VENN DIAGRAM AND SET THEORY

VENN DIAGRAM

Venn diagram, also known as Euler-Venn diagram is a simple representation of sets by diagrams. The usual depiction makes use of a rectangle as the universal set and circles for the sets under consideration.

Let's take a look at some basic formulas for Venn diagrams of two and three elements.

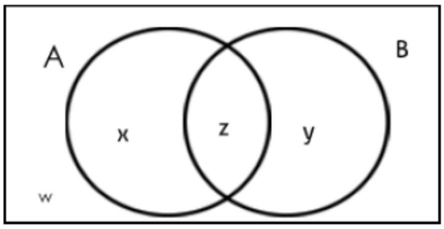
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

And so on, where n(A) = number of elements in set A.

Once you understand the concept of Venn diagram with the help of diagrams, you don't have to memorize these formulas.

Venn Diagram in case of two elements



Where;

X = number of elements that belong to set A only

Y = number of elements that belong to set B only

Z = number of elements that belong to set A and B both (A□B)

W = number of elements that belong to none of the sets A or B

From the above figure, it is clear that

$n(A) = x + z$;

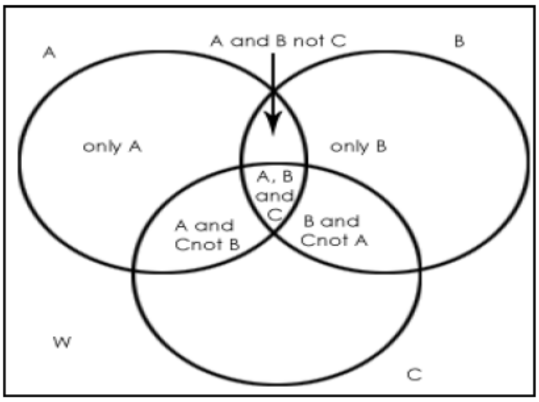
$n(B) = y + z$;

$n(A \cap B) = z$;

$n(A \cup B) = x + y + z$.

Total number of elements = $x + y + z + w$

Venn Diagram in case of three elements



Where, W = number of elements that belong to none of the sets A, B or C

Note: Always start filling values in the Venn diagram from the innermost value

SET THEORY

Set Theory

A Set is defined as a group of objects, known as elements. These objects could be anything conceivable, including numbers, letters, colors, even set themselves. However, none of the objects of the set can be the set itself.

Set Notation

We write sets using braces and denote them with capital letters. The most natural way to describe sets is by listing all its members. For example,

$A = \{1,2,3,\ldots,10\}$ is the set of the first 10 counting numbers, or naturals,

$B = \{$Red, Blue, Green$\}$ is the set of primary colors

Well defined Set

Well-defined means, it must be absolutely clear that which object belongs to the set and which does not. Some common examples of well-defined sets are:

The collection of vowels in English alphabets. This set contains five elements, namely, a, e, i, o, u

$N = \{1,2,3,\ldots\}$ is the set of counting numbers, or naturals.

$Z = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$ is the set of integers.

Definition of Subset:

If A and B are two sets, and every element of set A is also an element of set B, then A is called a subset of B and we write it as $A \subseteq B$ or $B \supseteq A$.

The symbol $\subset$ stands for 'is a subset of' or 'is contained in'

- Every set is a subset of itself, i.e., $A \subset A$, $B \subset B$.
- Empty set is a subset of every set.
- $A \subseteq B$ means A is a subset of B or A is contained in B.
- $B \subseteq A$ means B contains A.

For example:

Let $A = \{2, 4, 6\}$ and $B = \{6, 4, 8, 2\}$

Here A is a subset of B

Since, all the elements of set A are contained in set B.

But B is not the subset of A

Since, all the elements of set B are not contained in set A.

Number of Subsets of a given Set:

If a set contains 'n' elements, then the number of subsets of the set is 2^n.

Number of Proper Subsets of the Set:

If a set contains 'n' elements, then the number of proper subsets of the set is $2^n - 1$.

If $A = \{p, q\}$ the proper subsets of A are $[\{\ \}, \{p\}, \{q\}]$

$\Rightarrow$ Number of proper subsets of A are $= 2^2 - 1 = 4 - 1$

In general, number of proper subsets of a given set = 2m - 1, where m is the number of elements.

Types of Sets

1. Null set or Empty Set: A set which does not contain any element is called an empty set, or the null set or the void set and it is denoted by ∅

Eg: The set of whole numbers less than 0.

Let A = {x : 2 < x < 3, x is a natural number}

Here A is an empty set because there is no natural number between 2 and 3.

2. Singleton Set: A set which contains only one element is called a singleton set.

Eg: A = {x : x is neither prime nor composite}

It is a singleton set containing one element, i.e., 1.

