4.8.2. Foucault test

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• Foucault test aberrations

Invented by the French scientist Leon Foucault in 1858, this ingenious test uses point source of light placed at the center of curvature of a concave mirror (in practice, slightly to the side, so that the mirror focus is separated from the source, and focusing light can be intercepted without cutting off source of illumination), as illustrated on **FIG. 52**. The combination of simplicity and accuracy has made it the single most used test in the amateur telescope makers' circles.

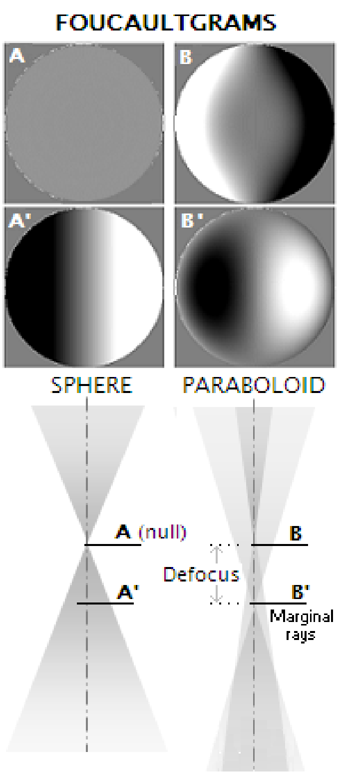


FIGURE 52: The principle of the Foucault test. Light reflected from mirror surface carries the information on geometric properties of the reflecting surface: if it is perfect spherical, the light from the entire surface will converge to a single aberration-free focus. If surface deviates from spherical, focus location will vary with the zonal height. An opaque thin plate with straight, sharp edge (usually some type of metal blade, called knife edge, or **KE** for short) moving perpendicularly across the focusing light in the proximity of focus location produces a shadow moving across the surface; shape of the shadow tells instantly whether a surface is spherical, with one unique focus for the entire surface, or not.

If it is a sphere, a straight-edge shadow moves over the surface as the KE cuts through the light converging to or diverging from the focus (**A'**); or the uniform, light-grayish shadow spreads over the entire surface when the KE intercepts converging cone at the focus, producing so called null (**A**). This makes the test particularly well suited for quick and reliable tests of spherical reflecting surfaces. Non-spherical surfaces produce defocused, commensurate to their conic, creating various shadow forms as the KE moves through the converging light near aberrated focus (**B**), with only a single zone nulled, and the rest of surface area split between darker and brighter areas (**B'**, edge zone nulled, patterns generated by Mike Lindner's Foucault Simulator).

While long ago replaced with newer testing technologies in the professional circles, Foucault test is still widely used by the amateurs. The standard reference is How to make a telescope by Texereau; more recent, A manual for amateur telescope makers, Leclaire. Online, David Harbour's description is among the most detailed of the Foucault test. Programs for analyzing Foucault test data, such as SIXTEST by Jim Burrows, are computer-era enhancement to the test's proven value. More recent variants of the test include replacing KE with a wire (wire test), the Foucault zonal mask by (Everest) pin-stick, or the eye by a camera, taking shots of the entire mirror surface for selected blade locations, which then can be analyzed with computer software (automated, or Robo-Foucault, such as the "semi-automated" version detailed by Mike Peck).

Due to inherent aberrations of the spherical mirror surface in imaging distant objects, paraboloid is preferred surface for telescope mirrors. Consequently, Foucault test is most often used in making paraboloidal mirrors. Ideally, paraboloid should be tested with collimated beam (i.e. for object at infinity), which is a null test for this conic. However, due to practical difficulty of producing collimated beam, it is usually tested with light source at the center of curvature (vertex radius). Since for non-spherical surface in such setup every zone has somewhat different focus location, the test consist in extracting the degree of deviation of the actual measured zonal foci from those corresponding to a perfect paraboloid.

Longitudinal aberration for object (source) at the center of curvature (fixed source, moving knife edge) is given by:

$$LA = KD\rho^2/8F = -K(\rho d)^2/R,$$

where **K** is the conic, **D** mirror diameter, **ρ** the zonal (radius) height normalized to 1 for D/2=**d**, and **F** the mirror focal ratio f/D (not to confuse with the actual test focal ratio, which is 2F), **f** being the focal length, half the vertex radius of curvature **R** (note that this applies only for paraboloid, i.e. K=-1; f<R/2 for K>-1 and f>R/2 for K<-1). It is measured with respect to paraxial focus, with the minus sign for K<0 indicating that outer zones focus farther away than the central zone.

