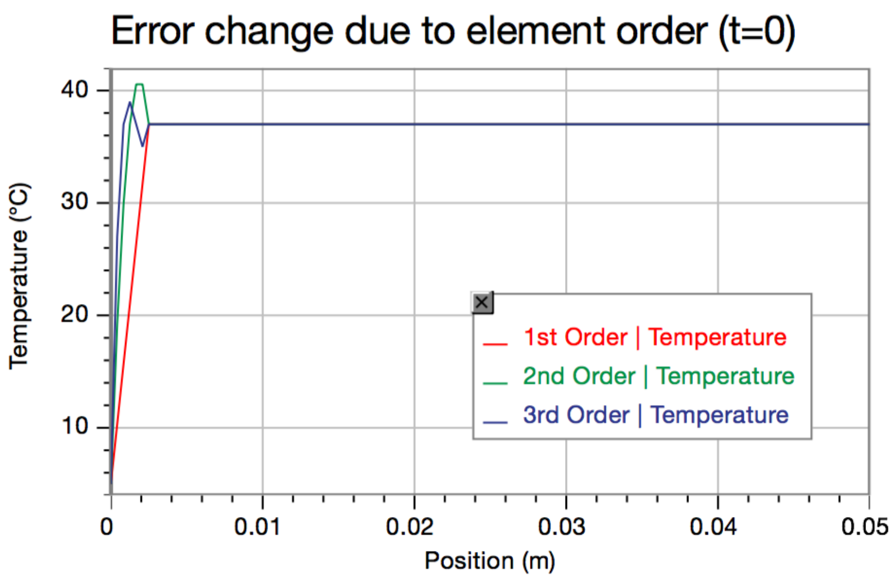
Why do the unphysical values seem to disappear at later time? In comparison with the analytical solution, which region has the highest absolute error in the computed values? Can you reduce the error without increasing the number of elements?
At later times, the physical process tends to equilibrium and the unusual high heat gets dissipated. The highest error is in that region, from the constant temperature boundary to about .5cm inward. It is possible to reduce the error by redistributing the elements to have a higher node density in that region, as shown below.

Change the distribution of the elements where the temperature is unphysical.

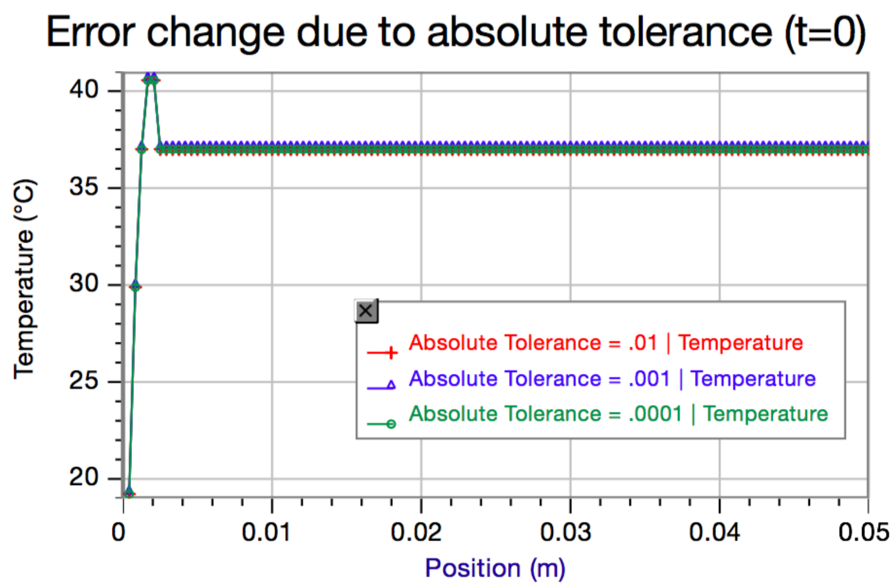
Did this reduce the error? Show a plot of the new mesh and a plot of the new temperature versus the old.
Yes. Although for this particular mesh the values are still unphysical, it is possible to see that the temperature spike would be further smoothed out with the distribution of more mesh points in the relevant region.
Note: The distributed mesh shown below has more total elements than the fixed-width mesh. However, since by x=.1cm the temperature values are constant, the tail elements could easily be discarded to create a 21-node distributed mesh without affecting the results.

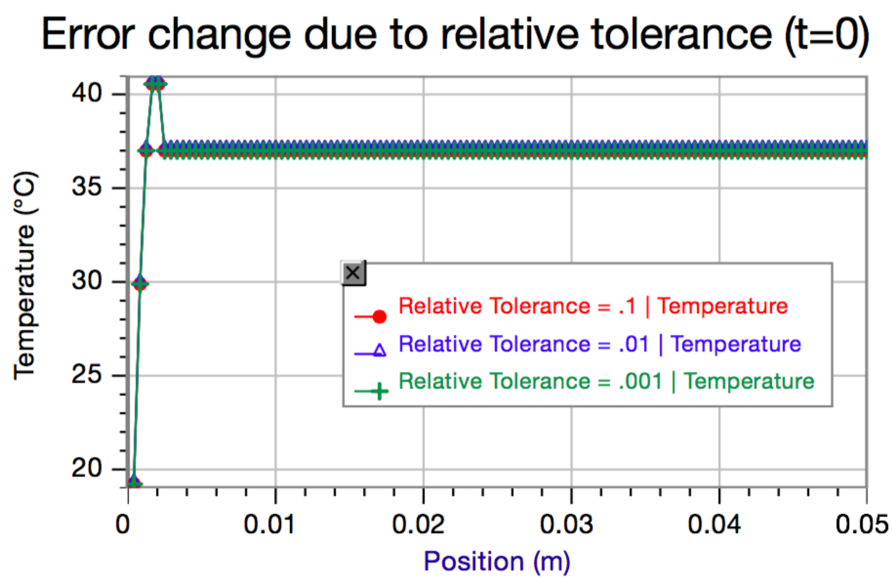


Does changing the order of the element reduce the error? Show the temperature profile for 1st, 2nd, and 3rd order elements.
Yes. Increasing order reduces error.



Does reducing the relative or absolute tolerance reduce the error? Show the temperature plot for different values of both.
No; neither makes a difference.





Did any of these 3 methods reduce the overall error? Which reduced error the most and which reduced it least?

Yes. Increasing the order reduced error the most and changes to tolerance reduced it least.