선형대수 4강 역행렬과 전치행렬

1.6. Inverse & Transpose

• **Inverse** 역행렬

$$A_{n\times n}A^{-1} = A^{-1}A = I$$

→ not all A has its inverse → $\det(A) \neq 0$

1) The inverse (A^{-1}) exists
iff Gauss elimination produces n pivots.

2) The inverse is unique!

3) If A^{-1} exists, $Ax = b$, $\cancel{A^{-1}A}\,x = A^{-1}b$.
$x \rightarrow$ unique!

4) Assume that there is a non-zero vector x such that
$Ax = 0$ $(b=0)$
- $x=0 \longrightarrow$ trivial solution
- $A^{-1}0 = x = 0$

⇒ then A^{-1} does not exist.

5) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

det $\neq 0$

6) Diagonal Matrix

$\begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ & & \ddots \\ 0 & & d_n \end{bmatrix}$ $\prod_{i=1} d_i \neq 0$ (∏는 모두 곱) $\xrightarrow{\text{inverse}}$ $\begin{bmatrix} \frac{1}{d_1} & & 0 \\ & \frac{1}{d_2} & \\ & & \ddots \\ 0 & & \frac{1}{d_n} \end{bmatrix}$

7) The inverse comes in the reverse order.

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

- A^{-1} : Gauss-Jordan Method

$$Ax = b$$

$$AB = A[b_1\ b_2 \cdots b_n] = [Ab_1\ Ab_2 \cdots Ab_n]$$

$$AA^{-1} = I, \quad A^{-1} = [x_1\ x_2 \cdots x_n]$$

$$[Ax_1\ Ax_2 \cdots Ax_n] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & & 1 \end{bmatrix} \Rightarrow Ax_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ Ax_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \cdots$$

- $A \xrightarrow{GE} U$

$$\underbrace{E_{32}E_{31}E_{21}}_{L^{-1}} A \longrightarrow U \quad \Rightarrow \quad \begin{aligned} AA^{-1} &= I \\ \cancel{U^{-1}U} A^{-1} &= U^{-1}L^{-1} \end{aligned}$$

ex) $A \ A^{-1} = I$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 4 & -6 & 0 & 0 & 1 & 0 \\ -2 & 7 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -8 & -2 & -2 & 1 & 0 \\ 0 & 8 & 3 & 1 & 0 & 1 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -8 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\longrightarrow$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 2 & -1 & -1 \\ 0 & -8 & 0 & -4 & 3 & 2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \uparrow$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{3}{2} & -\frac{5}{8} & -\frac{3}{4} \\ 0 & -8 & 0 & -4 & 3 & 2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{4} & -\frac{5}{16} & -\frac{3}{8} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{3}{8} & -\frac{1}{4} \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \longrightarrow IA^{-1}$$

- **Transpose 전치행렬**

$$a_{ij} \xrightarrow{A^T} a_{ji}$$

$$\begin{cases} (A+B)^T = A^T + B^T \\ (A+B)^{-1} \neq A^{-1} + B^{-1} \end{cases} \qquad \begin{cases} (AB)^T = B^T A^T \\ (AB)^{-1} = B^{-1} A^{-1} \end{cases} \qquad \cdot\ (A^{-1})^T = (A^T)^{-1}$$

- **Symmetric Matrix 대칭행렬**

$A^T = A \quad a_{ij} = a_{ji}$

- if A is symmetric and invertible, then A^{-1} is too. $\quad (A^{-1})^T = A^{-1}$

- $A = LU = LDU$ (U pivot all 1)

$A^T = A$ 대칭 행렬

$\Rightarrow$ **$LDU = U^T D L^T$**

- Correlation Matrix

$R = A^T A$ $= \begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \\ \text{—} \end{bmatrix} \begin{bmatrix} | & | & | & | \end{bmatrix} = \begin{bmatrix} x_1^T x_1 & x_1^T x_2 & \cdots & x_1^T x_n \\ & \ddots & & \\ & & & x_n^T x_n \end{bmatrix}$

$R^T = R$