3. Finite Set: A set which contains a definite number of elements is called a finite set. Empty set is also called a finite set.

Eg: The set of all colors in the rainbow.

N = {x : x ∈ N, x < 7}

4. Infinite Set: The set whose elements cannot be listed, i.e., set containing never-ending elements is called an infinite set.

Eg: Set of all points in a plane

A = {x : x ∈ N, x > 1}

5. Difference of Sets: The difference of sets A and B, written as A-B, is the set of elements belonging to set A and NOT to set B.

Eg: A = {1,2,3,4,5}, B = {2,3,5}

The difference of A and B (i.e. A-B) is {1,4}

NOTE: A-B ≠ B-A

6. Disjoint Sets: If two sets A and B should have no common elements or we can say that the intersection of any two sets A and B is the empty set, then these sets are known as disjoint sets i.e. A ∩ B = ϕ.

Eg: A = {1,2,3}, B = {4,5}

A ∩ B = 0.

Therefore, these sets A and B are disjoint sets.

7. Equality of Two Sets or Equal Sets: Two sets are said to be equal or identical to each other, if they contain the same elements. The sets P and Q is said to be equal, if P ⊆ Q and Q ⊆ P, then we will write as P = Q.

Eg: If A = {1,2,3} and B = {1,2,3}, then A = B.

Let P = {a, e, i, o, u} and B = {a, e, i, o, u, v}, then P ≠Q, since set Q has element v as the extra element.

8. Cardinal Number or Cardinality of a Set: The number of distinct elements in a given set A is called the cardinal number of A. It is denoted by n(A).

Eg: A {x : x ∈ N, x < 5} i.e. A = {1, 2, 3, 4}

Therefore, n(A) = 4

9. Equivalent sets: Two sets which have the same number of elements, i.e. same cardinality are equivalent sets.

Eg: P = {p. q. r, s, t} and Q = {a, e, i, o, u}

Since the two sets P and Q contain the same number of elements 5, therefore they are equivalent sets.

10. Super Set: Whenever a set A is a subset of set B, we say the B is a superset of A and we write, B $\supseteq$ A. Symbol $\supseteq$ is used to denote 'is a super set of'

Eg: A = {a, e, i, o, u} andB = {a, b, c, ..., z}

Here A $\subseteq$ B i.e., A is a subset of B but B $\supseteq$ A i.e., B is a super set of A

11. Proper Subset: If A and B are two sets, then A is called the proper subset of B if A $\subseteq$ B but B $\supseteq$ A i.e., A $\neq$ B. The symbol '$\subset$' is used to denote proper subset. Symbolically, we write A $\subset$ B.

Eg: A = {1, 2, 3, 4} , Here n(A) = 4

B = {1, 2, 3, 4, 5} , Here n(B) = 5

We observe that, all the elements of A are present in B but the element '5' of B is not present in A.

So, we say that A is a proper subset of B i.e. A $\subset$ B

Note:

1. No set is a proper subset of itself.

2. Null set or ∅ is a proper subset of every set.

12. Power Set: The collection of all subsets of set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set.

Eg: If A = {p, q} then all the subsets of A will be

P(A) = {∅, {p}, {q}, {p, q}}

Number of elements of P(A) = n[P(A)] = 4 = 2^2

In general, Power Set =n[P(A)] = 2m where m is the number of elements in set A.

13. Universal Set: A set which contains all the elements of other given sets is called a universal set. The symbol for denoting a universal set is U or ξ.

Eg: If A = {1, 2, 3}   B = {2, 3, 4}   C = {3, 5, 7}

then U = {1, 2, 3, 4, 5, 7}

Operations on Sets

When two or more sets combine together to form one set under the given conditions, then operations on sets are carried out.

1. Union of Sets: The union of sets A and B, written as AUB, is the set of elements that appear in either A OR B.

Eg: A = {1,2,3,4,5}, B = {2,4,6,8,10}

The union of A and B (i.e. AUB) is {1, 2, 3, 4, 5, 6, 8, 10}

2. Intersection of Sets: The intersection of sets A and B, denoted as A ∩ B, is the set of elements common to both A AND B.

Eg: A = {1,2,3,4,5}, B = {2,4,6,8,10}

The intersection of A and B (i.e. A∩B) is simply {2, 4}

3. Cartesian Product of Sets: The Cartesian product of sets A and B, written A x B, is expressed as:

A x B = {(a,b) | a is every element in A, b is every element in B}

Eg: A = {1,2}, B = {4,5,6}

The Cartesian product of A and B (i.e. A x B) is {(1,4), (1,5), (1,6), (2,4), (2,5), (2,6)}

4. Complement of a Set: In complement of a set if U be the universal set and A a subset ofU, then the complement of A is the set of all elements of U which are not the elements of A. We denote the complement of A as A'.