For source moving with the knife edge the LA is half as large as for fixed source, i.e.

$$LA = KD\rho^2/16F = -K(\rho d)^2/2R$$

These expressions are valid for most amateur mirrors. For very fast mirrors, inclusion of correction terms may be needed to maintain test accuracy.

Specific manner in which the data obtained by measuring deviations from perfect zonal foci is used varies somewhat from one source to another. In general, the row data - the actual measured deviations from the zonal foci of a corresponding paraboloid - are first offset by the longitudinal aberration of a perfect paraboloid. The result is called *residuals*. Specifically, if the longitudinal aberration of a paraboloid for four zonal heights are L_{1P}, L_{2P}, L_{3P} and L_{4P}, and actual measured values L_{1A}, L_{2A}, L_{3A} and L_{4A}, the residuals are (L_{1A}-L_{1P}), (L_{2A}-L_{2P}), (L_{3A}-L_{3P}) and (L_{4A}-L_{4P}).

The purpose of deriving residuals is to filter out the error in placing the reference point for the measurements vs. actual mirror center of curvature, which is hard to pinpoint. The next step is so called *reduction*, which is deducting residual of a selected zone from the rest of residual values. Its purpose is to reduce the value of residuals to an actual longitudinal differential between the measurements. Selection of the zonal value to be deducted is arbitrary; it is usually either the inner zone, the ~70% (ρ~0.7) or the outer zone.

Obviously, the reduced LA value for selected zone is zero, which means that it is assumed to be a part of the perfect reference surface. If the actual measurements are identical to those for the perfect paraboloid, except for the reference point differential, the residuals for all zones would equal the reference point differential, and the reduced LA value would be zero for all.

The reduced value L_R are used to obtain *relative transverse aberration*, from RTA=-ρL_R/2F. It is termed *relative* because it is based on longitudinal aberration relative to a chosen zonal focus. It is usually measured against, or expressed in units of the Airy disc diameter. The minus sign determines RTA value as positive for the longitudinal aberration $\mathbf{L}_R$ relative to the reference focus extending away from the mirror, and vice versa.

RTA is not to be confused with the transverse spherical aberration, because the two are generally different, i.e. relative transverse aberration in Airy disc diameters is generally smaller, possibly significantly, than the transverse spherical aberration for given value of conic deviation.

Consider 300mm ï¿½/5 prolate ellipsoid with K=-0.5 tested at the center of curvature. Taking, for simplicity, median zonal heights (**ρ**) as 0.9, 0.7, 0.5 and 0.3, the measured zonal foci will deviate from those of the perfect paraboloid by half its longitudinal aberration, which from LA=KDρ^2/8F, comes to 3.04mm, 1.84mm, 0.89mm and 0.34mm, respectively.

Choosing the inner zone for the reduction gives the respective reduced values 2.7mm, 1.5mm, 0.54mm and zero. They give the respective RTA as -0.243mm, -0.105mm, -0.045mm and zero. In units of the Airy disc diameter (2.44λF, or 0.0134mm for 550nm wavelength and the effective F=10), it is 18.1, 10, 3.6 and zero, in the same order. For comparison, full longitudinal spherical aberration for this mirror (for infinity focus) is -3.75mm, and its paraxial blur diameter is 0.375mm, or 27.9 Airy disc diameters.

If the 70% zone is chosen for reduction, the relative longitudinal aberration values are 1.2mm, zero, -0.95mm and -1.5mm, and RTA is 0.108mm, zero, 0.048mm and 0.045mm (8, zero, 3.5, and 3.4 in Airy disc diameters), respectively. Nominally much better, despite being for the same surface. Obviously, the choice of reduction zone significantly influences nominal indicators of surface quality with the Foucault test, and the two test results are comparable only if using identical reduction zone. Choice of a different reduction zone will also give a different indication of which portion of mirror surface needs to be addressed. Choosing central zone indicates that all (other) zones focus short of the reference paraboloid, the higher zone the more so, hence that the surface needs to be corrected by polishing off nearly the entire surface, increasing toward the outer area. On the other hand, with reduction relative to the 70% zone, relative longitudinal aberration indicates the outer zone focusing shorter, and the two inner zones focusing longer. That indicates that the outer and inner zones need to be worked on, leaving 70% zone area out.

