

Calculating collision time of ith ball with jth cushion's collision line

with (LinearAlgebra) :

The line is defined by vectors

$$p1 := \begin{bmatrix} p1x \\ p1y \\ h \end{bmatrix} \qquad \begin{bmatrix} p1x \\ p1y \\ h \end{bmatrix} \qquad (1)$$

$$p2 := \begin{bmatrix} p2x \\ p2y \\ h \end{bmatrix} \qquad \begin{bmatrix} p2x \\ p2y \\ h \end{bmatrix} \qquad (2)$$

The ball trajectory is defined by

$$r := \begin{bmatrix} ax t^2 + bx t + cx \\ ay t^2 + byy t + cy \\ az t^2 + bz t + cz \end{bmatrix} \qquad \begin{bmatrix} ax t^2 + bx t + cx \\ ay t^2 + byy t + cy \\ az t^2 + bz t + cz \end{bmatrix} \qquad (3)$$

Using equation for distance of point to line (<https://mathworld.wolfram.com/Point-LineDistance3-Dimensional.html>),

and setting distance to R yields

$$0 = \frac{\text{Norm}((r - p1) \times (r - p2), 2)}{\text{Norm}(p2 - p1, 2)} - R$$

$$0 = 1 /$$

(4)

$$\begin{aligned}
& (|-p2x + plx|^2 + |ply \\
& - p2y|^2) \\
&^{1/2} \left(|(ay t^2 + byy t + cy - ply) (az t^2 + bz t + cz - h) - (az t^2 + bz t + cz \right. \\
& - h) (ay t^2 + byy t + cy - p2y)|^2 + |(az t^2 + bz t + cz - h) (ax t^2 + bx t + cx \\
& - p2x) - (ax t^2 + bx t + cx - plx) (az t^2 + bz t + cz - h)|^2 + |(ax t^2 + bx t + cx \\
& - plx) (ay t^2 + byy t + cy - p2y) - (ay t^2 + byy t + cy - ply) (ax t^2 + bx t + cx \\
& - p2x)|^2 \Big)^{1/2} - R
\end{aligned}$$

$\xrightarrow{\text{simplify symbolic}}$

0

(5)

$$\begin{aligned}
& = \left((ply^2 az^2 t^4 + ply^2 bz^2 t^2 + az^2 t^4 p2y^2 + bz^2 t^2 p2y^2 + az^2 t^4 p2x^2 + bz^2 t^2 p2x^2 \right. \\
& + plx^2 az^2 t^4 + plx^2 bz^2 t^2 + ax^2 t^4 p2y^2 + bx^2 t^2 p2y^2 + plx^2 ay^2 t^4 + plx^2 byy^2 t^2 \\
& + ay^2 t^4 p2x^2 + byy^2 t^2 p2x^2 + ply^2 ax^2 t^4 + ply^2 bx^2 t^2 + 4 ply cz h p2y + 4 cz p2x plx h \\
& + 2 cx p2y plx cy - 2 cx p2y cy p2x + 2 plx p2y cy p2x - 2 ply cx plx cy \\
& + 2 ply cx plx p2y + 2 ply cx cy p2x + 2 ply p2x cx p2y + 2 ply p2x plx cy \\
& - 2 ply p2x plx p2y - 2 ply^2 cx p2x - 2 ply cx^2 p2y - 2 ply p2x^2 cy - 2 cx p2y^2 plx \\
& - 2 plx^2 cy p2y - 2 plx cy^2 p2x - 2 ply^2 cz h - 2 ply cz^2 p2y - 2 ply h^2 p2y \\
& - 2 cz p2y^2 h + ply^2 cx^2 + ply^2 p2x^2 + cx^2 p2y^2 + plx^2 cy^2 + plx^2 p2y^2 + cy^2 p2x^2 \\
& + ply^2 cz^2 + ply^2 h^2 + cz^2 p2y^2 + h^2 p2y^2 + cz^2 p2x^2 + h^2 p2x^2 + plx^2 cz^2 + plx^2 h^2 \\
& - 2 cz p2x^2 h - 2 cz^2 p2x plx - 2 h^2 p2x plx - 2 plx^2 cz h + 2 ply^2 az t^3 bz \\
& - 2 ply az^2 t^4 p2y + 2 ply^2 az t^2 cz - 2 ply^2 az t^2 h - 2 ply bz^2 t^2 p2y + 2 ply^2 bz t cz \\
& - 2 ply^2 bz t h + 2 az t^3 p2y^2 bz + 2 az t^2 p2y^2 cz - 2 az t^2 p2y^2 h + 2 bz t p2y^2 cz \\
& - 2 bz t p2y^2 h + 2 az t^3 p2x^2 bz - 2 az^2 t^4 p2x plx + 2 az t^2 p2x^2 cz - 2 az t^2 p2x^2 h \\
& - 2 bz^2 t^2 p2x plx + 2 bz t p2x^2 cz - 2 bz t p2x^2 h + 2 plx^2 az t^3 bz + 2 plx^2 az t^2 cz \\
& - 2 plx^2 az t^2 h + 2 plx^2 bz t cz - 2 plx^2 bz t h - 2 ay t^2 p2x^2 ply + 2 ay t^2 p2x^2 cy \\
& - 2 byy t p2x^2 ply + 2 byy t p2x^2 cy + 2 ply^2 ax t^3 bx + 2 ply^2 ax t^2 cx - 2 ply^2 ax t^2 p2x \\
& + 2 ply^2 bx t cx - 2 ply^2 bx t p2x + 2 ax t^3 p2y^2 bx - 2 ax^2 t^4 p2y ply + 2 ax t^2 p2y^2 cx
\end{aligned}$$

$$\begin{aligned}
& -2axt^2p2y^2plx - 2bx^2t^2p2yply + 2bxtp2y^2cx - 2bxtp2y^2plx + 2plx^2ayt^3byy \\
& -2plxay^2t^4p2x + 2plx^2ayt^2cy - 2plx^2ayt^2p2y - 2plxbyy^2t^2p2x \\
& + 2plx^2byyt^2cy - 2plx^2byyt^2p2y + 2ayt^3p2x^2byy + 2ayt^4p2xplyax \\
& + 2ayt^3p2xplybx + 2ayt^2p2xplycx - 2ayt^2p2xcxp2y + 2ayt^2p2xplx p2y \\
& + 2byyt^3p2xplyax + 2byyt^2p2xplybx + 2byyt^2p2xplycx - 2byyt^2p2xcxp2y \\
& + 2byyt^2p2xplx p2y - 2plyaxt^2plx cy + 2plyaxt^2plx p2y + 2plyaxt^2cy p2x \\
& - 2plybx tplx cy + 2plybx tplx p2y + 2plybx tcy p2x + 2axt^4p2yplx ay \\
& + 2axt^3p2yplx byy - 2axt^4p2yay p2x - 2axt^3p2ybyy p2x - 4axt^3p2yply bx \\
& - 4axt^2p2yply cx + 2axt^2p2yply p2x + 2axt^2p2yplx cy - 2axt^2p2y cy p2x \\
& + 2bxt^3p2yplx ay + 2bxt^2p2yplx byy - 2bxt^3p2yay p2x - 2bxt^2p2ybyy p2x \\
& - 4bxt^2p2yply cx + 2bxt^2p2yply p2x + 2bxt^2p2yplx cy - 2bxt^2p2y cy p2x \\
& - 4plxayt^3byy p2x - 2plxayt^4ply ax - 2plxayt^3ply bx - 2plxayt^2ply cx \\
& + 2plxayt^2ply p2x + 2plxayt^2cx p2y - 4plxayt^2cy p2x - 2plxbyyt^3ply ax \\
& - 2plxbyyt^2ply bx - 2plxbyyt^2ply cx + 2plxbyyt^2ply p2x + 2plxbyyt^2cx p2y \\
& - 4plxbyyt^2cy p2x - 4plyazt^3bz p2y - 4plyazt^2cz p2y + 4plyazt^2h p2y \\
& - 4plybz tcz p2y + 4plybz th p2y - 4azt^3p2xplx bz - 4azt^2p2xplx cz \\
& + 4azt^2p2xplx h - 4bz t p2xplx cz + 4bz t p2xplx h)^{1/2} \\
& - R \sqrt{-2plyp2y + p2y^2 + ply^2 - 2plx p2x + plx^2 + p2x^2} \Big) / \\
& \sqrt{-2plyp2y + p2y^2 + ply^2 - 2plx p2x + plx^2 + p2x^2}
\end{aligned}$$

That's certainly a mouthful. Let's assume

$$\text{sqrt}(-2plyp2y + p2y^2 + ply^2 - 2plx p2x + plx^2 + p2x^2)$$

is not 0. That may be provable if point 1 is not point 2. Not sure. Then we can multiply both sides by it, simplifying things. We also bring $R \sqrt{-2plyp2y + p2y^2 + ply^2 - 2plx p2x + plx^2 + p2x^2}$ to the LHS, and square both sides.

