Announcements

Environments for Higher-Order Functions

Environments Enable Higher-Order Functions

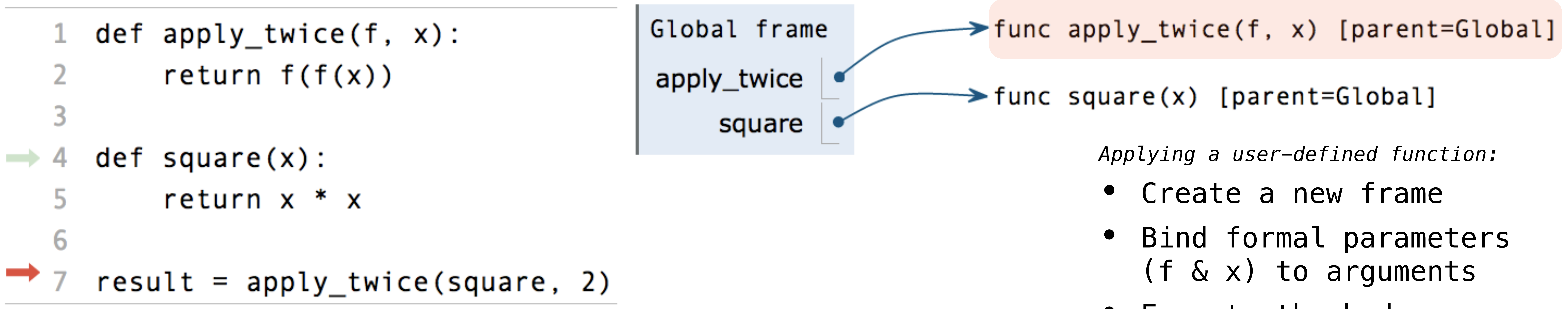
Functions are first-class: Functions are values in our programming language

Higher-order function: A function that takes a function as an argument value **or**
A function that returns a function as a return value

Environment diagrams describe how higher-order functions work!

(Demo1)

Names can be Bound to Functional Arguments

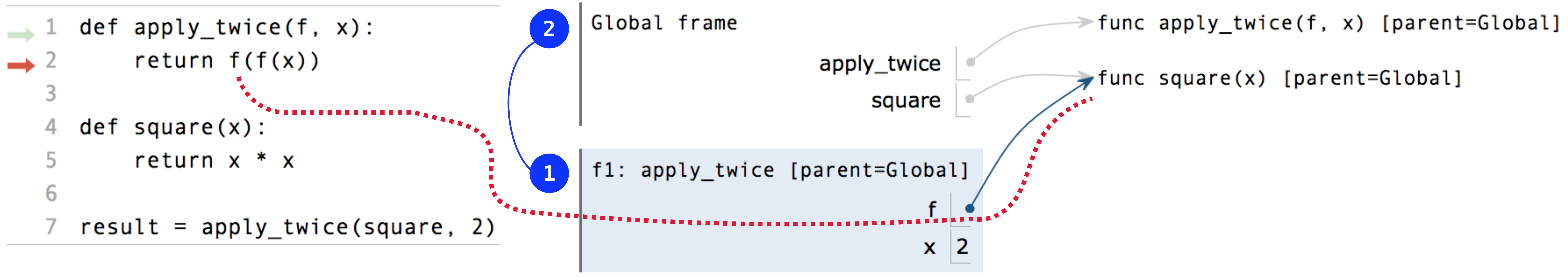


```
def apply_twice(f, x):
    return f(f(x))

def square(x):
    return x * x

result = apply_twice(square, 2)
```

Global frame
apply_twice
square
func apply_twice(f, x) [parent=Global]
func square(x) [parent=Global]

Applying a user-defined function:

- Create a new frame
- Bind formal parameters (f & x) to arguments
- Execute the body: return f(f(x))



```
def apply_twice(f, x):
    return f(f(x))

def square(x):
    return x * x

result = apply_twice(square, 2)
```

