1st class

Proposition :- (statement).

proposition :- A proposition or statement is a declarative or ~~statement~~ sentence which is either true or false but not both.

$\underbrace{2+3=7}_{\text{false} \rightarrow \text{prop.}}$ $\underbrace{2+3=5}_{\text{True} \rightarrow \text{prop.}}$

$x+3=7.$

T if $x=4$ F if $x \neq 4$ } not a prop.

$\underbrace{x+3=7;\ x=8}_{\text{false} \longrightarrow \text{prop}}$

LPU is in Chandigarh.
false $\longrightarrow$ prop.

★ True or false is called Truth value.
(T & F = Truth value)

★ Proposition is denoted by small letter.

p : $2+3=7$

p : $2+3=5$

q : $x+3=7;\ x=2$

propositional variable

★ Declaring the fact $\longrightarrow$ prop.

Ex - what is your name?
$\rightarrow$ Introgative / Not Declarative
$\Downarrow$
Not a prop.

? Not Declarative then it is not a proposition

Compound Proposition :- A prop. i.e obtain from the Combination of two or more atomic (primary) proposition by mean of some words and pharase is Called a compound proposition.

* word of Pharase
* Logical operator / connective

combine {combination}

⇒ The word and pharases are used to perform a Compound proposition is Called logical operator.

there are five (5) Basic logical operator.

(1) Conjunction :- Any two prop P, q Combine with the word "And" to form a Compound proposition is Called conjunction of P, q.

Ex :- P and q ( )

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

if Both P, q are true then $P \wedge q$ is true otherwise false.

Ex :- p : It is cold

q : It is raining

$p \wedge q$ = p and q (It is cold and raining)

(2) **Disjunction** :- p, q ("or") :- Any two prop. combined with the word "or" to perform a compound prop.

Ex :- p, q , p or q

$p \vee q$, p or q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$p \vee q$ is true if atleast one of p, q is true.

Ex :-

Ex :- p : He will go to Delhi.

q : He will go to Chandigarh

$p \vee q$: He will go ~~to~~ Delhi or chandigarh.

Conjunction ⟶ And

Disjunction ⟶ or

③ Negation :- The negation of prop. p is the statement " It is not the case of p". "Not p"

Negation ⇒ Deny , $\sim p$, $\neg p$

p	$\sim p$
T	F
F	T

④ Conditional statement :- (operator) if the compound statement of the form " p then q"

written as $p \to q$

as $p \Rightarrow q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \to q$ is false if p is True & q is false otherwise it is True.

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p : Hypothesis

q : Conclusion

(5) <u>Biconditional</u> :- Compound prop. of the form "p if and only if q".

"p iff q" $p \leftrightarrow q$

$\boxed{p \Leftrightarrow q}$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$p \leftrightarrow q$

$p \to q$, & $q \to p$

$(p \to q) \wedge (q \to p)$

*

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

$p \oplus q$

<u>XOR operator</u>

operator Name	Connectory word	Symbol	Symbolic form
conjunction	And	$\wedge$	$p \wedge q$
Disjunction	or	$\vee$	$p \vee q$
Negation	Not	$\sim, \neg$	$\sim p$
Conditional	if–then–	$\rightarrow$	$p \rightarrow q$
Biconditional	if and only if	$\leftrightarrow$	$p \leftrightarrow q$
	XOR	$\oplus$	$p \oplus q$

set	prop logic	Boolen
union OR $A \cup B$	Disjunction $\vee$ $p \vee q$	OR $+$ $x+y$
Intersection And $A \cap B$	Conjunction $\wedge$ $p \wedge q$	And $\cdot$ $x \cdot y$
Compliment $\bar{A}$, A' not A	Negation $\sim p$, $\neg p$	$\bar{x}$
	XOR $\oplus$	$x \oplus y$

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the other conditional

$p \to q$

1. if p then q
2. if p, q
3. p implies q
4. p only if q
5. p is sufficient for q
6. q if p
7. q whenever p
8. q follows from p
9. q is Necessary for p
10. q unless $\sim p$

Ist clear Type

2nd clear hypothesis

3rd conclusion

solving Approach

★ Converse, Contrapositive & Inverse of a conditional statement

Let p and q ($p \to q$) be the given conditional statement then

① ~~p→q~~ (if q then p) is $q \to p$ called converse of $p \to q$

{Back geear}

② $\sim q \to \sim p$ (If $\sim q$ then $\sim p$) .... Contrapositive of $p \to$

③ $\sim p \to \sim q$ (If $\sim p$ then $\sim q$) .... Inverse of $p \to q$

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Q. Find converse, contrapositive and inverse of the conditional statement
" If it rains, the crops will grow".

Soln :-
Statement :- " If it rains, the crops will grow"
which is the form of if p, q.

$\therefore$ p : It rains

q : the crops will grow

(i). Converse :- If the crops will grow then It rains.

(ii). Contrapositive :- ~~contrapo~~ $\sim q \rightarrow \sim p$

If $\sim p \rightarrow \sim q$

then i.e If $\sim q$ then $\sim p$

$\Rightarrow$ If the crops will not grow then It doesn't rain.

(iii) Inverse :- $\sim p \rightarrow \sim q$

If It do not rain then crops will not grow.

Q. Find converse, contrapositive and inverse of the statement " I go to the beach whenever It is sunny summer day".

Soln :- Given that

Statement :- " I go to the beach whenever It is sunny summer day".

which follow $\rightarrow$ q whenever p.

$\therefore$ p : It is a sunny summer day.

q : I go to the beach.

Now

$\sim p$: It is not a sunny summer day

$\sim q$: I do not go to the beach.

(i). <u>Converse :-</u> It is a sunny summer day whenever I go to the beach.

Logical Equivalence or equal :- Two Compound prop.

P and q P(p, q, r ......) and Q(p, q, r ......)
where p, q, r are propositional variable
are called logical equivelence or equal.
if they have same truth value for every
Possible.

written as $P \approx Q$, $P = Q$

★ **Note** :- Now of Row in Truth table $= 2^n$

$$\boxed{\text{No. of Row} = 2^n}$$

n = no. of propositional variable.

Tautology :- A compound statement P is said to be if it is Always true for every possible value of p, q, r ....

Contradiction :- A compound statement P is said to be Contradition if it is Always false for every possible value of p, q, r......

Containjency :- If it is Always some True and some false.

Q. prove that logically :-

$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r).$$

Q. Prove that logically

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Sol^n :-

No. of prop variable = 3 (p, q, r)

$\therefore$ No. of Row in T.T $= 2^n$

$= 2^3$

$= 8$

Now $\frac{N}{2} = \frac{8}{2} = 4$ for p | for q $\frac{4}{2} = 2$ | for r $\frac{2}{2} = 1$

Let P: $p \wedge (q \vee r)$

q: $(p \wedge q) \vee (p \wedge r)$

I	II	III	IV	V	VI	VII	VIII
p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

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∵ Here the column V^{th} & $VIII^{th}$ of T.T are identical

$\boxed{\therefore P \approxeq Q}$ or $\boxed{P = Q}$

proved

****

<u>Note:-</u> Two compound prop. P, q are logically equal if their ~~Bgco~~ Biconditional statement i.e $P \leftrightarrow Q$ is a Tautology.

Q. Discuss the equality and inequality

$(p \rightarrow \sim q) \vee (q \rightarrow \sim r)$

and $(p \rightarrow \sim q) \wedge (p \leftrightarrow r)$

wether they are equal or not.

<u>Sol^n</u> :-

Given that

No. of prop. variable = 3(P, q, r)

∴ No. of Row in T.T $= 2^n$

$= 2^3$

$= 8$

for P	for q	for r
$\frac{N}{2} = \frac{8}{2} = 4$	$\frac{4}{2} = 2$	$\frac{2}{2} = 1$

Law of propositional logics :-

① Identify law :-

$p \land T = p$ T : Tautology

$p \lor F = p$ F : Contradiction

② Domination law :-

$p \lor T = T$

$p \land F = F$

③ Idempotent law :-

$p \lor p = p$

$p \land p = p$

④ Double Negation law :-

$\sim(\sim p) = p$

⑤ Commutative law :-

$p \lor q = q \lor p$

$p \land q = q \land p$

⑥ Associative law :-

$p \lor (q \lor r) = (p \lor q) \lor r)$

$p \land (q \land r) = (p \land q) \land r$

⑦ Distributive law :-

$p \lor (q \land r) = (p \lor q) \land (p \lor r)$

$p \land (q \lor r) = (p \land q) \lor (p \land r)$

(8) De-morgens law :-

$$\sim(p \vee q) = \sim p \wedge \sim q$$

$$\sim(p \wedge q) = \sim p \vee \sim q$$

(9) Absorption law :-

$$p \vee (p \wedge q) = p$$

$$p \wedge (p \vee q)\ p$$

(10) Negation law :-

$$p \vee \sim p = T$$

$$p \wedge \sim p = F$$

Logical equivalence involving condition statement:-

① $p \rightarrow q \equiv \sim p \vee q$

② $p \rightarrow q \equiv \sim q \rightarrow \sim p$

③ $p \vee q \equiv \sim p \rightarrow q$

④ $p \wedge q \equiv \sim (p \rightarrow \sim q)$

⑤ $\sim (p \rightarrow q) \equiv p \wedge \sim q$

⑥ $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

⑦ $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

⑧ $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

⑨ $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

★ Hypothesis = same → sign no change

★ Conclusion = same → sign change

Logical equivalence with Bi-conditional statement

① $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

② $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

③ $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

④ $\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

predicate logic :-

Actually it is the extension of ~~predicate~~ logic.

propositional logic

" $x > 5$ "

" x is greater than 5 "

Variable called Subject of the statement

Domain

Condition 'x'

predicate

p: greater than 5.

$p(x)$: x is greater than 5 ; $x \in$ Domain

↳ function of x

$x = a$: $p(a)$ is a proposition.

★ **predicate logic :-** predicate logic is an extension of propositional ~~variable~~ logic.

→ It add the concept of predicate and quantifiers to better capture, the meaning that can not be expressed by prop. logic.

predicate refers to a properties that a subject of the statement can have.

In above example

Ex- $p(x)$: x is greater than 5

'p': Denote the predicate
" greater than 5" of variable x.
or subject x.

The predicate can be considered as a subject x and it tells us the truth value of the statement $p(x)$ at x.

once a value is assign to variable x;
$p(x)$ becomes a proposition and has truth value.

Ex - $p(x)$: '$x+3=7$'

Now $P(3) = \underbrace{3+3=7}$ which is false
& Become a prop.
which truth value = F

$P(5)$: $5+3=7$ —— F

$P(4)$: $4+3=7$ —— T

Ex :- $Q(x,y) : x+y>2$; $x, y \in I$

Let $Q(1,2) : 1+2>2$
$= 3>2$ —— T

Again, $Q(0,1) : 0+1>2$
$1>2$ —— F

Quantifiers :-

Some student of section DOC13 are non-series and intelligent.

Quantifiers

⇒ Quantifiers :- Quantifiers are the words that refers to quantity.

i.e Some, few, all, every and tells us Howmany elements are therefore that predicate i.e Express the extend to which a pridicate is true over the domain of subject x.

→ Using a Quantifiers to create a such type of prop. is called Quantification.

All
Every

Ex- Some Student of DOC13 are Non-Serie and intelligent.

★ Note :- $D \in$ section, student $\in$ subject

Types of Quantifiers :-

① **Universal Quantifiers** :- A universal Quantification of $p(x)$ is the statement "$p(x)\ \forall x$ in the domain".

or "$p(x)$ is True $\forall$ .... "

and denoted by $\forall x \in D;\ p(x)$

$\therefore \forall$ is called universal Quantifiers.

Note :- An element for which $p(x)$ is false is called Counter example of $\forall x : p(x)$

Q. Find the truth value of the universal quantification $\forall x : p(x)$

where $p(x)$ be the statement $x > 5$ and the domain consist of integer.

Soln :- $p(x) : x > 5$

domain : set of integer
$x \in I$

If let $x = 4 \in I$

$p(4) = 4 > 5$
which is false

$\therefore$ the universal quantification .... $\forall x : p(x)$ is false.

Counter example

$P(x): x+1 > x : x \in R$

for every real no. x

$\therefore x+1 > x$ Always True.

$\therefore \forall x : P(x)$ is True.

② **Existential Quantifiers** :- the existential quantification of $P(x)$

" there exist an element x such that $P(x)$

$\exists x : P(x)$ where $\exists$ is called Existential quantier.

{ $\exists$ mean some, few, atleast one }

Q. What is the truth value of quantification where $\exists x : P(x)$ where $P(x)$ is $x > 5$ where Domain $\in I$

Soln :- $P(x) : x > 5 ; x \in I$

take $x = 6$, $P(x) : x > 5$ where it is True.

$\therefore \exists x : P(x)$ is True

When the all the elements in the domain are listed $x_1, x_2, x_3, \ldots\ldots x_n$

(i) the universal quantifiers = conjunction of $P(x_1), P(x_2) \ldots P(x_n)$

$\forall x : P(x) \approxeq P(x_1) \wedge P(x_2) \wedge P(x_3) \ldots\ldots \wedge P(x_n)$

(ii). $\exists x : P(x) \approx P(x_1) \vee P(x_2) \vee P(x_3) \ldots \ldots \vee P(x_n)$

↳ Disjunction of $P(x_1) \ldots P(x_n)$

Example :- What is the truth value

(i) $\forall x : P(x)$ (ii) $\exists x : P(x)$; $P(x) : x^2 < 10$

where Domain contain not exceeding 5

i.e 1, 2, 3, 4

Soln :- $P(x)$ $x^2 < 10$

D = +ve integer, Not exceeding 5

i.e $D = \{1, 2, 3, 4, 5\}$

x	$P(x)$	
1	$P(1)$ $1^2 < 10$	T
2	$P(2)$ $2^2 < 10$	T
3	$P(3)$ $3^2 < 10$ $9 < 10$	T
4	$P(4)$ $4^2 < 10$ $16 < 10$	F
5	$P(5)$ $5^2 < 10$ $25 < 10$	F

(i) Truth value of

$\forall x : P(x) =$

$T \wedge T \wedge T \wedge F \wedge F = F$

(ii) Truth value of

$\exists x : P(x) =$

$T \vee T \vee T \vee F \vee F = T$

③ Negation of Quantification :— (De-Morgan's Law)

(i) ★ $\sim(\forall x : p(x)) \approx \sim\forall x : \sim p(x)$

The Negation of universal quantification become a existential quantification of Negation of predicate.

(ii) ★ $\sim(\exists x : p(x)) \approx \forall x : \sim P(x)$

The Negation of exential quantification become a universal quantification of Negation of predicate.

Statement :- "All students of DOC13 are intelligent"

⇒ there exit some student of DOC13 are not intelligent.

① $\sim(\forall x : p(x) \vee Q(x)) \approx \exists x : \sim(p(x) \vee Q(x))$

$\approx \exists x : \sim p(x) \wedge \sim Q(x)$

② $\sim(\forall x : P(x) \wedge Q(x)) \approx$

③ $\sim(\exists x : P(x) \vee Q(x)) \approx$

④ $\sim(\exists x : p(x) \wedge Q(x)) \approx$

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④ **Nested Quantifiers :-** if we use a Quantifiers that appears within a scope of another quantifier called nested Quantifiers or Multiple Quantifiers.

$\forall x, \forall y : p(x,y)$

$\forall x, \exists y : p(x,y)$

$\exists x, \forall y : p(x,y)$

$\exists x, \exists y : p(x,y)$

★ Note :- $\exists x, \forall y : p(x,y) \neq \forall x, \exists y : p(x,y)$

Ex:- Statement :- " All Rabits are faster than all tortoise".

D_1 = set of Rabits

D_2 = set of Tortoise

predicate p : faster than

$p(x,y)$: Rabit x is faster than tortoise y.   predicate

$x \in D_1$, $y \in D_2$

$\forall x, \forall y : p(x,y)$

$\forall x \in D_1, \forall y \in D_2 : p(x,y)$

#

Statement :- " There are some Rabits who is faster than all tortoise".

$\exists x \in D_1, \forall y \in D_2 : p(x,y)$

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Logical Equivalence :- The statement involving predicate and quantifiers are logically equivalence if they have same truth value no matter which predicates are used (Sub into the statement and which domain is used.

Argument :- By an argument means a sequence of statement called premises that ends with it statement called conclusion.

$p_1, p_2, p_3 \ldots\ldots p_n$ are the premises.

and q be the conclusion of Argument.

$$\boxed{p_1, p_2, p_3 \ldots\ldots p_n \vdash q}$$

gt Valid Argument means that the conclusion of an argument must follows from the truth of the premises of the Argument.

If all the premises are true then the conclusion is also True.

Note :- the Argument $p_1, p_2, p_3 \ldots\ldots p_n \vdash q$ to q is valid if the conditional statement

$p_1 \wedge p_2 \wedge p_3 \ldots\ldots \wedge p_n \longrightarrow q$ is a Tautology.

Note :- if $p_1 \wedge p_2 \wedge p_3 \ldots\ldots p_n$ is ~~conditional~~ contradiction then argument is valid.

Q. Check the validity of the Argument
"if it rains, I drive to university",
"if it rains" so "I drive to university".

Soln :- "If it rains, I drive to university,
"If rains" so "I derive to university".

Let p: It rains

q : I drive to university

$(s_1 \wedge s_2) \Rightarrow s_3$

$\therefore$ The premises are $s_1 : p \rightarrow q$

$2(p, q)$

$s_2 : p$

$2^n = 2^2 = 4$

p	q
$\frac{4}{2} = 2$	$\frac{2}{2} = 1$

Conclusion $s_3 : q$

p	q	s_1 $p \rightarrow q$	s_2 (p)	$s_1 \wedge s_2$ $(p \rightarrow q) \wedge p$	$s_3(q)$	$(s_1 \wedge s_2) \rightarrow s_3$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	F	F	F	~~F~~ T

$\because (s_1 \wedge s_2) \rightarrow s_3$ is a Tautology.

$\therefore$ the Argument is Valid

8. Statement :- "If sherya work hard then she will sucessful "if she is successful she will be happy therefore the hard working leads to happyness.

10marks

CA.MT ET

Sol^n

$S_1 : p \to q$

$S_2 : q \to r$

$S_3 : p \to r$

$3(p,q,r)$
$= 2^n$
$= 2^3 = 8$
$\frac{8}{2} = 4|2|1$

$(S_1 \wedge S_2) \to S_3$ is a Tautology

I	II	III	IV	V	VI	VII	VIII
p	q	r	S_1 $p \to q$	S_2 $q \to r$	$S_1 \wedge S_2$	S_3 $p \to r$	$(S_1 \wedge S_2) \to S_3$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore (s_1 \wedge s_2) \to s_3$ is a Tautology

$\therefore$ Argument is Valid.

Q. Discuss the Tautology of an argument "if I am sick their will be no lecture today". "Either I am not sick or the Student will be happy" the sudent are not happy therefore I am not sick".

3 prop. variable

Soln :-

p : I am sick.

q : there will be class today.

r : Student are happy.

Premises

$s_1 : p \to \sim q$

$s_2 : \sim p \vee r$

$s_3 : \sim r$

Coln $\Rightarrow s_4 : \sim p$

To prove

$(s_1 \wedge s_2 \wedge s_3) \to s_4$

is a Tautology

Check S6

p	q	r	s_1	s_2	s_3	$(s_1 \wedge s_2 \wedge s_3)$	s_4 =	$\sim p$	$\sim q$	$\sim r$
T	T	T	F	T	F	F		F	F	F
T	T	F	F	T	T	F		F	F	T
T	F	T	T	T	F	F		F	T	F
T	F	F	T	T	T	T		F	T	T
F	T	T	T	T	F	F		T	F	F
F	T	F	T	T	T	T		T	F	T
F	F	T	T	T	F	F		T	T	F
F	F	F	T	T	T	T		T	T	T

the Rule of ~~interference~~ inference :-

The rules of inference are some basic valid arguments which can be used as a building block to construct a complex argument.

① Modus ponens :-

Law of Detachment

Law of Affirming

premises : $p \rightarrow q$, p

conclusion : q

$$\boxed{\begin{array}{r} p \rightarrow q \\ p \\ \hline \therefore q \end{array}}$$

$[(p \rightarrow q) \land p] \rightarrow q$ is a Tatulogy

② Modus Tollens :-

$$\boxed{\begin{array}{r} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}} \qquad \boxed{\begin{array}{r} p \rightarrow \sim q \\ q \\ \hline \therefore p \end{array}}$$

③ Hypothetical syllogism :-

$$\boxed{\begin{array}{r} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}}$$

$[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a Tautology.

④ Disjunctive Syllogism :-

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array} \qquad \begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$

⑤ Addition :-

~~only one premises prim~~
only one premises.

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

$p \rightarrow (p \vee q)$ is a Tautology

⑥ Simplification :-

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array} \qquad \begin{array}{l} p \wedge q \\ \hline \therefore q \end{array}$$

$(p \wedge q) \rightarrow p$ is a Tautology

⑦ Conjunction :-

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

⑧ Resolusion :-

$$\begin{array}{l} p \vee r \\ \sim p \vee r \\ \hline \therefore q \vee r \end{array}$$

Q. By using inference of Rule Discuss the validity of argument — "if I am sick there will be no lecture today.

'Either there will be class or student are Happy'.

"The student are not happy therefore I am not sick".

p : I am sick

q : there will be lecture today

r : student are happy

the premises are

$S_1 : p \rightarrow \sim q$

$S_2 : p \vee r$

$S_3 : \sim r$

The conclusion is $S_4 : \sim p$

① By Disjunctive law

$$\begin{array}{l} S_2 : q \vee r \\ S_3 : \sim r \\ \hline \therefore q \end{array}$$

② By Modus tollens

$$\begin{array}{l} p \rightarrow \sim q \\ q \\ \hline \therefore \sim p \end{array}$$

$\therefore$ the Conclusion is $\sim p = S_4$

the Conclusion is I am not sick.

$\therefore$ the Argument is valid.

☆ Duality law :-

Introduction To proofs :-

* proof:- A proof is a valid argument that, stablish the truthness of Mathematical Statement of Called theorem.

theorem :- theorem is a Mathematical statement that we can proof.

Method Ist (1) Direct Method) :- the direct method of a mathematical condition $p \to q$ is contruct when the Ist steps is asumption (we assum) hypothesis p is true.

and next step is contructive by using rule of inference.

It is showing that conclusion statement q is also true. lastly.

Q. Using Method of direct proof show that "if n is odd integer then n^2 is odd"

Soln :- the given statement is —

"If n is odd integer then n^2 is odd"

$$\forall n \in I,\ p(n) \to q(n)$$

where $p(n)$: n is odd

$q(n)$: n^2 is odd

We Assumed that $p(n)$ is True

(Let $p(n)$ is True)

$\therefore$ n is odd Integer

$\therefore$ there existance Integer m s.t

We can write $\boxed{n = 2m+1}$

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Now $q(x) = x^2$

$$\Rightarrow (2m+1)^2$$
$$= (2m)^2 + 2(2m)(1) + (1)^2$$
$$= 4m^2 + 4m + 1$$
$$= 2(2m^2+2m)+1$$
$$= 2t+1 \quad (\text{where } (2m^2+2m) = t \text{ Put})$$

$\therefore$ $q(x)$ is odd Number

$\therefore$ $q(x)$ is also True.

Hence the Result is ~~proofed~~ proved by Direct Method.

(2) Contraposition Method :-

for the proof of the theorem
$$\forall x : p(x) \longrightarrow q(x)$$

We will show that the Contrapositive Statement $\sim q(x) \rightarrow \sim p(x)$ of the Conditional Statement $p(x) \longrightarrow q(x)$ is True

If $\sim q(x)$ is True then $\sim p(x)$ is also True.

8. prove that "If n is an integer and $(3n+2)$ is odd then n is odd".

Soln :- $\forall n \in I$, $P(n) \rightarrow q(n)$.

where $P(x)$: $3n+2$ is odd

$q(x)$: n is odd

Direct method

Let $3n+2$ = odd of form $2m+1$

$\Rightarrow 3n+2 = 2m+1$

$\Rightarrow 3n-2m = 1-2$

$\Rightarrow n$

$\notin I$ false.

Contrapositive Method

Soln :- $\forall n \in I$, $P(n) \longrightarrow q(n)$

where $P(n)$: $3n+2$ is odd

$q(n)$ = n is odd

$\sim P(n)$: $3n+2$ is not odd

$\sim q(n)$: n is not odd

Let $\sim q(x)$ is True

$\therefore$ n is not odd

$\therefore$ n is even

$\therefore$ there exist Integer m s.t

$$\boxed{n = 2m}$$

Now $3n+2 = 3(2m)+2$

$= 6m+2$

$= 2(3m+1)$

$= 2t$ (where we put $3m+1 = t \in I$)

= Even no.

$\therefore$ $3n+2$ is even No.

$3n+2$ is Not odd.

$\therefore$ $\sim p(n)$ is True.

If $\sim q(n)$ is True then $\sim p(n)$ is True

$\therefore$ By the method of contraposition proof

If $p(n)$ is true then $q(n)$ is also True.

$\therefore$ If $3n+2$ is odd then n is odd $\therefore$

proved

③ Contradiction Method :-

Ans→ To prove the statement p is true we can find contradiction q such that $\sim p \rightarrow q$ is True.

Because q is always false and $\sim p \rightarrow q$ is true, we can conclude that $\sim p$ is false then p is true.

Q. prove that $\sqrt{2}$ is Irrational by giving a proof of Contradiction

let p be the given proposition

p: $\sqrt{2}$ is Irrational

proof By contradition

$\boxed{\text{let } \sim p \text{ is True}}$

$\sqrt{2}$ is Not Irrational is True

So $\sqrt{2}$ is Rational

$\therefore$ there exist Integer p, q

$q \neq 0$ and $(p, q) = 1$ s.t

$$\sqrt{2} = \frac{p}{q}$$

Now squaring on the both sides we get,

$$\Rightarrow (\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$$

$$\Rightarrow 2 = \frac{p^2}{q^2}$$

$\Rightarrow 2q^2 = p^2$ —— (1)

$\Rightarrow p^2 = 2q^2 \quad \Rightarrow p = \sqrt{2q^2} \Rightarrow p = \sqrt{2}q \Rightarrow p = 2m$

$\Rightarrow p^2$ is even

$\Rightarrow p$ is also even

$\therefore$ there exist $m \in I$

s.t $\boxed{p = 2m}$ —— (2)

(1) $\Rightarrow (2m)^2 = 2q^2$ GCD

$\Rightarrow 4m^2 = 2q^2$

$\Rightarrow q^2 = 2m^2$

$\Rightarrow q^2$ is even

$\therefore \exists$ integer n

s.t $\boxed{q = 2n}$ —— (3)

from (2) and (3) both p, q are even

$\therefore (p, q) = 2$

which contradict the given hypothesis as $(p, q) = 1$

.So our Assuption is woorong / false

$\therefore \sim p$ leads to a contradiction / false

$\therefore p$ is True

$\therefore \sqrt{2}$ is Irrational <u>no proved</u>

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Mistakes In proof :-

$a = b$ (Let)

$\Rightarrow a \cdot a = a \cdot b$

$\Rightarrow a^2 = ab$

$\Rightarrow a^2 - b^2 = ab - b^2$

$\Rightarrow (a+b)\cancel{(a-b)} = b\cancel{(a-b)}$

$\Rightarrow a + b = b$

$\Rightarrow b + b = b$

$\Rightarrow 2\cancel{b} = \cancel{b}\,1$

$\Rightarrow \boxed{2 = 1}$

$i = \sqrt{-1}$

$i^2 = -1$

$LHS = i^2$

$= i \cdot i$

$= \sqrt{-1} \cdot \sqrt{-1}$

$= \sqrt{(-1)(-1)}$

$= \sqrt{1} = 1$

So $\boxed{1 = -1}$

AD. Unit of Set Theory :-

① Roster form :- Ex $A = \{1, 2, 3, 4, 5\}$
Tabular

② Set Builder :- Ex $A = \{x : x \in N ; x \leq 5\}$
Property
or $\{x : x \in N ; x < 6\}$

* Subset :- A is subset of B
A part of
$\boxed{A \subseteq B}$

* proper subset A is proper subset of B
$\boxed{A \subset B}$

$\boxed{\text{No. of Subset of a set} = 2^{n(A)}}$
n = No. of element in A

ϕ = Null / Empty / Void set.
$\phi = \{\ \}$

ϕ is subset of every set. $\phi \subseteq A$

A

$\boxed{A \subseteq A}$

$\boxed{\text{* non empty subset} = 2^n - 1}$

① **Union** $(A \cup B)$ A or B

$A \subseteq A \cup B$

$B \subseteq A \cup B$

② **Intersection** $(A \cap B)$ A & B

Common Element

③ **Compliment** ($\bar{A}$, not A , A')

$\bar{A} = X - A$

$A' = X - A$

④ **A−B** :- from A Remove the element of B. as

⑤ **B−A** :- from B Remove the element of A.