With the outer zone being chosen for reduction, the indication would be that surface deviation increases toward mirror center.

Since RTA can be used to construct the wavefront profile in zonal segments, with the wavefront zonal angular deviation from the reference wavefront's slope closely approximated by β=RTA/2R, **R** being the mirror radius of curvature, and the corresponding linear deviation of the aberrated wavefront Zβ, **Z** being the zonal width, the choice of reduction zone will also determine the resulting wavefront profile. Consequently, the best fit parabola for the outer, 70% and inner zone, will be one centered at the marginal focus (approximately), best focus and paraxial focus (approximately).

The following is a graphic example of the main Foucault test parameters and relationships, illustrated with the actual surface being a sphere. If the sphere is, for instance, 150mm ï¿½/8 mirror, thus with radius of curvature R=2400mm, its longitudinal aberration at infinity focus is D/32F=0.586mm. With the object at the center of curvature, the wavefront incident on mirror surface coincides with it, and it is reflected straight back to the center of curvature. Thus, the longitudinal aberration is zero. However, if paraboloidal surface is desired, then the reference is a paraboloid of identical vertex radius, which would produce longitudinal aberration four times greater than that of the sphere at infinity focus, or 2.34mm. It varies with the square of zonal height, so taking for simplicity three off-center zones, 30-50, 50-65 and 65-75, hence of the respective zonal widths z_1=20mm, z_2=15mm and z_3=10mm, with median heights (**h**) of 40, 57.5 and 70mm, their longitudinal aberrations (**LA**) are 0.67, 1.38 and 2.04mm, respectively.