2
1
Global frame
apply_twice
square
func apply_twice(f, x) [parent=Global]
func square(x) [parent=Global]
f1: apply_twice [parent=Global]
f
x 2

pythontutor.com/composingprograms.html#code=def%20apply_twice%28f,%20x%29%3A%0A%20%20%20%20return%20f%28f%28x%29%29%0A%20%20%20%20%0Adef%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%20%20%20%20%0Aresult%20%3D%20apply_twice%28square,%202%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

Functions as Return Values

Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

A function that returns a function

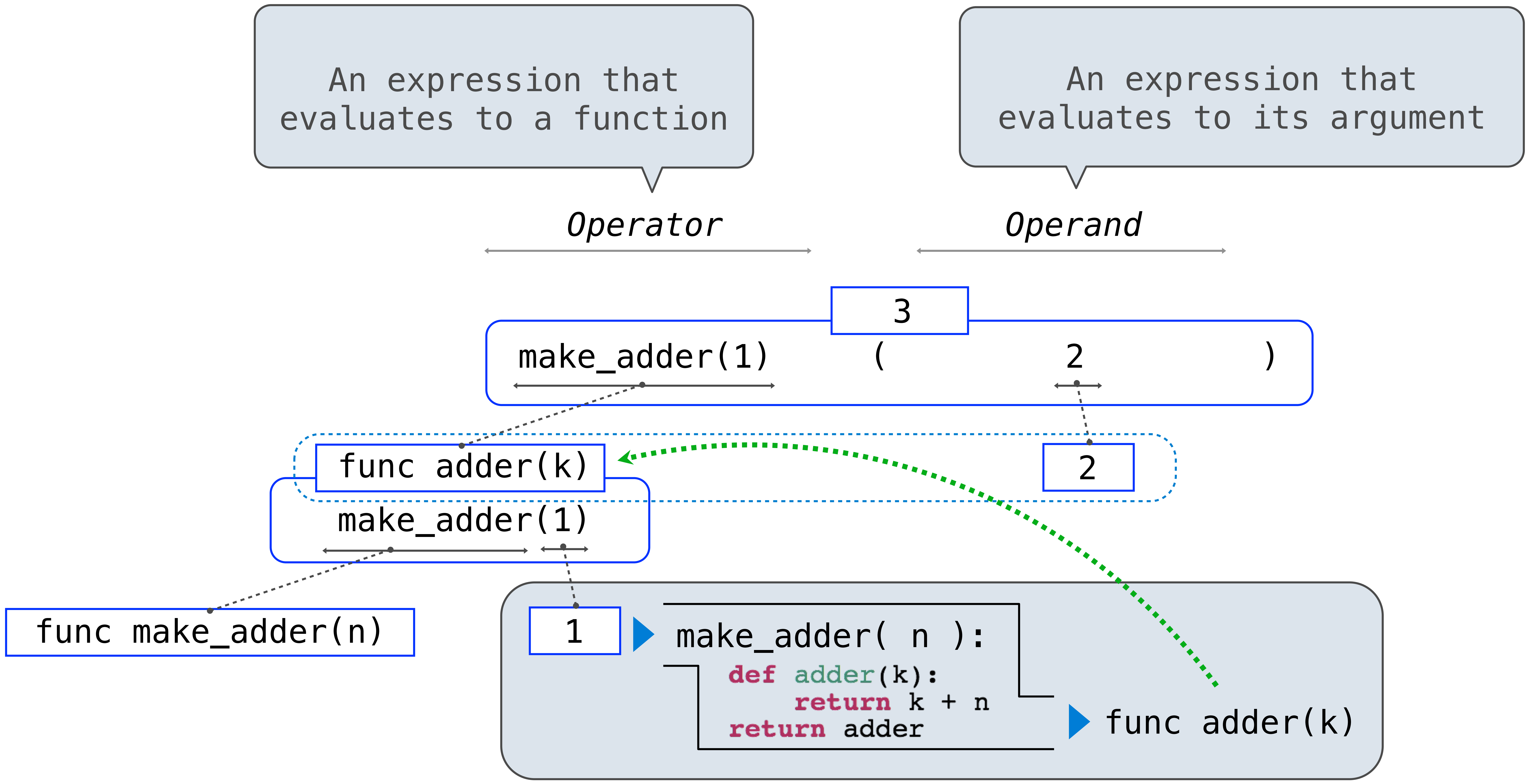
```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

The name add_three is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions

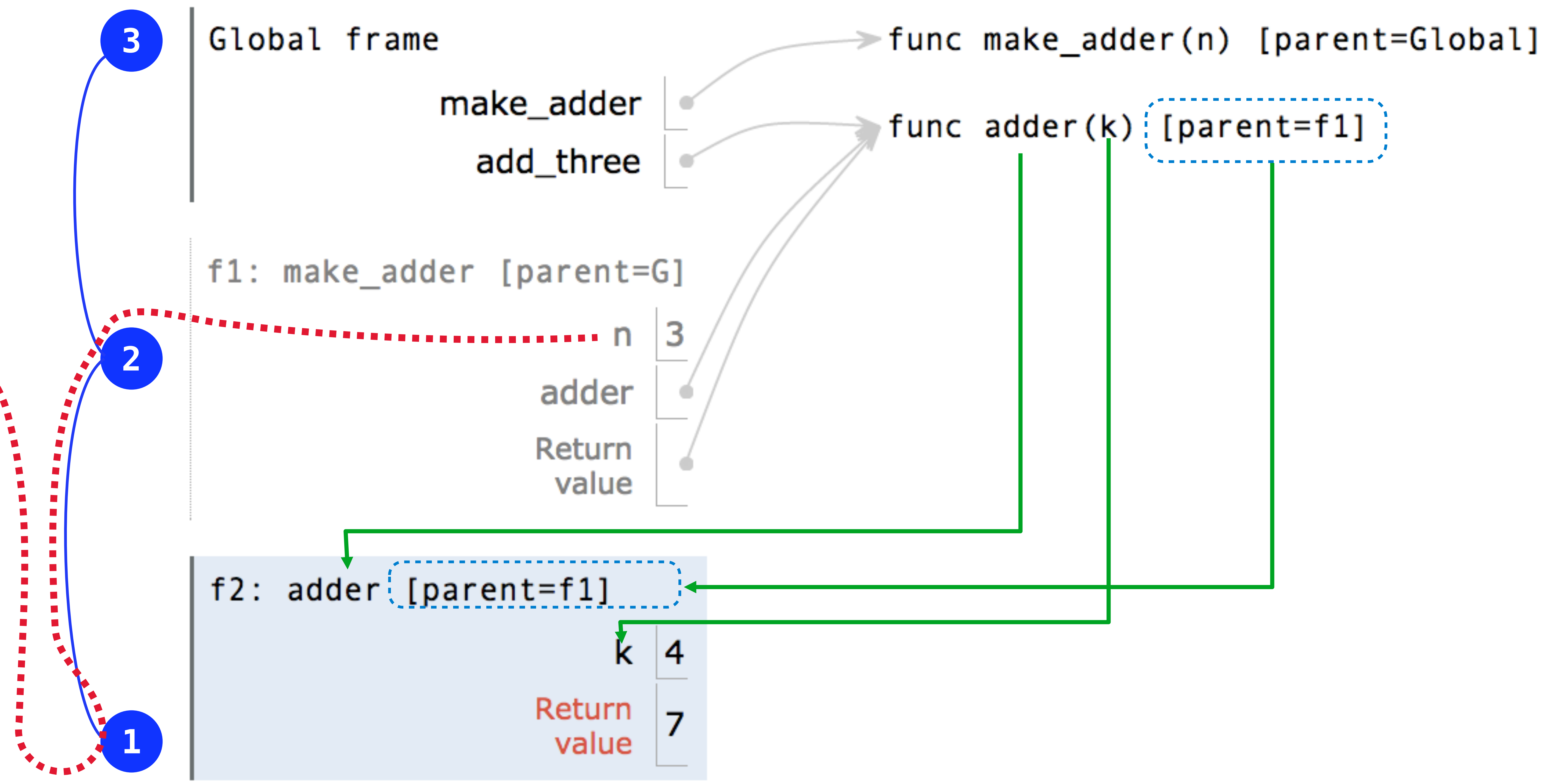