Cartesian Product of sets :-

A, B non-empty set

$$A \times B = \{(x, y) : x \in A, y \in B\}$$

$$B \times A = \{(b, a) : b \in B, a \in A\}$$

$$\boxed{A \times B \neq B \times A}$$

set of order pair (we can get from Cartesian product)

Ex : $A = \{1,2,3\}$; $B = \{a,b\}$

$A \times B = \{1,2,3\} \times \{a,b\}$

$= \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$

$B \times A = \{a,b\} \times \{1,2,3\}$

$= \{(a,1)(a,2)(a,3)(b,1)(b,2)(b,3)\}$

$\boxed{A \times B \neq B \times A}$

$\boxed{n(A \times B) = n(A) \cdot n(B)}$

$\boxed{\text{no. of elements in } A \times B = n(A) \times n(B)}$

$= 3 \times 2 = 6$

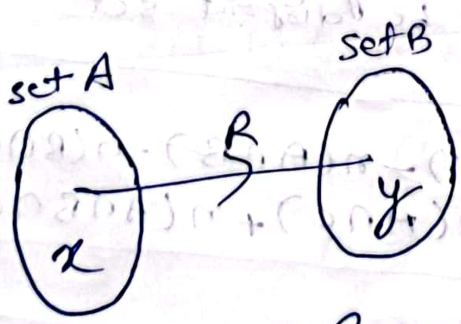
$2 \times 3 = 6$

No. of subset of $A \times B = 2^{n(A \times B)}$

$= 2^6$

$= 64$

Relation :-



R is a Relation from set A to set B

$R : A \longrightarrow B$

$(x,y) \in R$

$x R y$

x is related to y $\boxed{R \subseteq A \times B}$

Symmetric difference of two set :-

$\Rightarrow A \Delta B$

$$\boxed{A \Delta B = (A-B) \cup (B-A)}$$

Ex :- $A = \{1, 2, 5, 7, 9\}$
$B = \{2, 3, 5, 7, 11\}$

$A \Delta B = (A-B) \cup (B-A)$
$= (1, 9) \cup (3, 11)$
$= (1, 9, 3, 11)$ Ans

Distributive law :-

(i). $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's law :-

$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

$\boxed{n(A \cup B) = n(A) + n(B)}$ only when A & B will be disjoint set

$$\boxed{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)}$$

Power set P(A)

if $A = \{1, 2, 3\}$
$P(A) = \text{power set} = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Type of Relation :-

~~② Reflexive~~

① **Reflexive Relation :-** A Relation R is said to Reflexive Relation when a set A :

$$\forall x \in A, (x,x) \in R$$

every element of set a related to itself

Ex :- $A = \{1,2,3\}$

$$R = \{(1,1),(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$$

$\therefore (1,1), (2,2)\ (3,3) \in R$

$\therefore$ R is Reflexive Relation

② **Non-Reflexive Relation :-**

A Relation R on a set A is said to be non-Reflexive if for some $x \in A$, $xRx\ ((x,x) \in R)$

$$\exists x \in A \ ; (x,x) \in R$$

Some element of A are Related to itself.

$$R = \{(1,1)(2,1)(3,2)(3,3)\}$$

Here $(1,1)\ (3,3) \in R$

But $(2,2) \notin R$

③ **Irreflexive Relation :-**

If no element of A is Related to itself is Called irreflexive Relation.

$$\forall x \in A \ ; (x,x) \notin R$$

$R = \{(1,2)(1,3)(2,3)\}$ $(1,1) \notin R$ $(2,2) \notin R$ $(3,3) \notin R$

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④ Symmetric Relation :-

A Relation R on the set A is Called symmetric if

$$(x,y) \in R \Leftrightarrow (y,x) \in R$$
$$\forall (x,y) \in A$$

Ex :-
$$R = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)$$
$$(3,1)(3,2)(3,3)\}$$

$\therefore (2,2) \notin R$ R is non-Reflexive

$(1,2) \in R \Rightarrow (2,1) \in R$
$(1,3) \in R \Rightarrow (3,1) \in R$
$(2,3) \in R \Rightarrow (3,2) \in R$

} symm.

⑤ Asymmetric Relation :-

A Relation R on a set A is called Asymmetric if

$$(x,y) \in R \Rightarrow (y,x) \notin R$$
$$x, y \in A$$

⑥ Antisymmetric Relation :-

A Relation R on a set A is said to be Anti-symmetic Relation if

$$(x,y) \in R \text{ and } (y,x) \in R \Leftrightarrow x = y$$
$$\Rightarrow xRy \text{ and } yRx \Leftrightarrow x = y$$

(7) Transitive :-

$(x,y),(y,z) \in R \Rightarrow (x,z) \in R$

$xRy, yRz \Rightarrow xRz$

(8) Equivalence Relation :- A Relation R on a set A is called equivalence Relation if R is Reflexive, Symmetric and transitive.

$R = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

(i). $(1,1)\ (2,2)\ (3,3) \in R$ → Reflexive ✓

(ii). $(1,2) \in R \Rightarrow (2,1) \in R$

$(1,3) \in R \Rightarrow (3,1) \in R$

$(2,3) \in R \Rightarrow (3,2) \in R$

Symmetric ✓

(iii). $(1,2)\ (2,1) \Rightarrow (1,1) \in R$

$(1,2)\ (2,3) \Rightarrow (1,3) \in R$

$(1,3)\ (3,1) \Rightarrow (1,1) \in R$

$(1,3)\ (3,2) \Rightarrow (1,2) \in R$

Transitive ✓

Hence R is an Equivalence Relation.

Power set :-

The set formed by all the sub-set of a given set 'A' is called Power set of A. denoted by P(A).

if $A = \{1,2,3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

power set

$\boxed{\text{POSET}}$:- (partially orderd set)

★ Partially ordered Relation :- A relation R on a set A is said be be partially orderering Relation, if R is Reflexsive antisymmetric and transitive.

Reflexsive, Antisymmetric & Transitive

(i). $\forall x \in A$; $(x,x) \in R$

(ii). $(x,y) \in R$, $(y,x) \in R \Leftrightarrow x=y$

(iii). $(x,y) \in R \cdot (y,z) \in R \Rightarrow (x,z) \in R$

then the set 'A' together with partially ordered relation R is called partially ordered set.

OR $\boxed{\text{POSET}}$ (A, R)

★ set of ordered pairs are called Binary Relation.

① $A \subseteq A$ for any set A

② $A \subseteq B$ and $B \subseteq A$ then $A = B$

③ $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

} if A follow all then it is Poset

$\boxed{\begin{array}{c}(a,b) \in R \\ a \preceq b\end{array}}$

$a \prec b$; $a \neq b$

$\boxed{\begin{array}{c}(x,y) \in R \\ x \preceq R\end{array}}$

$x \prec y$, $x \neq y$

(10 marks)

Q. Show that the Relation
(i) "$\le$" (ii) "$\ge$" is a partially ordering relation on the set of Integer Z

OR

Q. Show that (i) $(Z, \le)$ (ii) $(Z, \ge)$ is a Poset where Z is set of Integer

Soln :- Given that

set 'Z' : set of integers

Q. (i) Relation : "$\le$"

$$R = \{(x, y) : x \le y,\ x, y \in Z\}$$

(i). Reflexive

$\forall x \in Z,\ x \le x$

$\therefore \forall x \in Z,\ xRx\ ;\ (x, x) \in R$

R is Reflexive

$2 \le 2$
$3 \le 3$
$-1 \le -1$
$-7 \le -7$

(ii) Antisymmetric

$x, y \in Z$

and $x \le y,\ y \le x \Rightarrow x = y$

$\therefore xRy,\ yRx \Leftrightarrow x = y$

(iii) Transitive

$(x, y) \in Z,\ (y, z) \in Z \Rightarrow (x, z) \in Z$

$x \le y,\ y \le z \Rightarrow x \le z$

$xRy,\ yRz \Rightarrow xRz$

Q(ii) Relation "$\geq$"

① Reflexive :-

$\forall x \in Z, \quad x \geq x$

$\therefore \forall x \in Z, \quad xRx \rightarrow (x,x) \in R$

$\therefore$ R is Reflexive

② Anti-Symmetric

$x, y \in Z$

$x \geq y, \; y \geq x \Leftrightarrow x = y \quad (x,y) \in Z$

$\therefore xRy, \; yRx \Leftrightarrow x = y$

$\therefore$ R is anti-symmetric

③ Transitive

$x, y \in Z \; ; \; y, z \in Z$

$x \geq y, \; y \geq z \quad \forall \; x, y, z \in Z$

$\therefore x \geq z \quad \forall (x, z) \in Z$

$\therefore xRy, \; yRz \Rightarrow xRz \quad \forall \; x, z \in R$

$\forall (x,y) \in Z, \; (y,z) \in Z, \; (x,z) \in R$

$\therefore$ R is Transitive

Note :- the symbol $\preceq$ is used to denote the Relation in any poset (A, R)

$x \preceq y \Rightarrow (x, y) \in R$

$x \preceq y$ i.e $x, y \in R$

OR

$x \prec y$ But $x \neq b$

Poset$(A, \preceq)$ are called Comparable.

if either $a \preceq b$ or $b \preceq a$.

if neither $a \npreceq b$ or $b \npreceq a \rightarrow$ InComparable

* * * * * * * * *

Note :- The partial order relation are often denoted by $\preceq$

$x \preceq y$ means x preceeds y

$x \prec y$ means x strictly

Comparable :- two elements (x, y) of a POSET ~~eg~~ (x, y) $(A, \preceq)$ are said to be Comparable if either $x \preceq y$ or $y \preceq x$

and if Neither $x \preceq y$ nor $y \preceq x$ then they are called InComparable Not comparable.

Ex :- In Poset $(Z^+, /)$

the element 3, 9 are Comparable

$\because 3/9$

But 2; 7 are not Comparable.

$2 \nmid 7$ and $7 \nmid 2$

Total ordering Relation :- A Relation R on a set A is called total ordering relation if R is a partially orderd relation and also satisfy Dechotomy law.

i.e Every pair of the element of A are comparable

$\forall x, y \in A$, either $x \preceq y$ or $y \preceq x$

Immedate Predecessor & Immedate Successor :

let $(A, \preceq)$ be a POSET and $x, y \in A$ then x is called Immedate predecessor of y

y is called successor of x

If $x \preceq y$ and there is No element of set A lies in between x, y

HASSE Diagram :-

A graphical representation of POSET (partially ordering Relation) in which all the Arrow heads are understoods to be pointing in upward direction is called Hasse Diagram.

★ **working Rule :-**

let (A, R) is a given Poset where R is Partially orderd relation

(i). R in the Roster form.

(ii). Draw the Diagraph of R

(iii). Delete all the loops from Diagraph

(iv). Eliminate all the Edges that are implided by transitive property of the relation

(v). Replace the circle with dots and also omits the Arrow

8. Draw the Hasse Diagram of a Poset $(S, \preceq)$ (Partially orderd) where

$S = \{3, 4, 6, 12, 24, 48, 72\}$

$\preceq$ is defined as

"$x \preceq y$ if x divides y"

or x is a factor of y.

Given that

$S = \{3, 4, 6, 12, 24, 48, 72\}$

Relation R $(\preceq)$

$R = \{(x, y) : x \preceq y ; x, y \in S\}$

$= \{(x, y) : x$ divides $, x, y \in S\}$

$\therefore R = \{(3,3)(3,6)(3,12)(3,24)(3,48)(3,72)$
$(4,4)(4,12)(4,24)(4,48)(4,72)$
$(6,6)(6,12)(6,24)(6,48)(6,72)$
$(12,12)(12,24)(12,48)(12,72)$
$(24,24)(24,48)(24,72)$
$(48,48)(72,72)\}$

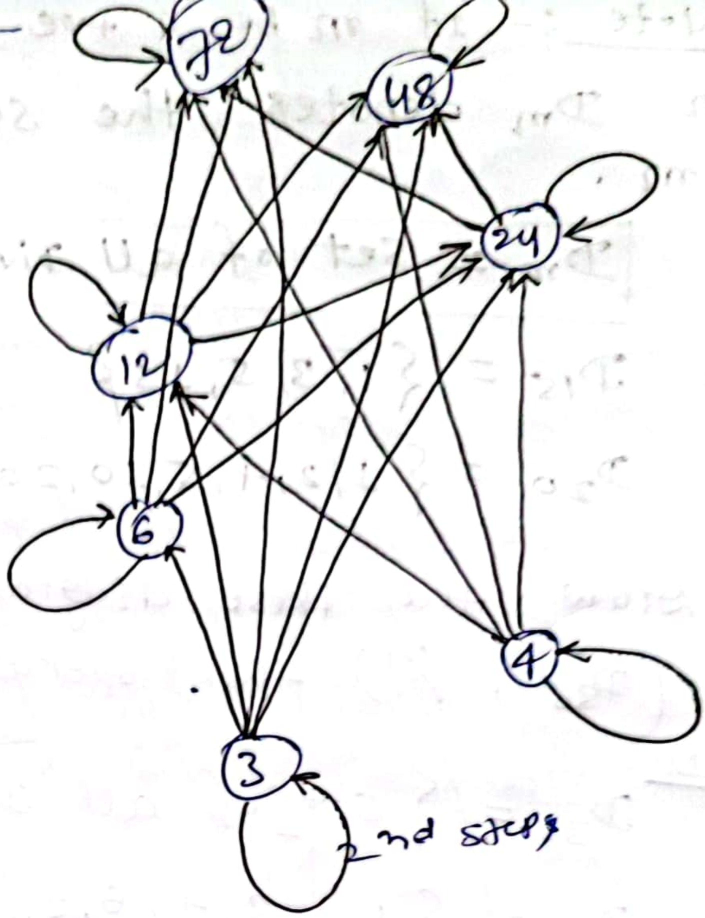
1st steps

the Diagraph is above for R:

Now for finding transitive edges

3rd Step for transition element

(3,6)(6,12) ⇒ (3,12)
(3,6)(6,24) ⇒ (3,24)
(3,6)(6,48) ⇒ (3,48)
(3,6)(6,72) ⇒ (3,72)



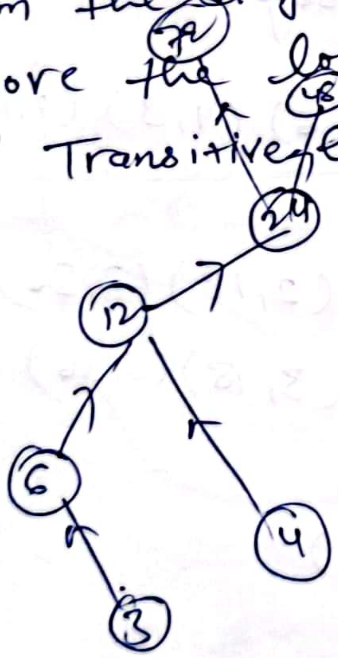
(4,12)(12,24) ⇒ (4,24)
(4,12)(12,48) ⇒ (4,48)
(4,12)(12,72) ⇒ (4,72)

(6,12)(12,24) ⇒ (6,24)
(6,12)(12,48) ⇒ (6,48)
(6,12)(12,72) ⇒ (6,72)

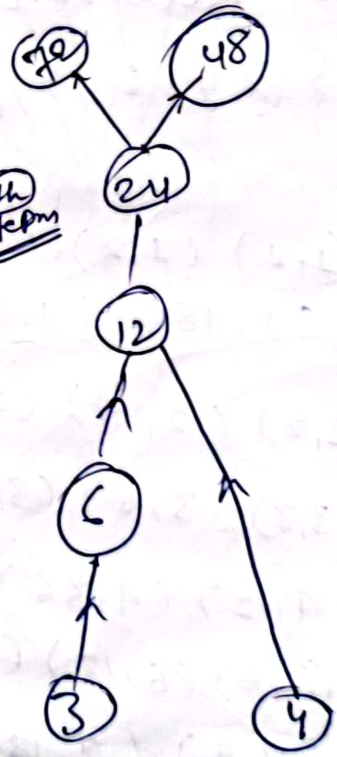
(12,24)(24,48) ⇒ (12,48)
(12,24)(24,72) ⇒ (12,72)

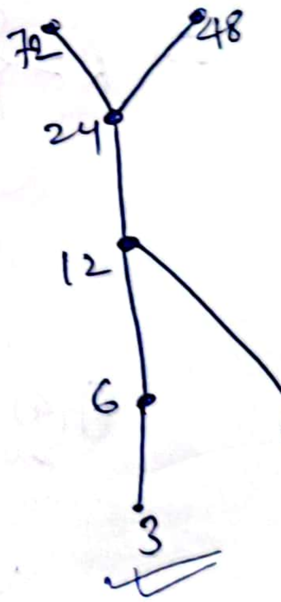
from the diagraph
remove the loops
and Transitive element

4th S



5th Steps





6th Steps

Note :- If m be a +ve integer (Natural no.) then D_m denotes the set of all divisor of m.

$$\boxed{D_m = \text{set of all divisor of } m}$$

$D_{15} = \{1, 3, 5, 15\}$

$D_{20} = \{1, 2, 4, 5, 10, 20\}$

Q Draw the Hase diagram of the poset $(D_{36}, /)$ the operation of divisibility

Given

$D_{36} = \{$ set of all divisor of $36\}$

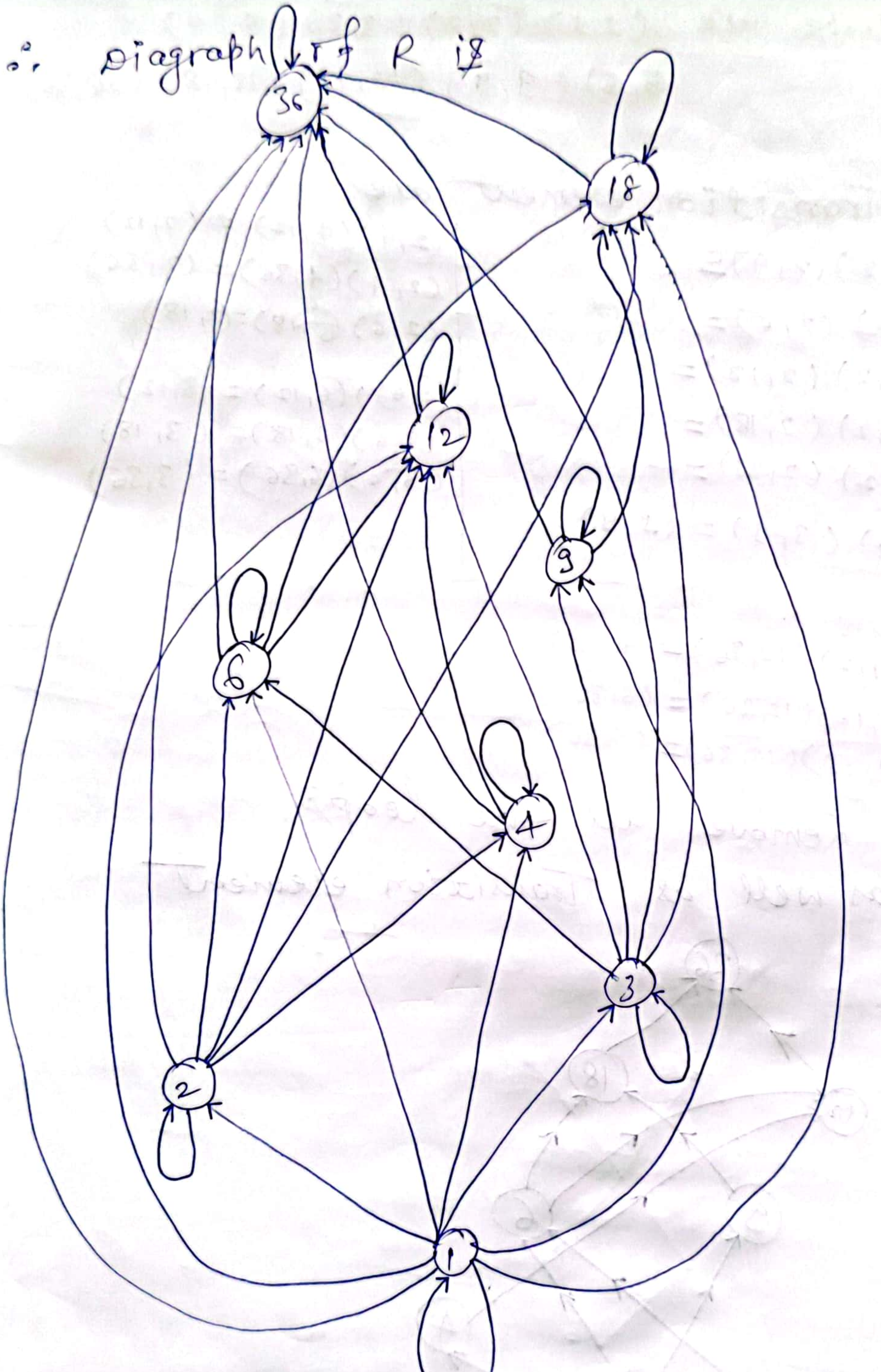
So $D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

and The Relation R be defined as

$R = \{(x, y) : x/y \;\; x, y \in D_{36}\}$

$\therefore R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,9)\ (1,12)$
$(1,18)\ (1,36)$
$(2,2)\ (2,4)\ (2,6)\ (2,12)\ (2,18)\ (2,36)$
$(3,3)\ (3,6)\ (3,9)\ (3,12)\ (3,18)\ (3,36)$
$(4,4)\ (4,12)\ (4,36)\ ($
$(6,6)\ (6,12)\ (6,18)\ (6,36)$
$(9,9)\ (9,18)\ (9,36)$
$(12,12)\ (12,36)\ (18,18)\ (18,36)\ (36,36)\}$

∴ Diagraph of R is



Diagraph of R

the loops are (1,1) (2,2) (3,3) (4,4) (6,6) (9,9) (12,12) (18,18) (36,36)

and Transition element are

(1,2)(2,4)=

(1,2) (2,6)=

(1,2) (2,12)=

(1,2) (2,18)=

(1,2) (2,36)=

(1,3) (3,9) = (1,9)

(2,4)(4,12) = (2,12)

(2,4)(4,36) = (2,36)

(2,6)(6,18) = (2,18)

(3,6)(6,12) = (3,12)

(3,6)(6,18) = (3,18)

(3,6)(6,36) = (3,36)

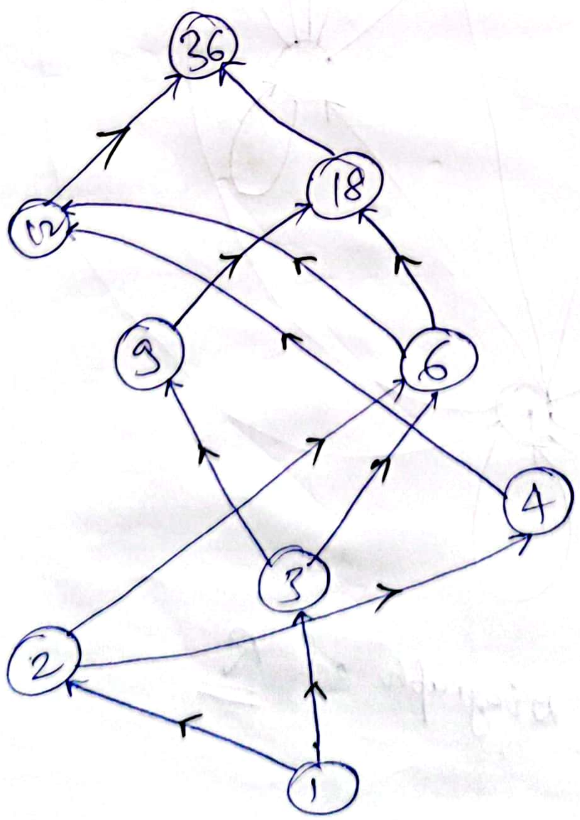
Again

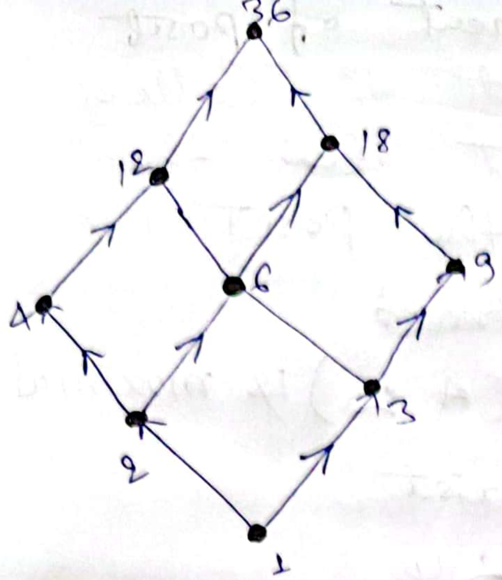
(4,12)(12,36) = (4,36)

(6,12)(12,36) = (6,36)

(9,18)(18,36) = (9,36)

Now Remove all the loops as well as Transition element




36
12
18
4
6
9
2
3
1

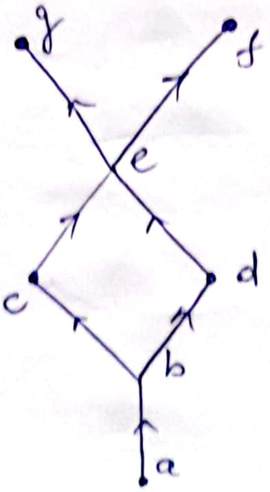
Maximal and Minimal element of poset

An element of a poset is called poset if it is not less than any other element of the poset i.e

an element x is ~~maximal~~ an element x is maximal if $x \in A$ of poset $(A \preceq)$ is maximal if there exist No $y \in A$ s.t $x \preceq y$

and $x \in A$ is called Minimal :- if it is not greater than any element i.e $\exists$ no $y \in A$ s.t $y \preceq x$

<u>Note</u> :- In the Hasse diagram the Top and Buttom elements are the Maximal and minimal elements.

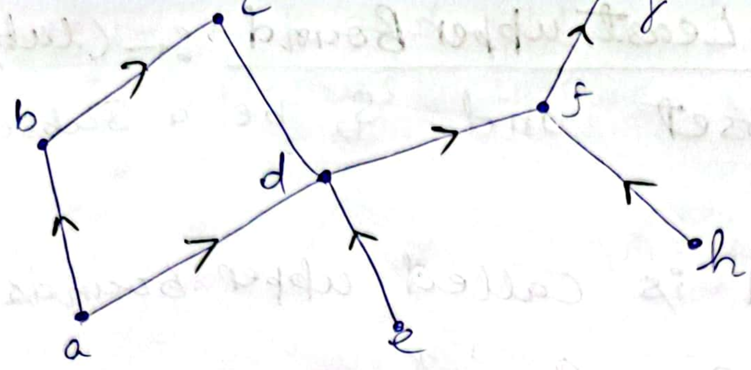


Maximal :- f, g

Minimal :- a

No Element before :- Minimal

No Element ~~bo~~ after :- Maximal



Always ↑up ward Direction (Arrow).

Maximal :- c, g

Minimal :- a, e, h

Greast and least Element :-

An element $x \in A$ is called greast element of the poset $(A, \preceq)$ if $\forall a \in A$, $a \preceq x$

and is called Least element if $\forall a \in A$, $x \preceq a$

Note :- the greast & Least element is exists then it is unique.

Upper Bound And Least upper Bound :- (lub)

Let A be a poset and S be a Subset of set A.

an element $x \in A$ is called upper bounds of S if $\forall a \in S$, $a \preceq x$

and is called least upper bound (lub) or Suprimum of S if $x \preceq y$ $\forall$ upper bound y of S.