Eg: If U = {1, 2, 3, 4, 5, 6, 7}

A = {1, 3, 7}

We observe that 2, 4, 5, 6 are the only elements of U which do not belong to A.

Therefore, A' = {2, 4, 5, 6}

Note:

The complement of a universal set is an empty set.

The complement of an empty set is a universal set.
The set and its complement are disjoint sets.

Some properties of complement sets
(i) A ∪ A' = A' ∪ A = U (Complement law)

(ii) (A ∩ B') = ϕ (Complement law)

(iii) (A ∪ B) = A' ∩ B' (De Morgan's law)

(iv) (A ∩ B)' = A' ∪ B' (De Morgan's law)

(v) (A')' = A (Law of complementation)

(vi) ϕ' = U (Law of empty set

(vii) U' = ϕ and universal set)

Laws of Sets

1. Commutative Laws: For any two finite sets A and B;

(i) A U B = B U A

(ii) A ∩ B = B ∩ A

2. Associative Laws: For any three finite sets A, B and C;

(i) (A U B) U C = A U (B U C)

(ii) (A ∩ B) ∩ C = A ∩ (B ∩ C)

Thus, union and intersection are associative.

3. Idempotent Laws: For any finite set A;

(i) $A \cup A = A$

(ii) $A \cap A = A$

4. Distributive Laws: For any three finite sets A, B and C;

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Thus, union and intersection are distributive over intersection and union respectively.

5. De Morgan's Laws: For any two finite sets A and B;

(i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

De Morgan's Laws can also we written as:

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

More laws of sets:

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

(iii) $A - B = A \Leftrightarrow A \cap B = \emptyset$

(iv) $(A - B) \cup B = A \cup B$

(v) $(A - B) \cap B = \emptyset$

(vi) $A \subseteq B \Leftrightarrow B' \subseteq A'$

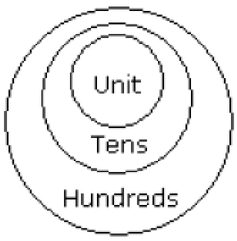
(vii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Also If A and B are two sets then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

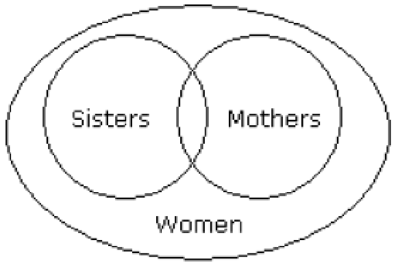
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(A \cap B) - (B \cap C) + n(A \cap B \cap C)$

Example 1: If the first word is related to second word and second word is related to third word. Then they will be shown by diagram as given below.
Unit, Tens, Hundreds



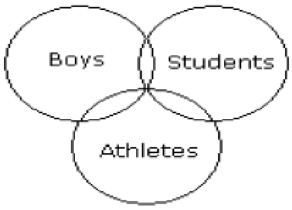
Ten units together make one Tens or in one tens, whole unit is available and ten tens together make one hundreds.

Example 2: If there is some relation between two items and these two items are completely related to a third item they will be shown as given below.
Women, Sisters, Mothers



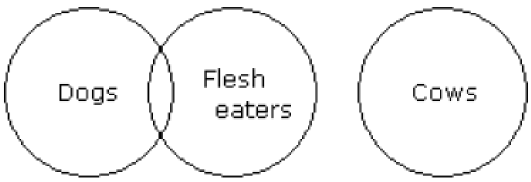
Some sisters may be mothers and vice-versa. Similarly some mothers may not be sisters and vice-versa. But all the sisters and all the mothers belong to women group.

Example 3: All the three items are related to one another but to some extent not completely.
Boys, Students, Athletes



Some boys may be students and vice-versa. Similarly some boys may be athletes and vice-versa. Some students may be athletes and vice-versa.

Example 4: First item is partially related to second but third is entirely different from the first two.
Dogs, Flesh-eaters, Cows



Some dogs are flesh-eaters but not all while any dog or any flesh-eater cannot be cow.

Example 5: If a set A = {3, 6, 9, 10, 13, 18}. State whether the following statements are 'true' or 'false':

(i) 7 ∈ A

(ii) 12 ∉ A

(iii) 13 ∈ A

(iv) 9, 12 ∈ A

(v) 12, 14, 15 ∈ A

Solution: (i) 7 ∈ A

False, since the element 7 does not belongs to the given set A.

(ii) $10 \notin A$

False, since the element 10 belongs to the given set A.

(iii) $13 \in A$

True, since the element 13 belongs to the given set A.