Then move it back over to the RHS and you get the following polynomial:

$$\begin{aligned}
g := & -R^2 \left(-2plyp2y + p2y^2 + ply^2 - 2plx p2x + plx^2 + p2x^2 \right) + ply^2 az^2 t^4 + ply^2 bz^2 t^2 \\
& + az^2 t^4 p2y^2 + bz^2 t^2 p2y^2 + az^2 t^4 p2x^2 + bz^2 t^2 p2x^2 + plx^2 az^2 t^4 + plx^2 bz^2 t^2 + ax^2 t^4 p2y^2 \\
& + bx^2 t^2 p2y^2 + plx^2 ay^2 t^4 + plx^2 byy^2 t^2 + ay^2 t^4 p2x^2 + byy^2 t^2 p2x^2 + ply^2 ax^2 t^4 \\
& + ply^2 bx^2 t^2 + 4ply cz h p2y + 4 cz p2x plx h + 2 cx p2y plx cy - 2 cx p2y cy p2x \\
& + 2 plx p2y cy p2x - 2 ply cx plx cy + 2 ply cx plx p2y + 2 ply cx cy p2x + 2 ply p2x cx p2y \\
& + 2 ply p2x plx cy - 2 ply p2x plx p2y - 2 ply^2 cx p2x - 2 ply cx^2 p2y - 2 ply p2x^2 cy \\
& - 2 cx p2y^2 plx - 2 plx^2 cy p2y - 2 plx cy^2 p2x - 2 ply^2 cz h - 2 ply cz^2 p2y - 2 ply h^2 p2y \\
& - 2 cz p2y^2 h + ply^2 cx^2 + ply^2 p2x^2 + cx^2 p2y^2 + plx^2 cy^2 + plx^2 p2y^2 + cy^2 p2x^2 + ply^2 cz^2 \\
& + ply^2 h^2 + cz^2 p2y^2 + h^2 p2y^2 + cz^2 p2x^2 + h^2 p2x^2 + plx^2 cz^2 + plx^2 h^2 - 2 cz p2x^2 h \\
& - 2 cz^2 p2x plx - 2 h^2 p2x plx - 2 plx^2 cz h + 2 ply^2 az t^3 bz - 2 ply az^2 t^4 p2y \\
& + 2 ply^2 az t^2 cz - 2 ply^2 az t^2 h - 2 ply bz^2 t^2 p2y + 2 ply^2 bz t cz - 2 ply^2 bz t h \\
& + 2 az t^3 p2y^2 bz + 2 az t^2 p2y^2 cz - 2 az t^2 p2y^2 h + 2 bz t p2y^2 cz - 2 bz t p2y^2 h \\
& + 2 az t^3 p2x^2 bz - 2 az^2 t^4 p2x plx + 2 az t^2 p2x^2 cz - 2 az t^2 p2x^2 h - 2 bz^2 t^2 p2x plx \\
& + 2 bz t p2x^2 cz - 2 bz t p2x^2 h + 2 plx^2 az t^3 bz + 2 plx^2 az t^2 cz - 2 plx^2 az t^2 h \\
& + 2 plx^2 bz t cz - 2 plx^2 bz t h - 2 ay t^2 p2x^2 ply + 2 ay t^2 p2x^2 cy - 2 byy t p2x^2 ply \\
& + 2 byy t p2x^2 cy + 2 ply^2 ax t^3 bx + 2 ply^2 ax t^2 cx - 2 ply^2 ax t^2 p2x + 2 ply^2 bx t cx \\
& - 2 ply^2 bx t p2x + 2 ax t^3 p2y^2 bx - 2 ax^2 t^4 p2y ply + 2 ax t^2 p2y^2 cx - 2 ax t^2 p2y^2 plx \\
& - 2 bx^2 t^2 p2y ply + 2 bx t p2y^2 cx - 2 bx t p2y^2 plx + 2 plx^2 ay t^3 byy - 2 plx ay^2 t^4 p2x \\
& + 2 plx^2 ay t^2 cy - 2 plx^2 ay t^2 p2y - 2 plx byy^2 t^2 p2x + 2 plx^2 byy t cy - 2 plx^2 byy t p2y \\
& + 2 ay t^3 p2x^2 byy + 2 ay t^4 p2x ply ax + 2 ay t^3 p2x ply bx + 2 ay t^2 p2x ply cx \\
& - 2 ay t^2 p2x cx p2y + 2 ay t^2 p2x plx p2y + 2 byy t^3 p2x ply ax + 2 byy t^2 p2x ply bx \\
& + 2 byy t p2x ply cx - 2 byy t p2x cx p2y + 2 byy t p2x plx p2y - 2 ply ax t^2 plx cy \\
& + 2 ply ax t^2 plx p2y + 2 ply ax t^2 cy p2x - 2 ply bx t plx cy + 2 ply bx t plx p2y \\
& + 2 ply bx t cy p2x + 2 ax t^4 p2y plx ay + 2 ax t^3 p2y plx byy - 2 ax t^4 p2y ay p2x \\
& - 2 ax t^3 p2y byy p2x - 4 ax t^3 p2y ply bx - 4 ax t^2 p2y ply cx + 2 ax t^2 p2y ply p2x \\
& + 2 ax t^2 p2y plx cy - 2 ax t^2 p2y cy p2x + 2 bx t^3 p2y plx ay + 2 bx t^2 p2y plx byy \\
& - 2 bx t^3 p2y ay p2x - 2 bx t^2 p2y byy p2x - 4 bx t p2y ply cx + 2 bx t p2y ply p2x \\
& + 2 bx t p2y plx cy - 2 bx t p2y cy p2x - 4 plx ay t^3 byy p2x - 2 plx ay t^4 ply ax \\
& - 2 plx ay t^3 ply bx - 2 plx ay t^2 ply cx + 2 plx ay t^2 ply p2x + 2 plx ay t^2 cx p2y \\
& - 4 plx ay t^2 cy p2x - 2 plx byy t^3 ply ax - 2 plx byy t^2 ply bx - 2 plx byy t ply cx \\
& + 2 plx byy t ply p2x + 2 plx byy t cx p2y - 4 plx byy t cy p2x - 4 ply az t^3 bz p2y \\
& - 4 ply az t^2 cz p2y + 4 ply az t^2 h p2y - 4 ply bz t cz p2y + 4 ply bz t h p2y - 4 az t^3 p2x plx bz \\
& - 4 az t^2 p2x plx cz + 4 az t^2 p2x plx h - 4 bz t p2x plx cz + 4 bz t p2x plx h \\
& ply^2 az^2 t^4 + ply^2 bz^2 t^2 + az^2 t^4 p2y^2 + bz^2 t^2 p2y^2 + az^2 t^4 p2x^2 + bz^2 t^2 p2x^2 + plx^2 az^2 t^4 \\
& + plx^2 bz^2 t^2 + ax^2 t^4 p2y^2 + bx^2 t^2 p2y^2 + plx^2 ay^2 t^4 + plx^2 byy^2 t^2 + ay^2 t^4 p2x^2 \\
& + byy^2 t^2 p2x^2 + ply^2 ax^2 t^4 + ply^2 bx^2 t^2 + 4ply cz h p2y + 4 cz p2x plx h \\
& + 2 cx p2y plx cy - 2 cx p2y cy p2x + 2 plx p2y cy p2x - 2 ply cx plx cy \\
& + 2 ply cx plx p2y + 2 ply cx cy p2x + 2 ply p2x cx p2y + 2 ply p2x plx cy
\end{aligned} \tag{6}$$