```
def adder(k):
    return k + n
return adder
```

Environment Diagrams for Nested Def Statements

Nested def

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder

add_three = make_adder(3)
add_three(4)
```



- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)
- The parent of a frame is the parent of the function called

http://pythontutor.com/composingprograms.html#code=def%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Athree_more_than%20%3D%20make_adder%283%29%0Aresult%20%3D%20three_more_than%284%29&cumulative=false&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

How to Draw an Environment Diagram

When a function is defined:

Create a function value: func <name>(<formal parameters>) [parent=<label>]

Its parent is the current frame.

```
f1: make_adder        func adder(k) [parent=f1]
```

Bind <name> to the function value in the current frame

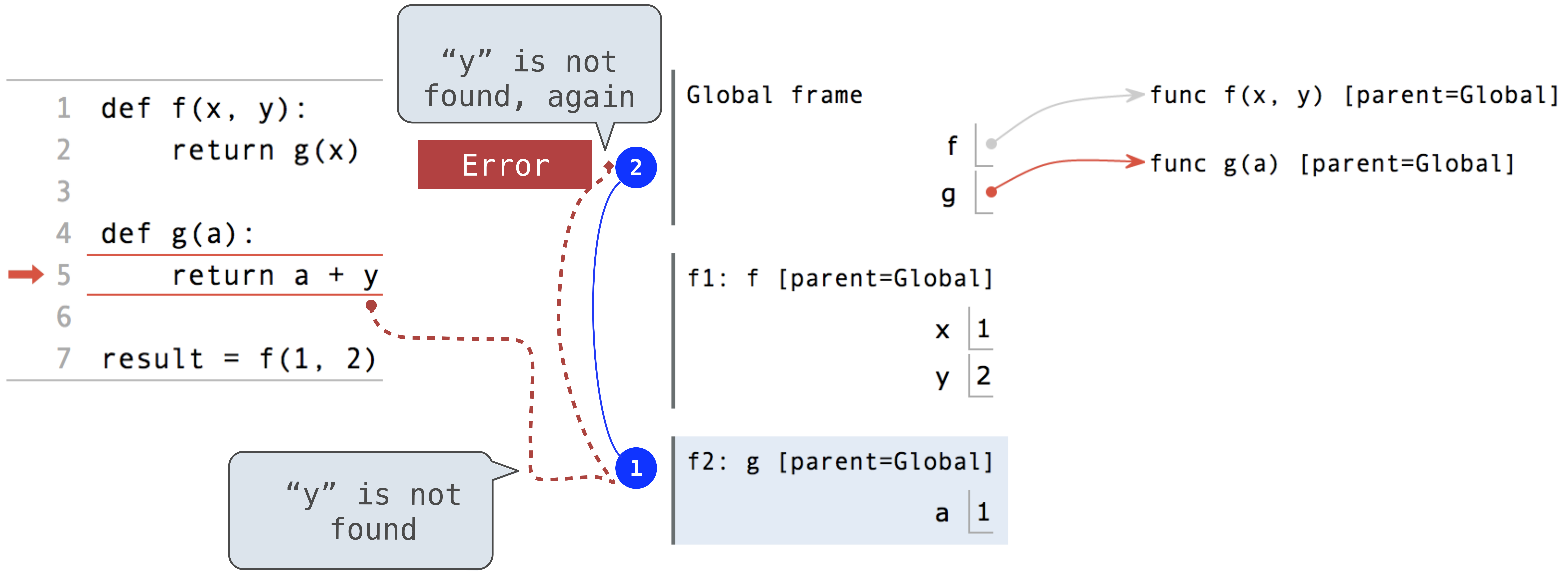
When a function is called:

1. Add a local frame, titled with the <name> of the function being called.
2. Copy the parent of the function to the local frame: [parent=<label>]
3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

```
(Demo2)
```

Local Names are not Visible to Other (Non-Nested) Functions



```
def f(x, y):
    return g(x)

def g(a):
    return a + y

result = f(1, 2)
```

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,%20y%29%3A%0A%20%20%20%20return%20g%28x%29%0A%0Adef%20g%28a%29%3A%0A%20%20%20%20return%20a%20%2B%20y%0A%20%20%20%20%0Aresult%20%3D%20f%281,%202%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

Lambda Expressions

λ

`(Demo4)`

Lambda Expressions

```
>>> x = 10
```

An expression: this one evaluates to a number

```
>>> square = x * x
```

Also an expression: evaluates to a function

```
>>> square = lambda x: x * x
```

Important: No "return" keyword!

A function

with formal parameter x

that returns the value of "x * x"

Must be a single expression

```
>>> square(4)
16
```

Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

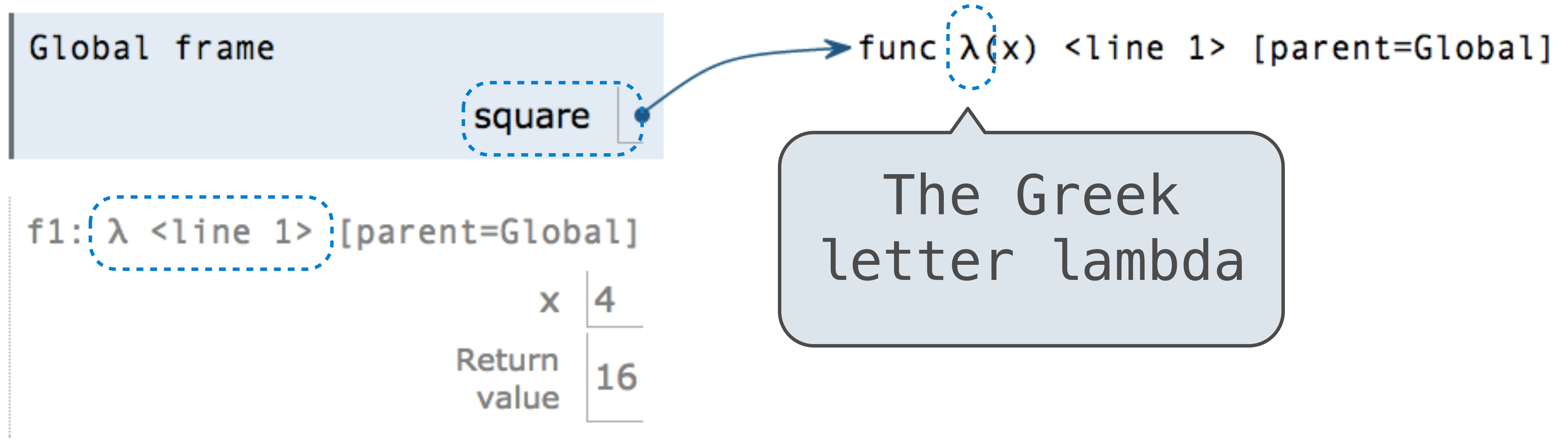
Lambda Expressions Versus Def Statements

```
square = lambda x: x * x
```

