선형 대수 1강 선형성 정의 및 1차 연립방정식의 의미

· 선형성 만족 조건

1) Superposition : $f(x_1+x_2)=f(x_1)+f(x_2)$

2) Homogeniety : $f(ax)=af(x)$

∨ $y=mx+n$ 원점을 지나지 않는 직선은 선형성 X

· 행렬 Matrix

$$\boxed{A}\,\boxed{x}=\boxed{y} \qquad A(a_1X_1+a_2X_2)=a_1AX_1+a_2AX_2$$

· Basic Notations of Matrix

Vector : ~~$v=(a,b,c)$~~ → $v=\begin{bmatrix}a\\b\\c\end{bmatrix}$ → column vector

Transpose (전치) : row ⟷ column

$$\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}^T \rightarrow \begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}$$

Linear Combination (선형결합) :

$$v=\begin{bmatrix}a_1\\b_1\\c_1\end{bmatrix},\quad w=\begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix} \qquad \alpha v+\beta w=\alpha\begin{bmatrix}a_1\\b_1\\c_1\end{bmatrix}+\beta\begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix}$$

$$[v\ w]=\begin{bmatrix}a_1&a_2\\b_1&b_2\\c_1&c_2\end{bmatrix} \qquad =\begin{bmatrix}\alpha a_1+\beta a_2\\ \alpha b_1+\beta b_2\\ \alpha c_1+\beta c_2\end{bmatrix}$$

$$\begin{bmatrix}a_1&a_2\\b_1&b_2\\c_1&c_2\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=$$

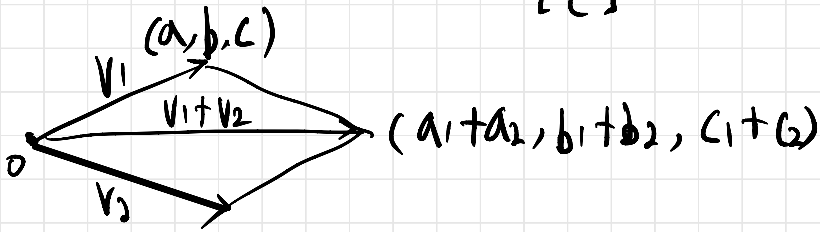
∨ 행렬과 벡터의 곱으로 linear combination 표현 가능

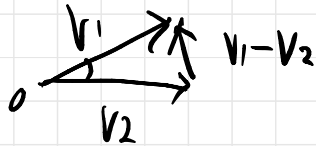
· Matrix

$$AB\neq BA \qquad AI=IA=A \qquad I=\begin{bmatrix}1&0\\0&\ddots\end{bmatrix} \qquad AA^{-1}=A^{-1}A=I$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

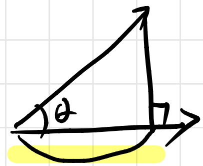
- vector $\quad v = (a, b, c)^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$





- inner product 내적

$$V_1 \cdot V_2 = |V_1||V_2|\cos\theta = (a_1a_2 + b_1b_2 + c_1c_2)$$



- 함수의 내적

$$(f_1(t), f_2(t)) = \sum_{k=-\infty}^{\infty} f_1(t_k)f_2(t_k) \longrightarrow \int f_1(t)f_2(t)\,dt$$

Hilbert space

Chap 1. Gauss Elimination 가우스 소거법

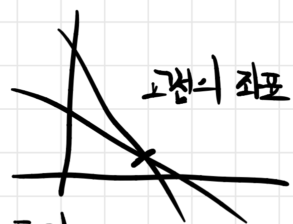
선형 연립 방정식 풀이

$$\begin{cases} x + 2y = 3 \\ 4x + 5y = 6 \end{cases}$$

intersection

$\xrightarrow{①} \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$



linear combination

$\xrightarrow{②} x\begin{bmatrix} 1 \\ 4 \end{bmatrix} + y\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$