$$\begin{aligned}
& -2plyp2xp1xp2y - 2ply^2cxp2x - 2plycx^2p2y - 2plyp2x^2cy - 2cxp2y^2p1x \\
& - 2p1x^2cyp2y - 2p1xcy^2p2x - 2ply^2czh - 2plycz^2p2y - 2plyh^2p2y \\
& - 2czp2y^2h + ply^2cx^2 + ply^2p2x^2 + cx^2p2y^2 + p1x^2cy^2 + p1x^2p2y^2 + cy^2p2x^2 \\
& + ply^2cz^2 + ply^2h^2 + cz^2p2y^2 + h^2p2y^2 + cz^2p2x^2 + h^2p2x^2 + p1x^2cz^2 + p1x^2h^2 \\
& - 2czp2x^2h - 2cz^2p2xp1x - 2h^2p2xp1x - 2p1x^2czh + 2ply^2azt^3bz \\
& - 2plyaz^2t^4p2y + 2ply^2azt^2cz - 2ply^2azt^2h - 2plybz^2t^2p2y + 2ply^2bzt^2cz \\
& - 2ply^2bzt^2h + 2azt^3p2y^2bz + 2azt^2p2y^2cz - 2azt^2p2y^2h + 2bzt^2p2y^2cz \\
& - 2bzt^2p2y^2h + 2azt^3p2x^2bz - 2az^2t^4p2xp1x + 2azt^2p2x^2cz - 2azt^2p2x^2h \\
& - 2bz^2t^2p2xp1x + 2bzt^2p2x^2cz - 2bzt^2p2x^2h + 2p1x^2azt^3bz + 2p1x^2azt^2cz \\
& - 2p1x^2azt^2h + 2p1x^2bzt^2cz - 2p1x^2bzt^2h - 2ayt^2p2x^2ply + 2ayt^2p2x^2cy \\
& - 2byyt^2p2x^2ply + 2byyt^2p2x^2cy + 2ply^2axt^3bx + 2ply^2axt^2cx - 2ply^2axt^2p2x \\
& + 2ply^2bxt^2cx - 2ply^2bxt^2p2x + 2axt^3p2y^2bx - 2ax^2t^4p2yply + 2axt^2p2y^2cx \\
& - 2axt^2p2y^2p1x - 2bxt^2p2yply + 2bxt^2p2y^2cx - 2bxt^2p2y^2p1x + 2p1x^2ayt^3byy \\
& - 2p1xay^2t^4p2x + 2p1x^2ayt^2cy - 2p1x^2ayt^2p2y - 2p1xbyy^2t^2p2x \\
& + 2p1x^2byyt^2cy - 2p1x^2byyt^2p2y + 2ayt^3p2x^2byy - R^2(-2plyp2y + p2y^2 + ply^2 \\
& - 2p1xp2x + p1x^2 + p2x^2) + 2ayt^4p2xp1xax + 2ayt^3p2xp1xbx + 2ayt^2p2xp1xcy \\
& - 2ayt^2p2xcx p2y + 2ayt^2p2xp1xp2y + 2byyt^3p2xp1xax + 2byyt^2p2xp1ybx \\
& + 2byyt^2p2xp1y cy - 2byyt^2p2xcx p2y + 2byyt^2p2xp1xp2y - 2plyaxt^2p1xcy \\
& + 2plyaxt^2p1xp2y + 2plyaxt^2cyp2x - 2plybxt^2p1xcy + 2plybxt^2p1xp2y \\
& + 2plybxt^2cyp2x + 2axt^4p2yp1xay + 2axt^3p2yp1xbyy - 2axt^4p2yayp2x \\
& - 2axt^3p2ybyyp2x - 4axt^3p2yp1ybx - 4axt^2p2yp1y cy + 2axt^2p2yp1yp2x \\
& + 2axt^2p2yp1x cy - 2axt^2p2ycyp2x + 2bxt^3p2yp1xay + 2bxt^2p2yp1xbyy \\
& - 2bxt^3p2yayp2x - 2bxt^2p2ybyyp2x - 4bxt^2p2yp1y cy + 2bxt^2p2yp1yp2x \\
& + 2bxt^2p2yp1x cy - 2bxt^2p2ycyp2x - 4p1xayt^3byyp2x - 2p1xayt^4plyax \\
& - 2p1xayt^3plybx - 2p1xayt^2ply cy + 2p1xayt^2plyp2x + 2p1xayt^2cx p2y \\
& - 4p1xayt^2cyp2x - 2p1xbyyt^3plyax - 2p1xbyyt^2plybx - 2p1xbyyt^2ply cy \\
& + 2p1xbyyt^2plyp2x + 2p1xbyyt^2cx p2y - 4p1xbyyt^2cyp2x - 4plyazt^3bz p2y \\
& - 4plyazt^2cz p2y + 4plyazt^2h p2y - 4plybzt^2cz p2y + 4plybzt^2h p2y \\
& - 4azt^3p2xp1xbz - 4azt^2p2xp1xcz + 4azt^2p2xp1xh - 4bzt^2p2xp1xcz \\
& + 4bzt^2p2xp1xh
\end{aligned}$$

to LaTeX

$$\begin{aligned}
& \{ply\}^2\{az\}^2\{t\}^4 + \{ply\}^2\{bz\}^2\{t\}^2 \\
& + \{az\}^2\{t\}^4\{p2y\}^2 + \{bz\}^2\{t\}^2\{it
\end{aligned}$$

[illegible]

$$\begin{aligned}
& 2\{\text{it bz}\}\backslash,\text{th}+2\backslash,\{\text{it az}\}\backslash,\{t\}^3\{\{\text{it p2y}\}\}^2\{\text{it bz}\}+2\backslash,\{\text{it az}\} \\
& \backslash,\{t\}^2\{\{\text{it p2y}\}\}^2\{\text{it cz}\}-2\backslash,\{\text{it az}\}\backslash,\{t\}^2\{\{\text{it p2y}\}\}^2h+ \\
& 2\backslash,\{\text{it bz}\}\backslash,t\{\{\text{it p2y}\}\}^2\{\text{it cz}\}-2\backslash,\{\text{it bz}\}\backslash,t\{\{\text{it p2y}\}\}^2h+2 \\
& \backslash,\{\text{it az}\}\backslash,\{t\}^3\{\{\text{it p2x}\}\}^2\{\text{it bz}\}-2\backslash,\{\{\text{it az}\}\}^2\{t\}^4\{ \\
& \{\text{it p2x}\}\backslash,\{\text{it plx}\}+2\backslash,\{\text{it az}\}\backslash,\{t\}^2\{\{\text{it p2x}\}\}^2\{\text{it cz}\}-2\backslash, \\
& \{ \\
& \{\text{it az}\}\backslash,\{t\}^2\{\{\text{it p2x}\}\}^2h-2\backslash,\{\{\text{it bz}\}\}^2\{t\}^2\{\text{it p2x}\} \\
& \backslash,\{ \\
& \{\text{it plx}\}+2\backslash,\{\text{it bz}\}\backslash,t\{\{\text{it p2x}\}\}^2\{\text{it cz}\}-2\backslash,\{\text{it bz}\}\backslash,t\{\{\text{it p2x}\} \\
& \}^2h+2\backslash,\{\{\text{it plx}\}\}^2\{\text{it az}\}\backslash,\{t\}^3\{\text{it bz}\}+2\backslash,\{\{\text{it plx}\}\}^2 \\
& \{\text{it az}\}\backslash,\{t\}^2\{\text{it cz}\}-2\backslash,\{\{\text{it plx}\}\}^2\{\text{it az}\}\backslash,\{t\}^2h+2\backslash,\{ \\
& \{ \\
& \{\text{it plx}\}\}^2\{\text{it bz}\}\backslash,t\{\text{it cz}\}-2\backslash,\{\{\text{it plx}\}\}^2\{\text{it bz}\}\backslash,\text{th}-2\backslash, \\
& \{ \\
& \{\text{it ay}\}\backslash,\{t\}^2\{\{\text{it p2x}\}\}^2\{\text{it ply}\}+2\backslash,\{\text{it ay}\}\backslash,\{t\}^2\{\{\text{it p2x}\} \\
& \}^2\{\text{it cy}\}-2\backslash,\{\text{it byy}\}\backslash,t\{\{\text{it p2x}\}\}^2\{\text{it ply}\}+2\backslash,\{\text{it byy}\} \\
& \backslash,t \\
& \{\{\text{it p2x}\}\}^2\{\text{it cy}\}+2\backslash,\{\{\text{it ply}\}\}^2\{\text{it ax}\}\backslash,\{t\}^3\{\text{it bx}\} \\
& +2 \\
& \backslash,\{\{\text{it ply}\}\}^2\{\text{it ax}\}\backslash,\{t\}^2\{\text{it cx}\}-2\backslash,\{\{\text{it ply}\}\}^2\{\text{it ax}\} \\
& \backslash,\{t\}^2\{\text{it p2x}\}+2\backslash,\{\{\text{it ply}\}\}^2\{\text{it bx}\}\backslash,t\{\text{it cx}\}-2\backslash,\{\{\text{it ply}\} \\
& \}^2\{\text{it bx}\}\backslash,t\{\text{it p2x}\}+2\backslash,\{\text{it ax}\}\backslash,\{t\}^3\{\{\text{it p2y}\}\}^2\{\text{it bx}\}- \\
& 2\backslash,\{\{\text{it ax}\}\}^2\{t\}^4\{\text{it p2y}\}\backslash,\{\text{it ply}\}+2\backslash,\{\text{it ax}\}\backslash,\{t\}^2\{\{\text{it p2y}\}\}^2\{\text{it cx}\}-2\backslash,\{\text{it ax}\}\backslash,\{t\}^2\{\{\text{it p2y}\}\}^2\{\text{it plx}\}-2\backslash,\{\{\text{it bx}\}\}^2\{t\}^2\{\text{it p2y}\}\backslash,\{\text{it ply}\}+2\backslash,\{\text{it bx}\}\backslash,t\{\{\text{it p2y}\}\}^2\{\text{it cx}\}-2\backslash,\{\text{it bx}\}\backslash,t\{\{\text{it p2y}\}\}^2\{\text{it plx}\}+2\backslash,\{\{\text{it plx}\}\}^2\{\text{it ay}\}\backslash,\{t\}^3\{\text{it byy}\}-2\backslash,\{\text{it plx}\}\backslash,\{\{\text{it ay}\}\}^2\{t\}^4\{\text{it p2x}\}+2\backslash,\{\{\text{it plx}\}\}^2\{\text{it ay}\}\backslash,\{t\}^2\{\text{it cy}\}-2\backslash,\{\{\text{it plx}\}\}^2\{\text{it ay}\}\backslash,\{t\}^2\{\text{it p2y}\}-2\backslash,\{\text{it plx}\}\backslash,\{\{\text{it byy}\}\}^2\{t\}^2\{\text{it p2x}\}+2\backslash,\{\{\text{it plx}\}\}^2\{\text{it byy}\}\backslash,t\{\text{it cy}\}-2\backslash,\{\{\text{it plx}\}\}^2\{\text{it byy}\}\backslash,t\{\text{it p2y}\}+2\backslash,\{\text{it ay}\}\backslash,\{t\}^3\{\{\text{it p2x}\}\}^2\{\text{it byy}\}-\{R\}^2\left(-2\backslash,\{\text{it ply}\}\backslash,\{\text{it p2y}\}+\{\{\text{it p2y}\}\}^2+\{\{\text{it ply}\}\}^2-2\backslash,\{\text{it plx}\}\backslash,\{\text{it p2x}\}+\{\{\text{it plx}\}\}^2+\{\{\text{it p2x}\}\}^2\right)+2\backslash,\{\text{it ay}\}\backslash,\{t\}^4\{\text{it p2x}\}\backslash,\{\text{it ply}\}\backslash,\{\text{it ax}\}+2\backslash,\{\text{it ay}\}\backslash,\{t\}^3\{\text{it p2x}\}\backslash,\{\text{it ply}\}\backslash,\{\text{it bx}\}+2\backslash,\{\text{it ay}\}\backslash,\{t\}^2\{\text{it p2x}\}\backslash,\{\text{it ply}\}\backslash,\{\text{it cx}\}-2\backslash,\{\text{it ay}\}\backslash,\{t\}^2\{\text{it p2x}\}\backslash,\{\text{it cx}\}\backslash,\{\text{it p2y}\}+2\backslash,\{\text{it ay}\}\backslash,\{t\}^2\{ \\
& \{
\end{aligned}$$