$\boxed{y \Rightarrow \text{upper bound}}$ (preceeding)

Greast lower bound (lower bound) :- (glb)

Let A be a poset and S be a Subset of A

then an element $x \in A$ is called greast lower bound of S if

$\forall a \in S, x \preceq a$

~~Great least bo~~

Greast lower bound or g.l.b or Infimum

if $y \preceq x$, ... lower bound of y of S

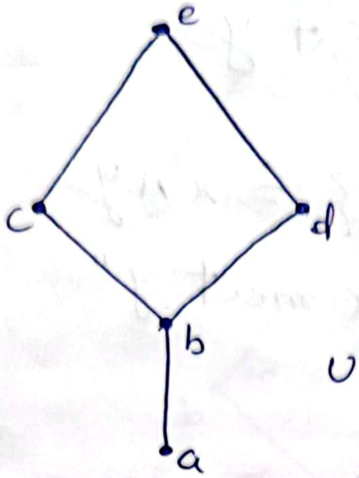
$\boxed{lub \ (\vee) \ \& \ glb \ (\wedge)}$

$\boxed{\text{Superimum or Infimum}}$

$\vee$ lub, $\wedge$ glb

Note :- l.u.b and g.l.b of a set is existing is always unique (one → single)

Ex :-



$A = \{a, b, c, d, e\}$

upper Bound of $\{a, b\} = (b, c, d, e)$

l.u.b of $\{a, b\} = b$ (Common of upper bond of both)

g.l.b of $\{a, b\} = a$ (Common of lower bond of both)

l.u.b = Intersection of upper bound of both

g.l.b = Intersection of lower bound of both

Note :- For a Poset A with partial orderd relation of $(A, \preceq)$ element of set of A is natural numbers (+ve integer)

$\preceq$ is "devides" ("factor of")

l.u.b of (x, y) = LCM $\{x, y\}$

l.u.b of (x, y, z) = LCM $\{x, y, z\}$

g.l.b $\{x, y\}$ = HCF $\{x, y\}$
GCD $\{x, y\}$

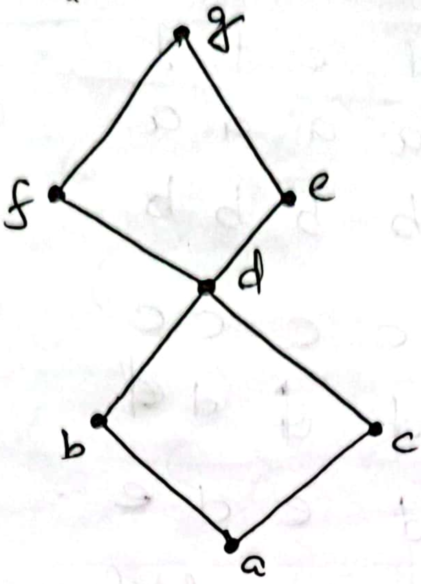
② lub $\{x, y\}$ = $\sup\{x, y\} = x \vee y$
$= x$ joint y.

glb of $\{x, y\}$ = $\inf\{x, y\} = x \wedge y$.
"x meet y"

Lattice :- A poset $(A, \preceq)$ is said to be lattice if every pair of element in A has unique lub and glb.

$\forall x, y \in A$, $\sup\{x, y\}$ & and $\inf\{x, y\}$ Exist in A

Q. Find wether the Poset whose Diagram following Determine Lattice or not



Sol^n :- Let's find the Closure table for Sup/lub and Inf/glb

See greatest

अगर level same आया तो Just upper देखना

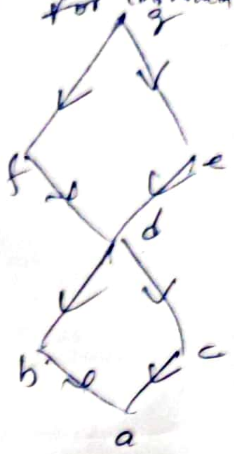
(i) For Supremum (l.u.b) {$\vee$} :-

$\vee$	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b	b	b	d	d	e	f	g
c	c	d	c	d	e	f	g
d	d	d	d	d	e	f	g
e	e	e	e	e	e	g	g
f	f	f	f	f	g	f	g
g	g	g	g	g	g	g	g

(ii) Closure Table for Imfimum (g.l.b = ∧)

∧	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	b	b	b	b
c	a	a	c	c	c	c	c
d	a	b	c	d	d	d	d
e	a	b	c	d	e	d	e
f	a	b	c	d	d	f	f
g	a	b	c	d	e	f	g

Rough for Infimum



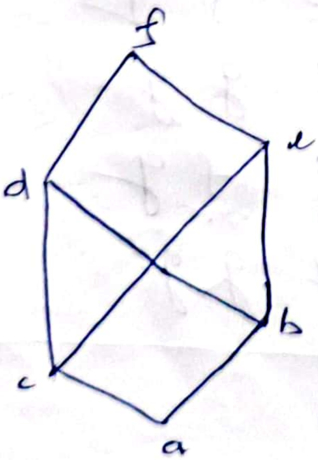
अगर level same आया तो just lower इसका

From the tables we concude that the Supa and Inf of every pair of element exist.

∴ the poset $(A, \preceq)$ is a lattice proved

Checked

Q.



$(A, \preceq)$; $A = \{a, b, c, d, e, f\}$

b : b, d, e, f

c : c, d, e, f

∨ = (d e) Two Dont exist / so

Theorey only unique el Doesn not exist

	a	b	c	d	e	f
a	a	b	c			
b	b	b	—			
c						
d						
e						
f						

So missing entries

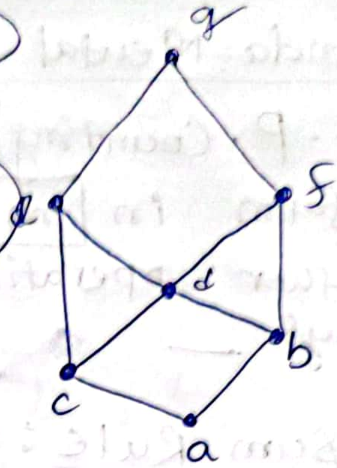
So Does not exists

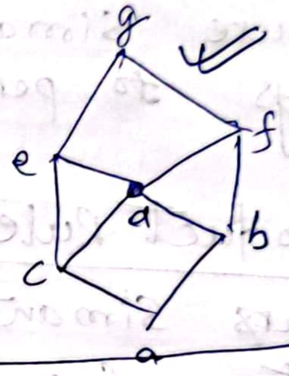
lattice ✗

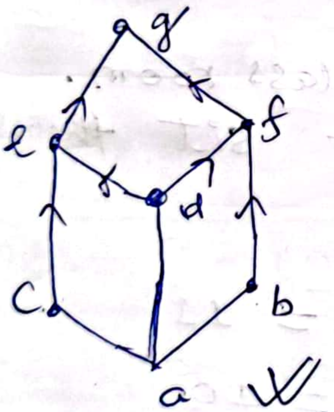
Supremum doesn't exist

अगर level same आया तो just upper/lower देखना

Same है तो Dono को same देखा







∨	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b	b	b	~~g~~	~~g~~	e	f	g
c	c	d	c	—	e	—	g
d	d	—	—	d			
e							
f							
g							

~~Not~~ a lattice

b : b, f, g
c : c, e, g

g

Always Check

b : b, f, g
d : f, g

b : b, f, g
e : e, g

c

b : f, f, g
f : f, g

at very first upper bound

Remember

Join ↑
meet ↓

③ Techniques of Counting :-

Funda Mental Principal of Counting :-

(F.P. Counting) :- If an operation can be perform in $\boxed{m\text{-different}}$ ways and another operation can perform in $\boxed{n\text{-different}}$ ways —

① **Sum Rule** :- If two operation can not occure Simanteneously then total no. of ways to perform the operation = $\boxed{m+n}$

② **Product Rule** :- If both the operation occurs Simanteneously then total no. of way that perform the operation = $\boxed{m \cdot n}$

Q. There are 11 seats in a class room. In How many ways can it sit themself

11

No. of ways for 1st student = 11
———————— 2nd ———— = 10
———————— 3rd ———— = 9
———————— 4th ———— = 8
———————— 5th ———— = 7

By F.P.C

Total no. of ways = $11 \times 10 \times 9 \times 8 \times 7$

Arranging ↓ Permutation	sellection ↓ Combination

3. Fundamental principal of Counting :-

$\lfloor 8 = 8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 40320$

$\lfloor 7 = 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 5040$

$\lfloor 6 = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

$\lfloor 5 = 5\cdot4\cdot3\cdot2\cdot1 = 120$

$\lfloor 4 = 4\cdot3\cdot2\cdot1 = 24$

$\lfloor 3 = 3\cdot2\cdot1 = 6$

$\lfloor 2 = 2\cdot1 = 2$

$\lfloor 1 = 1$

$\boxed{\lfloor 0 = 1}$ $\boxed{\lfloor -ve = \text{Not exists}}$

$\boxed{\lfloor\ ,\ !}$ Both are same

$\lfloor 8 = 8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

$= 8\cdot7!$

$\lfloor 6 = 6\cdot5!$

$$\boxed{n! = n(n-1)!}$$

Q. $\lfloor n+2 = 2550 \lfloor n$

$(n+2)(n+1)\cancel{\lfloor n} = 2550\cancel{\lfloor n}$

$\Rightarrow n^2+3n+2 = 2550$

$\Rightarrow n^2+3n-2548=0$

$\Rightarrow n^2+51n-50n-2548=0$

$\Rightarrow n(n+51)-50(n+51)=0$

$n=49$ Solve

Permutation (Arrangment) :-

Different Arrangement which can be make out given no. of things some and all of them then at a time, is called permutation.

permutation of n- different things taking r ($1 \le r \le n$)

is denoted by $\boxed{{}^{n}P_{r} \text{ , } P(n,r)}$

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

Ex :- 11 seats, How many way that sit 5 at a time $= {}^{11}P_{5}$

$$= \frac{11!}{(11-5)!} = \frac{11!}{6!}$$

$$= \frac{11\times 10\times 9\times 8\times 7\times \cancel{6!}}{\cancel{6!}}$$

$$= 11\times 10\times 9\times 8\times 7$$

Ans

Q. In How many ways can be letter of a word "TRIANGLE" be arranged : = 8!

How many of these

(i). Begain with 'A'

(ii). end with 'L'

(iii). Begain with 'A' & End with 'L'

(iv). A, L occupy end places

(v). All vowels are Togethers

(vi). No. 2 vowels are Together.

(vii) No all vowel together

(viii). Vowel odd places occupy

(ix). Vowel even places occupy

The no. of letters in the words TRIANGLE = 8 (All are different)

Arrangement

Total no. of arrangment of words = $^8P_8$

$$= \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1}$$

$(\because 0! = 1) \Rightarrow 8!$

① Begain with 'A'

$\boxed{A}$ × × × × × × × 7 ways

Since the word Begain with A therefore We fix 'A' at 1st position

No. of ways of Arranging 'A' = 1

No of ways of Arranging 7 letters = $^7P_7$
$= 7! = 5040$

By F.P.C

total no. of words $= 1 \times 5040$
$= 5040$ Ans

② End with 'L'

× × × × × × × $\boxed{L}$
fix

No. of ways of Arranging $L = 1! = 1$

No. of ________ 7 letters $= {}^7P_7$
$= 7! = 5040$

By F.P.C

to total no. of word that ends with 'L' $= 1 \times 5040$
$= 5040$ Ans

③ Start with A and End with L

$\boxed{A}$ × × × × × × $\boxed{L}$
Fix Fix

$AL = 1$ ways

Remaining 6 letter Arrangement $= {}^6P_6$
$= 6!$
$= 720$

By F.P.C

Total No. of ways that letter can be Arranged which start with A & End with L
$= 1 \times 720 = 720$ Ans

④ A, L occupy end places

$\boxed{A}$ × × × × × × $\boxed{L}$ → 720

$\boxed{L}$ × × × × × × $\boxed{A}$ → 720

Two posibilities

(Ans → 2 of 720 = 1440)

No. of ways to Arrange A, L = 2 ways

∴ No. of way to Arrange $\boxed{A}$ — $\boxed{L}$

$= {}^6P_6 = 6!$

$= 720$

No. of way to Arranging $\boxed{L}$ — $\boxed{A}$

$= {}^6P_6$ (Also)

$= 6! = 720$

By F.P.C

∴ Total no. of way of Arranging the letter TRIANGLE which A, L occupy end places $= 2 \times 720$

$= 1440$ Ans

⑤ All vowels are together (TRAINGLE)

Vowels = 3 (A, I, E)

Consonant = 5

$\boxed{(A, E, I)}$ × × × × ×

1 ways, 5 ways

$\underbrace{\qquad}_{6 \text{ ways}} = 6! = 720$

$^3P_3 = 3! = 6$

By F.P.C $= 6 \times 720$

⑥ No Two vowels are together

Vowels $= 3 (A, I, E)$

Consonant $= 5$

$* T * R * N * G * L *$

$\boxed{*}$ → मे vowel को रख सकते है। 6 places है। तो एक साथ नहीं होगा।

No. of two vowels are together

$= \cancel{{}^8P_3 = 8 \cdot 120}$

$= {}^6P_{3(A,I,E)} = \frac{6!}{(6-3)!}$

$= \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$

$= 120$

No. of ways of Constant ${}^5P_5$

$= 5! = 120$

By F.P.C

Total no. of ways $= 120 \times 120$

$= 14400$

⑦ No. all vowels are together

$= (\text{Total}) - (\text{Two volews are togeher})$

$= 40320 - 4320$

$= 36000$

⑧ vowels at odd places

① 2 ③ 4 ⑤ 6 ⑦ 8

No. of odd places $= 4$

volvels $= 3 (A, I, E)$

No. of ways of Arrang volvel $= {}^4P_3 = 24$

No of ways of Cosont $= {}^5P_5$

$= 5! = 120$

By F.P.C $= 24 \times 120$

$= 2880$ Ans

* The permutation of things not all different set.

\# The no. of way of permutation of $\boxed{n\text{-things}}$ taken all at a time of which

n-things

p are of one kind

q are of 2nd kind

t are of 3rd kind

are given by

$$\boxed{\frac{\lfloor n}{\lfloor p \cdot \lfloor q \cdot \lfloor t}}$$

Ex Apple

$n=5$

$\frac{5!}{2!}$

Q. How many word can be forom the letter of the word "MATHEMATICS"

Soln:- Total no. of words = 11

when A occurs 2 times

T occurs 2 times

M occrse 2 time

$\therefore$ No. of words $= \frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2}$ Ans

Eg Apple

$n=5$

$= \frac{5!}{2!}$

Eg Egglessuutmt

$n=12$

$g=2$, $s=2$, $u=2$, $t=2$

$\frac{12!}{2!\,2!\,2!\,2!}$

Circular Permutation :-

The no. of ways that we can arrange the object in circular

$= \boxed{\lfloor n-1} \Leftrightarrow (n-1)!$

Neackless with n different units and gramants

$= \boxed{\frac{1}{2}\lfloor n-1} \Leftrightarrow \frac{1}{2}(n-1)!$

Same 2 parts

In How many ways 6 men and 4 women can Arrany so that No Two women sit together.

(a) in a line — simple

(b) In a circle — use formula

जो ज्यादा हो उसे Arrange करते हैं।

① Arranging in a line ~~(combination)~~

$* M_1 * M_2 * M_3 * M_4 * M_5 * M_6 *$

No. of ways of Arranging 6 Men $= {}^6P_6$

$= \lfloor 6$

$= 720$

No. of ways of Arranging 4 women

$= {}^7P_4 = \frac{\lfloor 7}{\lfloor 3}$

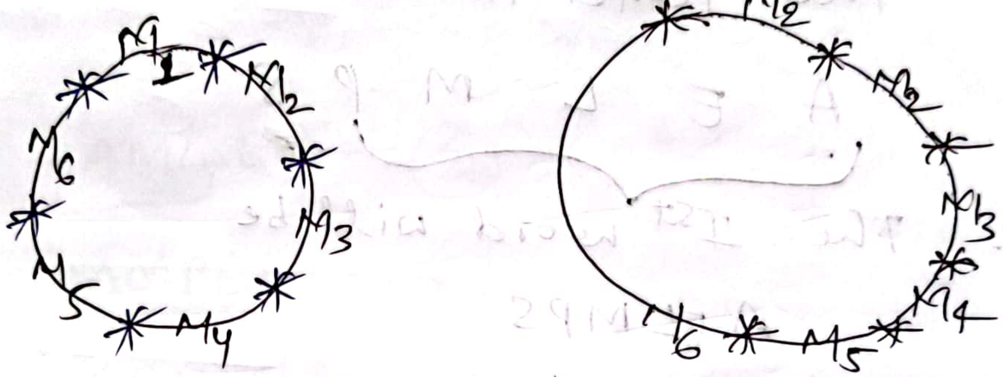
$= 7 \times 6 \times 5 \times 4 = 840$

By F.P.C total No. of ways

$= 720 \times 840 =$ ______

Note :-

(5) Arranging in a circle :-



No. of ways Arranging 6 men in a circle $= \lfloor 6-1 = \lfloor 5$
$= 120$

No. of ways of Arranging 4 women
$= {}^6P_4 = \dfrac{\lfloor 6}{\lfloor 2}$
$= \dfrac{720}{2} = 360$

By FPC

Total No. of ways
$= 120 \times 360$
$= 43200$

Q. If the letter of the word "SAMPLE" We arranged in a dictionary then what is the rank of the word Sample.

Soln No. of letters = 6

the letter in order

$\underbrace{A \quad E \quad L \quad M \quad P}$ S

∴ The Ist word will be

AELMPS

A No. of words stand with A

$= 1 \times {}^5P_5$

$= 1 \times \lfloor 5 = 120$

120

E No. of word stand with E

$= 1 \times {}^5P_5 = 120$

120

L No. of word stand with L = 120

= 12

120

M No. of words stand with M = 120

120

P No. of words stand with P = 120

~~= 120~~

120

Next word will be

$\boxed{120 \times 6}$

SAELMP

No. of words start with SAE $= 1 \times {}^3P_3$

$= 1 \times \lfloor 3 = 6$

Next word will be SAL EMP

No. of words with SAL = 6

Next word SAMELP

No. of words with SAME $= {}^2P_2 = \lfloor 2$

$= 2$

SAMLEP — 2

SAMPEL — 1

SAMPLE

$\therefore$ No. of words Before the word

SAMPLE $= 120+120+120+120+120+6+6+2+2+1$

$= 600+17$

$= 617$

Q. SACHIN (Rank in Dictonary)

A C H I N S

A → ${}^5P_5 = \lfloor 5 = 120$

C → 120

H → 120

I → 120

N → 120

600

SACHIN

120×5

= 600

+1

601

Combination : The Combination is a sellection or Grouping of things are taking at a time.

⇒ The Combination of n different things are taking at a time is denoted by ^nC_r, $C(n,r)$ $\binom{n}{r}$.

and defined as $\boxed{^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}}$

$\boxed{\text{permutation} > \text{Combination}}$

(1) $^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r} \rightarrow {}^nP_r$

$^nC_r = \frac{^nP_r}{\lfloor r}$

$\boxed{\lfloor r \cdot {}^nC_r = {}^nP_r}$

$\boxed{^nC_r = \frac{n!}{r!(n-r)!}}$

(2) $\boxed{{}^{n}C_r = {}^{n}C_{n-r}}$

(3) If ${}^{n}C_p = {}^{n}C_q$ then

either $p = q$ or $p + q = n$

(4) $\boxed{{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r}$

Q. In How many ways can a team of 6-players can be selected out of 11-players. & How many of them

4 (a) always have do particular player

4 (b) Always have not particular player.

$\therefore$ No. of ways $= {}^{11}C_6$

$$= \frac{11!}{(11-6)!\,6!} = \frac{11!}{5!\,6!}$$

$$= \frac{11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times \cancel{6!}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{6!}}$$

$42 \times 11 = 462$

42
42
462

(a) Two Particular players are selected :—

$11 - 2 = 9$
$6 - 2 = 4$

Since Two

$\therefore$ No. of ways of selecting 2 part player $= 1$

No. of ways of selecting Remain 4 Players out of $9 = {}^{9}C_4$

$= {}^{9}C_{4}$

$= \dfrac{9!}{(9-4)!\,4!}$

$= \dfrac{9!}{5!\,4!} = \dfrac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!} \times \cancel{4} \times \cancel{3} \times \cancel{2}}$

$= \cancel{126}\ 9 \times 2 \times 7$

$= \underline{\underline{126}}$

(b) Since Two particular players are never selected so.

No. of ways of total players $= 11 - 2$
$= 9$ players

at wich 6 has to selected
So no. of ways $= 6 - 2 = \underline{\underline{4}}$

$\therefore$ No. of ways of selecting ~~Rema~~ 6 players out of 9 $= {}^{9}C_{6}$

$= {}^{9}C_{3} = \dfrac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7}{\cancel{3} \times \cancel{2}}$

$= 84$

Note :- If there are $\boxed{n\text{-Points}}$ in P-collinear
(Lines on the same lines)

(1) Total No. of lines formed $= \boxed{{}^nC_2 - {}^PC_2 + 1}$

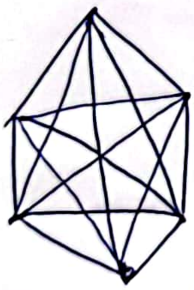
(2) Total No. of Triangles $= \boxed{{}^nC_3 - {}^PC_3}$

(3) No. of r side figure $= \boxed{{}^nC_r - {}^PC_r}$

(4) No. of Diagonals $= \boxed{{}^nC_2 - n = \frac{n^2 - 3n}{2} = \boxed{\frac{n(n-3)}{2}}}$

where n = no. of points
= vertices
= sides of polygon.

Eg



Diagonals = ?
$n = 6$
$= \frac{n(n-3)}{2}$
$= \frac{6(6-3)}{2}$
$= 3 \times 3$
$= 9$

Q. Find the no. diagonal in octagon.
$n = 8 \quad = \frac{n(n-3)}{2} = \frac{8(8-3)}{2}$
$= 4 \times 5 = 20$

Q. Find the no. of diagonal of sides of a polygon which has 47 diagonal
$\because$ diagonal $= \frac{n(n-3)}{2}$
$\Rightarrow 47 = \frac{n^2 - 3n}{2} \Rightarrow 94 = n^2 - 3n$
$\Rightarrow n^2 - 3n - 94 = 0$ ✗ not possible

Q. Find the no. of ~~diagonal~~ sides of diagonal is 44.
11 Ans

$\because$ Diagonal $= \frac{n(n-3)}{2}$
$\Rightarrow 44 = \frac{n^2 - 3n}{2} \Rightarrow 88 = n^2 - 3n = 0$
$\Rightarrow n^2 - 3n - 88 = 0$
$\Rightarrow n^2 - 11n + 8n - 88 = 0$
$\Rightarrow n(n-11) + 8(n-11) = 0$

$(n-11)(n+8) = 0$
$n = 11$ OR $n = -8$ ✗
$n = 11$ Ans

$\boxed{{}^nP_r \text{ Arrangement}}$ $\boxed{{}^nC_r \text{ (Selection) Combination}}$

$\boxed{n \geq r}$ Always

$${}^nP_r = \frac{n!}{(n-r)!}$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$\Rightarrow {}^nC_r = \frac{1}{r!}\,{}^nP_r$$

$$\Rightarrow \boxed{r!\,{}^nC_r = {}^nP_r}$$

$$\boxed{{}^nP_r = r!\,{}^nC_r}$$

Think If $\boxed{n < r}$

the came the concept

of $\boxed{\text{Pigeonhole Principal}}$

Pigeonhole principale :—

If n Pigenos are assigned to m Pigeons holes where $\boxed{n > m}$ then at least one pigon hole contain two or more pigeons.

Note :- n pigeons are assigned to m pigeon hole where $n > m$ then atleast one pigeon holes contain $\boxed{\left[\frac{n}{m}+1\right]}$

go with it.

OR $\left[\frac{n-1}{m}\right]+1$

Q. Find the minimum no. of students needed to guarantee that five of them belong to same class (Freshman, Sophomore, Junior, Senior)

Soln :- $n = 4$ classes are the pigeon holes

and $k+1=5$

$\Rightarrow 4+1=5$ i.e $k=4$

thus among any $kn+1$
$= 4\times4+1$
$= 16+1$
$= 17$

$17n$

$\left[\frac{5}{4}+1\right]$

$[1.w+1]$

$[2.]$

2

Total no. of Triangle = $^nC_3 - {}^pC_3$

Q. Ex – ① Total 12 points i.e $n = 12$

Collinear points = 5

$\therefore p = 5$

(i) Δ formed

(ii) Total no. of Lines formed

(i). To. no. of Δ form = $^{12}C_3 - {}^5C_3$

$= \frac{12!}{3!\,9!} - \frac{5!}{(5-3)!\,3!}$

$= \frac{12 \times 11 \times 10 \times \cancel{9!}}{3! \times \cancel{9!}} - \frac{5!}{2!\,3!}$

$= \frac{\overset{2}{\cancel{12}} \times 11 \times 10}{\cancel{3 \times 2}} - \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{2} \times \cancel{3!}}$

$= 220 - 10$

$= 210$ Ans

(ii) Total no.'s of Lines formed

$= {}^nC_2 - {}^pC_2 + 1$

$= {}^{12}C_2 - {}^5C_2 + 1$

$= \frac{\overset{6}{\cancel{12}} \times 11}{\cancel{2}} - \frac{5 \times \overset{2}{\cancel{4}}}{\cancel{2}} + 1$

$= 66 - 10 + 1$

$= 67 - 10 = 57$

Q. Total 18 points
collinear point = 5

(i) No. of Δ formed = ${}^{n}C_3 - {}^{P}C_3$

where n = total point $(n) = 18$
collinear point $= P = 5$

So $\quad {}^{18}C_3 - {}^{5}C_3$

$$= \frac{\cancel{18}^{3} \times 17 \times 16}{\cancel{3} \times \cancel{2}} - \frac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2}_{1}}$$

$$= 51 \times 16 - 10$$

$$= 816 \text{ Ans}$$

$51 \times 16 = 306 + 51 = 816$

(ii) No. of lines formed $= {}^{n}C_2 - {}^{P}C_2 + 1$

$$= {}^{18}C_2 - {}^{5}C_2 + 1$$

$$= \frac{\cancel{18}^{9} \times 17}{\cancel{2}} - \frac{5 \times \cancel{4}^{2}}{\cancel{2}} + 1$$

$$= 153 - 10 + 1$$

$$= 154 - 10$$

$$= 144 \text{ Ans}$$

$\boxed{\begin{array}{l} n = \text{total no. of point} \\ P = \text{collinear points} \end{array}}$

No. of Non-Reflexive Relation $= 2^{n^2} - 2^{n^2-n}$

$\boxed{\text{No of Reflexive Relation} = 2^{n^2-n}}$

No of Irreflexive Relation $= 2^{n(n-1)} = 2^{n^2-n}$

$\boxed{\text{No. of Symmetric Relation} = 2^{\frac{n(n+1)}{2}}}$

No. of Asymmetric Relation $= 3^{\frac{n(n-1)}{2}}$

No. of Anti-symetric Relation $= 2^n \times 3^{\frac{n(n-1)}{2}}$

No. of Transitive Relation = there is no formula

$\boxed{\text{Total no. of Relation in Symmetric \& Reflexive} = 2^{\frac{n(n-1)}{2}}}$

$\boxed{\text{Total no. of element in Relation} = 2^{n^2}}$

n^2/n

$2^{n^2}/2^n$

total $= n^2$

(1,1) (1,2)

(2,1) (2,2)

(3,1) (3,2) (3,3)

n.

* Diagonal $= n$

* Non diagonal $= n^2 - n$

CamScanner

Pigeon hole principle :- $[\frac{n}{m}+1]$

n pegeon are assigned for m Pigeon hole

① if 'n' pigeon hole are occupied by $\boxed{kn+1}$ or more pigeons then atleast one Pigeonholes is occupied by $\boxed{k+1}$ or more pigeon.

Q. Find the minimum number of teacher for a college to be sure that four of them are born in the same month.

Given :- 4 teacher $\Rightarrow k+1 = 3+1$

$\Rightarrow k = 3$

Born in same month

month = 12

i.e $n = 12$

~~KOI~~ $\Rightarrow kn+1$

$\Rightarrow 3 \times 12 + 1$

$\Rightarrow 36 + 1$

$\Rightarrow$ 37 Ans

② A box Contain 10 blue ball, 20 red balls 8 green balls, 15 yellow balls & 25 white balls. How many balls must we choose to ensure that we have 12 ball of the same color.