(iv) $9, 10 \in A$

True, since the elements 9 and 12 both belong to the given set A.

(v) $10, 13, 14 \in A$

False, since the element 14 does not belongs to the given set A.

Example 6: If A {1, 3, 5}, then write all the possible subsets of A. Find their numbers.

Solution: The subset of A containing no elements - { }

The subset of A containing one element each - {1} {3} {5}

The subset of A containing two elements each - {1, 3} {1, 5} {3, 5}

The subset of A containing three elements - {1, 3, 5)

All possible subsets of A are { }, {1}, {3}, {5}, {1, 3}, {3, 5}, {1,5},{1,3,5}

Therefore, number of all possible subsets of A is 8 which is equal 23.

Proper subsets are = {1}, {3}, {5}, {1, 3}, {3, 5}, {1,5}, {1, 3, 5}

Number of proper subsets are 7 = 8 - 1 = 23 – 1

Example 7: Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Solution: Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

then $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 20 + 28 - 36 = 48 - 36 = 12$

Example 8: In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

Solution: Let A = Set of people who like cold drinks B = Set of people who like hot drinks Given, $(A \cup B) = 60$ $n(A) = 27$ $n(B) = 42$ then;

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 27 + 42 - 60$

$= 69 - 60 = 9$

Therefore, 9 people like both tea and coffee.

Example 9: A survey was conducted of 100 people to find out whether they had read recent issues of Golmal, a monthly magazine. The summarized information regarding readership in 3 months is given below:

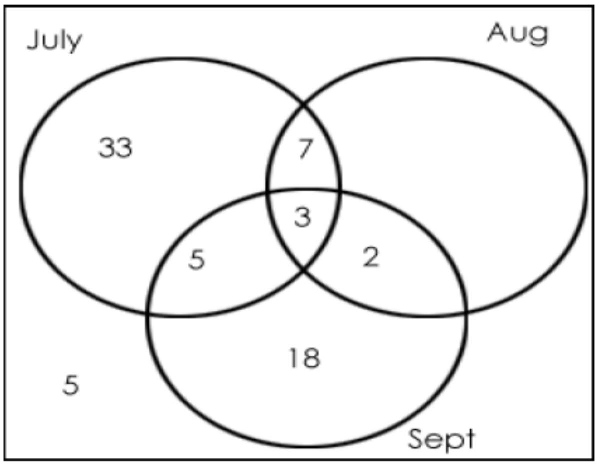
Only September: 18; September: 28; None of the three months: 24.

September but not August: 23; July: 48; September and July: 8; July and August: 10

What is the number of surveyed people who have read exactly two consecutive issues (out of the three)?

A. 7 B. 9 C. 12 D. 14 E. 17

Solution:



So, exactly two consecutive issues will be in July-August and August-September.

So, the answer is 7+2=9 i.e. option B.

Example 10: In a survey of 500 students of a college, it was found that 49% liked watching football, 53% liked watching hockey and 62% liked watching basketball. Also, 27% liked watching football and hockey both, 29% liked watching basketball and hockey both and 28% liked watching football and basketball both. 5% liked watching none of these games.

How many students like watching all the three games?

Find the ratio of number of students who like watching only football to those who like watching only hockey.

Find the number of students who like watching only one of the three given games.

Find the number of students who like watching at least two of the given games.

Solution: n(F) = percentage of students who like watching football = 49%

n(H) = percentage of students who like watching hockey = 53%

n(B)= percentage of students who like watching basketball = 62%

$n(F \cap H) = 27\%$; $n(B \cap H) = 29\%$; $n(F \cap B) = 28\%$

Since 5% like watching none of the given games so, $n(F \cup H \cup B) = 95\%$.

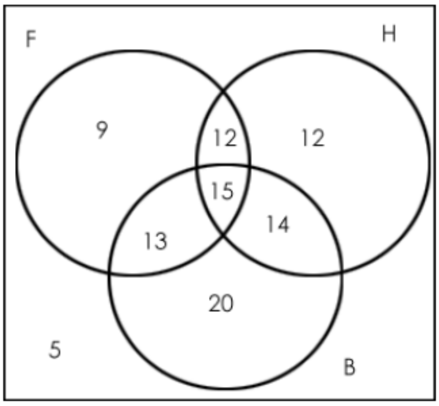
Now applying the basic formula,

$95\% = 49\% + 53\% + 62\% - 27\% - 29\% - 28\% + n(F \cap H \cap B)$

Solving, you get $n(F \cap H \cap B) = 15\%$.

Now, make the Venn diagram as per the information given.

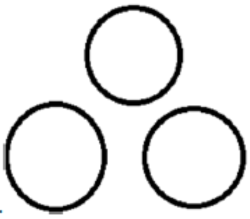
Note: All values in the Venn diagram are in percentage.