$\{p_2x\}, \{p_1x\}, \{p_2y+2\}, \{byy\}, \{t^3\{p_2x\}, \{p_1y\}, \{ax\}+2\}, \{byy\}, \{t^2\{p_2x\}, \{p_1y\}, \{bx\}+2\},$
 $\{byy\}, t\{p_2x\}, \{p_1y\}, \{cx\}-2\}, \{byy\}, t\{p_2x\}$
 $\{$
 $\{cx\}, \{p_2y+2\}, \{byy\}, t\{p_2x\}, \{p_1x\}, \{p_2y$
 $-2\}, \{$
 $\{p_1y\}, \{ax\}, \{t^2\{p_1x\}, \{cy\}+2\}, \{p_1y\}, \{ax$
 $\}, \{t^2\{p_1x\}, \{p_2y+2\}, \{p_1y\}, \{ax\}, \{t^2\{it$
 $cy\}$
 $\}, \{p_2x\}-2\}, \{p_1y\}, \{bx\}, t\{p_1x\}, \{cy\}+2\}, \{it$
 $p_1y\}$
 $\}, \{bx\}, t\{p_1x\}, \{p_2y+2\}, \{p_1y\}, \{bx\}, t\{cy\}$
 $\}, \{$
 $\{p_2x\}+2\}, \{ax\}, \{t^4\{p_2y\}, \{p_1x\}, \{ay\}+2\}, \{it$
 ax
 $\}, \{t^3\{p_2y\}, \{p_1x\}, \{byy\}-2\}, \{ax\}, \{t^4\{it$
 p_2y
 $\}, \{ay\}, \{p_2x\}-2\}, \{ax\}, \{t^3\{p_2y\}, \{byy\},$
 $\{it$
 $p_2x\}-4\}, \{ax\}, \{t^3\{p_2y\}, \{p_1y\}, \{bx\}-4\}, \{ax\}$
 $\}, \{$
 $t^2\{p_2y\}, \{p_1y\}, \{cx\}+2\}, \{ax\}, \{t^2\{p_2y\}$
 $\}, \{$
 $\{p_1y\}, \{p_2x\}+2\}, \{ax\}, \{t^2\{p_2y\}, \{p_1x\}, \{it$
 $cy\}$
 $-2\}, \{ax\}, \{t^2\{p_2y\}, \{cy\}, \{p_2x\}+2\}, \{bx\},$
 $\{t^{\wedge}\{$
 $3\{p_2y\}, \{p_1x\}, \{ay\}+2\}, \{bx\}, \{t^2\{p_2y\},$
 $\{it$
 $p_1x\}, \{byy\}-2\}, \{bx\}, \{t^3\{p_2y\}, \{ay\}, \{p_2x\}$
 $-2\},$
 $\{bx\}, \{t^2\{p_2y\}, \{byy\}, \{p_2x\}-4\}, \{bx\}, t\{it$
 $p_2y\}, \{p_1y\}, \{cx\}+2\}, \{bx\}, t\{p_2y\}, \{p_1y\}, \{it$
 p_2x
 $\}+2\}, \{bx\}, t\{p_2y\}, \{p_1x\}, \{cy\}-2\}, \{bx\}, t\{it$
 $p_2y\}$
 $\}, \{cy\}, \{p_2x\}-4\}, \{p_1x\}, \{ay\}, \{t^3\{byy\},$
 $\{it$
 $p_2x\}-2\}, \{p_1x\}, \{ay\}, \{t^4\{p_1y\}, \{ax\}-2\}, \{it$
 $p_1x\},$
 $\{ay\}, \{t^3\{p_1y\}, \{bx\}-2\}, \{p_1x\}, \{ay\}, \{t^{\wedge}$
 $\{2\}\{$
 $\{p_1y\}, \{cx\}+2\}, \{p_1x\}, \{ay\}, \{t^2\{p_1y\}, \{it$
 $p_2x\}$
 $+2\}, \{p_1x\}, \{ay\}, \{t^2\{cx\}, \{p_2y\}-4\}, \{p_1x\}, \{$
 $\{ay\}, \{t^2\{cy\}, \{p_2x\}-2\}, \{p_1x\}, \{byy\}, \{t^{\wedge}$
 $\{3\}\{$
 $\{p_1y\}, \{ax\}-2\}, \{p_1x\}, \{byy\}, \{t^2\{p_1y\}, \{it$
 $bx\}$
 $-2\}, \{p_1x\}, \{byy\}, t\{p_1y\}, \{cx\}+2\}, \{p_1x\}, \{it$
 $byy\}$
 $\}, t\{p_1y\}, \{p_2x\}+2\}, \{p_1x\}, \{byy\}, t\{cx\}, \{it$
 $p_2y\}-$

$4\backslash,\{\textit{plx}\}\backslash,\{\textit{byy}\}\backslash,t\{\textit{cy}\}\backslash,\{\textit{p2x}\}-4\backslash,\{\textit{ply}\}\backslash,\{\textit{az}\}\backslash,$
 $\{t\}^{\{3\}}\{\textit{bz}\}\backslash,\{\textit{p2y}\}-4\backslash,\{\textit{ply}\}\backslash,\{\textit{az}\}\backslash,\{t\}^{\{2\}}\{\textit{cz}\}$
 $\backslash,\{\textit{p2y}\}+4\backslash,\{\textit{ply}\}\backslash,\{\textit{az}\}\backslash,\{t\}^{\{2\}}h\{\textit{p2y}\}-4\backslash,\{\textit{ply}\}\backslash,$
 $\{\textit{bz}\}\backslash,t\{\textit{cz}\}\backslash,\{\textit{p2y}\}+4\backslash,\{\textit{ply}\}\backslash,\{\textit{bz}\}\backslash,th\{\textit{p2y}\}-4\backslash,$
 $\{\textit{az}\}\backslash,\{t\}^{\{3\}}\{\textit{p2x}\}\backslash,\{\textit{plx}\}\backslash,\{\textit{bz}\}-4\backslash,\{\textit{az}\}\backslash,\{t\}^{\{2\}}\{\textit{p2x}\}$
 $\backslash,\{\textit{plx}\}\backslash,\{\textit{cz}\}+4\backslash,\{\textit{az}\}\backslash,\{t\}^{\{2\}}\{\textit{p2x}\}\backslash,\{\textit{plx}\}$
 $\backslash,h-$
 $4\backslash,\{\textit{bz}\}\backslash,t\{\textit{p2x}\}\backslash,\{\textit{plx}\}\backslash,\{\textit{cz}\}+4\backslash,\{\textit{bz}\}\backslash,t\{\textit{p2x}\}\backslash,$
 $\{\textit{plx}\}\backslash,h$

