선형대수 3강 LU 분할

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ -2 & 7 & 2 \end{bmatrix} \quad E_{21}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ 0 & 8 & 3 \end{bmatrix} \quad E_{31}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 8 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{32}$$

$$E_{32}E_{31}E_{21}A \rightarrow U$$

- Elementary Matrix in G.E.

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -l_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ② - ① \times l_{21} \Rightarrow E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Uparrow$$

$$E_{21}A^{②} \rightarrow ②'$$

$E_{21}^{-1}A' \rightarrow A^{②}$ 가우스 소거법의 역과정 : 부호만 바뀜

$$A = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U$$

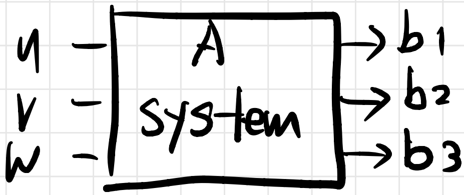
Lower triangular Mat.

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_{31} & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & l_{32} & 1 \end{bmatrix}U = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}U$$

$$= LU$$

1.5 Triangular Factors

$A = LU$: LU factorization (decomposition)



$$Ax = b$$

$$\underline{L^{-1}A}x = L^{-1}b = c$$

$$Ux = c$$

$$A = LU \qquad Lc = b$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

- $\det(A) = U$ $\underset{\text{pivot}}{\underline{\text{대각선 값}}}$들의 곱

대각행렬

$$U = \begin{bmatrix} d_1 & u_{12} & u_{13} & \cdots \\ & d_2 & & \\ 0 & & \ddots & \\ & & & d_n \end{bmatrix} = \begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ 0 & & \ddots \; d_n \end{bmatrix} \begin{bmatrix} 1 & \frac{u_{12}}{d_1} & \frac{u_{13}}{d_1} & \cdots \\ & 1 & \frac{u_{23}}{d_2} & \\ & & 1 & \\ & & & \ddots \end{bmatrix} = DU$$

$A = LU \Rightarrow LDU$

$$D^n = \begin{bmatrix} d_1^n & & 0 \\ & d_2^n & \\ 0 & & \ddots \; d_n^n \end{bmatrix}$$

- LU factorization is unique!

- Row Exchange (Pivoting)

$\Rightarrow$ Permutation P

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -6 & 0 \\ 2 & 1 & 1 \\ -2 & 7 & 2 \end{bmatrix}$$

P_{21}

- Permutation Matrix has the same rows with I

$\rightarrow$ There is a single "1" in every row and column

$$\overleftarrow{P_{32} P_{21}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\neq P_{21} P_{32}$$

$PA = LU$

$P_{21} \rightarrow P_{21}^{-1}$

$P^{-1} = P^T$

$A = P^T L U$