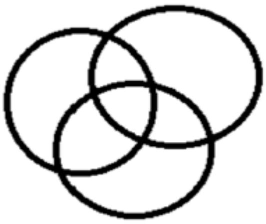
1. Number of students who like watching all the three games = 15 % of 500 = 75.
2. Ratio of the number of students who like only football to those who like only hockey = (9% of 500)/(12% of 500) = 9/12 = 3:4.
3. The number of students who like watching only one of the three given games = (9% + 12% + 20%) of 500 = 205
4. The number of students who like watching at least two of the given games=(number of students who like watching only two of the games) +(number of students who like watching all the three games)= (12 + 13 + 14 + 15)% i.e. 54% of 500 = 270.

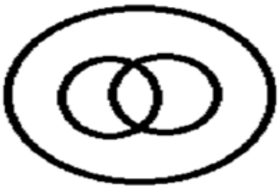
LEVEL - I

Directions(1-10): Which of the following venn diagrams correctly represents relations for the following:

A. 

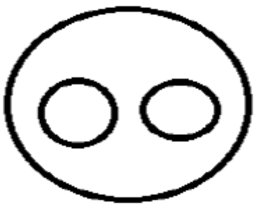
B. 

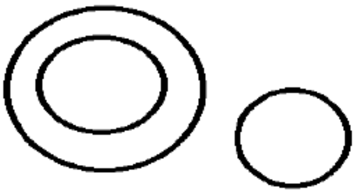
C. 

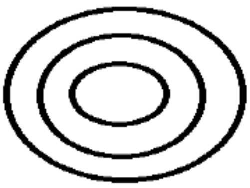
D. 

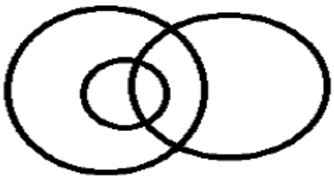
1. Yak, Zebra, Bear
2. Citizens, Educated, Men
3. Dog, Animal, Pet
4. Men, Authors, Teachers
5. Boys, Students, Athletes
6. Whales, Fishes, Crocodiles
7. Tennis fans, Cricket Players, Students
8. Mountains, Forests, Earth
9. Flowers, Cloths, White
10. Examination, Questions and Practice

Directions(11-20): Choose the correct diagram from the below mentioned venn diagrams for the following relations:

A. 

B. 

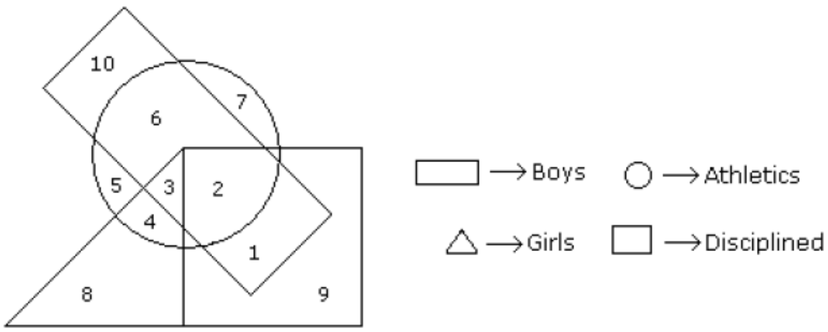
C. 

D. 

11. Diseases, T.B., Scurvy
12. Sun, Moon, Stars
13. Animals, Men, Plants
14. Factory, Product and Machinery
15. Doctors, Lawyers, Professionals
16. Triangles, Four-sided figure, Square
17. Human, girls and boys
18. Musicians, Instrumentalist, Violinists
19. Sparrows, Birds, Mice
20. Elected house, M.P., M.L.A.

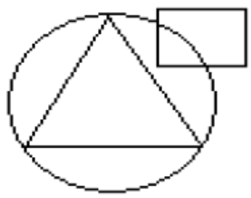
LEVEL - II

1. In the following diagram the boys who are athletic and are disciplined are indicated by which number?



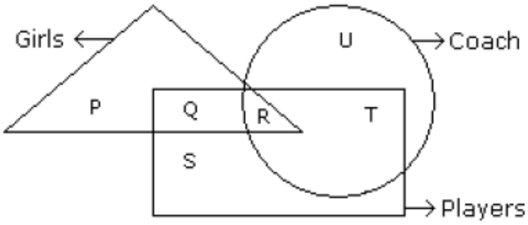
A. 1 B. 2 C. 10 D. 6

2. In an organization of pollution control board, engineers are represented by a circle, legal experts by a square and environmentalist by a triangle. Who is most represented in the board as shown in the following figure ?



