선형대수 7강 벡터의 선형독립과 기저벡터

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Echeln form

$Ux = 0$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row Reduced form

$Rx = 0$

(pivot 위.아래 모두 0)

$$\Rightarrow \underset{\text{free val.}}{v}\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + z\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

special sol.

- $Ax = b \, (\neq 0)$

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \overset{G.E.}{\Rightarrow} \left[\begin{array}{cccc|c} 1 & 3 & 3 & 2 & b_1 \\ 0 & 0 & 3 & 3 & b_2-2b_1 \\ 0 & 0 & 0 & 0 & b_3-2b_2+5b_1 \end{array}\right]$$

$b_3-2b_2+5b_1=0$ 일 경우만 Solution 존재.

$b \in C(A)$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & -2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} u = -3v+z-2 \\ w = -z+1 \end{cases}$$

Pivot val에 소거법을 적용

$$\Rightarrow \begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \underbrace{v\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + z\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}}_{x_n} + \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

원점을 기준으로 x_p 만큼 평행이동

$x = x_n + x_p$ (상수벡터, particular sol.)

- $Ax = A(x_n + x_p)$

$$= \underbrace{Ax_n}_{\text{null space} \atop 0} + Ax_p = b$$

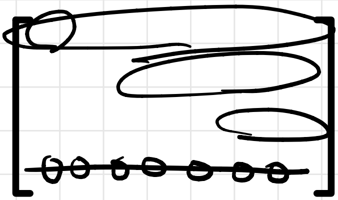
2.3. Linear Independence

$$c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0$$

* only $c_1 = c_2 = \cdots = c_n = 0$

ex)
$$c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- If G.E. of A generates m non-zero rows,



} m independent column vecters in A

- Rank of A = # of independent column vecters
  = # of independent row vecters
  = # of pivots in G.E. (counting pivot)
  = Dim of C(A)

ex)
$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow x.y \text{ 평면}$$
(2차원)

- Spanning : all linear combinations of vectors
  $\{v_1, v_2, \cdots, v_n\}$ constant a vector space
  $= \{v_1, v_2, \cdots, v_n\}$ span vector space

ex) span 조합 다양

$c_1 = c_2 = 2$ $c_1 = 0 \,.\, c_2 = 2$

$$2\text{차원} \Rightarrow \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

- Basis (vectors)

: # of minimum linearly independent vectors to span the vector span

: linear combination is unique from basis.

- Basis is not unique for a vector space

ex) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{cases} \text{두 case 모두 basis} \\ c_1 . c_2 \rightarrow \text{unique} \end{cases}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$