We collect everything in terms of t

$collect(g, t)$

$$\begin{aligned}
& 4ply\,cz\,h\,p2y + 4cz\,p2x\,plx\,h + 2cx\,p2y\,plx\,cy - 2cx\,p2y\,cy\,p2x + 2plx\,p2y\,cy\,p2x \\
& - 2ply\,cx\,plx\,cy + 2ply\,cx\,plx\,p2y + 2ply\,cx\,cy\,p2x + 2ply\,p2x\,cx\,p2y \\
& + 2ply\,p2x\,plx\,cy - 2ply\,p2x\,plx\,p2y - 2ply^2\,cx\,p2x - 2ply\,cx^2\,p2y - 2ply\,p2x^2\,cy \\
& - 2cx\,p2y^2\,plx - 2plx^2\,cy\,p2y - 2plx\,cy^2\,p2x - 2ply^2\,cz\,h - 2ply\,cz^2\,p2y \\
& - 2ply\,h^2\,p2y - 2cz\,p2y^2\,h + ply^2\,cx^2 + ply^2\,p2x^2 + cx^2\,p2y^2 + plx^2\,cy^2 + plx^2\,p2y^2 \\
& + cy^2\,p2x^2 + ply^2\,cz^2 + ply^2\,h^2 + cz^2\,p2y^2 + h^2\,p2y^2 + cz^2\,p2x^2 + h^2\,p2x^2 + plx^2\,cz^2 \\
& + plx^2\,h^2 - 2cz\,p2x^2\,h - 2cz^2\,p2x\,plx - 2h^2\,p2x\,plx - 2plx^2\,cz\,h - R^2\,(-2ply\,p2y \\
& + p2y^2 + ply^2 - 2plx\,p2x + plx^2 + p2x^2) + (az^2\,p2x^2 + plx^2\,ay^2 - 2ply\,az^2\,p2y \\
& - 2az^2\,p2x\,plx + ay^2\,p2x^2 - 2plx\,ay^2\,p2x + plx^2\,az^2 - 2plx\,ay\,ply\,ax \\
& + 2ay\,p2x\,ply\,ax - 2ax^2\,p2y\,ply + ax^2\,p2y^2 + az^2\,p2y^2 + 2ax\,p2y\,plx\,ay + ply^2\,az^2 \\
& - 2ax\,p2y\,ay\,p2x + ply^2\,ax^2) \, t^4 + (-2bx\,p2y\,ay\,p2x + 2plx^2\,ay\,byy + 2bx\,p2y\,plx\,ay \\
& - 4ax\,p2y\,ply\,bx + 2ply^2\,az\,bz - 4plx\,ay\,byy\,p2x + 2ay\,p2x^2\,byy + 2az\,p2y^2\,bz \\
& + 2plx^2\,az\,bz + 2az\,p2x^2\,bz + 2byy\,p2x\,ply\,ax - 2ax\,p2y\,byy\,p2x - 2plx\,byy\,ply\,ax \\
& - 4ply\,az\,bz\,p2y + 2ply^2\,ax\,bx + 2ax\,p2y^2\,bx + 2ay\,p2x\,ply\,bx - 4az\,p2x\,plx\,bz \\
& - 2plx\,ay\,ply\,bx + 2ax\,p2y\,plx\,byy) \, t^3 + (2ply^2\,cx\,ax - 2ply\,p2x^2\,ay - 2ply\,bz^2\,p2y \\
& + 2ply^2\,cz\,az - 2ply^2\,h\,az + 2cz\,p2y^2\,az - 2h\,p2y^2\,az - 2bz^2\,p2x\,plx + 2cz\,p2x^2\,az \\
& - 2h\,p2x^2\,az + 2plx^2\,cz\,az - 2plx^2\,h\,az - 2ply^2\,p2x\,ax + 2cx\,p2y^2\,ax + 2plx^2\,cy\,ay \\
& - 2plx\,p2y^2\,ax - 2plx^2\,p2y\,ay + 2cy\,p2x^2\,ay - 2bx^2\,p2y\,ply - 2plx\,byy^2\,p2x \\
& + bx^2\,p2y^2 + plx^2\,byy^2 + byy^2\,p2x^2 + ply^2\,bx^2 + ply^2\,bz^2 + bz^2\,p2y^2 + bz^2\,p2x^2 \\
& + plx^2\,bz^2 + 2bx\,p2y\,plx\,byy - 2bx\,p2y\,byy\,p2x - 2plx\,byy\,ply\,bx - 4ply\,cx\,ax\,p2y \\
& - 2ply\,cx\,plx\,ay + 2ply\,cx\,ay\,p2x + 2ply\,p2x\,ax\,p2y + 2ply\,p2x\,plx\,ay
\end{aligned} \tag{7}$$

$$\begin{aligned}
& + 2 cx p2y plx ay - 2 cx p2y ay p2x + 2 plx cy ax p2y - 4 plx cy ay p2x - 2 plx cy ply ax \\
& + 2 plx p2y ay p2x + 2 plx p2y ply ax - 4 ply cz az p2y + 4 ply h az p2y \\
& - 4 cz p2x plx az + 4 h p2x plx az + 2 byy p2x ply bx - 2 cy p2x ax p2y \\
& + 2 cy p2x ply ax) t^2 + (2 ply^2 cx bx - 2 ply p2x^2 byy + 2 ply^2 cz bz - 2 ply^2 h bz \\
& + 2 cz p2y^2 bz - 2 h p2y^2 bz + 2 cz p2x^2 bz - 2 h p2x^2 bz + 2 plx^2 cz bz - 2 plx^2 h bz \\
& - 2 ply^2 p2x bx + 2 cx p2y^2 bx + 2 plx^2 cy byy - 2 plx p2y^2 bx - 2 plx^2 p2y byy \\
& + 2 cy p2x^2 byy - 4 ply cx bx p2y - 2 ply cx plx byy + 2 ply cx byy p2x \\
& + 2 ply p2x bx p2y + 2 ply p2x plx byy + 2 cx p2y plx byy - 2 cx p2y byy p2x \\
& + 2 plx cy bx p2y - 4 plx cy byy p2x - 2 plx cy ply bx + 2 plx p2y byy p2x \\
& + 2 plx p2y ply bx - 4 ply cz bz p2y + 4 ply h bz p2y - 4 cz p2x plx bz + 4 h p2x plx bz \\
& - 2 cy p2x bx p2y + 2 cy p2x ply bx) t
\end{aligned}$$

This is a polynomial

$$At^4 + Bt^3 + Ct^2 + DDt + E = 0$$

where

$$\begin{aligned}
A := & az^2 p2x^2 + plx^2 ay^2 - 2 ply az^2 p2y - 2 az^2 p2x plx + ay^2 p2x^2 - 2 plx ay^2 p2x + plx^2 az^2 \\
& - 2 plx ay ply ax + 2 ay p2x ply ax - 2 ax^2 p2y ply + ax^2 p2y^2 + az^2 p2y^2 + 2 ax p2y plx ay \\
& + ply^2 az^2 - 2 ax p2y ay p2x + ply^2 ax^2 :
\end{aligned}$$

$$\begin{aligned}
B := & -2 bx p2y ay p2x + 2 plx^2 ay byy + 2 bx p2y plx ay - 4 ax p2y ply bx + 2 ply^2 az bz \\
& - 4 plx ay byy p2x + 2 ay p2x^2 byy + 2 az p2y^2 bz + 2 plx^2 az bz + 2 az p2x^2 bz \\
& + 2 byy p2x ply ax - 2 ax p2y byy p2x - 2 plx byy ply ax - 4 ply az bz p2y + 2 ply^2 ax bx \\
& + 2 ax p2y^2 bx + 2 ay p2x ply bx - 4 az p2x plx bz - 2 plx ay ply bx + 2 ax p2y plx byy :
\end{aligned}$$

$$\begin{aligned}
C := & 2 ply^2 cx ax - 2 ply p2x^2 ay - 2 ply bz^2 p2y + 2 ply^2 cz az - 2 ply^2 h az + 2 cz p2y^2 az \\
& - 2 h p2y^2 az - 2 bz^2 p2x plx + 2 cz p2x^2 az - 2 h p2x^2 az + 2 plx^2 cz az - 2 plx^2 h az \\
& - 2 ply^2 p2x ax + 2 cx p2y^2 ax + 2 plx^2 cy ay - 2 plx p2y^2 ax - 2 plx^2 p2y ay + 2 cy p2x^2 ay \\
& - 2 bx^2 p2y ply - 2 plx byy^2 p2x + bx^2 p2y^2 + plx^2 byy^2 + byy^2 p2x^2 + ply^2 bx^2 + ply^2 bz^2 \\
& + bz^2 p2y^2 + bz^2 p2x^2 + plx^2 bz^2 + 2 bx p2y plx byy - 2 bx p2y byy p2x - 2 plx byy ply bx \\
& - 4 ply cx ax p2y - 2 ply cx plx ay + 2 ply cx ay p2x + 2 ply p2x ax p2y + 2 ply p2x plx ay \\
& + 2 cx p2y plx ay - 2 cx p2y ay p2x + 2 plx cy ax p2y - 4 plx cy ay p2x - 2 plx cy ply ax \\
& + 2 plx p2y ay p2x + 2 plx p2y ply ax - 4 ply cz az p2y + 4 ply h az p2y - 4 cz p2x plx az \\
& + 4 h p2x plx az + 2 byy p2x ply bx - 2 cy p2x ax p2y + 2 cy p2x ply ax :
\end{aligned}$$

$$\begin{aligned}
DD := & 2 ply^2 cx bx - 2 ply p2x^2 byy + 2 ply^2 cz bz - 2 ply^2 h bz + 2 cz p2y^2 bz - 2 h p2y^2 bz \\
& + 2 cz p2x^2 bz - 2 h p2x^2 bz + 2 plx^2 cz bz - 2 plx^2 h bz - 2 ply^2 p2x bx + 2 cx p2y^2 bx
\end{aligned}$$

