선형대수 6강 영벡터 공간과 해집합

$Ax = b$; $A_{n \times n}$

$x = A^{-1}b$

if A^{-1} exists, always $b \in C(A)$ ← column space, whole space

whole space is constructed by linear combinations → span

· **Null Space of A ($N(A)$)**

: Set of vectors such that $Ax = 0$

$$N(A) = \{ x \mid Ax = 0 \}$$

1) Closed under addition for $Ax_1 = 0$, $Ax_2 = 0$

$x_1 + x_2 \in N(A)$, $A(x_1 + x_2) = 0$?

$Ax_1 + Ax_2 = 0$

2) Closed under scalar mul. for $Ax = 0$ for any c.

$cx \in N(A)$, $A(cx) = 0$?

$cAx = 0$

3) 원점 포함

2.2. Solving $Ax = 0$ & $Ax = b$

$$A_{m \times n} \, x = b \qquad (m < n)$$

· **Echelon form U**

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(3x4; the equation is crossed out) 항상 0이므로

$C(A) \subset R^m$

$N(A) \subset R^n$

$\Rightarrow \begin{bmatrix} \textcircled{1} & 3 & 3 & 2 \\ 0 & 0 & \textcircled{3} & 3 \\ 0 & 0 & 6 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ all pivot 1 $\Longrightarrow \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ · Row Reduced from R

$Ax=0 \rightarrow Ux=0 \rightarrow Rx=0$ (정형화된 null space)

$\begin{cases} u+3v-z=0 \\ w+z=0 \end{cases} \rightarrow$ pivot variables : u. w.
free variables : v. z.

$\downarrow$

$\begin{cases} u=-3v+z \\ w=-z \end{cases} \rightarrow \begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \begin{bmatrix} -3v+z \\ v \\ -z \\ z \end{bmatrix} = v\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + z\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ ← N(A) special solution

(평면 위 점만 Null space)

$Ax=0$

$N(A) = \left\{ \begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} \middle| c_1\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$ ─ Dim(N(A)) = 2

· Dimension of vector Space

$\begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix}$ in R^4

Dim(N(A)) = # of independent special solutions vectors