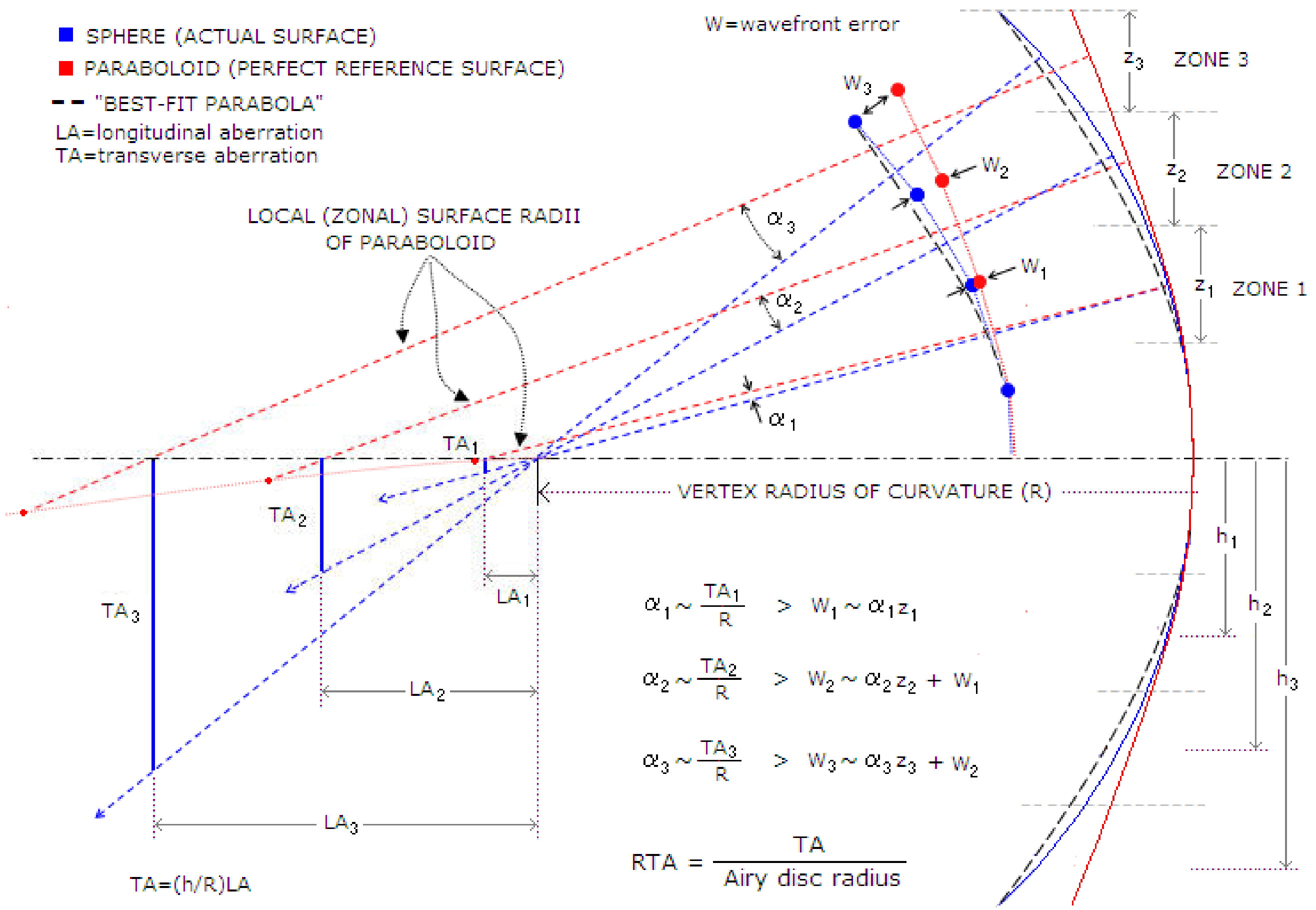


FIGURE 53: Geometry of the Foucault test in determining how close is mirror surface to that of a paraboloid. Unlike sphere (blue), where normal to the surface at any zonal height leads to the same point - the center of curvature - thus every zone focuses point-object placed at the center of curvature back onto itself, paraboloid (red) flattens out toward the edge, with the surface normals, coinciding with local, or zonal radius of curvature (defined as a radius of the osculating circle for the zonal point) spreading out as the zonal height increases. For given vertex radius, paraboloid's zonal foci will have specific deviations - longitudinal aberration, or LA - from the paraxial focus. The measured linear LA directly determines transverse zonal aberration which, expressed in units of Airy disc diameter, defines the relative transverse aberration (RTA). Transverse aberration can be used to express slope of the zonal section of the wavefront α, which in turn determines the deviation **w** of the sphere-produced wavefront from that produced by paraboloid (blue and red dots, respectively). These zonal sections can be used to construct an approximate shape of the actual wavefront. Since every conic other than sphere will generate specific longitudinal aberration, the numbers will closely specify the actual surface shape (note that the scheme assumes moving source, which directly measures the zonal radius).

Since the measured zonal sections of the sphere are perpendicular to the mid-zone radius, and all the radii project into the center of curvature, the shape of reconstructed wavefront approximates mirror surface. However, if the actual surface is desired paraboloid, projected zonal segments are perpendicular to the mean zonal radius (i.e. local radius of curvature of the paraboloid) do not converge to a single point (with moving source, Foucault test directly measures foci locations of these local, i.e. mean zonal radii, which in turn determine averaged zonal slope at the surface). As a result, sections of wavefront from any single point do not reflect off simultaneously. Since paraboloid's surface spreads out toward the edge, the incident wavefront originating, for instance, at the center of curvature, reaches outer zones latter, and those sections lag behind the inner wavefront portion. In effect, the wavefront reflected off the paraboloid flattens out relative to paraboloid's surface, and the differential between it and the wavefront produced by a sphere is double their zonal sagitta differential.

There is a direct relationship between the degree of deviation of these two wavefronts (the actual one, and that of a perfect reference surface) and measured zonal deviations.

From TA=(h/R)LA, the corresponding transverse aberrations for the three zonal LA readings with the above sphere, in the same order, are 0.011, 0.033 and 0.0595mm, and their zonal slope deviations, s=(TA/R)z, are 0.0000917, 0.000206 and 0.00025mm. In units of λ=0.00055mm wavelengths, it is 0.17, 0.037 and 0.45λ. The total deviation is a simple sum of the three, or 0.99λ.