$$\begin{aligned}
& + 2 plx^2 cy byy - 2 plx p2y^2 bx - 2 plx^2 p2y byy + 2 cy p2x^2 byy - 4 ply cx bx p2y \\
& - 2 ply cx plx byy + 2 ply cx byy p2x + 2 ply p2x bx p2y + 2 ply p2x plx byy + 2 cx p2y plx byy \\
& - 2 cx p2y byy p2x + 2 plx cy bx p2y - 4 plx cy byy p2x - 2 plx cy ply bx + 2 plx p2y byy p2x \\
& + 2 plx p2y ply bx - 4 ply cz bz p2y + 4 ply h bz p2y - 4 cz p2x plx bz + 4 h p2x plx bz \\
& - 2 cy p2x bx p2y + 2 cy p2x ply bx :
\end{aligned}$$

$$\begin{aligned}
E := & 4 ply cz h p2y + 4 cz p2x plx h + 2 cx p2y plx cy - 2 cx p2y cy p2x + 2 plx p2y cy p2x \\
& - 2 ply cx plx cy + 2 ply cx plx p2y + 2 ply cx cy p2x + 2 ply p2x cx p2y + 2 ply p2x plx cy \\
& - 2 ply p2x plx p2y - 2 ply^2 cx p2x - 2 ply cx^2 p2y - 2 ply p2x^2 cy - 2 cx p2y^2 plx \\
& - 2 plx^2 cy p2y - 2 plx cy^2 p2x - 2 ply^2 cz h - 2 ply cz^2 p2y - 2 ply h^2 p2y - 2 cz p2y^2 h \\
& + ply^2 cx^2 + ply^2 p2x^2 + cx^2 p2y^2 + plx^2 cy^2 + plx^2 p2y^2 + cy^2 p2x^2 + ply^2 cz^2 + ply^2 h^2 \\
& + cz^2 p2y^2 + h^2 p2y^2 + cz^2 p2x^2 + h^2 p2x^2 + plx^2 cz^2 + plx^2 h^2 - 2 cz p2x^2 h - 2 cz^2 p2x plx \\
& - 2 h^2 p2x plx - 2 plx^2 cz h - R^2 (-2 ply p2y + p2y^2 + ply^2 - 2 plx p2x + plx^2 + p2x^2) :
\end{aligned}$$

It would be great to reduce the size of these guys if possible. Here are the size-based simplifications but I don't think they do a great job, and they are less readable so I think I will opt to keep them as shown above.

$$\begin{aligned}
& \text{simplify}(A, \text{size}) \\
& (ply - p2y)^2 ax^2 - 2 ay (ply - p2y) (-p2x + plx) ax + (-p2x + plx)^2 ay^2 + az^2 (\\
& \quad -2 ply p2y + p2y^2 + ply^2 - 2 plx p2x + plx^2 + p2x^2)
\end{aligned} \tag{8}$$

$$\begin{aligned}
& \text{simplify}(B, \text{size}) \\
& (2 ay byy + 2 az bz) plx^2 + ((-2 ax byy - 2 ay bx) ply + (-4 az bz - 4 ay byy) p2x \\
& \quad + 2 p2y (ax byy + ay bx)) plx + (2 ax bx + 2 az bz) ply^2 + ((2 ay bx + 2 ax byy) p2x \\
& \quad - 4 p2y (ax bx + az bz)) ply + (2 ay byy + 2 az bz) p2x^2 - 2 p2y (ax byy + ay bx) p2x \\
& \quad + 2 p2y^2 (ax bx + az bz)
\end{aligned} \tag{9}$$

$$\begin{aligned}
& \text{simplify}(C, \text{size}) \\
& (-2 ay p2y + (-2 h + 2 cz) az + bz^2 + 2 cy ay + byy^2) plx^2 + ((-2 byy bx + 2 ax p2y \\
& \quad - 2 ax cy - 2 ay cx + 2 ay p2x) ply + (2 ay p2y + (4 h - 4 cz) az - 2 bz^2 - 2 byy^2 \\
& \quad - 4 cy ay) p2x + 2 p2y (byy bx - ax p2y + ax cy + ay cx)) plx + (-2 ax p2x + (-2 h \\
& \quad + 2 cz) az + 2 cx ax + bz^2 + bx^2) ply^2 + \left(-2 ay p2x^2 + (2 byy bx + 2 ay cx + 2 ax p2y \right. \\
& \quad \left. + 2 ax cy) p2x + 4 \left((-cz + h) az - cx ax - \frac{1}{2} bz^2 - \frac{1}{2} bx^2 \right) p2y \right) ply + ((-2 h \\
& \quad + 2 cz) az + bz^2 + 2 cy ay + byy^2) p2x^2 - 2 p2y (byy bx + ax cy + ay cx) p2x - 2 \left((-cz \right.
\end{aligned} \tag{10}$$

$$+ h) az - cx ax - \frac{1}{2} bz^2 - \frac{1}{2} bx^2 \Big) p2y^2$$

simplify(DD, size)

$$\begin{aligned} & (-2 p2y byy + (-2 h + 2 cz) bz + 2 byy cy) plx^2 + ((-2 byy cx + 2 bx p2y - 2 bx cy \\ & + 2 byy p2x) ply + (2 p2y byy + (4 h - 4 cz) bz - 4 byy cy) p2x - 2 p2y (-byy cx \\ & + bx p2y - bx cy)) plx + (-2 p2x bx + (-2 h + 2 cz) bz + 2 bx cx) ply^2 + (-2 p2x^2 byy \\ & + (2 bx p2y + 2 byy cx + 2 bx cy) p2x + 4 p2y ((-cz + h) bz - bx cx)) ply + ((-2 h \\ & + 2 cz) bz + 2 byy cy) p2x^2 - 2 p2y (byy cx + bx cy) p2x - 2 p2y^2 ((-cz + h) bz \\ & - bx cx) \end{aligned} \quad (11)$$

simplify(E, size)

$$\begin{aligned} & (cz^2 + cy^2 + h^2 - 2 p2y cy - 2 h cz - R^2 + p2y^2) plx^2 + (-2 (-p2y + cy) (cx - p2x) ply \\ & + (2 p2y cy - 2 cz^2 - 2 cy^2 + 4 h cz - 2 h^2 + 2 R^2) p2x + 2 cx p2y (-p2y + cy)) plx \\ & + (cz^2 - R^2 + cx^2 - 2 h cz - 2 p2x cx + p2x^2 + h^2) ply^2 + (-2 cy p2x^2 + 2 cx (p2y \\ & + cy) p2x + 2 p2y (-cz^2 - cx^2 + 2 h cz + R^2 - h^2)) ply + (h^2 - R^2 - 2 h cz + cy^2 \\ & + cz^2) p2x^2 - 2 cx p2y cy p2x - p2y^2 (-cz^2 - cx^2 + 2 h cz + R^2 - h^2) \end{aligned} \quad (12)$$

$A \xrightarrow{\text{to fortran}}$ `cg3 = az ** 2 * p2x ** 2 + plx ** 2 * ay ** 2 - 2 * ply *`
`az ** 2`
`#* p2y - 2 * az ** 2 * p2x * plx + ay ** 2 * p2x ** 2 - 2 * plx * a`
`#y ** 2 * p2x + plx ** 2 * az ** 2 - 2 * plx * ay * ply * ax + 2 *`
`#ay * p2x * ply * ax - 2 * ax ** 2 * p2y * ply + ax ** 2 * p2y ** 2`
`# + az ** 2 * p2y ** 2 + 2 * ax * p2y * plx * ay + ply ** 2 * az **`
`# 2 - 2 * ax * p2y * ay * p2x + ply ** 2 * ax ** 2`
 $\xrightarrow{\text{to LaTeX}}$ $\{(\text{it az})^2(\text{it p2x})^2+(\text{it plx})^2(\text{it ay})^2-2\,$
 $\{\text{it}$
 $\text{ply}\},\{(\text{it az})^2(\text{it p2y})-2\},\{(\text{it az})^2(\text{it p2x}),(\text{it plx})+(\text{it}$
 $\text{ay})^2(\text{it p2x})^2-2\},\{\text{it plx}\},\{(\text{it ay})^2(\text{it p2x})+(\text{it}$
 $\text{plx})^2(\text{it az})^2-2\},\{\text{it plx}\},\{\text{it ay}\},\{\text{it ply}\},\{\text{it ax}$
 $+2\},\{\text{it ay}\},\{\text{it p2x}\},\{\text{it ply}\},\{\text{it ax}-2\},\{(\text{it ax})^2(\text{it}$
 $\text{p2y})\},\{\text{it ply}+(\text{it ax})^2(\text{it p2y})^2+(\text{it az})^2(\text{it p2y})$
 $\}^2+2\},\{\text{it ax}\},\{\text{it p2y}\},\{\text{it plx}\},\{\text{it ay}+(\text{it ply})^2(\text{it}$
 $\text{az})^2-2\},\{\text{it ax}\},\{\text{it p2y}\},\{\text{it ay}\},\{\text{it p2x}+(\text{it ply})^2(\text{it}$
 $\text{ax})^2\}$