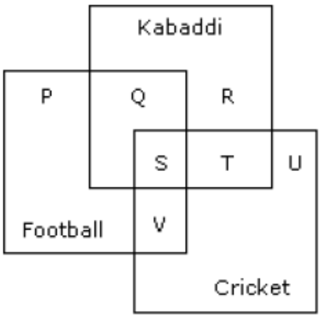
A. Environmentalists
B. Legal Experts
C. Engineers with legal background
D. Environmentalists with Engineering background

3. In the following figure triangle represents 'girls', square players and circle-coach. Which part of the diagram represents the girls who are player but not coach?



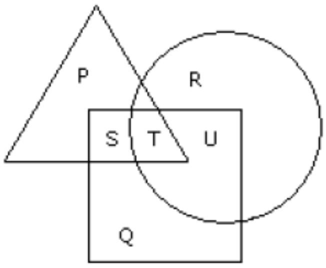
A. P B. Q C. R D. S

4. The diagram given below represents those students who play Cricket, Football and Kabaddi. Study the diagram and identify the students who play all the three games.



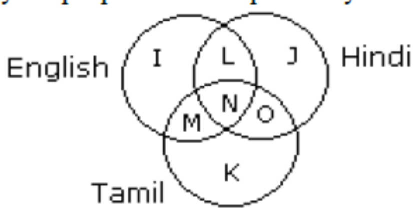
A. P + Q + R B. V + T C. S + T + V D. S

5. In the figure given below, square represents doctors, triangle represents ladies and circle represents surgeon. By which letter the ladies who doctor and surgeon both are represented ?



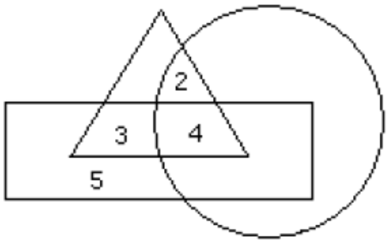
A. U B. T C. S D. P

6. Study the diagram and identify the people who can speak only one language.



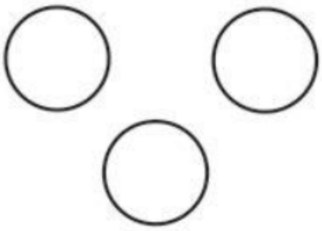
A. L + M + O B. K + J + I C. K D. I

7. In the given figure if Triangle represents healthy people, Square represents old persons and Circle represents men then What is the number of those men who are healthy but not old ?



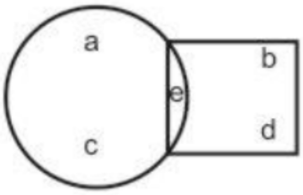
A. 3 B. 4 C. 6 D. 2

8. Which of the following groups of elements given in the alternatives is best represented by the diagram, given below:



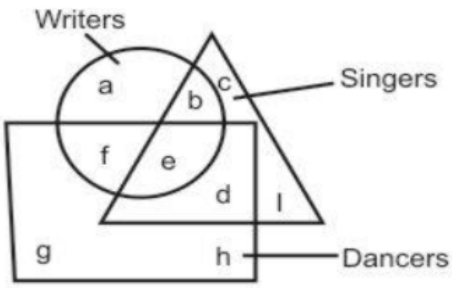
A. Barley, Mustard, Potato
B. Shoes, Garments, Clothes
C. Hand, Body, Feet
D. Bridge, Brick, Building

9. In the diagram given below, the circle represents the students qualified in General Awareness (GA) and the square represents the students qualified in Quantitative Aptitude (QA) test paper. Which of the following represents the students who passed in both the papers?



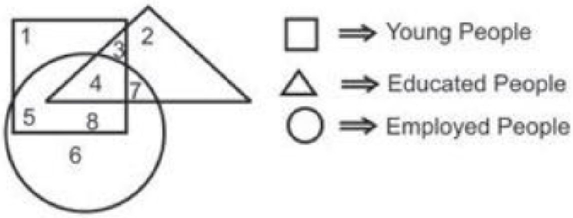
A. a and c B. b and d C. a, b, c, d and e all D. e only

10. In the following Venn-Diagram, find out the letters/ alphabet that represents the writer who can sing as well as dance. Rectangle represents dancers, triangle represents singers and the circle represents the writers.



A. f B. b C. e D. i

11. If square represents young people, triangle represents educated people and circle represents employed people then, which of the following numbers might represent those areas that represent young, uneducated but employed people?



A. 4 B. 5 C. 8 D. 5 and 8

12. In a town of 500 people, 285 read Hindu and 212 read Indian Express and 127 read Times of India, 20 read only Hindu and Times of India and 29 read only Hindu and Indian Express and 35 read only Times of India and Indian express. 50 read no newspaper. Then how many read only one paper?