Given that :- $(\overset{B}{10}+\overset{R}{20}+\overset{G}{8}+\overset{Y}{15}+\overset{W}{25}) = 7\cancel{8} \Rightarrow n=5$

$k+1=12$

$k=11$

$78 \times$ ... 858

12 ball of same color = $k+1$

$\Rightarrow 12 = 11+1 \Rightarrow k=11$

$= kn+1$

$= 11 \times \cancel{78} + 1 = \cancel{858}$ 56

(3) prove that 1,00,000 people there are two who have born on same time.

Soln :- born on same Time

i.e Time = 24 h

people = 1,00,000

In h $\rightarrow \left[\frac{100000}{24}\right] = 4167$

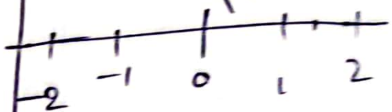
In m $\rightarrow \left[\frac{4167}{60}+1\right] = 70$

In Second $\rightarrow \left[\frac{70}{60}+1\right]$

$= [1.\cdots+1] = [2.\cdots]$

$= 2$

$[1.2] = 1$
$[2.9] = 2$
$[-2.3] = -3$



$[-1.2] - 1$

(3) Q. Show that 13 people, there are at least two people who were born in the same month.

month = 12 = m   Pigeon hole Principle $\left[\frac{n}{m}+1\right]$

people = 13 = n

$= \left[\frac{13}{12}+1\right]$

$= [1.8+1]$

$= [2.8] = 2$

(5) Show that If we allot 26 rooms to the student in a P.G hostel from numbered from 1 and 50 both inclusive Atleast two allotment are consicutively number

here $n = 50$

$m = 26$

Pigeon hole principle $= \left[\frac{n}{m} + 1\right]$

$= \left[\frac{50}{26} + 1\right]$

$= [1.92 + 1]$

$= [2.92]$

$= 2$ Ans

(1) - d
(2) - d
$\boxed{(3) - c}$
(4) - a
(5) - (d)
(6) - c
(7) - a
(8) - b
(9) - a
(10) - a
(11) - b
(12) - c

Graph Theory :- (I)

 # Graph :- A graph G Consist of two things

(i). A set $V = V(G)$ vertices, Points or Nodes of graph G

(ii). A set $E = E(G)$ of unordered points of vertices called edges of G

Denoted by $\boxed{G = G(V, E)}$

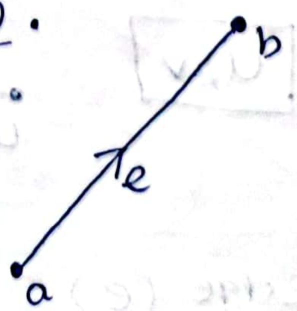
⇒ Each elements e of $E(G)$ is assign an unordered pair of (a, b) is called 'end vertices' of e

$$\boxed{\therefore\ e = (a, b)}$$

Two Types of Graph :-

1. Directed graph
2. Undirected graph

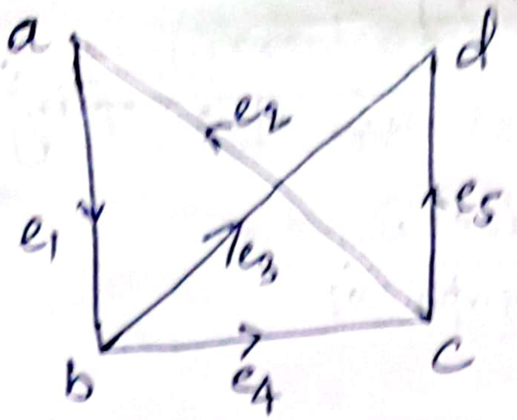
 # Directed Graph :- Directed graph is a graph in which each element of e of $E(G)$ is assign an ordered pair of (a, b) along with ~~ordere~~ Arrow.

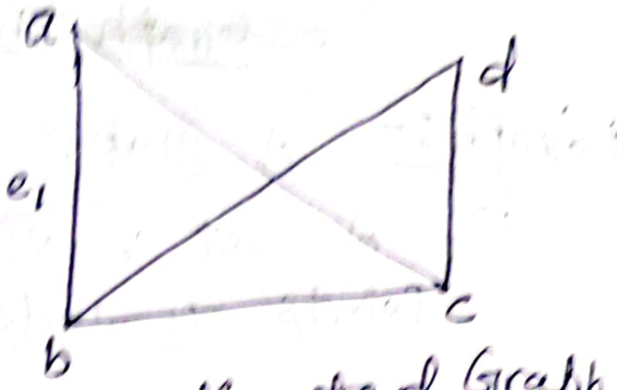


$e = (a, b)$

$\left\{ \begin{array}{l} a = \text{Initial vertex of } e \\ b = \text{Final vertex of } e \end{array} \right\}$

Final → Terminal





Directed Graph

$e_1 = (a, b)$

is an edge From a to b.

un-directed Graph

$e_1 = (a, b)$

OR $e_1 = (b, a)$.

e_1 is edge b/n a & b

No Matter direction Arrow.

Note:- In a Graph It doesn't Matter wether the line joining means any with two vertices is a straight line or any curve. (Longer or shorter).

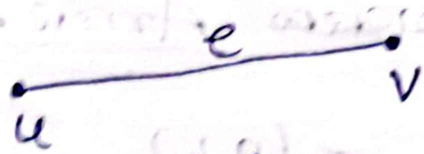
line joining → Straight Line

line joining → Curve

vertices and Edges :-

① Adjacent vertices :- Two vertices u, v are said to be adjacent if there is an edge (e) b/n them.

$\boxed{u, v}$



$e = (e, v)$

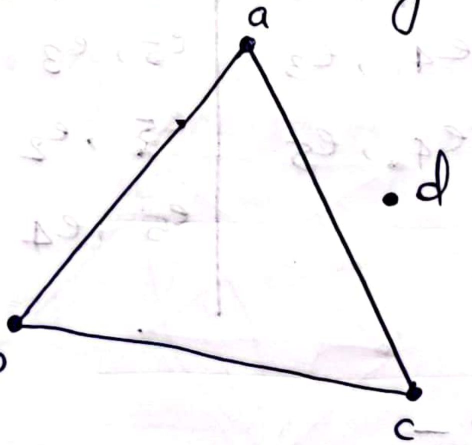
* The edge (e) is called incident on vertices (u, v)

② **Loop** :- An edge that incident from end into itself i.e to end at a same vertices is called loop.

e (u,u)
e(v,v)

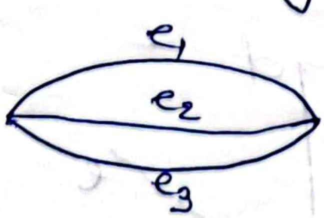
loops Represent the Reflexive Repersantation.
(OBious That)

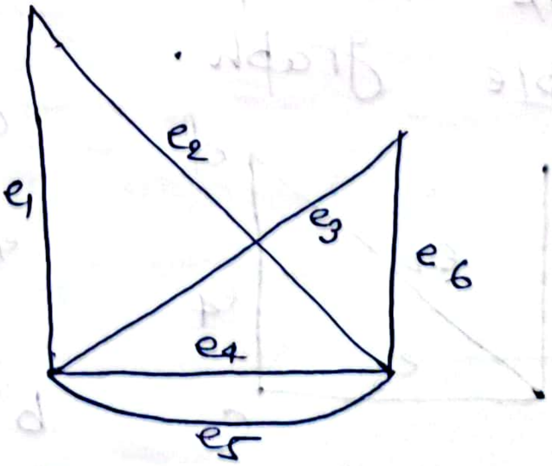
③ **Isolate vertex** :- A vertex of a Graph 'G' is called isolate vertex if it is not connected with any other vertices of G(V,E).



'd' is called 9solate vertex.

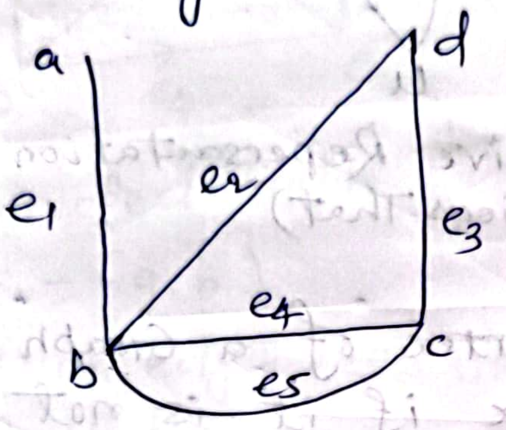
④ **Parallel Edge** :- if two or more edge that have same end vertices called Parallel edges.





e_4 and e_5 are Parallel edges.

⑤ **Adjacent edges** :- Two non parallel edges of a graph $G(V, E)$ are called adjacent if they have a common vertex.

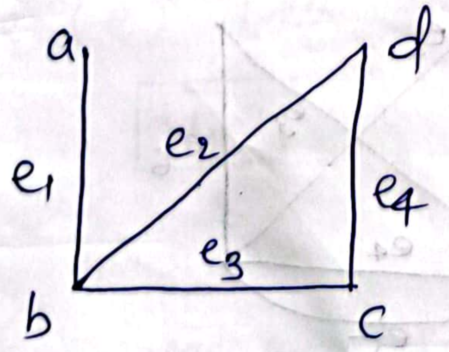


e_1, e_2
e_1, e_4
e_1, e_5 } Are Adjacent edges
$\because$ b is common pt.

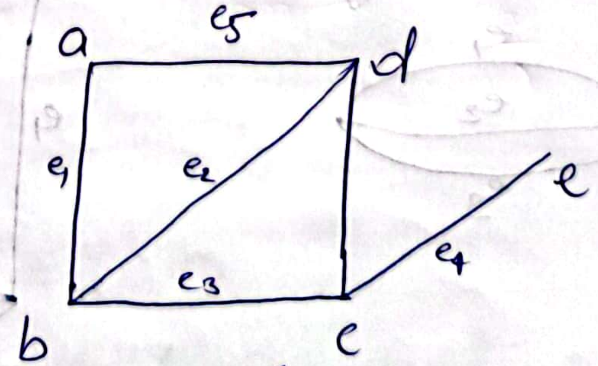
e_2, e_3	e_3, e_2	e_4, e_2	e_5, e_1
e_2, e_1	e_3, e_4	e_4, e_3	e_5, e_3
e_2, e_4	e_3, e_5	e_4, e_5	e_5, e_2
e_2, e_5			e_5, e_4

Types of Graph :-

① **Simple Graph** :- A Graph has nither loops nor parallel edges is called simple graph.

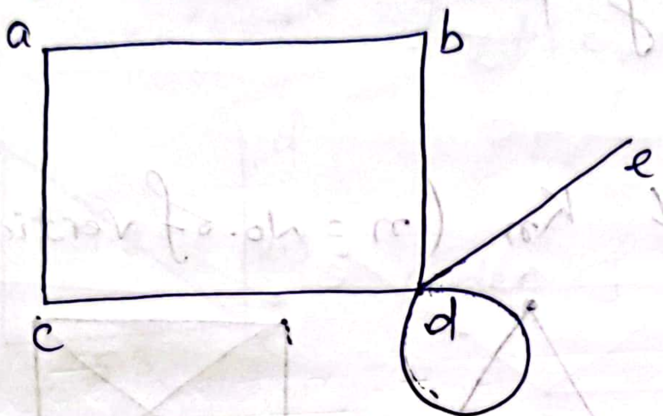


Simple Graph



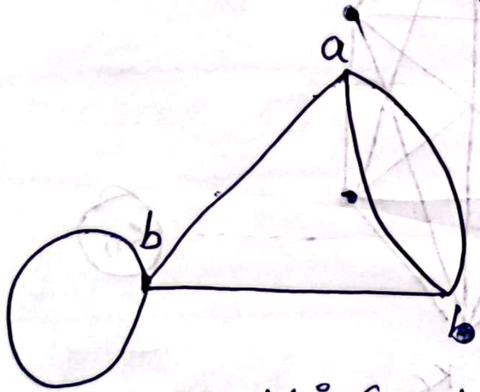
Simple Graph

② Multi-Graph :- A Graph that have either loop or parallel edge or both of them is called multi Graph.



Multi-Graph
or
General Graph

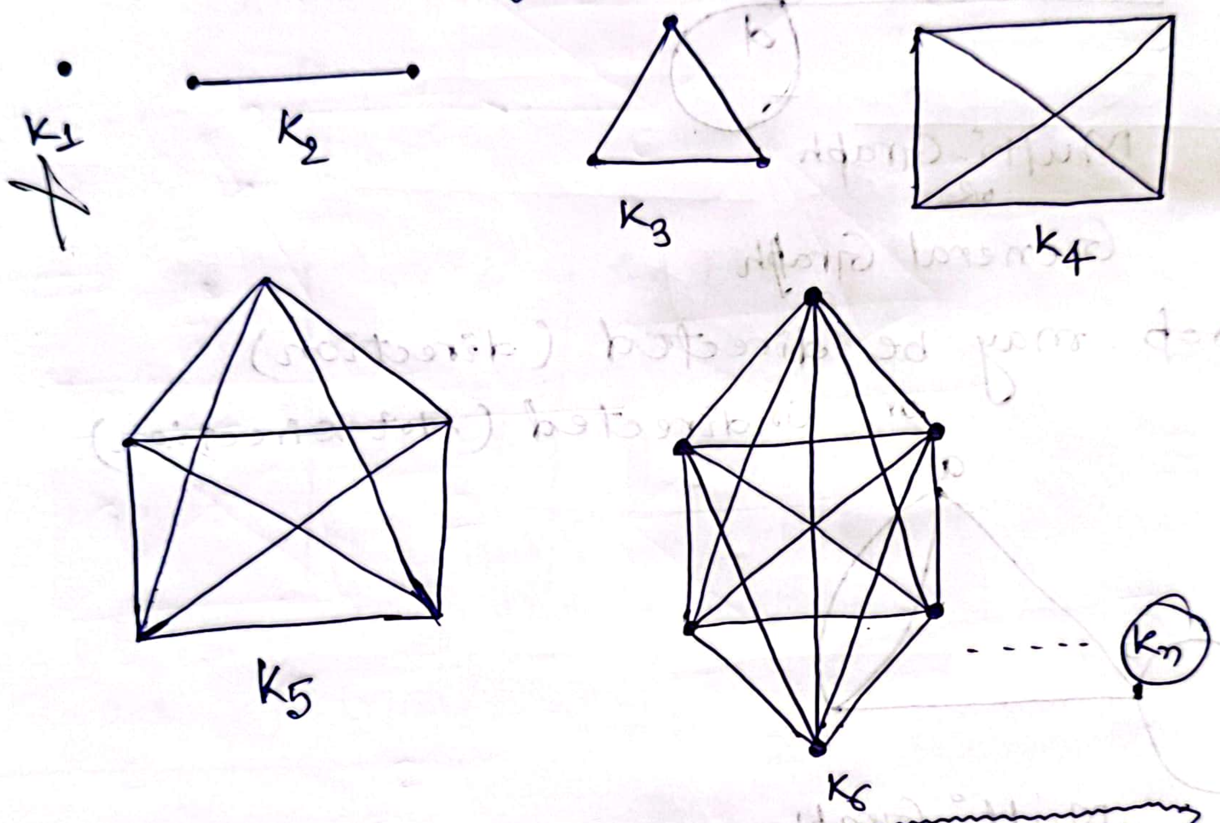
loop may be directed (direction)
or Undirected (Not direction)



Multi-Graph

③ <u>Complete graph</u> :- A Graph $G(V,E)$ is said to be complete, if there is an edge b/w every pair of vertices of edges.

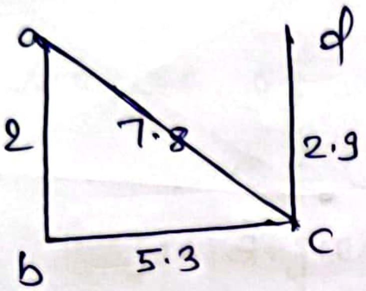
and denoted by K_n. (n = No. of vertices)



$$\boxed{{}^nC_2 = \frac{n(n-1)}{2}}$$

<u>Note</u> :- No. of Edges in $K_n = {}^nC_2$
(A Complete graph with n-vertices)

④ **weighted graph** :- Let G(V, E) be any graph, then G is said to be weighted graph if each edge is assigned a number called weight of the edge.



order of a graph $|V(G)$

Note :- The order of a graph is the no. of vertices in ~~which~~ vertex set of G. & denoted by V(G).

Degree of a vertex :-

① **In-degree** :- The in-degree of vertex x is defined the no. of edges for which 'V' is terminate (end) vertex.

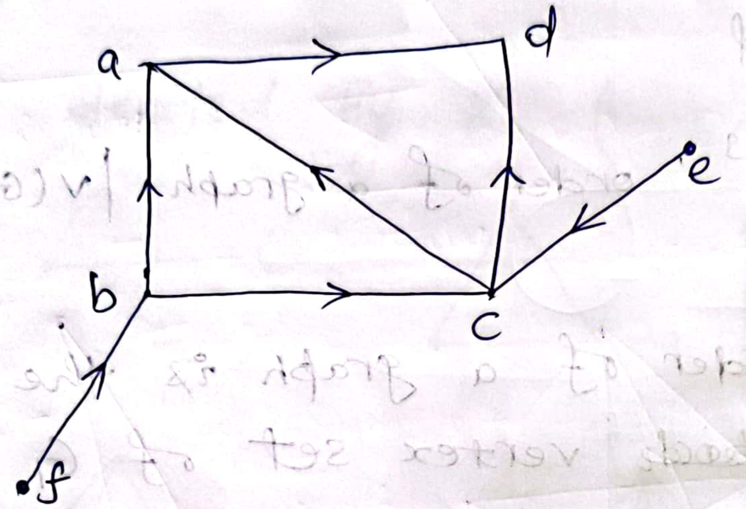
② **out-degree** :- The out degree of a vertex x is define the no. of edges for which 'V' is the initial vertex:

$$\boxed{deg(V) = \text{In-degree} + \text{out-degree}}$$ (Directed graph.)

for directed graph

For Un-directed graph :-

If G be a undirected graph, degree of vertex 'v' is the total no. of edges indicate on it.



Vertex	Indegree	outdegree	degree
a	2	1	3
b	1	2	3
c	2	2	4
d	2	0	2
e	0	1	1
f	0	1	1

★ The vertex whose indegee is 0 is called Source & the verstex whose out-degree is 0 is called Sink.

★ In privious fig e,f are Source.
d is Sink.

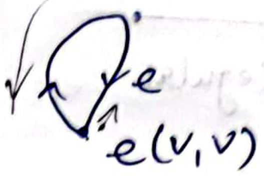
Pendent vertex :-

★ A vertex with degree one is called pendent.

★ e and f are pendent vertex.

Note :-

① Degree of a loop = 2



e = (v,v)

Indegree (v) = 1
out-degree (v) = 1

v e = (v,v) = 2

degree (v) = In + out
= 1 + 1
= 2

∴ degree of loop = 2

→ In a loop 1-vertex, one edge, degree = 2

② Degree of isolate vertex :- = 0

the degree of isolate vertex = 0

★ Because it doesn't connected to any vertex

Even or odd Parity :-

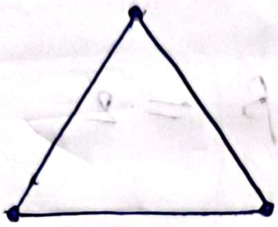
★ In a vertex of a Graph (G) is called even or odd . According

OR

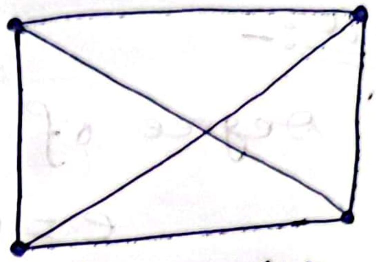
deg(v) is even or odd .

Regular Graph :- A Graph G in which the degree of ever vertex is same is called regular graph

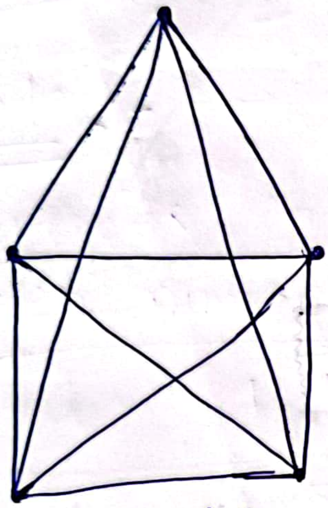
1-Regular



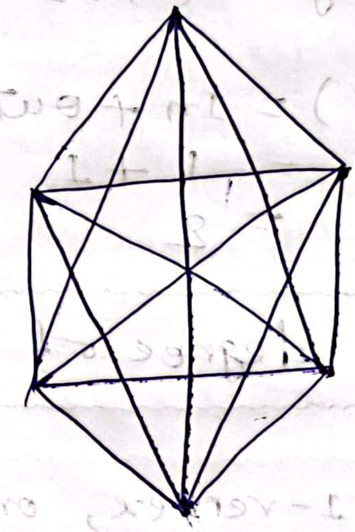
2-regular



3-regular



4-regular



5-Regular -- n

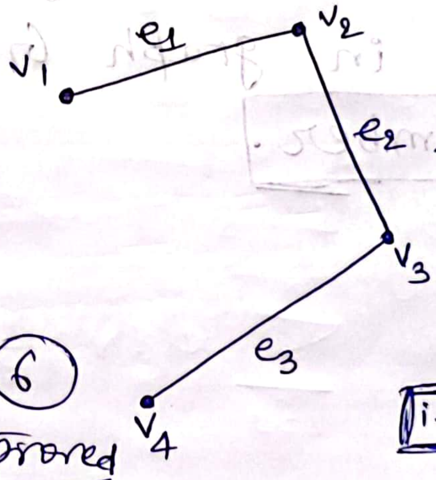
★ Degree of vertex in $K_n = (n-1)$.

★ Hand-shaking Theorem :- (H-S Theorem)

statement :- the sum of degree of the vertex in a graph G is equal to twice the number of edge.

$$\text{i.e}\quad \boxed{\sum_{i=1}^{n} d(v_i) = 2e}$$

proof :- let 'e' be the any edge in the graph G btn vertices V_1 and V_2.



$d(v_1) = 1(e_1)$

$d(v_2) = 2(e_1\ e_2)$

$d(v_3) = 2(e_2\ e_3)$

$d(v_4) = 1(e_3)$ $\Big|\; = 6$

Total degree = $\underline{\underline{6}}$ $\underline{\underline{\text{proved}}}$

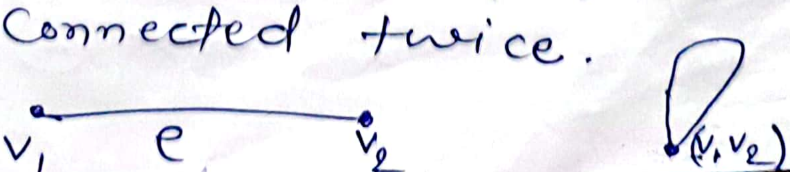
edge = 3
dege = 2·edge
= 2·3
= 6

$\boxed{\text{in-out}}$ Remember

★ when we Count the degree of vertex V_1 and V_2 then the edge is counted the twice.

→ once in degree of v_1 and again in degree of v_2

★ more over if V_1 and V_2 are identical then that will be a loop, then again e is connected twice.



* Hence every edge in the graph G is counted twice and so the sum of degree of all vertices is twice of edge.

i.e $$\boxed{\sum_{i=1}^{n} d(v_i) = 2e}$$

<u>Note</u>:- The sum of degree of all vertices in graph G is always even number.

Q. Prove that in a graph sum of odd degree vertex is even in number.

Proof :- Let $v_1, v_2, v_3 \ldots v_n$ be n-vertices and e be the no. of edges in graph G,

∴ By H-S Theorem

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$$\Rightarrow \sum_{\text{even}} d(v) + \sum_{\text{odd}} d(v) = 2e$$

$$\Rightarrow \text{Even} + \sum_{\text{odd}} d(v) = \text{even}$$

$$\Rightarrow \sum_{\text{odd}} d(v) = \text{even} - \text{even}$$

$$\Rightarrow \sum_{\text{odd}} d(\cancel{v}) = \text{even}$$

∴ Sum of odd degree vertex is even.

∴ the odd degree vertices is Even in number

proved

③ Prove that the maximum number of edges in a simple graph G with n-vertices is $\frac{n(n-1)}{2}$ $^nC_2$.

$$\left\{ e \leq \frac{n(n-1)}{2} \right\}$$

Proof :- let $v_1, v_2, v_3 ---- v_n$ be n-vertices of a simple graph G

and $e_1, e_2, --- e_n$ be the no. of edges in a graph G.

$\therefore$ By H-S theorem

$$\sum_{i=1}^{n} d(v_i) = 2e \quad \text{--- (1)}$$

$\because$ Graph is simple

$\therefore$ The maximum degree of a vertices $(n-1)$.

$\therefore$ the sum of max^m degree of all the n-vertices

$= (n-1) + (n-1) + ---$ n terms

$= n(n-1)$ --- (2)

From (1)

$$\sum_{i=1}^{n} d(v) = 2e$$

Now from (2)

$$\sum_{i=n}^{n} d(v) = n(n-1)$$

So By (1) and (2)

$$\Rightarrow 2e = n(n-1)$$

$$\Rightarrow \boxed{e = \frac{n(n-1)}{2}}$$

proved

Q. (4) prove that the no. of edges in a Complete graph G with n-vertices is $\frac{n(n-1)}{2}$.

proof :- Let $v_1, v_2 --- v_n$ be n-vertice and $e_1, e_2 —— e_n$ be n-edges of a graph G.

∴ By H-S Theorem

$$\sum_{i=1}^{n} d(v_i) = 2e \quad — (1)$$

∴ Graph is Complete

∴ the sum of degree of a vertex = (n-1)

∴ the sum of degree of all n-vertices

$= (n-1) + (n-1) + —— \text{upto } n \text{ terms}$

$= n(n-1) \quad — (11)$

From (1) & (11)

$$\Rightarrow n(n-1) = 2e$$

$$\boxed{e = \frac{n(n-1)}{2}}$$

proved.

∴ the no. of edges in complete graph

$G = \boxed{\frac{n(n-1)}{2}}$.

Q. Find the no. of edges in a graph G having 5-vertices with degree 3, 3, 3, 4, 1 also draw the graph

Sol^n :- Given that

$V = 5$

$d(v_1) = 3$, $d(v_2) = 3$, $d(v_3) = 3$

$d(v_4) = 4$, $d(v_5) = 1$

let $v_1, v_2, v_3 \text{ --- } v_n$ be n-vertices and e be the edge of graph G

∴ By H-S Theorem

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$$\Rightarrow \sum_{i=1}^{5} d(v_i) = 2e$$

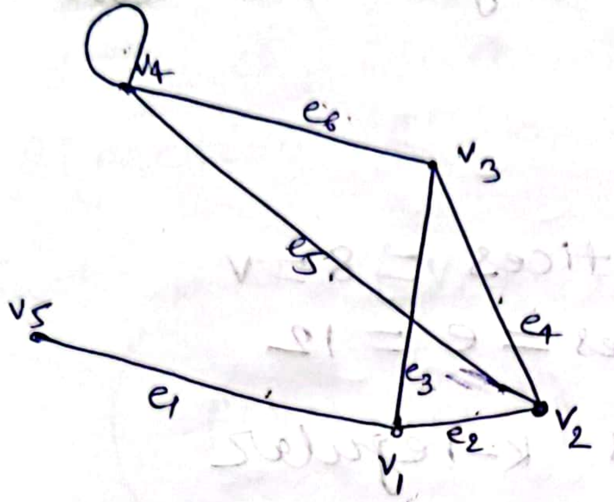
$$\Rightarrow (d(v_1) + d(v_2) - - + d(v_5)) = 2e$$

$$\Rightarrow 3+3+3+4+1 = 2e$$

$$\Rightarrow 14 = 2e \Rightarrow e = 7$$

CamScanner

$\therefore$ No. of edges $= e = 7$



loop has degree 2.

Q. A Graph has 21-edges, 3-vertices of degree 4 and remaining of degree 3. find the no. of vertices.

Soln :- Given that

No. of edges $e = 21$

let No. of vertex $= n$

3-vertices of degree 4 & Remaing of 3

$\therefore$ Remaing $(n-3)$ are of $d=3$.