$B \xrightarrow{\text{to fortran}}$ `cg4 = -2 * bx * p2y * ay * p2x + 2 * plx ** 2 * ay * byy`
`+ 2 * bx`
`#* p2y * plx * ay - 4 * ax * p2y * ply * bx + 2 * ply ** 2 * az * b`
`#z - 4 * plx * ay * byy * p2x + 2 * ay * p2x ** 2 * byy + 2 * az *`
`#p2y ** 2 * bz + 2 * plx ** 2 * az * bz + 2 * az * p2x ** 2 * bz +`
`#2 * byy * p2x * ply * ax - 2 * ax * p2y * byy * p2x - 2 * plx * by`
`#y * ply * ax - 4 * ply * az * bz * p2y + 2 * ply ** 2 * ax * bx +`
`#2 * ax * p2y ** 2 * bx + 2 * ay * p2x * ply * bx - 4 * az * p2x *`
`#plx * bz - 2 * plx * ay * ply * bx + 2 * ax * p2y * plx * byy`
 $\xrightarrow{\text{to LaTeX}}$

$$\begin{aligned}
& -2\backslash,\{\backslash\mathrm{it\,bx}\}\backslash,\{\backslash\mathrm{it\,p2y}\}\backslash,\{\backslash\mathrm{it\,ay}\}\backslash,\{\backslash\mathrm{it\,p2x}+2\backslash,\{\{\backslash\mathrm{it\,plx}\}\}^{\{2\}}\{\backslash\mathrm{it\,ay}} \\
& \backslash\backslash,\{\backslash\mathrm{it\,byy}+2\backslash,\{\backslash\mathrm{it\,bx}\}\backslash,\{\backslash\mathrm{it\,p2y}\}\backslash,\{\backslash\mathrm{it\,plx}\}\backslash,\{\backslash\mathrm{it\,ay}-4\backslash,\{\backslash\mathrm{it\,ax}}\backslash, \\
& \{\backslash\mathrm{it\,p2y}\}\backslash,\{\backslash\mathrm{it\,ply}\}\backslash,\{\backslash\mathrm{it\,bx}+2\backslash,\{\{\backslash\mathrm{it\,ply}\}\}^{\{2\}}\{\backslash\mathrm{it\,az}}\backslash,\{\backslash\mathrm{it\,bz}-4 \\
& \backslash,\{\backslash\mathrm{it\,plx}\}\backslash,\{\backslash\mathrm{it\,ay}\}\backslash,\{\backslash\mathrm{it\,byy}\}\backslash,\{\backslash\mathrm{it\,p2x}+2\backslash,\{\backslash\mathrm{it\,ay}\}\backslash,\{\{\backslash\mathrm{it\,p2x}\}\}^{\{2\}}\{\backslash\mathrm{it\,byy}}+2\backslash, \\
& \{\backslash\mathrm{it\,az}}\backslash,\{\{\backslash\mathrm{it\,p2y}\}\}^{\{2\}}\{\backslash\mathrm{it\,bz}+2\backslash,\{\{\backslash\mathrm{it\,plx}\}\}^{\{2\}}\{\backslash\mathrm{it\,az}}\backslash,\{\backslash\mathrm{it\,bz}+2\backslash,\{\backslash\mathrm{it\,az}}\backslash,\{\{\backslash\mathrm{it\,p2x}}\}^{\{2\}}\{\backslash\mathrm{it\,bz}}+2\backslash, \\
& \{\backslash\mathrm{it\,byy}\}\backslash,\{\backslash\mathrm{it\,p2x}}\backslash,\{\backslash\mathrm{it\,ply}\}\backslash,\{\backslash\mathrm{it\,ax}-2\backslash,\{\backslash\mathrm{it\,ax}}\backslash,\{\backslash\mathrm{it\,p2y}\}\backslash,\{\backslash\mathrm{it\,byy}}\backslash,\{\backslash\mathrm{it\,p2x}}-2\backslash, \\
& \{\backslash\mathrm{it\,plx}\}\backslash,\{\backslash\mathrm{it\,byy}}\backslash,\{\backslash\mathrm{it\,ply}\}\backslash,\{\backslash\mathrm{it\,ax}-4\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,az}}\backslash,\{\backslash\mathrm{it\,bz}}\backslash,\{\backslash\mathrm{it\,p2y}}+2\backslash, \\
& \{\{\backslash\mathrm{it\,ply}\}\}^{\{2\}}\{\backslash\mathrm{it\,ax}}\backslash,\{\backslash\mathrm{it\,bx}+2\backslash,\{\backslash\mathrm{it\,ay}}\backslash,\{\backslash\mathrm{it\,p2x}}\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,bx}}-4\backslash, \\
& \{\backslash\mathrm{it\,az}}\backslash,\{\backslash\mathrm{it\,p2x}}\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,bz}-2\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,ay}}\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,bx}}+2\backslash, \\
& \{\backslash\mathrm{it\,ax}}\backslash,\{\backslash\mathrm{it\,p2y}}\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,byy}}
\end{aligned}$$

$C \xrightarrow{\text{to fortran}}$

```

cg5 = 2 * ply ** 2 * cx * ax - 2 * ply * p2x ** 2 * ay -
2 * ply *
# bz ** 2 * p2y + 2 * ply ** 2 * cz * az - 2 * ply ** 2 * h * az +
# 2 * cz * p2y ** 2 * az - 2 * h * p2y ** 2 * az - 2 * bz ** 2 * p2x
# * plx + 2 * cz * p2x ** 2 * az - 2 * h * p2x ** 2 * az + 2 * plx
** 2 * cz * az - 2 * plx ** 2 * h * az - 2 * ply ** 2 * p2x * ax +
# 2 * cx * p2y ** 2 * ax + 2 * plx ** 2 * cy * ay - 2 * plx * p2y *
* 2 * ax - 2 * plx ** 2 * p2y * ay + 2 * cy * p2x ** 2 * ay - 2 *
#bx ** 2 * p2y * ply - 2 * plx * byy ** 2 * p2x + bx ** 2 * p2y **
# 2 + plx ** 2 * byy ** 2 + byy ** 2 * p2x ** 2 + ply ** 2 * bx ** 2
# + ply ** 2 * bz ** 2 + bz ** 2 * p2y ** 2 + bz ** 2 * p2x ** 2 +
#plx ** 2 * bz ** 2 + 2 * bx * p2y * plx * byy - 2 * bx * p2y * byy
# * p2x - 2 * plx * byy * ply * bx - 4 * ply * cx * ax * p2y - 2 *
#ply * cx * plx * ay + 2 * ply * cx * ay * p2x + 2 * ply * p2x * ax
# * p2y + 2 * ply * p2x * plx * ay + 2 * cx * p2y * plx * ay - 2 *
#cx * p2y * ay * p2x + 2 * plx * cy * ax * p2y - 4 * plx * cy * ay
* p2x - 2 * plx * cy * ply * ax + 2 * plx * p2y * ay * p2x + 2 * p
#lx * p2y * ply * ax - 4 * ply * cz * az * p2y + 4 * ply * h * az *
# p2y - 4 * cz * p2x * plx * az + 4 * h * p2x * plx * az + 2 * byy
* p2x * ply * bx - 2 * cy * p2x * ax * p2y + 2 * cy * p2x * ply *
#ax