A. 123 B. 231 C. 312 D. 321

13. Out of 120 students in a school, 5% can play all the three games Cricket, Chess and Carrom. If so happens that the number of players who can play any and only two games is 30. The number of students who can play the Cricket alone is 40. What is the total number of those who can play Chess alone or Carrom alone?

A. 45 B. 44 C. 46 D. 24

Directions(14-15): A college has 63 students studying Political Science, Chemistry and Botany. 33 students study Political Science, 25 Chemistry and 26 Botany. 10 study Political Science and Chemistry, 9 study Botany and Chemistry while 8 study both Political Science and Botany. Same numbers of students study all three subjects as those who learn none of the three.

14. How many students study all the three subjects?

A. 2 B. 3 C. 5 D. 7

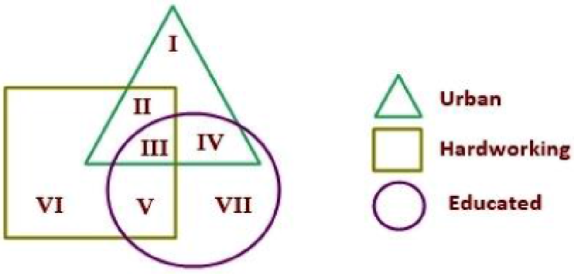
15. How many students study only one of the three subjects?

A. 21 B. 30 C. 39 D. 42

16. In a class,7 students like to play Basketball and 8 like to play Cricket. 3 students like to play on both Basketball and Cricket. How many students like to play Basketball or Cricket or both?

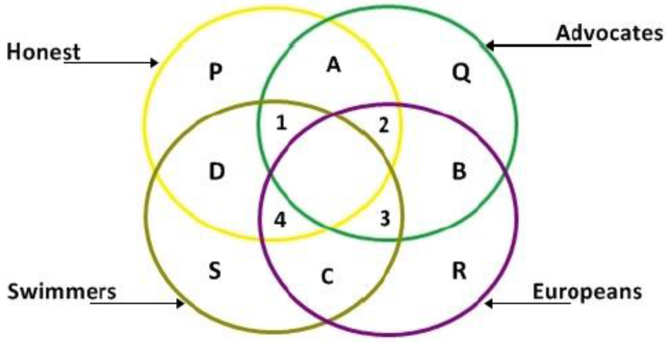
A. 12 B. 13 C. 15 D. 17

17. Which one of the area marked I – VII represents the urban educated who are not hardworking?



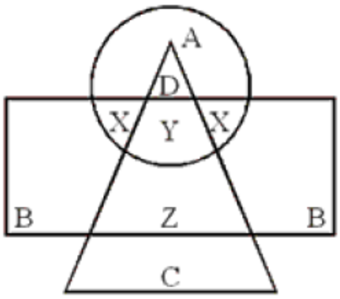
A. IV B. III C. II D. I

18. What does the area marked 1 in the figure given below represent?



A. All honest European swimmers
B. All honest advocates who are swimmers
C. All no-European advocates who are honest swimmers
D. All non-Europeans who are honest swimmers

Directions to Solve (19-20): In the following diagram, the circle represents College Professors, the triangle stands for Surgical Specialists, and Medical Specialists are represented by the rectangle.



19. College Professors who are also Surgical Specialists are represented by?

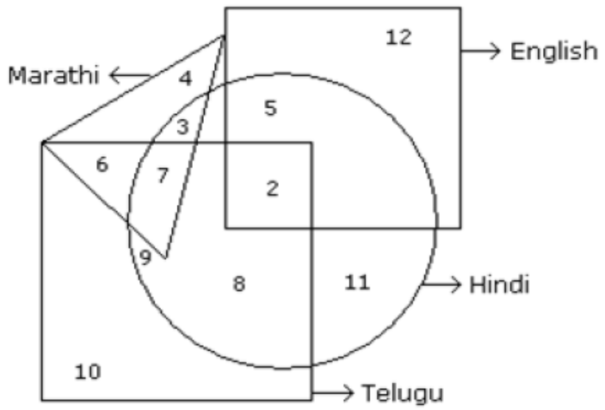
A. A B. B C. C D. D

20. B represents?
A. Professors who are neither Medical nor Surgical Specialists
B. Professors who are not Surgical Specialists
C. Medical Specialists who are neither Professors nor Surgical Specialists
D. Professors who are not Medical Specialists

LEVEL - III

Directions to Solve (1-5):

In the following figure small square represents the persons who know English, triangle to those who know Marathi, big square to those who know Telugu and circle to those who know Hindi. In the different regions of the figures from 1 to 12 are given.