$\therefore$ By H-S theorem

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$$\Rightarrow (4+4+4) + (3+3+ \cdots (n-3)\text{ terms}) = 2e$$

$$\Rightarrow 12 + 3(n-3) = 2 \times 21$$

$$\Rightarrow 12 + 3n - 9 = 42$$

$$\Rightarrow 3 + 3n = 42$$

$$\Rightarrow 3n = 39$$

$$\boxed{n = 13}$$

Q. Find K, if K-regular graph has 8-vertices and 12-edges and also draw the graph.

Soln :- Given that

No. of vertices $= 8 = v$

No. of edges $= e = 12$

$\because$ graph is K-regular

$\therefore$ degree of every vertex $= K$

$\therefore$ By H-S. Theorem

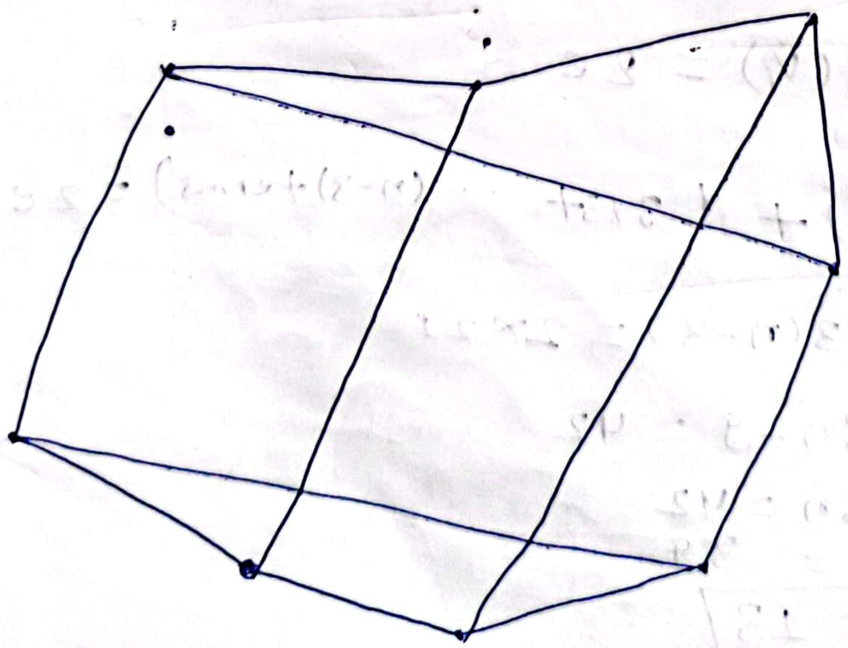
$$\sum_{i=1}^{n} d(v_i) = 2e$$

$\Rightarrow K + K + K$ — upto 8 terms $= 2e$

$\Rightarrow 8K = 2 \times 12$

$\Rightarrow 8K = 24$

$\Rightarrow \boxed{K = 3}$



Isomorphic graph :- Two graphs $G_1(V, E)$ and $G_2(V, E)$ are said to be Isomorphic to each-other if there exist Bijection 'f' from $V(G_1)$ onto $V(G_2)$

10 marks
m-term

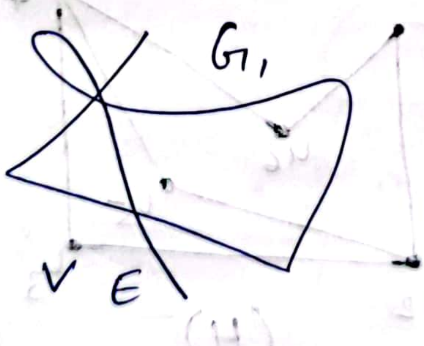
Such that $(v_i, v_j) \in E(G_1)$ iff

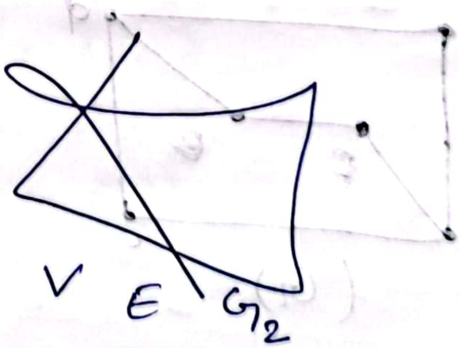
$$(f(v_i), f(v_j)) \in E(G_2)$$

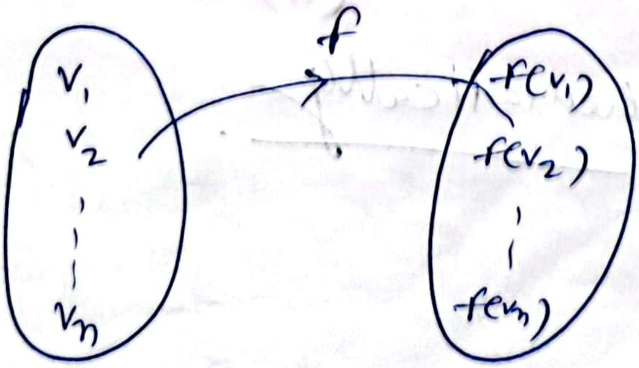
& denoted by $\boxed{G_1 \cong G_2}$

In other words:-

Two graph are Isomorphic to each-other if $\exists$ a one-one correspondence btn their vertices and edges such that incidence Relationship is preserved.







one-one $\longrightarrow$ 1-element with 1-img.

onto $\longrightarrow$ No extra img.
only 1-Image with 1e.

$$f : V(G_1) \longrightarrow V(G_2)$$

$\longrightarrow$ one-one

$(a,b) = e \in E(G_1)$

$(f(a), f(b)) = e' \in E(G_2)$

$\downarrow \quad \downarrow$

$u_1 \quad u_2$

edge to edge

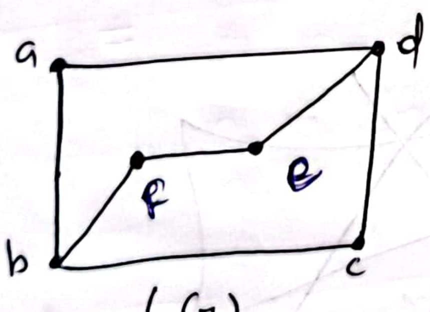
Note:- two graph which are Isomorphic to each other then

1. Same no. of vertices
2. Same no. of edges
3. Same degree sequences
4. Mapping (i) each pt degree with the other same pt (ii) Edges

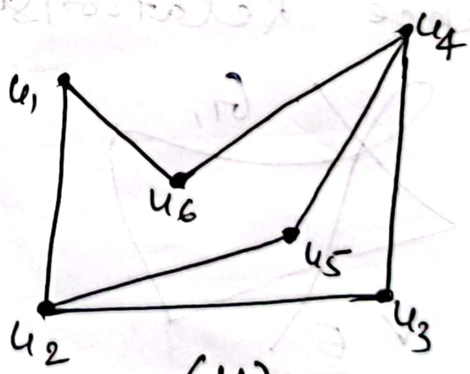
next page

Q. Show wether the graph is

(1) Isomorphic or not.



(G)



(H)

proof Mathematically.

Soln :- Given

	(G)	(H)
Vertex	6	6
Edge	7	7
degree	2, 2, 2, 2, 3, 3 a, c, f, e, b, d	2, 2, 2, 2, 3, 3 u_1 u_6 u_5 u_3 u_2 u_4

Since G, H have same vertices, edges and degree sequence. So G, H may or may not be isomorphic

for sure this

we define a mapping of f

From V(G) to V(H) as.

$f(a) = u_5$
$f(b) = u_2$
$f(c) = u_3$
$f(d) = u_4$
$f(e) = u_6$
$f(f) = u_1$

for mapping draw false as same graph.

To prove :- wether 'f' preserve incidence Relationship, we examine the ~~matrices~~ Adjacency matrices.

Now

Adjacency Matrix for graph G

	a	b	c	d	e	f
a	0	1	0	1	0	0
b	1	0	1	0	0	1
c	0	1	0	1	0	0
d	1	0	1	0	1	0
e	0	0	0	1	0	1
f	0	1	0	0	1	0

fill this value By seeing Edge b/w these two points

खेल खत्म

$$\therefore A_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

done

sames as end

Mapping	u_5	u_2	u_3	u_4	u_6	u_1
$f(a) = u_5$	0	1	0	1	0	0
$f(b) = u_2$	1	0	1	0	0	1
$f(c) = u_3$	0	1	0	1	0	0
$f(d) = u_4$	1	0	1	0	1	0
$f(e) = u_6$	0	0	0	1	0	1
$f(f) = u_1$	0	1	0	0	1	0

$$\therefore A_H = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}_{6\times 6}$$

Here Adjacency Matrices are same

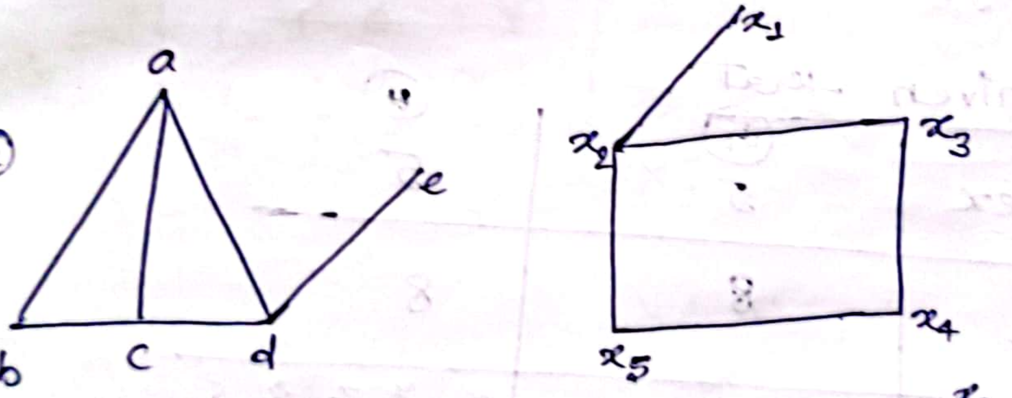
$$\therefore \boxed{A_G = A_H}$$

$\therefore f$ preserve the incidence Relation

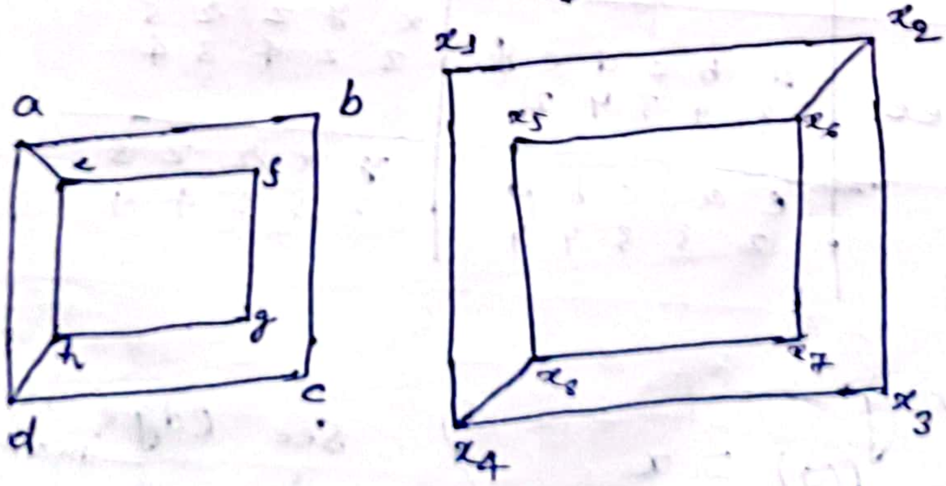
So $\boxed{G \cong H}$

proved

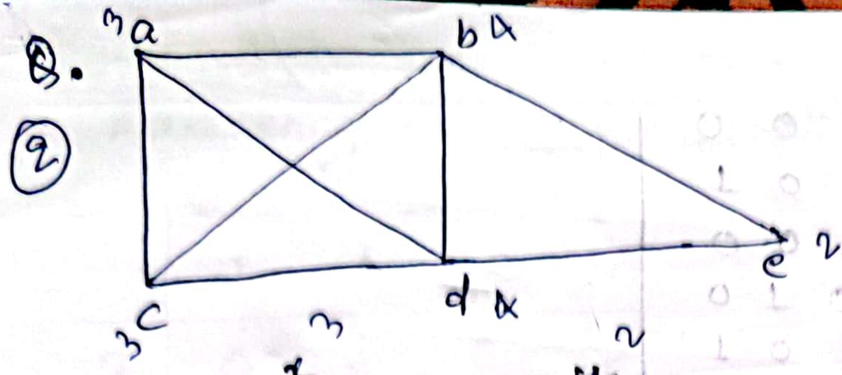
Q①

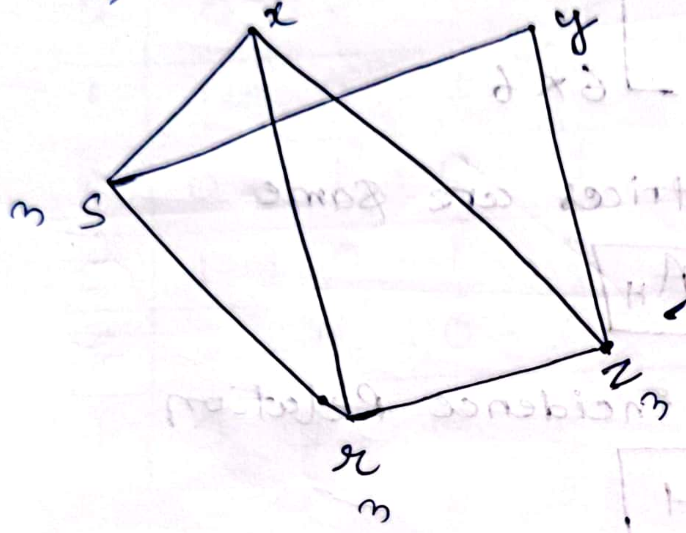


Q①

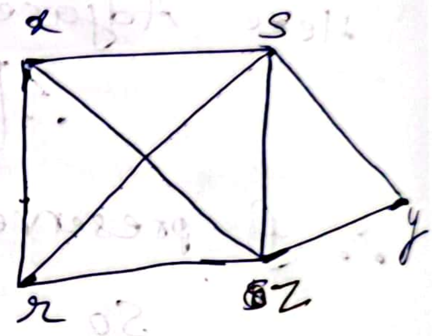


Q. ②





Rough fig



Soln :-

Given that

	G	H
vertex	5	5
Edge	8	8
Degree	a b c d e 3 4 3 4 2	x y z r s 3 2 4 3 4
	e a c b d 2 3 3 4 4	y x r z s 2 3 3 4 4

Now mapping —

$f(a) = x$
$f(b) = s$
$f(c) = r$
$f(d) = z$
$f(e) = y$

See edge point map

a | edge b — x | edge s (should be)

b | c — s | r

Now Adjacency Matrix for A_G

	a	b	c	d	e
a	0	1	1	1	0
b	1	0	1	1	1
c	1	1	0	1	0
d	1	1	1	0	1
e	0	1	0	1	0

$$\therefore A_G = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}_{5\times 5}$$

See it is symmetric matrix $\boxed{A = A'}$ that's why

Now Adjacency Matrix for H

	x	s	r	z	y
f(a) = x	0	1	1	1	0
f(b) = s	1	0	1	1	1
f(c) = r	1	1	0	1	0
f(d) = z	1	1	1	0	1
f(e) = y	0	1	0	1	0

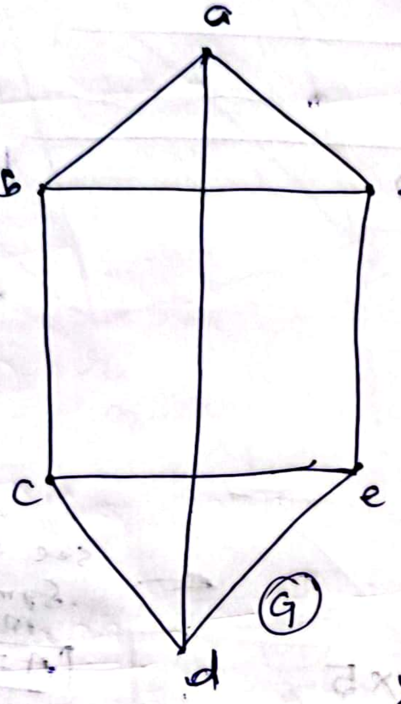
Here

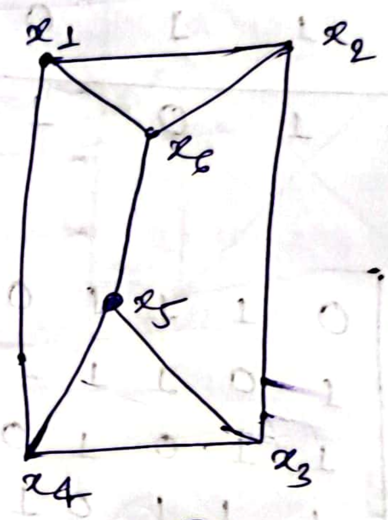
Adjacency Matrices are Same.

$\because A_G = A_H$

$\therefore \boxed{G \cong H}$ proved

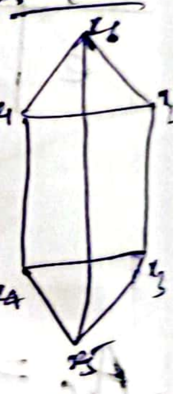
Q. 3





yes $G \cong H$

false



Soln :-

Given that

	G	H
vertex	6	6
Edge	9	9
Degree	a b c d e f 3 3 3 3 3 3	x_1 x_2 x_3 x_4 x_5 x_6 3 3 3 3 3 3
Mapping	$f(a) = x_6$ $f(b) = x_1$ $f(c) = x_4$	$f(d) = x_5$ $f(e) = x_3$ $f(f) = x_2$

Now Adjacency Matrix for graph G

	a	b	c	d	e	f
a	0	1	0	1	0	1
b	1	0	1	0	0	1
c	0	1	0	1	1	0
d	1	0	1	0	1	0
e	0	0	1	1	0	1
f	1	1	0	0	1	0

So

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}_{5\times 6}$$

Now Adjacency Matrix for graph H.

	x_6	x_1	x_4	x_5	x_3	x_2
$f(a)=x_6$	0	1	0	1	0	1
$f(b)=x_1$	1	0	1	0	0	1
$f(c)=x_4$	0	1	0	1	1	0
$f(d)=x_5$	1	0	1	0	1	0
$f(e)=x_3$	0	0	1	1	0	1
$f(f)=x_2$	1	1	0	0	1	0

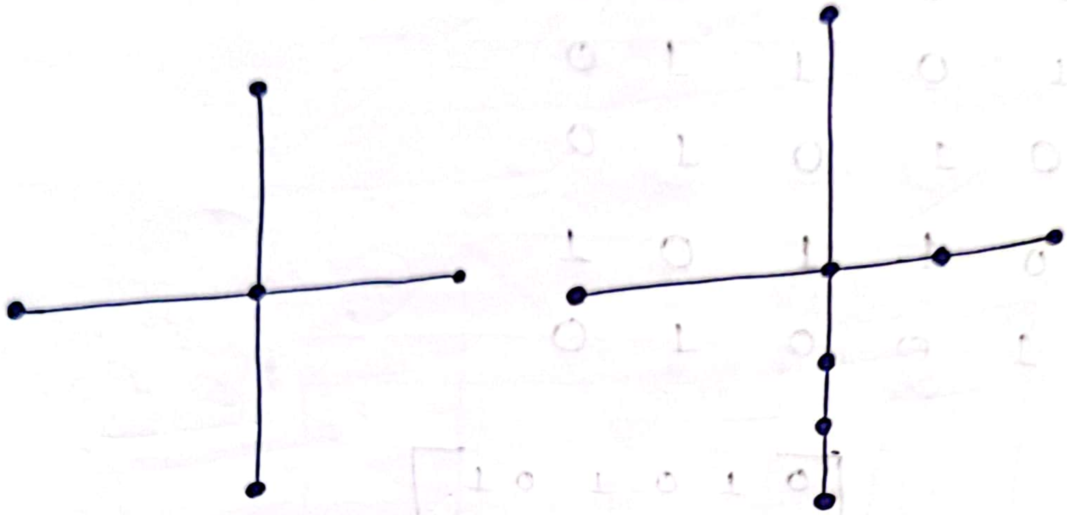
$$\therefore A_H = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}_{6\times 6}$$

$A_G = A_H$

$\therefore \boxed{G \cong H}$

proved

Homomorphic Graph :- Two Graph is said to be homoomorphic to each-other if one is



(a) (b)

(b) is homeomorphic to (a)

Sub-Graph :- Let $G(V,E)$ and $H(V,E)$ be the two graph

1. if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$

   then H is Sub-graph of G

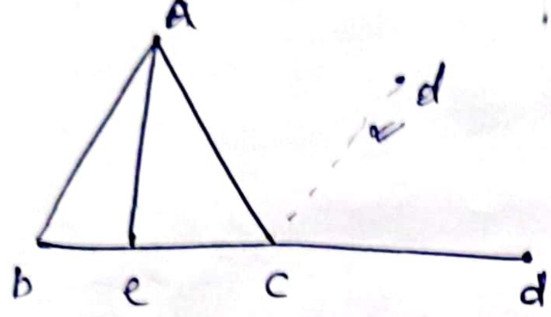
2. if $V(H) \subset V(G)$ and $E(H) \subset E(G)$

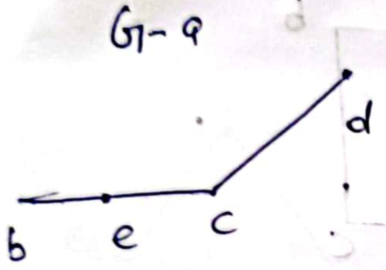
   $\therefore$ H is proper sub-graph

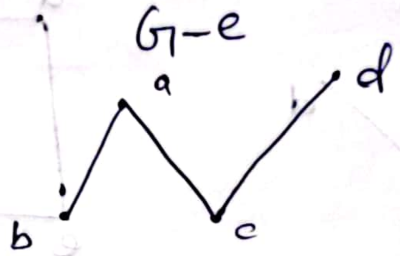
3. if $V(H) = V(G)$ and $E(H) \subset E(G)$

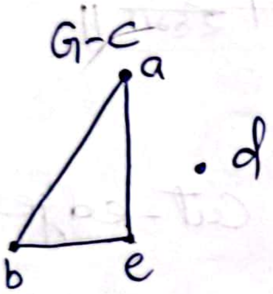
   H is spanning Subgraph of E

obtain by removing the vertex (V) from V(G) also the edge which are incidence of v



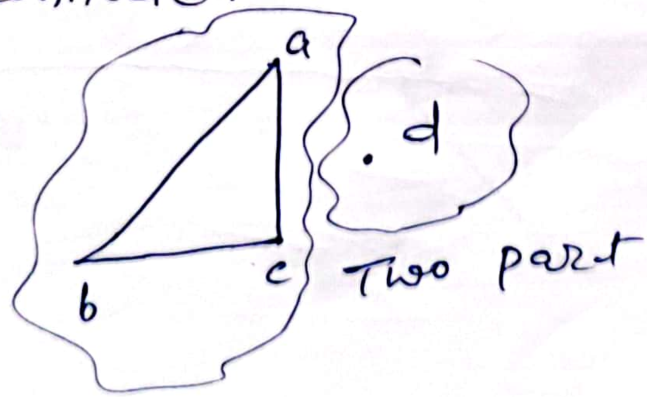




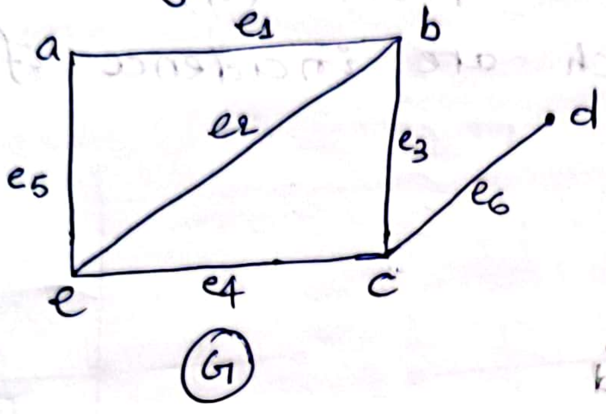


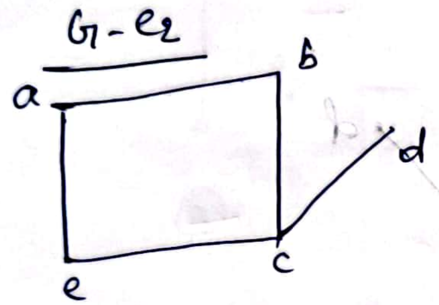
*** cut-vertex :-** A vertex v of a graph G cut vertex if G-v is disconnected graph

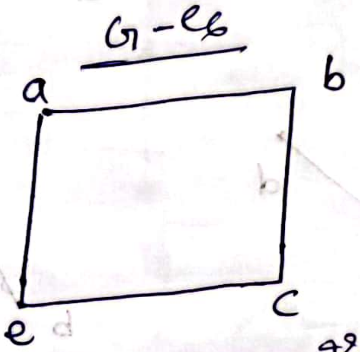
Disconnected → divided into 2-parts.



<u>G-e</u> : G-e is a subgraph of G obtaining by removing the edge 'e' from it.







याद रहे केवल edge को हटाना है / vertex रहेगा ॥

<u>Cut edge</u> :- An edge 'e' is disconnected if G-e is disconnected. e6 is cut-edge.

Operations of graph :-

⇒ Let $G_1(V,E)$ and $G_2(V,E)$ be two graph

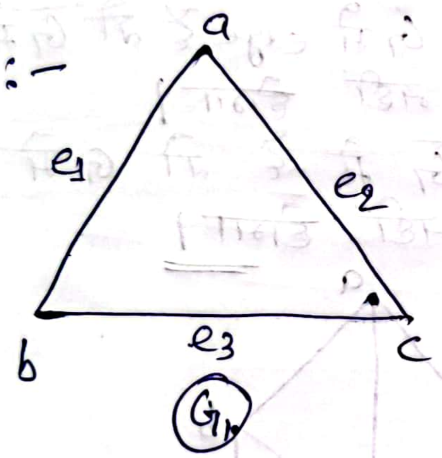
★ **Union** :-

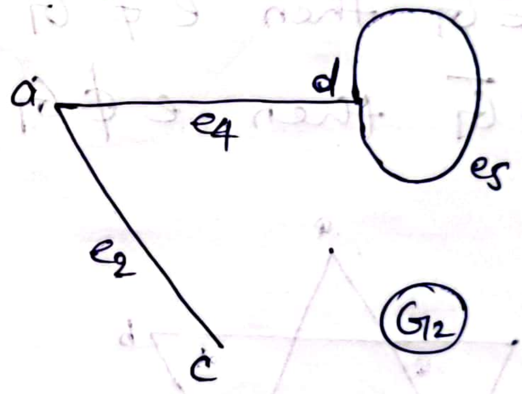
$$G_1 \cup G_2 = G\big(V(G_1) \cup V(G_2),\ E(G_1) \cup E(G_2)\big)$$

★ **Intersection** :-

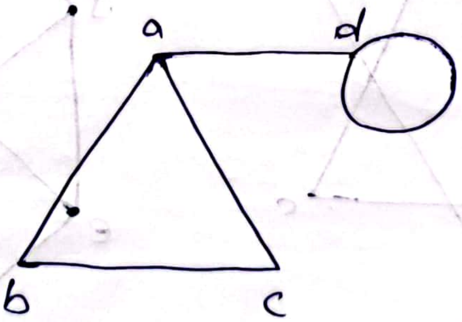
$$G_1 \cap G_2 = G\big(V(G_1) \cap V(G_2),\ E(G_1) \cap E(G_2)\big)$$

Ex :-

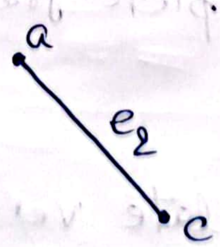




① $G_1 \cup G_2 =$



② $G_1 \cap G_2 =$

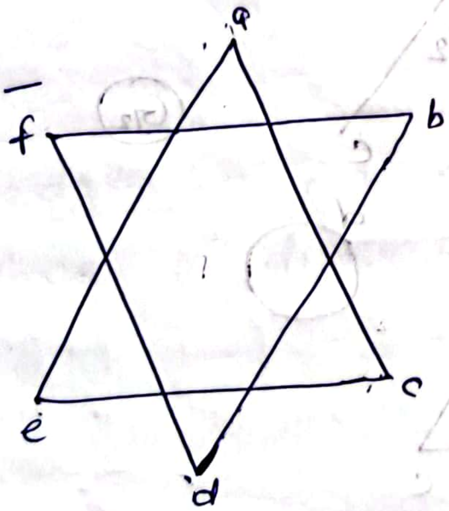


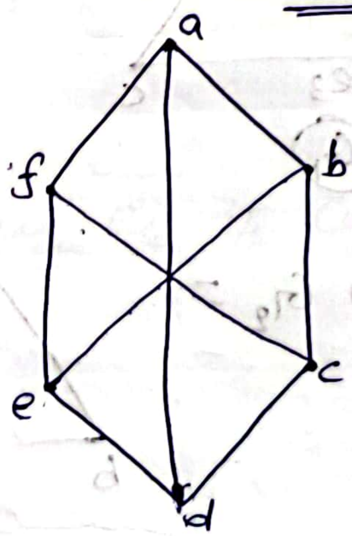
★ **Compliment** :- the Compliment of a graph G is denoted by $\overline{G}$.
and is defined as a simple graph having the same no. of vertices, together with the edges satisfying the property if there is an edge btn two vertices in G then there will be no edge btn in the $\overline{G}$. (vice-verse).

if $e \in G$ then $e \notin \overline{G}$
if $e \in \overline{G}$ then $e \notin G$

G मे edge है तो $\overline{G}$ मे नहीं होगा।
$\overline{G}$ मे है तो G मे नहीं होगा।

Eg :-





① $G_1 \cup G_2$ = Complete graph (K_n)

② $G_1 \cap G_2$ = Null

vertex → isolate vertex.