```

$\xrightarrow{\text{to LaTeX}}$

$$\begin{aligned}
& 2\backslash,\{\{\backslash\mathrm{it\,ply}\}\}^{\{2\}}\{\backslash\mathrm{it\,cx}}\backslash,\{\backslash\mathrm{it\,ax}}-2\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\{\backslash\mathrm{it\,p2x}}\}^{\{2\}}\{\backslash\mathrm{it\,ay}}-2\backslash, \\
& \{\backslash\mathrm{it\,ply}}\backslash,\{\{\backslash\mathrm{it\,bz}}\}^{\{2\}}\{\backslash\mathrm{it\,p2y}}+2\backslash,\{\{\backslash\mathrm{it\,ply}}\}^{\{2\}}\{\backslash\mathrm{it\,cz}}\backslash,\{\backslash\mathrm{it\,az}}-2\backslash, \\
& \{\{\backslash\mathrm{it\,ply}}\}^{\{2\}}h\{\backslash\mathrm{it\,az}}+2\backslash,\{\backslash\mathrm{it\,cz}}\backslash,\{\{\backslash\mathrm{it\,p2y}}\}^{\{2\}}\{\backslash\mathrm{it\,az}}-2\backslash, \\
& h\{\{\backslash\mathrm{it\,p2y}}\}^{\{2\}}\{\backslash\mathrm{it\,az}}-2\backslash,\{\{\backslash\mathrm{it\,bz}}\}^{\{2\}}\{\backslash\mathrm{it\,p2x}}\backslash,\{\backslash\mathrm{it\,plx}}+2\backslash, \\
& \{\backslash\mathrm{it\,cz}}\backslash,\{\{\backslash\mathrm{it\,p2x}}\}^{\{2\}}\{\backslash\mathrm{it\,az}}-2\backslash, h\{\{\backslash\mathrm{it\,p2x}}\}^{\{2\}}\{\backslash\mathrm{it\,az}}+2\backslash, \\
& \{\{\backslash\mathrm{it\,plx}}\}^{\{2\}}\{\backslash\mathrm{it\,cz}}\backslash,\{\backslash\mathrm{it\,az}}-2\backslash,\{\{\backslash\mathrm{it\,plx}}\}^{\{2\}}h\{\backslash\mathrm{it\,az}}-2\backslash, \\
& \{\{\backslash\mathrm{it\,ply}}\}^{\{2\}}\{\backslash\mathrm{it\,p2x}}\backslash,\{\backslash\mathrm{it\,ax}}+2\backslash,\{\backslash\mathrm{it\,cx}}\backslash,\{\{\backslash\mathrm{it\,p2y}}\}^{\{2\}}\{\backslash\mathrm{it\,ax}}+2\backslash, \\
& \{\{\backslash\mathrm{it\,plx}}\}^{\{2\}}\{\backslash\mathrm{it\,cy}}\backslash,\{\backslash\mathrm{it\,ay}}-2\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\{\backslash\mathrm{it\,p2y}}\}^{\{2\}}\{\backslash\mathrm{it\,ax}}-2\backslash, \\
& \{\{\backslash\mathrm{it\,plx}}\}^{\{2\}}\{\backslash\mathrm{it\,p2y}}\backslash,\{\backslash\mathrm{it\,ay}}+2\backslash,\{\backslash\mathrm{it\,cy}}\backslash,\{\{\backslash\mathrm{it\,p2x}}\}^{\{2\}}\{\backslash\mathrm{it\,ay}}-2\backslash, \\
& \{\{\backslash\mathrm{it\,bx}}\}^{\{2\}}\{\backslash\mathrm{it\,p2y}}\backslash,\{\backslash\mathrm{it\,ply}}-2\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\{\backslash\mathrm{it\,byy}}\}^{\{2\}}\{\backslash\mathrm{it\,p2x}}+ \\
& \{\{\backslash\mathrm{it\,bx}}\}^{\{2\}}\{\{\backslash\mathrm{it\,p2y}}\}^{\{2\}}+ \{\{\backslash\mathrm{it\,plx}}\}^{\{2\}}\{\{\backslash\mathrm{it\,byy}}\}^{\{2\}}+ \{\{\backslash\mathrm{it\,byy}}\}^{\{2\}}\{\{\backslash\mathrm{it\,p2x}}\}^{\{2\}}+ \\
& \{\{\backslash\mathrm{it\,ply}}\}^{\{2\}}\{\{\backslash\mathrm{it\,bx}}\}^{\{2\}}+ \{\{\backslash\mathrm{it\,ply}}\}^{\{2\}}\{\{\backslash\mathrm{it\,bz}}\}^{\{2\}}+ \{\{\backslash\mathrm{it\,bz}}\}^{\{2\}}\{\{\backslash\mathrm{it\,p2y}}\}^{\{2\}}+ \\
& \{\{\backslash\mathrm{it\,bz}}\}^{\{2\}}\{\{\backslash\mathrm{it\,p2x}}\}^{\{2\}}+ \{\{\backslash\mathrm{it\,plx}}\}^{\{2\}}\{\{\backslash\mathrm{it\,bz}}\}^{\{2\}}+ 2\backslash,\{\backslash\mathrm{it\,bx}}\backslash, \\
& \{\backslash\mathrm{it\,p2y}}\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,byy}}-2\backslash,\{\backslash\mathrm{it\,bx}}\backslash,\{\backslash\mathrm{it\,p2y}}\backslash,\{\backslash\mathrm{it\,byy}}\backslash, \\
& \{\backslash\mathrm{it\,p2x}}-2\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,byy}}\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,bx}}-4\backslash,\{\backslash\mathrm{it\,ply}}\backslash, \\
& \{\backslash\mathrm{it\,cx}}\backslash,\{\backslash\mathrm{it\,ax}}\backslash,\{\backslash\mathrm{it\,p2y}}-2\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,cx}}\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,ay}}+2\backslash, \\
& \{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,cx}}\backslash,\{\backslash\mathrm{it\,ay}}\backslash,\{\backslash\mathrm{it\,p2x}}+2\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,p2x}}\backslash, \\
& \{\backslash\mathrm{it\,ax}}\backslash,\{\backslash\mathrm{it\,p2y}}+2\backslash,\{\backslash\mathrm{it\,ply}}\backslash,\{\backslash\mathrm{it\,p2x}}\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,ay}}+2\backslash, \\
& \{\backslash\mathrm{it\,cx}}\backslash,\{\backslash\mathrm{it\,p2y}}\backslash,\{\backslash\mathrm{it\,plx}}\backslash,\{\backslash\mathrm{it\,ay}}-2\backslash,\{\backslash\mathrm{it\,cx}}\backslash,\{\backslash\mathrm{it\,p2y}}\backslash,\{\backslash\mathrm{it\,p2y}}\backslash,
\end{aligned}$$

$$E \xrightarrow{\text{LaTeX}} 4\backslash,\{\it ply\}\backslash,\{\it cz\}\backslash,h\{\it p2y\}+4\backslash,\{\it cz\}\backslash,\{\it p2x\}\backslash,\{\it plx\}\backslash,$$
$$h+2\backslash,\{\it cx\}\backslash,\{\it p2y\}\backslash,\{\it plx\}\backslash,\{\it cy\}-2\backslash,\{\it cx\}\backslash,\{\it p2y\}\backslash,$$
$$\{\it cy\}\backslash,\{\it p2x\}+2\backslash,\{\it plx\}\backslash,\{\it p2y\}\backslash,\{\it cy\}\backslash,\{\it p2x\}-2\backslash,\{\it ply\}\backslash,\{\it cx\}\backslash,\{\it plx\}\backslash,\{\it cy\}+2\backslash,\{\it ply\}\backslash,\{\it cx\}\backslash,\{\it$$
$$plx\}\backslash,\{\it p2y\}+2\backslash,\{\it ply\}\backslash,\{\it cx\}\backslash,\{\it cy\}\backslash,\{\it p2x\}+2\backslash,\{\it$$
$$ply\}\backslash,\{\it p2x\}\backslash,\{\it cx\}\backslash,\{\it p2y\}+2\backslash,\{\it ply\}\backslash,\{\it p2x\}\backslash,\{\it plx$$
$$\}\backslash,\{\it cy\}-2\backslash,\{\it ply\}\backslash,\{\it p2x\}\backslash,\{\it plx\}\backslash,\{\it p2y\}-2\backslash,\{\it ply$$
$$\}\wedge^2\{\it cx\}\backslash,\{\it p2x\}-2\backslash,\{\it ply\}\backslash,\{\it cx\}\wedge^2\{\it p2y\}-2\backslash,\{\it$$
$$ply\}\backslash,\{\it p2x\}\wedge^2\{\it cy\}-2\backslash,\{\it cx\}\backslash,\{\it p2y\}\wedge^2\{\it plx$$
$$$$p2x\}-2\backslash,\{\it ply\}\wedge^2\{\it cz\}\backslash,h-2\backslash,\{\it ply\}\backslash,\{\it cz\}\wedge^2\{\it$$
$$p2y\}-2\backslash,\{\it ply\}\backslash,\{h\}\wedge^2\{\it p2y\}-2\backslash,\{\it cz\}\backslash,\{\it p2y\}\wedge^2h+$$
$$\{\it ply\}\wedge^2\{\it cx\}\wedge^2+\{\it ply\}\wedge^2\{\it p2x\}\wedge^2+\{\it cx\}$$
$$\wedge^2\{\it p2y\}\wedge^2+\{\it plx\}\wedge^2\{\it cy\}\wedge^2+\{\it plx\}\wedge^2\{\it$$
$$p2y\}\wedge^2+\{\it cy\}\wedge^2\{\it p2x\}\wedge^2+\{\it ply\}\wedge^2\{\it cz\}\wedge^2$$
$$+\{\it ply\}\wedge^2\{h\}\wedge^2+\{\it cz\}\wedge^2\{\it p2y\}\wedge^2+\{h\}\wedge^2\{\it$$
$$p2y\}\wedge^2+\{\it cz\}\wedge^2\{\it p2x\}\wedge^2+\{h\}\wedge^2\{\it p2x\}\wedge^2+\{\it$$
$$plx\}\wedge^2\{\it cz\}\wedge^2+\{\it plx\}\wedge^2\{h\}\wedge^2-2\backslash,\{\it cz\}\backslash,\{\it p2x\}$$
$$\wedge^2h-2\backslash,\{\it cz\}\wedge^2\{\it p2x\}\backslash,\{\it plx\}-2\backslash,\{h\}\wedge^2\{\it p2x\}\backslash,\{\it$$
$$plx\}-2\backslash,\{\it plx\}\wedge^2\{\it cz\}\backslash,h-\mathrm{R}\wedge^2\left(-2\backslash,\{\it ply\}\backslash,\{\it$$
$$p2y\}+\{\it p2y\}\wedge^2+\{\it ply\}\wedge^2-2\backslash,\{\it plx\}\backslash,\{\it p2x\}+\{\it$$
$$plx\}\wedge^2+\{\it p2x\}\wedge^2\right)$$$$

