선형대수 2강 1차 연립 방정식과 가우스 소거법

- Singular Case
  - → No solution
  - → infinite solutions

(1) row form
- → parallel
- → overlap

(2) Column form
- → Column parallel
- → 

3차원 관점에서 원점과 평면과 a 표현

3. Gauss Elimination

↳pivot (non-zero)

$$\begin{cases} 2u+v+w=5 \\ 4u-6v=-2 \\ -2u+7v+2w=9 \end{cases} \Rightarrow \begin{cases} 2u+v+w=5 \\ -8v-2w=-12 & ② - ①\times 2 \\ 8v+3w=14 & ③ + ① \end{cases}$$

✓ all pivots are non-zero.

$$\Rightarrow \begin{cases} 2u+v+w=5 \\ -8v-2w=-12 \\ w=2 \end{cases} \begin{matrix} \text{대입} \\ ③+② \end{matrix}$$

⇒ G.E has unique sol.

$$\begin{bmatrix} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 8 & 3 & 14 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

upper triangular

- Breakdown : when a zero appears in a pivot position.
  G.E has to stop.
  the order of eqns has to be changed → pivoting

ex 1)

$$\begin{matrix} u+v+w=a \\ 2u+2v+5w=b \\ 4u+6v+8w=c \end{matrix} \Rightarrow \begin{matrix} u+v+w=a \\ 3w=b-2a \\ 2v+4w=c-4a \end{matrix} \xrightarrow{\text{pivoting}} \begin{matrix} u+v+w=a \\ 2v+4w=c-4a \\ 3w=b-2a \end{matrix}$$

ex 2)

$$\begin{matrix} u+v+w=a \\ 2u+2v+5w=b \\ 4u+4v+8w=c \end{matrix} \xrightarrow{G.E} \begin{matrix} u+v+w=a \\ 3w=b-2a \\ 4w=c-4a \end{matrix}$$

$w = \frac{b-2a}{3} = \frac{c-4a}{4}$ → 해가 무수히 많다 *

$\neq$ → 해가 없다

1.4. Matrix multiplication

$$\begin{cases} 2u+v+w=5 \\ 4u-6v=-2 \\ -2u+7v+2w=9 \end{cases}$$

linear combination

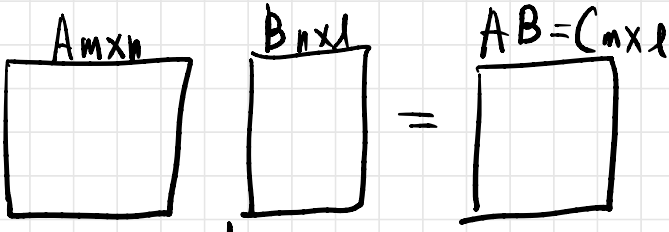
$$u\begin{bmatrix}2\\4\\-2\end{bmatrix}+v\begin{bmatrix}1\\-6\\7\end{bmatrix}+w\begin{bmatrix}1\\0\\2\end{bmatrix}=\begin{bmatrix}5\\2\\9\end{bmatrix}$$

① 미지수 = 방정식

② 미지수 < 방정식

③ 미지수 > 방정식

해가 없다



$$(AB)_{ij}=\sum_{k=1}^{n} a_{ik}b_{kj}$$

$$AB \neq BA$$

$$AB=A\begin{bmatrix}b_1 & b_2 & \cdots & b_l\end{bmatrix}=\begin{bmatrix}Ab_1 & Ab_2 & \cdots & Ab_l\end{bmatrix}$$