(3) If G be a graph with n-vertices (v-vertices) and $d(v)$ in G is 'k' then $d(v)$ is in $\overline{G}$ is $\boxed{n-k-1}$. $\boxed{V-d-1}$

n = vertices
k = degree.

* $d(c) = 2$ in G
and $n = 6$ (privious fig)

then in $\overline{G}$ $= n-k-1$
$= 6-2-1$
$= 6-3$
$= 3$

Q. Can a graph with 7 vertices isomorphic To a Compliment Justisfy your Answer -

Soln :-

Gin that

No. of vertices in Graph $G = 7$

i.e $n = 7$

Let $\overline{G}$ be the Compliment of G

$\therefore$ we have if $e \in G$ then $e \notin \overline{G}$

$\therefore$ Total no. of edges in G and $\overline{G}$

= No. of edges in Complete Graph

$$= \frac{n(n-1)}{2}$$

$$= \frac{7(7-1)}{2}$$

$$= 7 \times 3$$

$$= \underline{21}$$

$\Rightarrow$ No. of edges in $G \neq$ No. of edges in $\overline{G}$.

($\because$ 21 is odd)

{we can not divide it}

$\therefore$ G is not isomorphic to $\overline{G}$

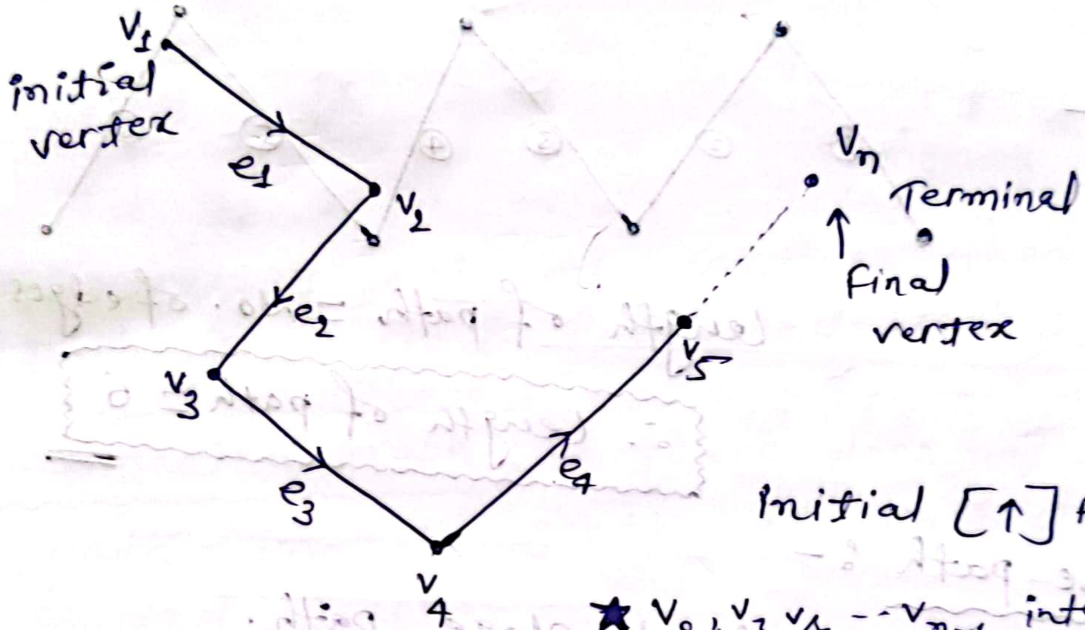
$$\boxed{G \ncong \overline{G}}$$

Justify

Walk :- A walk in graph G is a finite sequence whose terms are alternatively vertices and edges.

$$W = v_1, e_1, v_2, e_2, v_3, e_3 \cdots e_{n-1} v_n$$

= vertex, edge, rentex, edge ------



Initial [↑] final

★ $v_2, v_3, v_4 \cdots v_{n-1}$ internal vertices

Note :- In a walk every edge appear only once but a vertex may appears more than once.

[Edge Repeat – not walk
vertex – Repeat हो सकता है]

openwalk :- n vertices are different then it is called openwalk [$v_1 \neq v_n$]

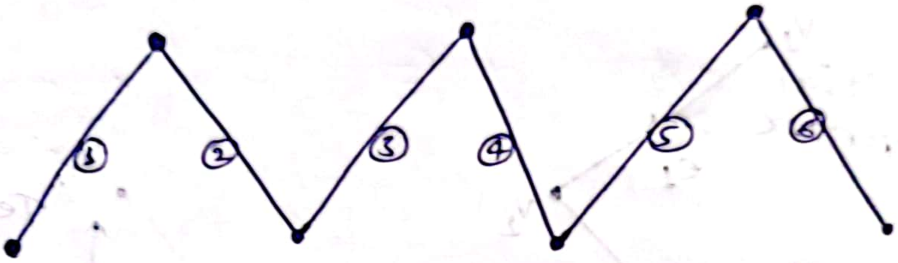
closewalk :- n vertices are same then called closewalk [$v_1 = v_n$]

Path :- An openwalk is called path in which every vertex appears once.

$\boxed{\text{vertex, edge} \longrightarrow \text{only once}}$

Length of path :- No. of edges in the sequence of path is called length of path.

Eg :-



length of path = No. of edges = 6

$\boxed{\therefore \text{Length of path} = 6}$

★ **close path :-**

A cycle is called close path.

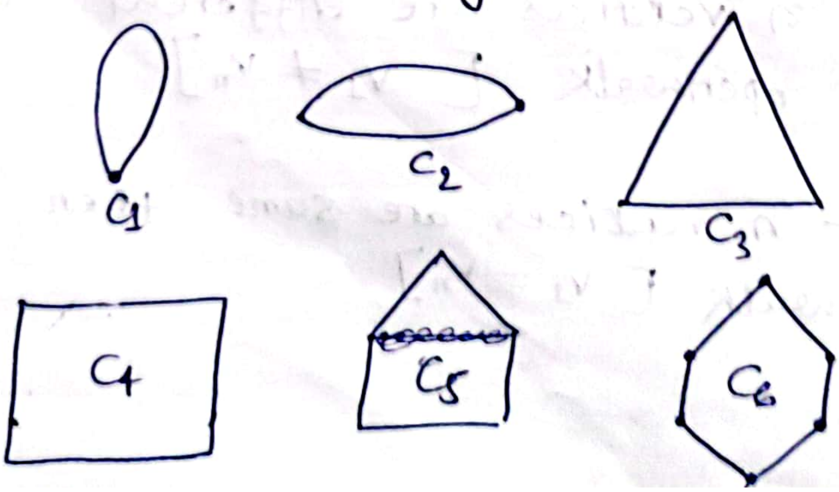
close path (cycle / circuit / polygon / ____)

n - cycle :-

a cycle with length n.

No. of edge in cycle $= n$

denoted by C_n



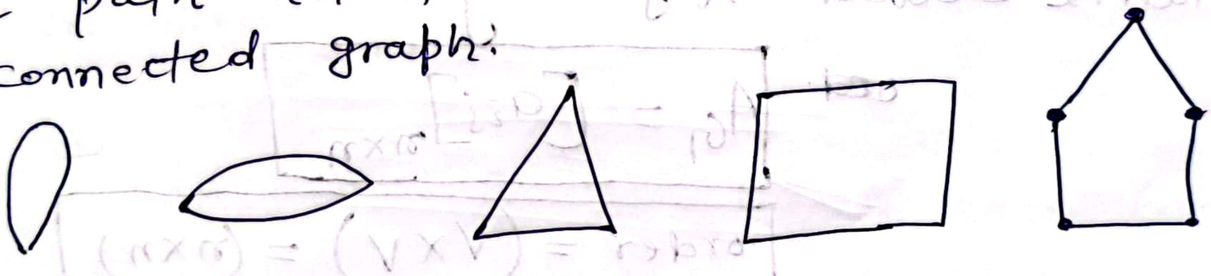
Note :- ① A self loop is a cycle of length 1.

② The degree of every vertex in a cycle is 2.

★ ~~connected path~~ :-

★ **Connected graph** :- A graph G is said to be connected if there exists atleast one path between every pair of vertices then the graph is called connected graph.

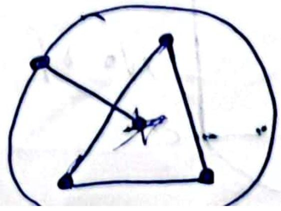
★ **Dis-connected graph** :- there does not exists any path in the graph is called disconnected graph.



← connected Graph →

← Dis connected graph →

Components (parts / टुकड़े) :- Each connected subgraph (part) of a disconnected graph are called components.



Component 2

Note :- A Connected graph has 1-Component i.e graph itself.

Matrix Representation of Graph :-

① Adjacency Matrix

② Incidence Matrix.

① Adjacency Matrix :- ①(undirected graph)

Let $G(E,V)$ be any graph having n-vertices then G is representing $n \times n$ matrix called Adjacency matrix.

$$\boxed{A_G = [a_{ij}]_{n \times n}}$$

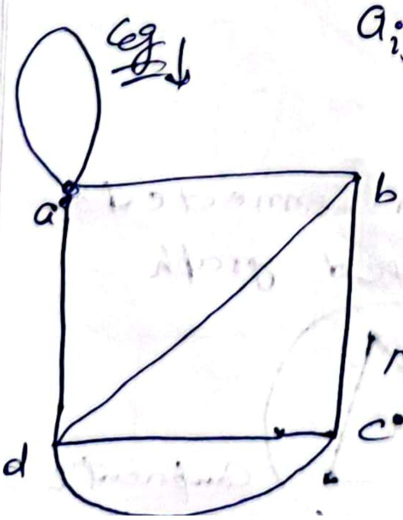
$$\boxed{\text{order} = (V \times V) = (n \times n)}$$

$$= (\text{vertices} \times \text{vertices})$$

★ Always A_G will be Sq. matrix.

$$a_{ij} = \begin{cases} k : k \text{ no. of edges b/w the vertices } a_i \text{ \& } a_j \\ 0 : \text{Otherwise.} \end{cases}$$

eg ↓



No. of vertices = V = 4

order = V×V = 4×4

= 4×4

	a	b	c	d
a	1	1	0	1
b	1	0	1	1
c	0	1	0	2
d	1	1	2	0

Creating of Table
logic is a—a মধ্যে edge,
i.e edge মধ্যে Two points

$$\therefore A_G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}_{4\times 4}$$

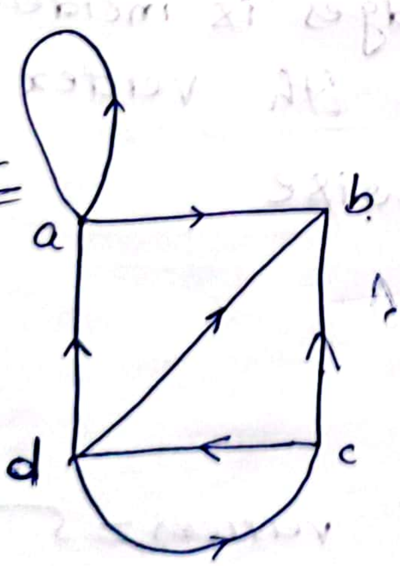
A_G is a Symmetric Matrix (Un-directed graph)

(2) Un-directed graph :-

$$A_G = [a_{ij}]_{n\times n}$$

$$a_{ij} = \begin{cases} K : K \text{ no. of edges from } a_i \text{ to } a_j \\ 0 : \text{otherwise.} \end{cases}$$

Eg



No. of vertices = 4
No. of A_G = 4×4
order = $v \times v$
= 4×4

	a	b	c	d
a	1	1	0	0
b	0	0	0	0
c	0	1	0	1
d	1	1	1	0

$$\therefore A_G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}_{4\times4}$$

★ In directed graph A_G will not Symmetric Graph.

② Incidence Matrix :–

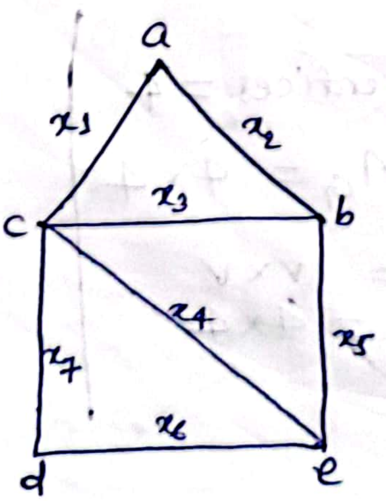
let G be a graph in which m–vertices and n– edges m x n matrix

$$\boxed{M = [a_{ij}]_{m\times n}}$$

$$\boxed{\text{order} = V \times E}$$

$$a_{ij} = \begin{cases} 1 : & \text{Ith edges is incident on ith vertex.} \\ 0 : & \text{otherwise} \end{cases}$$

Eg :–



No. of vertices = 5
No of edges = 7

order = m x n
= V x E = 5 x 7

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$d(V)$
a	1	1	0	0	0	0	0	2
b	0	1	1	0	1	0	0	3
c	1	0	1	1	0	0	1	4
d	0	0	0	0	0	1	1	2
e	0	0	0	1	1	1	0	3
Column Total	2	2	2	2	2	2	2	

$$M = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}_{5\times 7}$$

+1 out

−1 in

$M =$

Bipartite Graph :- if the vertex set of graph G can be partician in two disjoint set V_1 and V_2.

$$V(G) \to V_1, \; V_2$$

$$V_1 \cup V_2 = V$$

$$\star \; V_1 \cap V_2 = \phi \text{ (disjoint)}$$

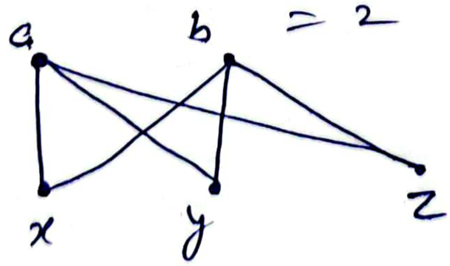
Such that every edge in G join a vertex in V_1 with a vertex in V_2 then graph is called Bipertite Graph.

Complete-Bipartite Graph :-

Every vertex in V_1 is connected with every vertex in V_2

and denoted by $K_{m,n}$.

$K_{2,3}$ means



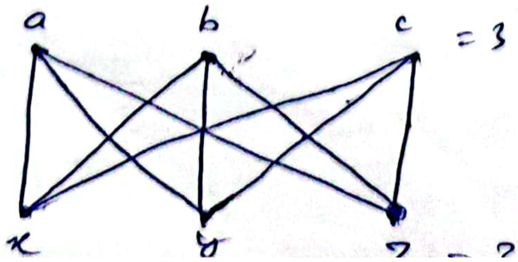
$V_1 = \{a, b, c\}$

$V_2 = \{u, v, x, y\}$

$V(G) = V_1 \cup V_2$

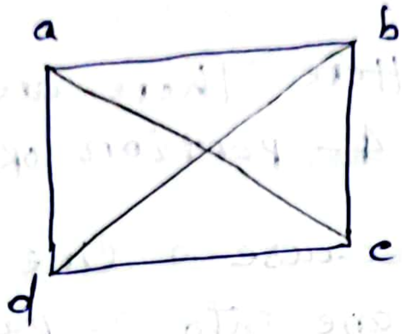
$= \{ \_\_\_\_ \}$

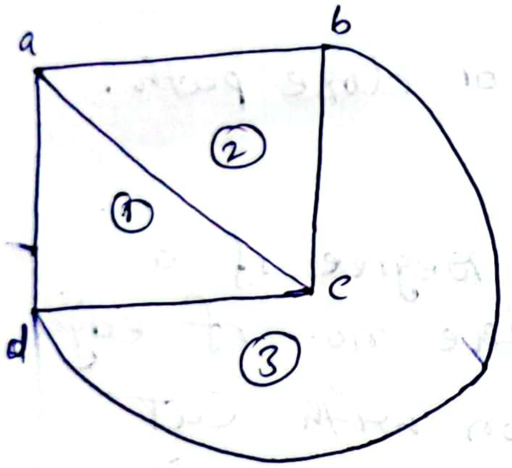
$K_{3,3}$ =



Planer graph :- A graph G is said to a planar graph if it can we re,drawn in such a way that no edge cross each other.

Ex :- K_4

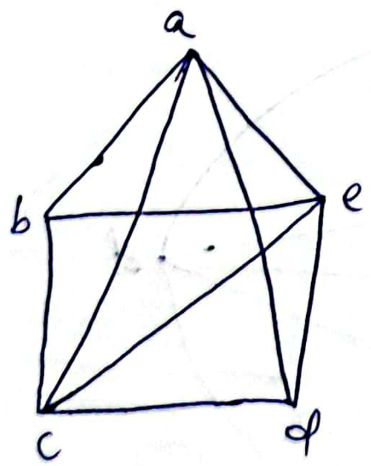


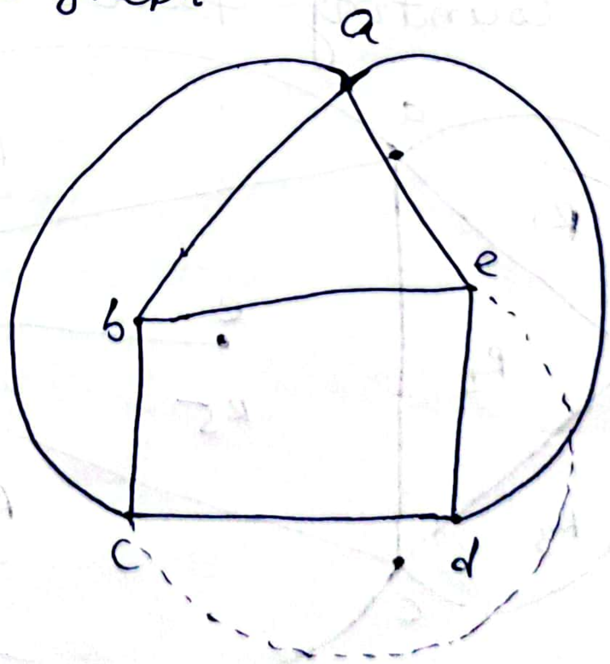


→ 4 portion

K_4 is planar graph

K_5



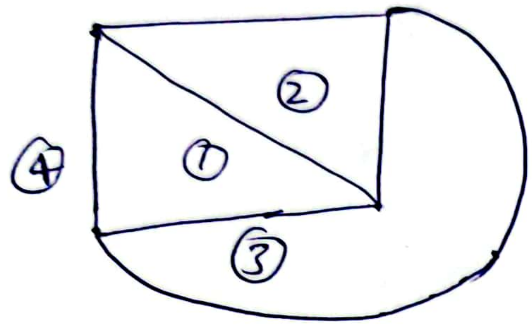


K_5 - Non-plonar graph

K_{33} - Non-planar graph.

① # Regions (face) → (portion) :-

A planar representation devided into 2-parts or no. of parts

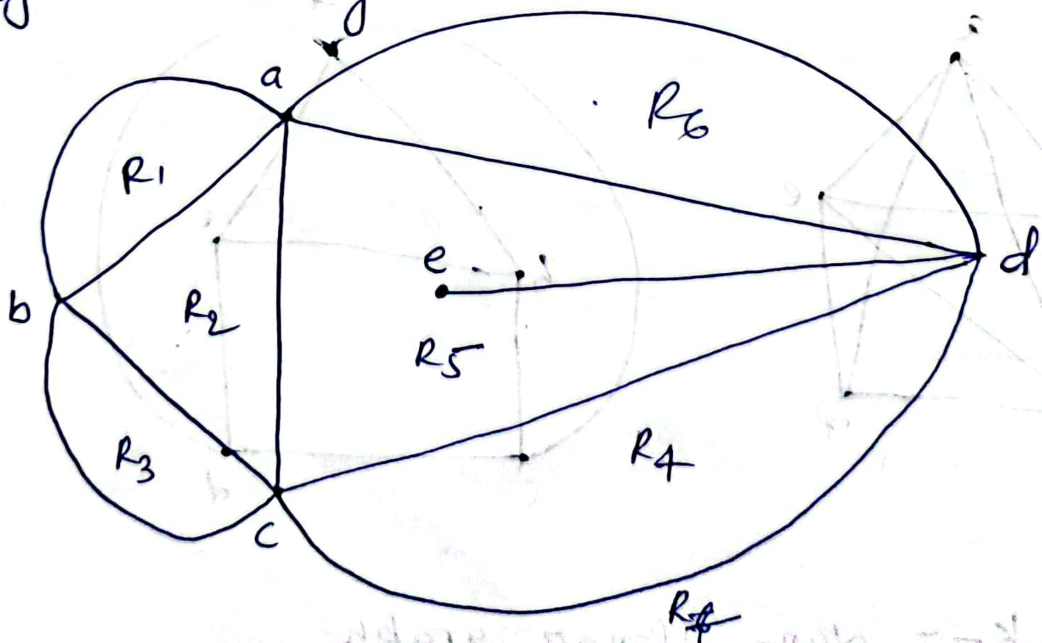


Here There are 4- portion OR 4-part

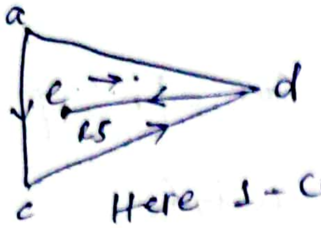
Because a line divide a plane into 2- part.

★ Every region is circuit or close path.

② **Degree of a region** :- Degree of a region is defined as the no. of edges in the boundary of region with cut edge counting twice.



$d(R_1) = 2$	$d(R_4) = 3$
$d(R_2) = 3$	$d(R_5) = 5$
$d(R_3) = 2$	



Here 1 - cut edge

(c,d) is a cut edge
Counted Twice.

$d(R_6) = 3$. out boundary
unbounded Regions.

theorem :-

* Euler - Theorem :-

If G will be a planar graph with v no. of vertices, e No. of edges and r no. of regions then $\boxed{e - v + 2 = r}$

$$\boxed{r = e - v + 2} \qquad \boxed{r + v - e = 2}$$

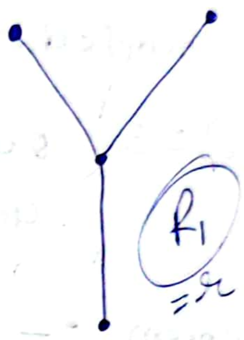
Proof :- We will prove that result or theorem by using principal of Mathematical induction or "r" No. of Region.

Mathematical induction has 3 steps
1. $r = 1$
2. $r = K$
3. $r = K + 1$

Now

we will prove the result for $\boxed{r=1}$

$r=1$, consider a graph G with 4-vertices and 3-edges.



So $v=4$, $e=3$

Now $e-v+2$

$= 3-4+2$

$= 5-4$

$= 1 = r$

It is true for $r=1$

Now Assume the result is True for $\boxed{r=K}$ i.e for a graph having for a K no. of regions.

$$\therefore \boxed{e-v+2=K} \quad \longrightarrow (I)$$

Let G be a connected planar graph having $(K+1)$ regions i.e $\boxed{r=K+1}$

lets check it for $r=K+1$ $\longrightarrow$ (II)

$e-v+2=K+1$

Let G' be a graph obtain from G by removing by an edge which is boundary of 2-regions

$\therefore$ The No. of regions in $G'=K$

We have $v' =$ No. of vertices in $G' = v$

$e' =$ No. of edges in $G' = e - 1$

$r' =$ No. of Regions in $G' = k + 1 - 1$

$= k$

By the II or eqn (1) the result is True for the graph having k - Region and so for G

$\therefore \quad \boxed{e' - v' + 2 = r'}$ —— (III)

Now

$(e-1) - (v) + 2 = k \qquad \left\{ \because e' = e-1,\ v' = v \right.$

$\Rightarrow e - v + 2 = k + 1$

$\Rightarrow \boxed{e - v + 2 = r}$

$\therefore$ By P.M.I proved.

$v = 3$

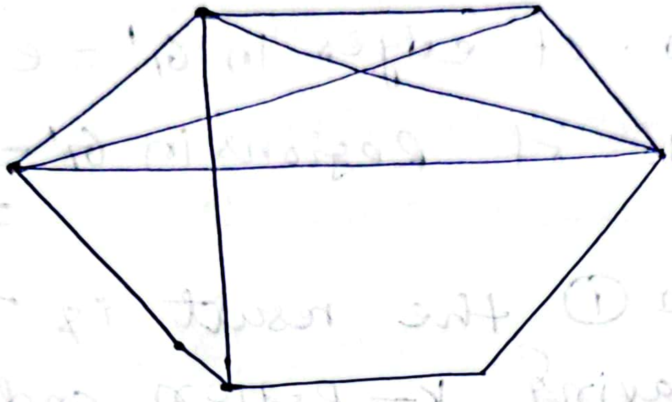
$R = 2$

$R = e - v + 2$

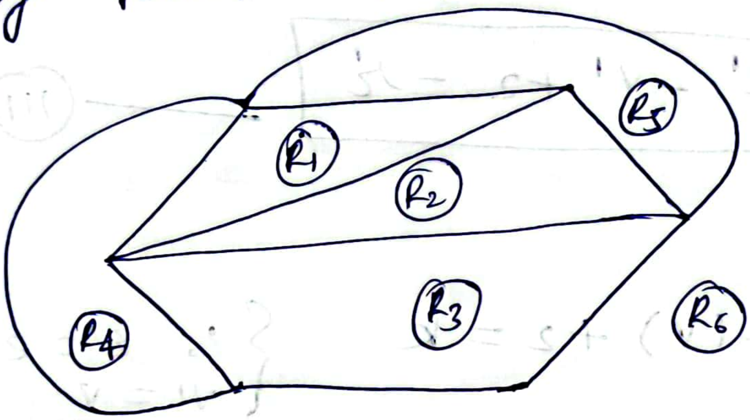
$= 3 - 3 + 2$

$= 5 - 3 = 2$

Q. Verify Euler formula for following



Istly Planar Representation



No. of vertices $= v = 6$

No. of edges $= e = 10$

No. of region $= r = 6$

Now $e - v + 2 = r$

$\Rightarrow$ LHS $= e - v + 2$

$= 10 - 6 + 2$

$= 12 - 6$

$= 6 \Rightarrow R_6 =$ Total Regions

Verify

8. Statement :-

If G be a simple connected planar graph with v-vertices, e-edges and r-Regions then prove that

① $\boxed{2e \geq 3r}$

② $\boxed{e \leq 3v-6}$

③ $\boxed{e \geq 2r}$ if G has no cycle of length 3

④ $\boxed{e \leq 2v-4}$ if G has no cycle of length 3

⑤ there is a vertex v of a graph G such that degree of v is less than equal to

$$\boxed{d(v) \leq 5}$$

v has degree not execding 5.