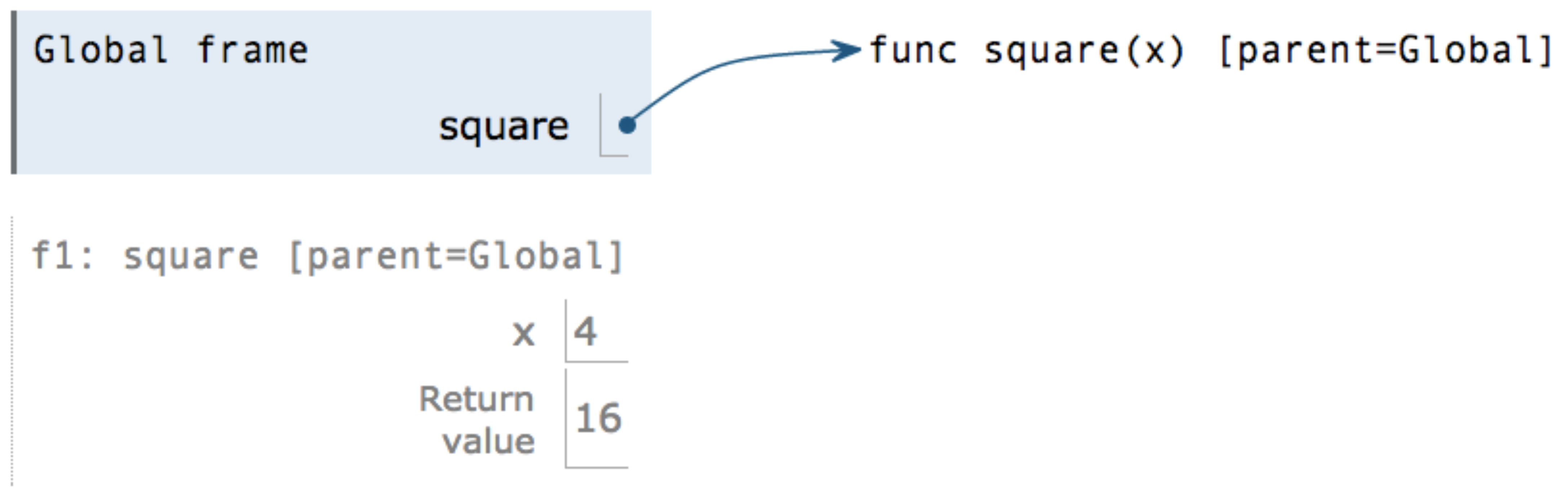
VS

```
def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).



Global frame
square
func λ(x) <line 1> [parent=Global]
f1: λ <line 1> [parent=Global]
x 4
Return value 16
The Greek letter lambda



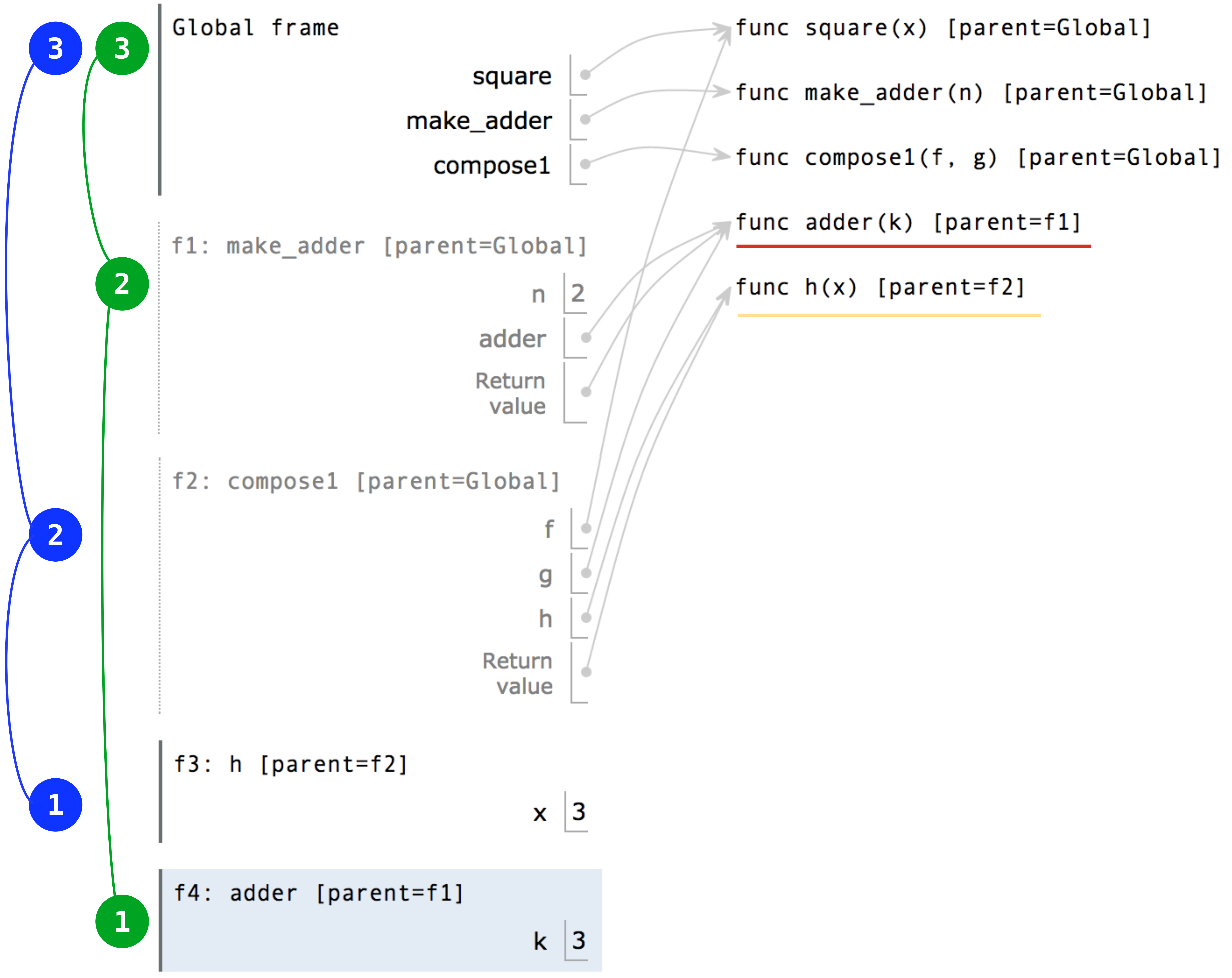
Function Composition

(Demo5)

The Environment Diagram for Function Composition

```
def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
    return adder

def compose1(f, g):
    def h(x):
        return f(g(x))
    return h

compose1(square, make_adder(2))(3)
```

Return value of make_adder is an argument to compose1



Global frame
square
make_adder
compose1
func square(x) [parent=Global]
func make_adder(n) [parent=Global]
func compose1(f, g) [parent=Global]
func adder(k) [parent=f1]
func h(x) [parent=f2]
f1: make_adder [parent=Global]
n 2
adder
Return value
f2: compose1 [parent=Global]
f
g
h
Return value
f3: h [parent=f2]
x 3
f4: adder [parent=f1]
k 3
1
2
3
1
2
3

http://pythontutor.com/composingprograms.html#code=def%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%0Adef%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Adef%20compose1%28f,%20g%29%3A%0A%20%20%20%20def%20h%28x%29%3A%0A%20%20%20%20%20%20%20%20return%20f%28g%28x%29%29%0A%20%20%20%20return%20h%0A%0Acompose1%28square,%20make_adder%282%29%29%283%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=%5B%5D&curInstr=