1. How many persons can speak English and Hindi both the languages only?

A. 5	B. 8	C. 7	D. 18

2. How many persons can speak Marathi and Telugu both?

A. 10	B. 11	C. 13	D. None of these

3. How many persons can speak only English ?

A. 9	B. 12	C. 7	D. 19

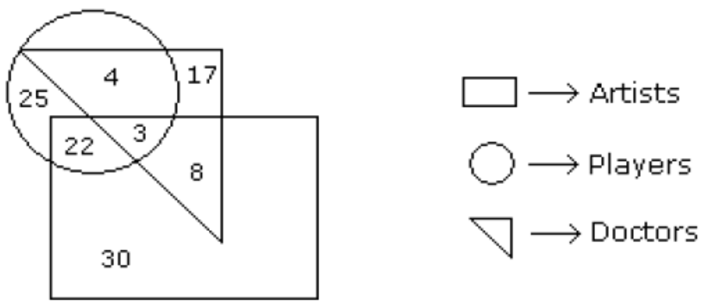
4. How many persons can speak English, Hindi and Telugu?

A. 8	B. 2	C. 7	D. None of these

5. How many persons can speak all the languages?

A. 1	B. 8	C. 2	D. None

Directions to Solve (6-10): Study the following figure and answer the questions given below.



6. How many doctors are neither artists nor players ?
A. 17 B. 5 C. 10 D. 30

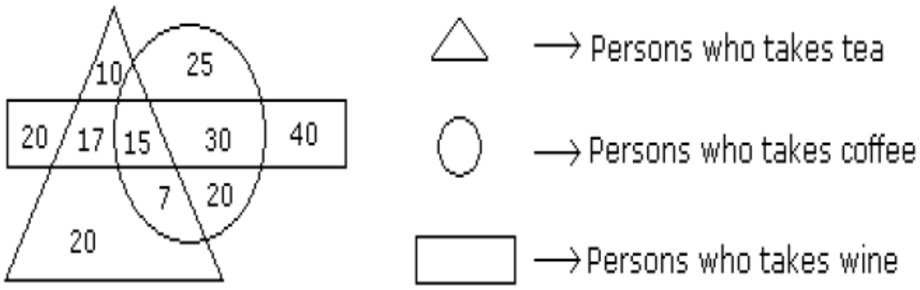
7. How many doctors are both players and artists ?
A. 22 B. 8 C. 3 D. 30

8. How many artists are players ?
A. 5 B. 8 C. 25 D. 16

9. How many players are neither artists nor doctors ?
A. 25 B. 17 C. 5 D. 10

10. How many artists are neither players nor doctors ?
A. 10 B. 17 C. 30 D. 15

Directions to Solve (11-15): Study the diagram given below and answer each of the following questions.



11. How many persons who take tea and wine but not coffee?
A. 20 B. 17 C. 25 D. 15

12. How many persons are there who take both tea and coffee but not wine?
A. 22 B. 17 C. 7 D. 20

13. How many persons take wine ?
A. 100 B. 82 C. 92 D. 122

14. How many persons are there who takes only coffee ?
A. 90 B. 45 C. 25 D. 20

15. How many persons take all the three?
A. 20 B. 17 C. 25 D. 15

Directions to Solve (16-20): In a college, 200 students are randomly selected. 140 like tea, 120 like coffee and 80 like both tea and coffee.

16. How many students like only tea?

A. 20	B. 40	C. 50	D. 60

17. How many students like only coffee?

A. 20	B. 40	C. 50	D. 60

18. How many students like neither tea nor coffee?

A. 20	B. 40	C. 50	D. 60

19. How many students like only one of tea or coffee?

A. 50	B. 70	C. 90	D. 100

20. How many students like at least one of the beverages?

A. 120	B. 170	C. 180	D. 150

Venn Diagram Set Theory Solutions

Level – I									
Q. No.	Answer	Q. No.	Answer	Q. No.	Answer	Q. No.	Answer	Q. No.	Answer
1	A	2	D	3	D	4	C	5	C
6	A	7	C	8	D	9	B	10	C
11	A	12	B	13	B	14	A	15	A
16	B	17	A	18	D	19	B	20	B
Level – II									
Q. No.	Answer	Q. No.	Answer	Q. No.	Answer	Q. No.	Answer	Q. No.	Answer
1	B	2	D	3	B	4	D	5	B
6	B	7	D	8	A	9	D	10	C
11	D	12	D	13	B	14	B	15	C
16	A	17	A	18	B	19	D	20	C
Level – III									
Q. No.	Answer	Q. No.	Answer	Q. No.	Answer	Q. No.	Answer	Q. No.	Answer
1	A	2	C	3	B	4	B	5	D
6	A	7	C	8	C	9	A	10	C
11	B	12	C	13	D	14	B	15	D
16	D	17	B	18	A	19	D	20	C