Soln :-

G is a simple connected planar graph In which no. of vertices $= v$

no. of edges $= e$

no of Regions $= r$

In a planar Represantation of a graph G a region contain atleast 3-edges.

$$\therefore \boxed{d(R_i) \geq 3}$$

Now By H-S Theorem
we know that sum of degree in all the region is twice the no. of edges.

$$\sum_{i=1}^{r} \deg(R_i) = 2e$$

$\Rightarrow 2e = \deg(R_1) + d(R_2) + \cdots d(R_r)$

$\Rightarrow 2e = 3+3+3+\cdots+$ upto r terms

$\Rightarrow 2e \geq 3(r)$

$\Rightarrow \boxed{2e \geq 3r}$ proved

or $\boxed{e \geq \frac{3}{2} r}$

or $\boxed{r \leq \frac{2}{3} e}$

prove $e \leq 3v-6$

(2) $\because$ G is connected planar graph

$\therefore 2e \geq 3r$

$$\boxed{r \leq \frac{2}{3}e} \quad \text{—— (1)}$$

Also By Euler threorem

$\Rightarrow e - v + 2 = r$

$\Rightarrow e - v + 2 = r \leq \frac{2}{3}e$

$\Rightarrow e - v + 2 \leq \frac{2}{3}e$

$\Rightarrow 3e - 3v + 6 \leq 2e$

$\Rightarrow 3e - 2e - 3v + 6 \leq 0$

$\Rightarrow e - 3v + 6 \leq 0$

$\Rightarrow \boxed{e \leq 3v - 6}$ proved

(3) G be connected planar graph having no cycle of length 3.

$\therefore$ Mim 4 edges are required for a region in a planar represantation of graph G.

$\therefore d(R_i) \geq 4$

By H-S Theorem

$$\sum_{i=1}^{r} deg(R_i) = 2e$$

$\Rightarrow 2e \geq deg(R_1) + deg(R_2) + \cdots deg(R_r)$

$\Rightarrow 2e \geq 4 + 4 + \cdots + 4$ up to r terms

$\Rightarrow \cancel{2}e \geq \cancel{4}r \Rightarrow \boxed{e \geq 2r}$ proved prove $e \le 2v-4$

(4) $\because$ G is connected planar graph having no. cycle of length 3.

$\therefore$ $\boxed{\because e \ge 2r}$ ——— (1)

$$r \le \frac{e}{2}$$

Now By Euler Theorem

$\because e - v + 2 = r$

$\Rightarrow e - v + 2 = r \le \frac{e}{2}$

$\Rightarrow e - v + 2 \le \frac{e}{2}$

$\Rightarrow 2e - 2v + 4 \le e$

$\Rightarrow 2e - e - 2v + 4 \le e - e \le 0$

$\Rightarrow e - 2v + 4 \le 0$

$\Rightarrow \boxed{e \le 2v - 4}$

proved

(5) To prove $d(v) \leq 5$

proof :- if possible let the degree of every vertex in graph G is greater than 5.

$$\deg(v_i) > 5$$

$$\Rightarrow \deg(v_i) \geq 6$$

Now By H-S Theorem

$$\sum_{i=1}^{v} d(v_i) = 2e$$

$\Rightarrow 2e = d(v_1) + d(v_2) + \dots v$ terms

$\Rightarrow 2e \geq 6 + 6 + 6 + \dots$ upto v terms

$\Rightarrow 2e \geq 6v$

$\Rightarrow e \geq 3v$

$\Rightarrow \frac{e}{3} \geq v$

$\Rightarrow \boxed{v \leq \frac{e}{3}}$ —— (1)

$\because G$ is planar graph then By Euler theorem

$$e - v + 2 = r$$

$\Rightarrow e - r + 2 = v$

$\Rightarrow e - r + 2 \leq \frac{e}{3}$

$\Rightarrow e \leq \frac{e}{3} + r - 2$

$\Rightarrow e \leq \frac{e}{3} + \frac{2}{3}e - 2$ $\Rightarrow$

$\Rightarrow$. $\Rightarrow$

$\Rightarrow e \le \frac{2}{3}e + \frac{1}{3}e - 2$

$\Rightarrow e \le \cancel{\frac{\text{[illegible]}}{6}}$

$\Rightarrow e \le e - 2$

$\Rightarrow e - e \le -2$

$\Rightarrow 0 \le -2$

$\Rightarrow \boxed{0 \ge 2}$

which is false.

So our Assumption is wrong

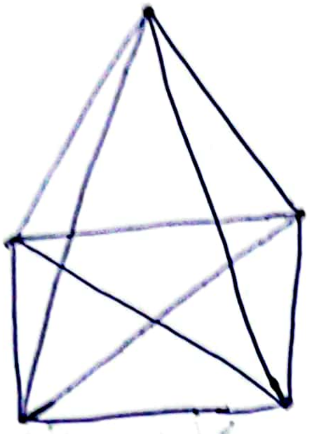
$\therefore \deg(v) \not> 5$

$$\boxed{\therefore \deg(v) \le 5}$$

proved

8. prove that K_5 is a non-planar graph.

proof :- K_5



No. of vertices $= v = 5$

No of edges $= e = 10$

If possible let K_5 is planar graph

We know that $\boxed{\therefore\ e \leq 3v-6}$

$\Rightarrow 10 \leq 3(5)-6$

$\Rightarrow 10 \leq 15-6$

$\Rightarrow 10 \leq 9$ false

which is not true.

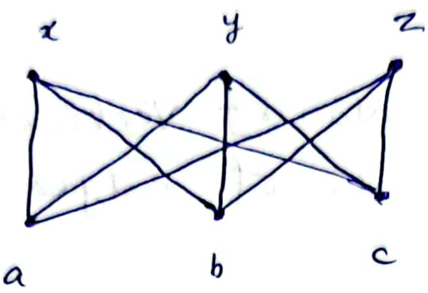
So our Assuption is wrong.

$\therefore$ K_5 is a Non-planar graph.

Q. prove that $K_{3,3}$ (a complete Bipertite graph) is a non-planar graph.

Sol^n

K_{33}



$K_{3,3}$

No. of vertices = 6 ⇒ $\boxed{v = 6}$

No. of edges = 9 ⇒ $\boxed{e=9}$

Assume (Let) $K_{3,3}$ is a planar graph.

∵ $K_{3,3}$ does not contain any cycle of length 3

∴ $\boxed{e \le 2v - 4}$

$\Rightarrow \quad 9 \le 2(6) - 4 \quad (\because v=6, e=9)$

$\Rightarrow \quad 9 \le 12 - 4$

$\Rightarrow \quad 9 \le 8$ false

⇒ which is ~~false~~ false

So our Assuption is wrong

So $K_{3.3}$ is a non-planar graph

proved

10April23

Graph Coloring :- A coloring is a Simple graph is the assignment to color or paint each vertex of the graph such that no Two adjacent vertex have same color.

* Adjacent vertex has different color

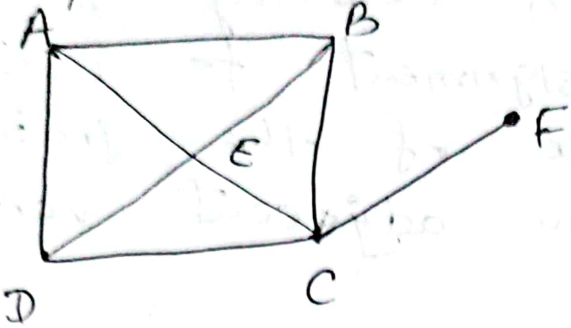
* vertex chromatic No :-
Chromatic Number :- The min^m no. of color to required to color all the vertices of graph G. Such that no two adjacent vertex have same no. of color. is called Chromatic No. of graph G.

and is denoted by $\chi(G)$ (chi)

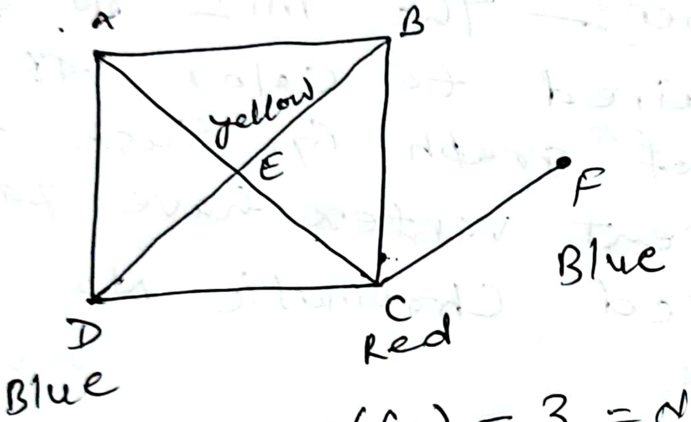
$\boxed{\text{Chromatic No.} = \text{vertex chromatic No.} \Rightarrow \chi(G)}$

* Edge chromatic No :-
No two adjacent edge have same color

8. Find the Chromatic No. of the graph



Sol^n :-



$\chi(G) = 3$ = No. of color

i.e Chromatic No = $\chi(G) = \underline{3}$

* * * * * * * * * * *

Welch Powell Algorithm :-

V.V.I
midterm
(10 marks)

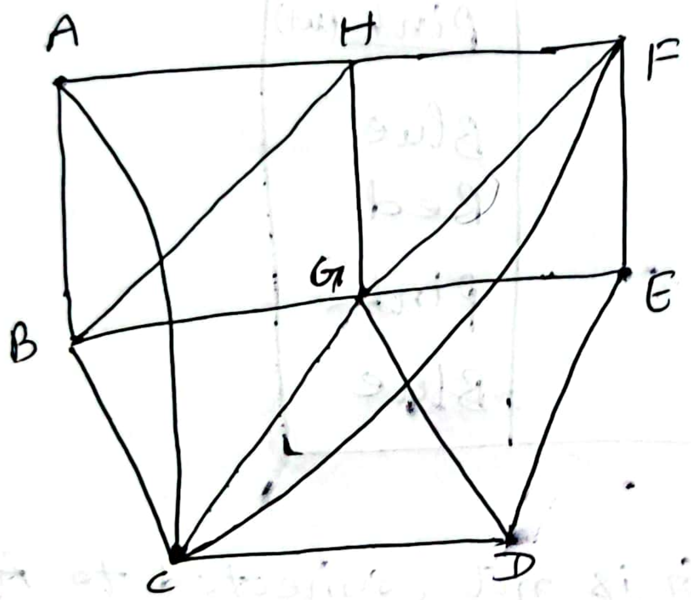
Setep 1 (1) :- Arrange the vertice of given graph G in descending order of their degree.

Stepno. 2 (2) :- Assign the first color C_1 to the 1st vertex (max degree)

and then to each vertex which are not adjacent to the privious vertices which was assign color C_1.

③ Repeate step ② while ~~Second~~ & so on

Q. Find the chromatic no. of the following graph by using welch powell Algorithm



[n = V = vertex]

Soln :- No. of vertices = n = 8

∴ Maximum degree that a vertex can have = n − 1
= 8 − 1 (A B C D E F G H = 8)
= 7

Now Degree of all the vertices

d(A) = 3	d(F) = 4
d(B) = 4	d(G) = 6
d(C) = 5	d(H) = 4
d(D) = 3	
d(E) = 3	

Arrange for decending order

Degree	vertex	Colour
6	G	Red (1st)
5	C	Blue (1st)
4	B	Pink
4	F	Pink (1st)
4	H	Blue
3	A	Red
3	D	Pink
3	E	Blue

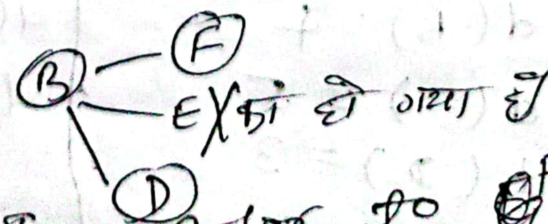
~~G is not~~

(i) $d(G)=6$ G is not connected to A and G, A are not adjacent Assian Same Red color to both (A, G) 1st

(ii). $d(C) = 5$, $\because$ C is not connected to E, H.

So give same color to E H, C
Because it is not adjacent

(iii). $d(B) = 4$



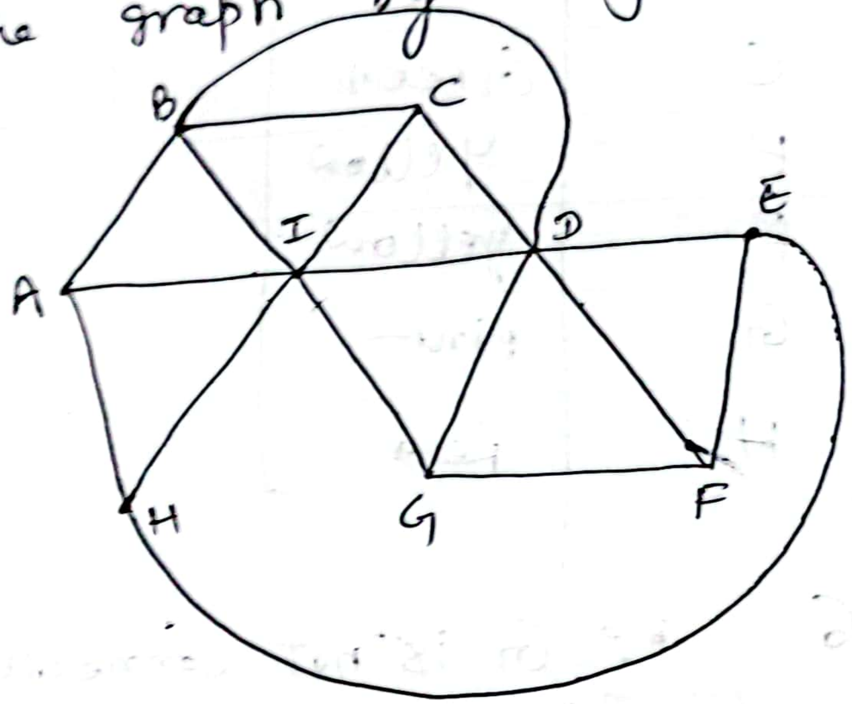
Give same color to ~~E~~ F, D & B
Because it is not adjacent

All Done

So $\chi(G) = 3$-color.

$\boxed{\therefore \chi(G) = 3}$ Ans

8. Find the Chromatic No. $\chi(G)$ of the graph by using welch powell Al.



No. of vertices = 9

$\therefore$ Max degree that a vertex can have $= n-1$
$= 9-1$
$= 8$

Now Degree of all vertices

$d(A) = 3$	$d(E) = 3$	$d(I) = 6$
$d(B) = 4$	$d(F) = 3$	
$d(C) = 3$	$d(G) = 3$	
$d(D) = 6$	$d(H) = 3$	

Degree	vertex	Color
6	D	Red
6	I	yellow
4	B	Red
3	A	Green
3	C	Green
3	E	yellow
3	F	yellow
3	G	Pink
3	H	Red

(i). $d(D) = 6$ ∵ G is not connected to 2 more vertex i.e H, B

Give same color to D, H, B - Red
∵ these are not adjacent

(ii) $d(I) = 6$ ∵ I is not connected to 2 more vertex i.e F, E (Not Direct connected)

Give same color to I, F, E = yellow

(iii) $d(B) = 4$ $\because$ B is not connected to 4 more vertices i.e D, E, G, H

So Give same color to
B, D, E, G, H = pink
✓ ✗ ✗ ✓ ✗

(iv) $d(A) = 3$ $\because$ A is not connected to 5 more vertex i.e D, E, not directly C, G, F

So give same color to each
A, D, E, C, G, F
✓ ✗ ✗ ✓ ✗ ✗

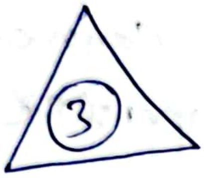
Color – Red, yellow, Gree, Pink — 4

$$\boxed{\chi(G) = 4}$$ Ans

Note :- ① Chromatic No. of a Complete graph with n-vertices :

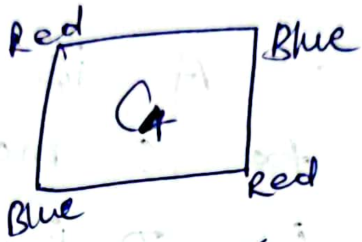
$$\boxed{\chi(K_n) = n.}$$

② Chromatic No. of cycle/polygon with n-vertices, $n \geq 3$

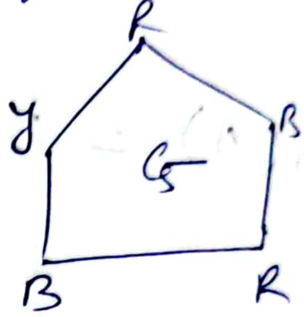


C_3.

3-color

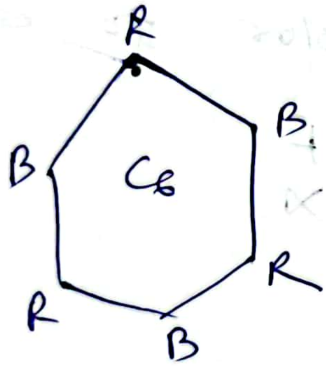


2-color



C_5

3-color



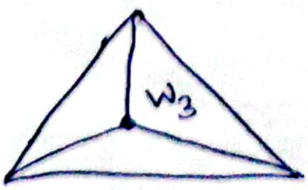
2-color

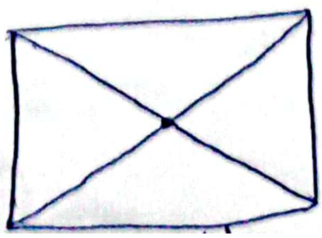
$$\chi(C_n) = \begin{cases} 2 : n \text{ is even} \\ 3 : n \text{ is odd} \end{cases}$$

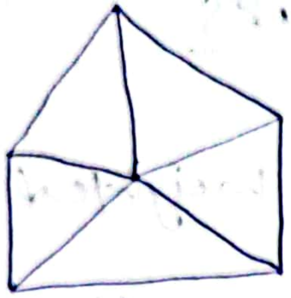
Chromatic ~~No.~~ Color depends on the the chromatic No. (even/odd).

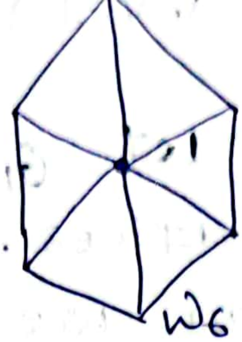
$$\boxed{\begin{array}{l} \text{even} - 2 \\ \text{odd} - 3 \end{array}}$$

③ Wheel graph W_n :-









W_n = wheel graph with $(n+1)$ vertices.

$$\chi(W_n) = \begin{cases} 4 : n \text{ is odd (No. of vertices are even)} \\ 3 : n \text{ is even (No. of vertices are odd)} \end{cases}$$

④ Chromatic No. of Bipertite Graph :-

$$\boxed{\chi(K_{m,n}) = 2}$$

Shortes path (V.V.I) 10 marks

Dij

let G be a weighted graph having n-vertices.
In this Algorithm we maintain a set of vertices whose shortest path from Source vertex is already known & if there is no path from source vertex then it is represented by ∞.

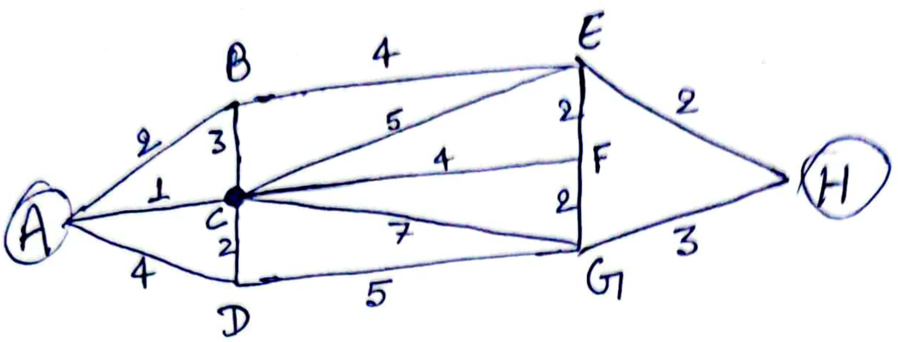
Steps

1. Initially there is no vertex in S.
2. Include a Source vertex V_s in set S
3. find All the path from V_s to all other vertices without going to any other vertex. [direct path]

3. Include that vertex in the set S which is nearest to V_s, find shortes path to all other vertices to this vertex

4. Repeate the process until (n-1) are not included in set S.

Q. Find the Shortest path from vertex to edge in the graph



① If the new distance is ≥ then use only previous

② No Direct path choose previous

③ ≤ (less than) then choose New

Solⁿ :- No. of vertex = 8

source	A	B	C	D	E	F	G	H
A	—	2(A)	$\boxed{1}$(A)	4(A)	∞	∞	∞	∞
AC	—	$\boxed{2}$(A)	—	3(AC)	6(AC)	5(AC)	8(AC)	∞
ACB	—	—	—	$\boxed{3}$(AC)	6(AC)	5(AC)	8(AC)	∞
ACBD	—	—	—	—	6(AC)	$\boxed{5}$(AC)	8(AC)	∞
ACBDF	—	—	—	—	$\boxed{6}$(AC)	—	7(ACD)	∞
ACBDFE	—	—	—	—	—	—	$\boxed{7}$(ACD)	8(ACE)
ACBDFEG	—	—	—	—	—	—	—	$\boxed{8(ACE)}$

{ □ less mark }

∴ the Shortest path is A-C-E-H

length of S.P = 8

Ans

Graph

Hamilton Graph

Hamilton Graph :-

vertices

Hamilton Path :- simple path that contain every vertex of the graph exactly 1. (once).

Hamilton Circuit :- Close H-Path

Hamilton Graph → H.Path
Hamilton Graph → H.Circuit

Ex G_1 : A – B – C – D (H.Path)

A – B – C – D – A (H.Circuit)

Euler graph :-

Edges

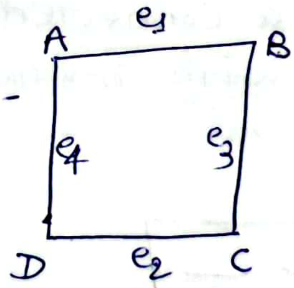
Euler path :- the simple path that contain every edges of the graph exectly once (1).

Euler circuit :- (close path)

:- initial vertex = final vertex

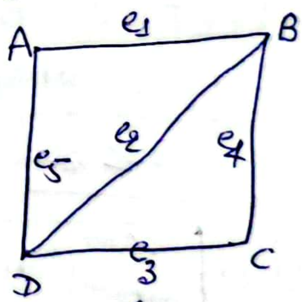
Euler Graph → Euler path
Euler Graph → Euler circuit

Ex :-



G_1

$\left(\begin{matrix} v = 4 \\ e = 4 \end{matrix}\right)$



G_2

$\left(\begin{matrix} v = 4 \\ e = 5 \end{matrix}\right)$

G_1 : $\underbrace{A\, e_1\, B\, e_3\, C\, e_2\, D\, e_4\, A}_{\text{same vertex}}$

Euler circuit

G_2 : $\underbrace{B\, e_4\, C\, e_1\, D\, e_2\, B\, e_1\, A\, e_5\, D}_{\text{different vertex}}$

Euler path

Himaliton Graph

① A ~~Ham~~ Hamilton Graph G contain either path or circuit or Both.

② In the Graph G has n-vertices then H. circuit contain n-edges.

$V = n = 4$ / edges $= 4$

$\boxed{\text{path} = (n-1)} = 4-1 = 3$

where n = no. of vertices

③ Let G be a connected simple graph with n-vertices

$\boxed{n \geq 3}$

(i). $\boxed{\text{if } d(V_i) \geq \frac{n}{2}}$

then 'G' is Hamiltonian.

Q. $n = 13$
Max degree of a vertex

(a) 5
(b) 6
(c) 6.5
(d) 7

Soln. $n = 13$

$\Rightarrow d(v_i) \geq \frac{n}{2}$

$\Rightarrow d(v_i) \geq \frac{7}{2}$

$\Rightarrow d(v_i) \geq 6.5$

$\Rightarrow \boxed{d(v_i) \geq 7}$

$n = 4$

$d(v_i) \geq \frac{4}{2}$

$\boxed{d(v_i) \geq 2}$

Euler Graph

① A Euler Graph Contain either euler path or circuit But not Both.

② A connected graph G is Euler graph and has euler circuit if the degree of every vertex is even.

A connected graph G is euler graph & has euler path if exactly two vertex has odd degree

Hamilton Graph

(ii). if $d(u)+d(v) \geq n$
where u, v are non-adjacent vertices, then G is Hamiltonion Graph.

$d(A)+d(C) = 2+2 = 4 = n$

(iii). Let m be the no. of edges in G
if $\boxed{m \geq \frac{1}{2}(n^2-3n+2)}$

m = no. of edges
n = no. of vertices

for graph G_1 and G_2

$n=4$

$m \geq \frac{1}{2}(4^2-3(4)+2)$

$m \geq \frac{1}{2}(16-12+2)$

$m \geq \frac{1}{2} \cdot \cancel{6}\,3$

$\boxed{m \geq 3}$

$G_1 \Rightarrow m=4$
$4>3$ ✓

$G_2 \Rightarrow m=5$
$5>3$ ✓

(iv) Complete graph K_n is always Hamiltonion Graph

$\frac{\underline{n-1}}{2}$ H. Circuit

$\boxed{\frac{(n-1)!}{2}}$

K_3, K_4, K_5 — — contain circuit.

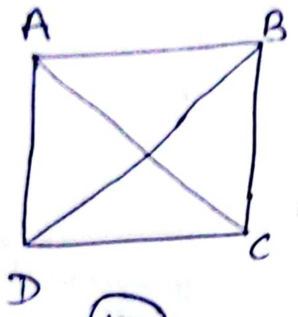
Ex ① K_4 ⇒ complete graph with 4 vertices

$\therefore n = 4$

No. of H. circuit $= \dfrac{(n-1)!}{2}$

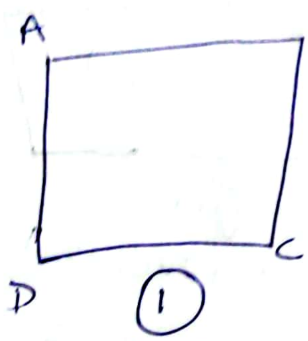
No. of (Hamilton circuit) $\dfrac{(n-1)!}{2}$

$= \dfrac{(4-1)!}{2} = \dfrac{3!}{2} = \dfrac{3 \times \cancel{2}}{\cancel{2}} = 3$

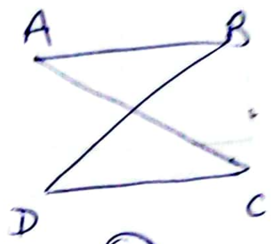


K_4

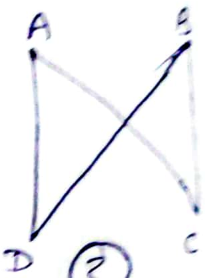
⇒



①



②



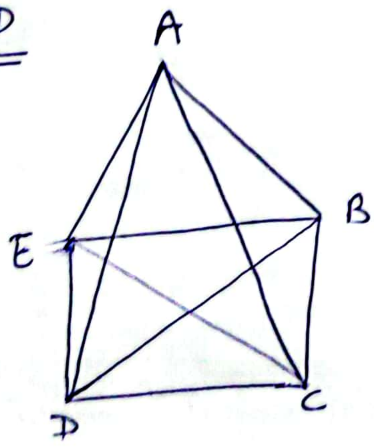
③

ACBD

BACD

CABD

② K_5



$n = 5$

No. of H. circuit $= \dfrac{(n-1)!}{2}$

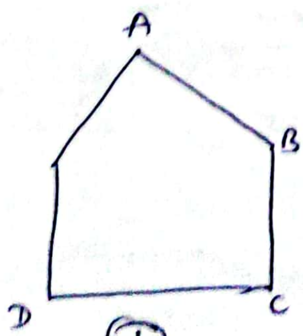
$\Rightarrow n = 5$

$\Rightarrow \dfrac{(5-1)!}{2}$

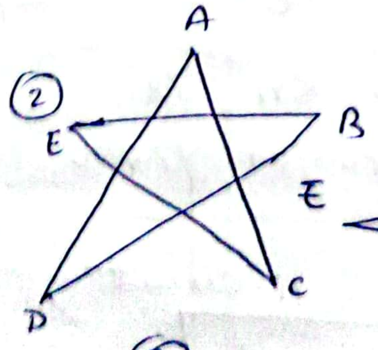
$\Rightarrow \dfrac{4!}{2} = \dfrac{4 \times 3 \times \cancel{2}}{\cancel{2}}$

$= 12$

Ans



①



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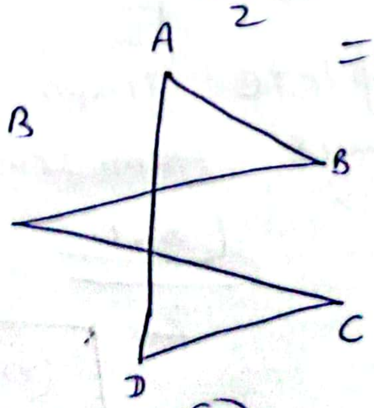

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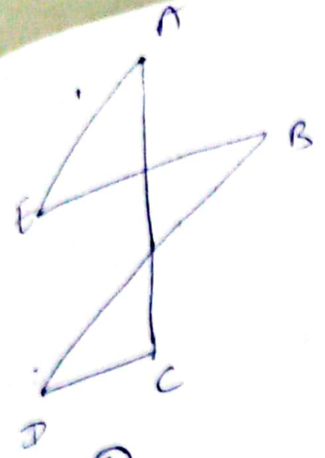
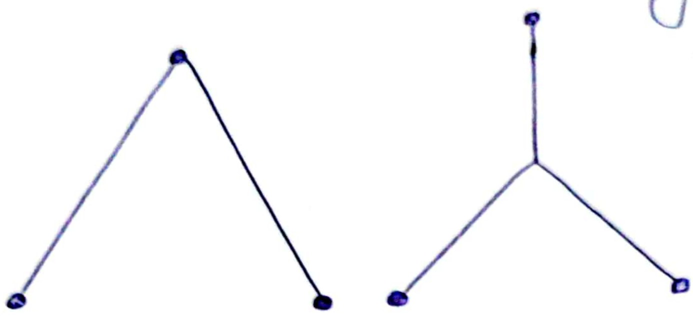


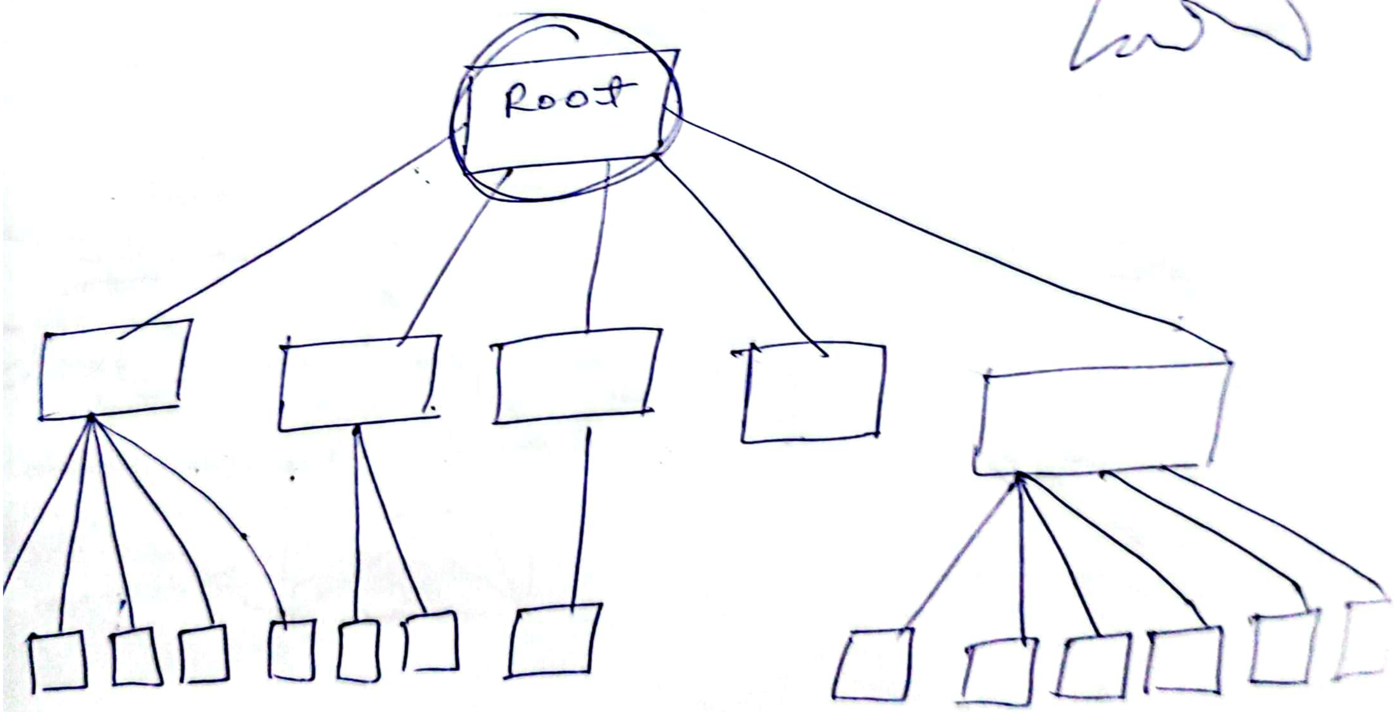
up to 12 fig (circuit)

4

⑥ Tree

A connected graph G having no cycle is called free graph or simply Tree.





$\boxed{\text{very } 1^{st} \text{ node} = \text{Root}}$

★ Tree graph arrows are always in downward direction

At the last — Leaves (Leaves node)

(Do not have anythings)

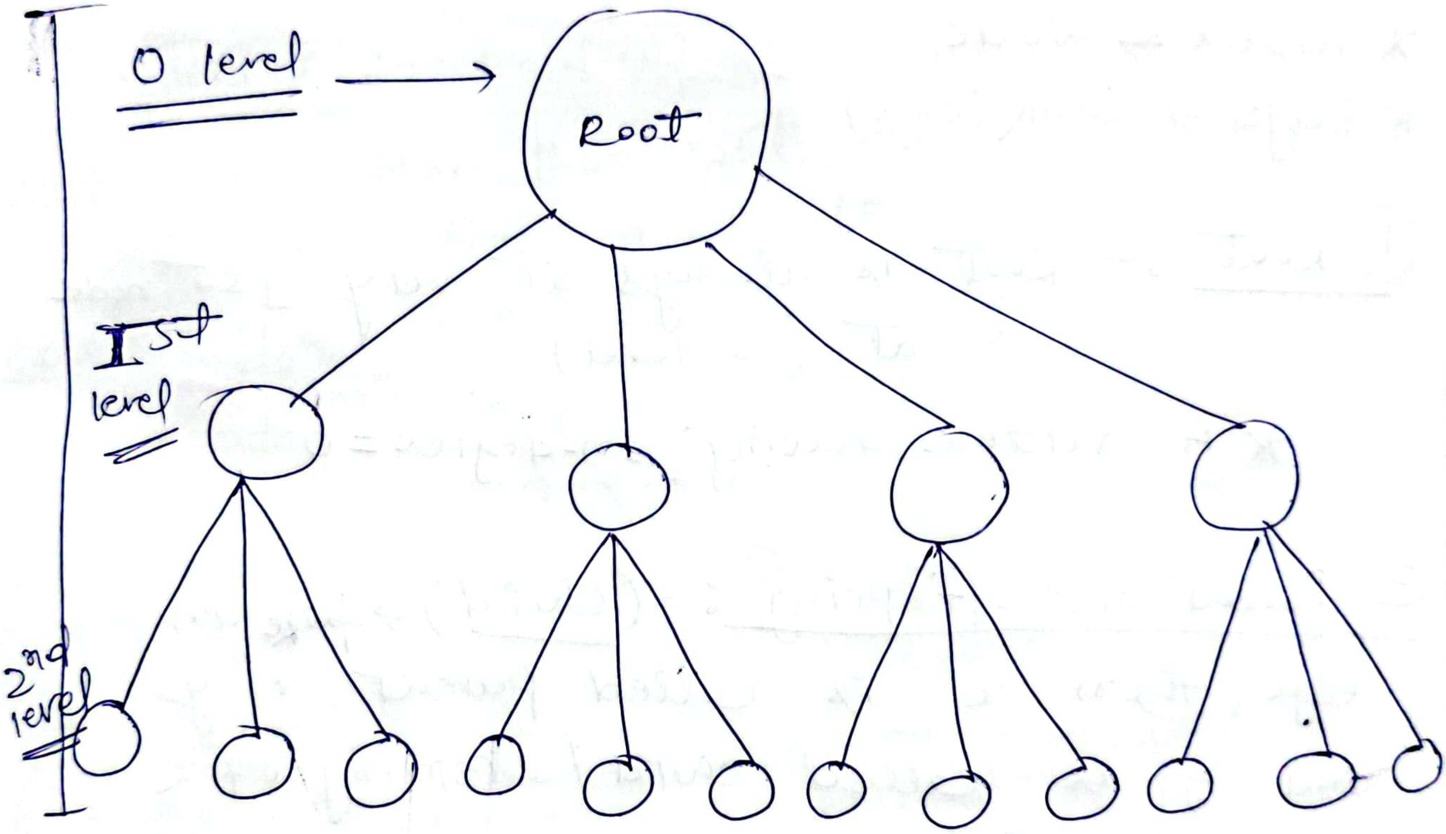
parent node
child node } use Also

↓ move downward direction

upper vertex = parent

lower vertex = child

⇒ ~~leaves~~ levels ⇒ Generations

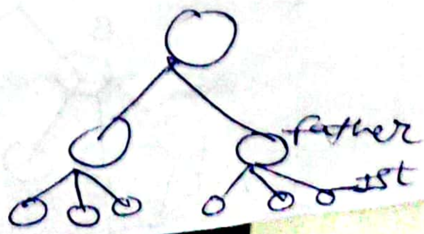


$\boxed{\text{Height of a Tree = Root to leaves}}$

at which level vertex is places.
height will be ↓ direction.

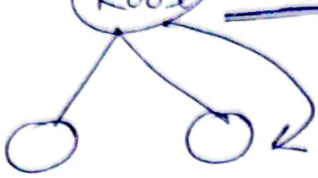
$\boxed{\text{Depth = leaves to root}}$ (see upper to lower)

father is Ist Anchestor
grandfather = 2nd Anchestor



decendent (आगे वाला) (अपने से आगे वाला)

Root at 0-level



← Decendent (is below that)

All are decendent of grandfather.

Definition :-

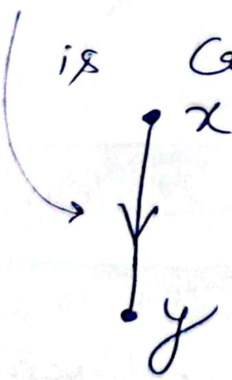
★ vertex ⇒ Node

★ Edge ⇒ Link (edge)

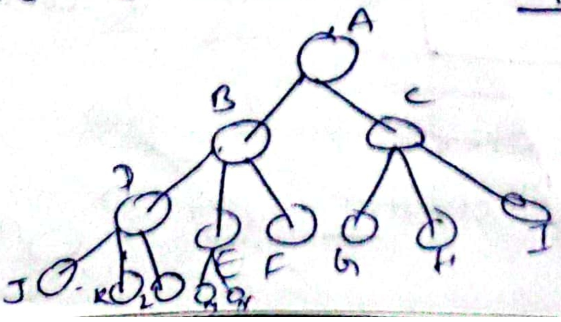
① Root :- Root is always at very 1st node i.e at (0-level)

★ A vertex having In-degree = 0

② Parent and offspring :- (Child) ⇒ ~~take~~ an edge, then x is called parent of y and y is called child / offspring of x



③ Leaves :- A vertex having out degree = 0 are called ~~leaves~~ leaves.



D, E, F G H I are leaves.

* Two or more node having same parent is called siblings.

4) **Siblings** :- D, E, F are siblings (parent B)

G, H, I are siblings (parent C)

B, C are siblings (parent A)

5) **Anchestor** :- (elders) :- Anchestor of a vertex be other than root are the vertices in the path from root to vertices V except the vertices (V).

Ancestor of F

path - A-B-E-F ✗

So Anchestor of F is B, A

1st Anchestor

2nd Anchestor

6) **Descendent** :- the descendent of a vertex V are those all vertices for which is an anchestor.

descendent of B

(B के निचे सारा)

[D, E, F]

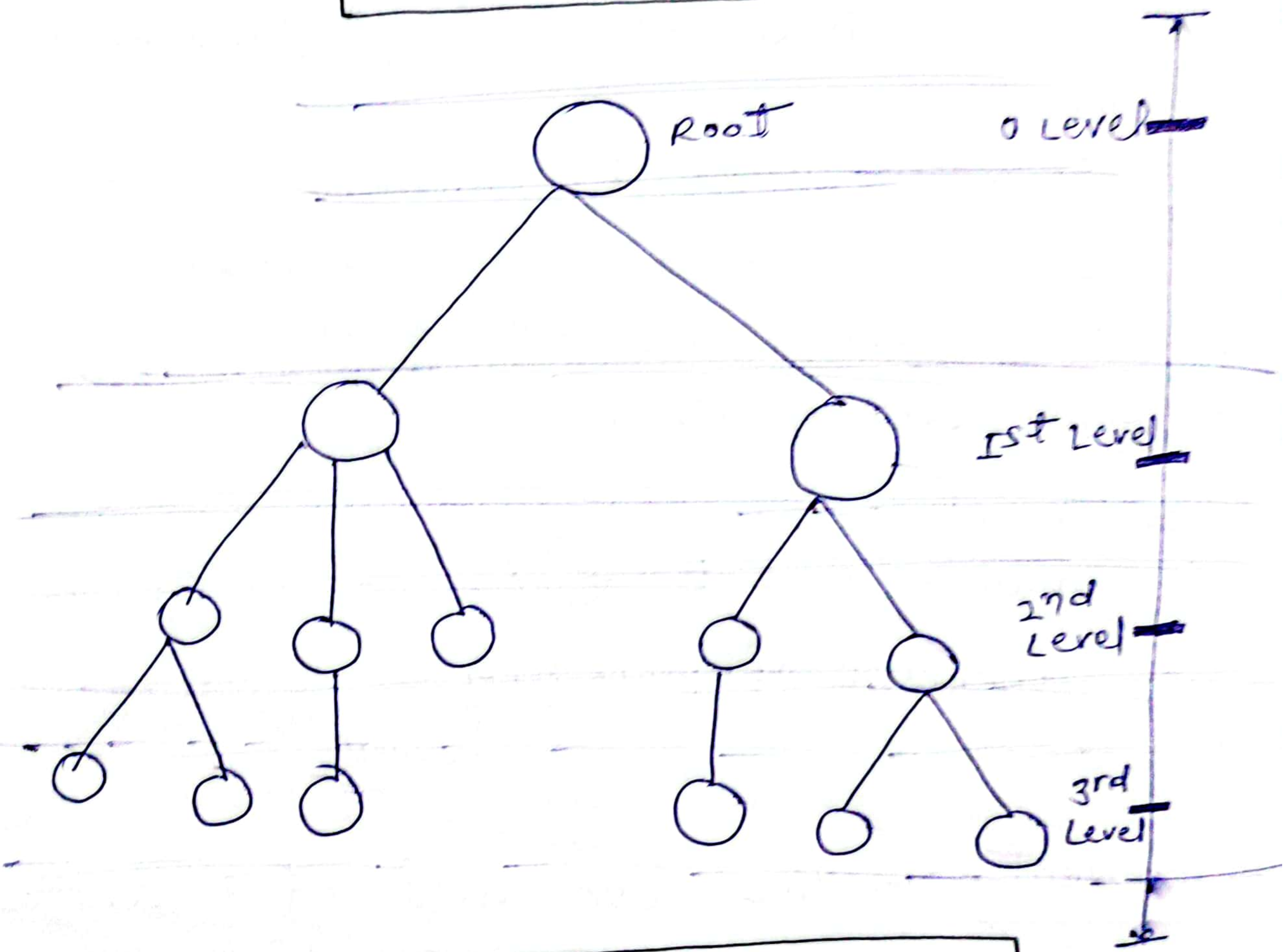
[J, K, L] [M, N] All these are descendent of B.

desendent of C (C के निचे पूरा)

G, H, I

⑦ level :- The level of a vertex of a node is an Integer value that measure the distance of the vertex from root.

Vertex from Root



Level of root = 0 (Always)

★ Root is always at zero Level.

⑧ Heigent :- (Heigent of a vertex) :- Heigent of a vertex is the length of the longest path from node to leave.

All the leaves are at zero(0) height.

Height (↑)

⑨ Depth :- Depth of a node (vertex) the length of the path from vertex→root to root

root to vertex ↑↓

vertex to root

i e that particular node (vertex) to root.

⑩ Sub-Tree :- A part of a Tree

⑪ Forest :- A graph whose components are all Tree.

Minimally connected graph :-

A Graph G said to be minimally connected that of every edge is cut edge, then graph is called minimally graph

properties of Tree :-

① there is only one or ~~more~~ only one path b/w every ~~vertex~~ pair of vertices in a tree.

② A Tree with n-vertices has (n-1) edges.

3) Tree is minimally Connected graph

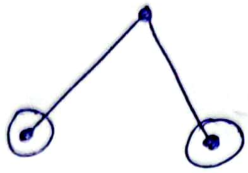
1) for a non-trivial tree
there are atleast two pendant vertex
(degree = 1)

very smallest Tree

$v = 2$
degree $= n-1$
$= 2-1$
$= 1$

Count edge $= 1$

$\underbrace{deg(v_1) = 1 = d(v_2)}$
Pendent



The no. of pendent vertex in a tree ≥ 2

Degree of leaves $= 1$

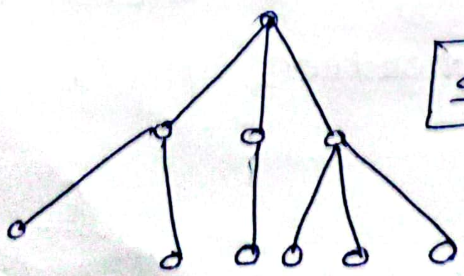
Leaves are pendent vertex (node)

m-Tree / m-ary tree :-

A Tree is said to be m-ary tree if every vertex has at most m-child

Complete m-tree :-

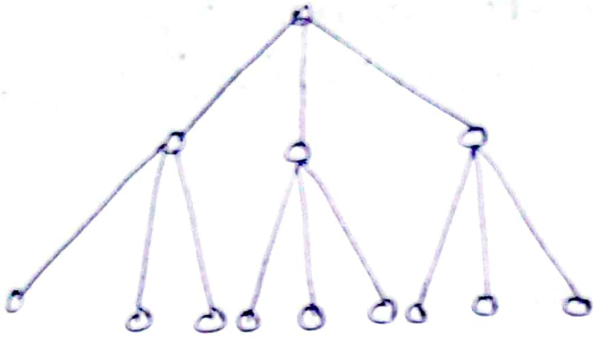
Every vertex has exactly m-childs.



$\boxed{\leq 3^n}$

Here maximum 3-child
So 3-ary Tree

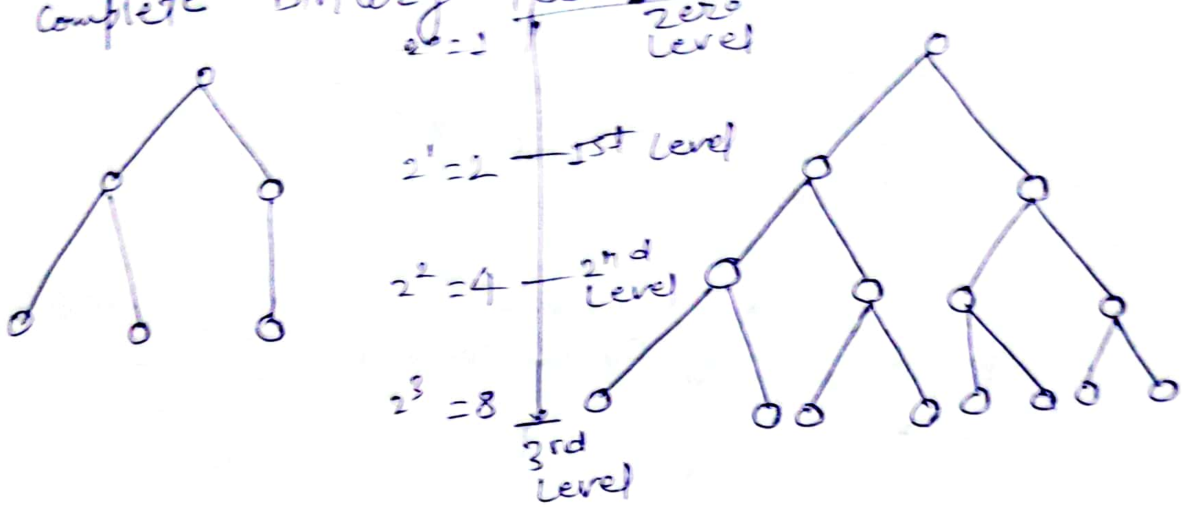
3-ary Tree



$\boxed{\Rightarrow 3^n}$

3-Tree (Complete)

Note:- ① If $m=2$ then it is called Binary Tree or Complete Binary Tree.



② No. of vertices in a Complete Binary Tree at nth level $= \boxed{2^n}$

③ No. of vertices in a Binary Tree $\boxed{\leq 2^n}$

④ No. of vertices in Complete m tree at nth level $= \boxed{m^n}$

⑤ No. of vertices in m-tree at nth level $\boxed{\leq m^n}$

Q. prove that the Graph Given by
$A = \{1, 2, 3, 4, 5\}$, $R = \{(1,1), (1,2) (1,3) (2,3) (3,3) (3,$
is not a Tree.

Soln :- No. of vertices = n = 5

$\therefore n = 5$

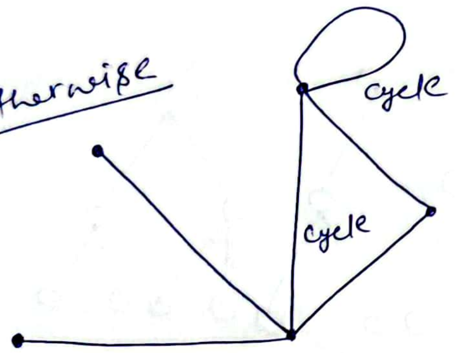
No. of edges $= 6 \neq (n-1)$

where n = vertices

So graph is not a tree

prove.

otherwise



without cycle or loop } होना चाहिए Tree के लिए।

So graph is not a Tree

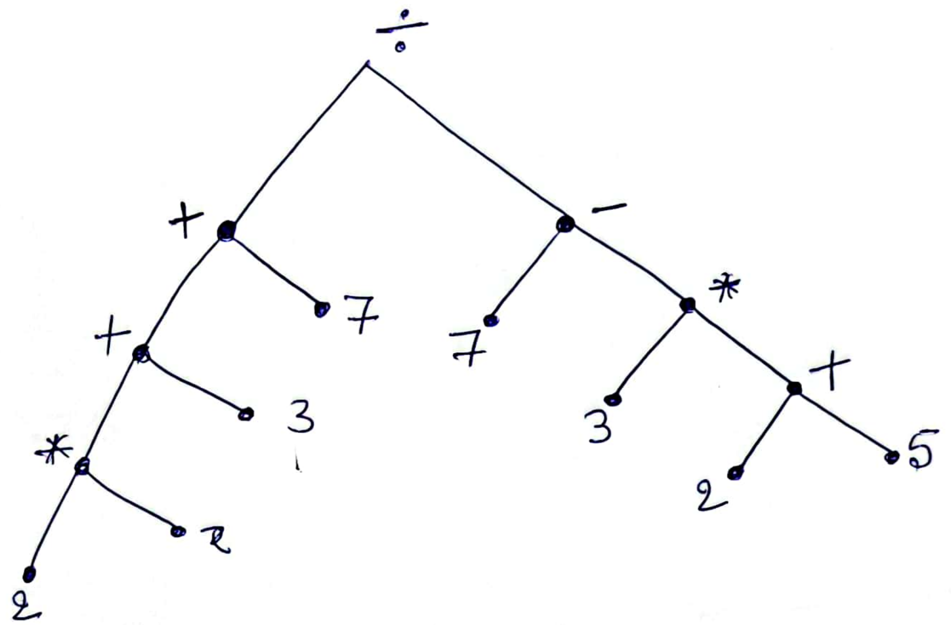
proved

Expression Tree :- An expression Tree is a label tree used to

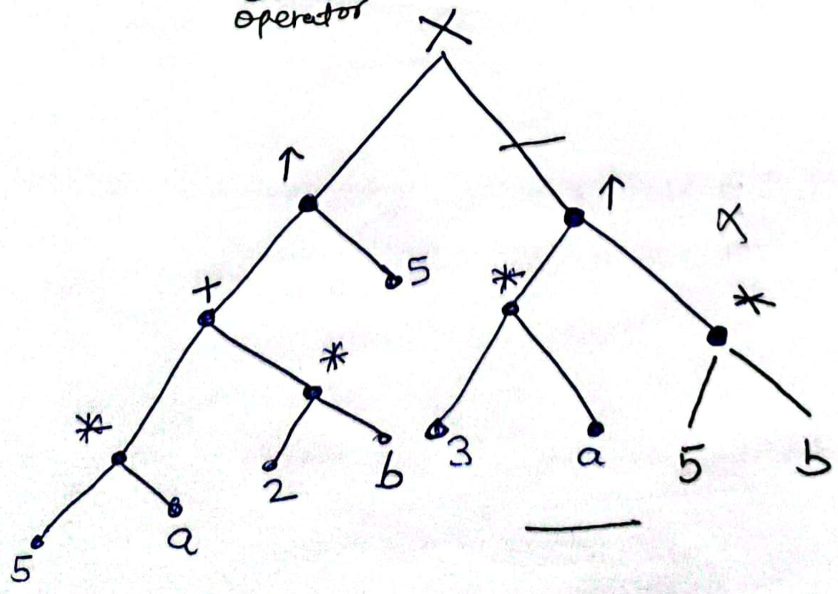
Q. Represent algebric expression into tree.

$$\underbrace{((2x+3)+7)}_{\text{Left Argument}} \underset{\substack{\uparrow \\ \text{Central} \\ \text{operator}}}{\div} \underbrace{(7-3(2+5))}_{\text{Right Argument}}$$

* Root is always be centeral operator.



Q. $$\underbrace{(5a+2b)^5}_{\text{Left Argument}} \underset{\substack{\uparrow \\ \text{Centeral} \\ \text{operator}}}{\times} \underbrace{(3a-5b)^2}_{\text{Right Argument}}$$

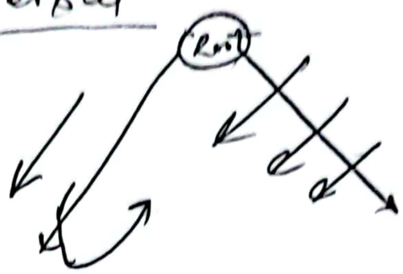


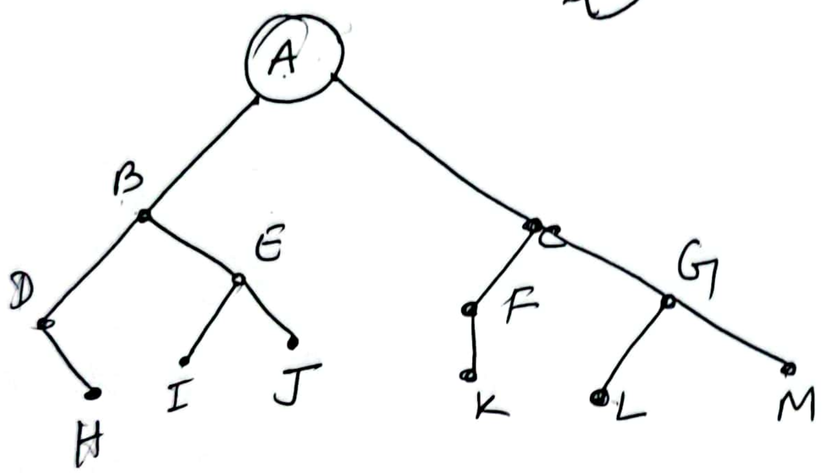
Traversal

1. preorder Traversal (Root L R)
2. post order Traversal (L R Root) }
3. Inorder Traversal (L Root R) } Start extreem last

pre-order Traversal

Root L R.





Pre-order ⇒ A, B, D, H, E, I, J
C, F, K, G, L, M

Post-order Traversal ⇒

Inorder Traversal ⇒ A B D H E I J
C F K G L M