This is nearly double the maximum surface differential between this sphere and its reference parabola of 0.52 wave (the figure is generally somewhat lower, due to the outer zone being averaged out). Assuming null at the zone 1, the corresponding relative transverse aberration is greatest at the zone 3, as $RTA_3=(TA_3-TA_1)/2.44\lambda F$ (for the doubled Airy disc with object at the center of curvature, with **F** being the infinity focal ratio) - a disastrous 4.5 Airy disc radii. But this mirror is, in fact, near diffraction-limited. Taking mid-zone for the null nearly reduces the RTA in half, as does nulling at the outer zone.

The 0.99λ wavefront error at the wavefront constructed from the measurements is not the actual error. In this case - and nearly any other - there is a parabola of slightly different radius, whose wavefront deviates less - often significantly so - from the constructed wavefront. It is called "best fit parabola". In this case, best fit paraboloid has slightly stronger curvature than one with vertex radius equal to that of the actual surface. It has identical sagitta to that of the sphere, touching the latter at the vertex and at the edge. The error vs. this parabola is reduced by a factor of 0.25, from 0.99λ down to 0.25λ. This is the final test result.

The only difference between sphere and a conic closer to a paraboloid is that the latter does not have zonal foci coinciding at the center of curvature. These foci are separated as for paraboloid, either in somewhat smaller (prolate ellipsoid) or greater degree (hyperboloid with conic less than -2), thus with smaller longitudinal aberration at each zone.

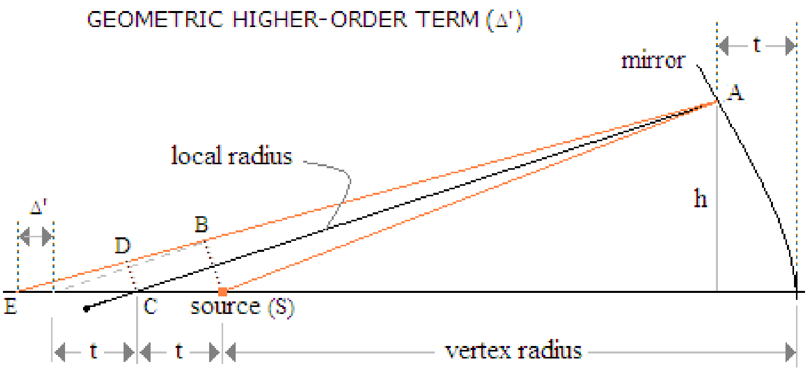
Correction terms for the longitudinal aberration

As the basic relations for longitudinal defocus imply, Foucault test can be used for any conic shape. Longitudinal zonal defocus with respect to paraxial focus relates to the conic **K** and the vertex radius of curvature **R** as

$$LA = -K(\rho d)^2/R,$$

with **ρ** being the zonal height normalized to 1 for the pupil radius **d**, for fixed light source, and half as much when both, source and edge, are moving together (the latter is the actual longitudinal aberration). The minus sign of defocus for K=-1 indicates that the outer zones focus farther from paraxial focus, the result of over-correction. It is arrived at by setting 2ψ=1 in Eq. 9.1, and using the resulting peak aberration coefficient, Eq. 9.3, in the relation for longitudinal aberration in Eq. 10 (note that when substituting for **F** in the latter, F=-R/D due to the doubled mirror-to-image separation). It is particularly convenient when testing prolate ellipsoids with reasonably close far focus, when the light source is placed at the near focus, and null is observed at the far focus. Obviously, this defocus figure is based on the lower-order surface approximation.

Correction factor for the Foucault measurements that may become significant with fast/large mirrors is related to the geometry of a ray originating at the vertex center of curvature and reflected back above paraboloid's local radius. This term is often called "higher order term" but has nothing to do with higher order spherical aberration, i.e. surface equation; it results from ray geometry, thus could be called "**geometric higher-order term**" as opposed to the "surface higher-order term", which is addressed next. While the incident and reflected angle of this ray vs. local radius are identical, the latter will intersect the axis slightly farther away from the local radius than the former, due to it being positioned above the local radius. In other words, its defocus vs. vertex center of curvature - where is located the source - will be slightly greater than the sagitta for the zonal height doubled. From ray geometry, similar triangles SBE and CDE imply near-equality $(t+\Delta')/(2t+\Delta')=(R+t)/2R$ solving to $\Delta'=h^4/2(R-t)R^2$, with the sagitta $t=h^2/2R$ for paraboloid, zonal height h=ρd, and the defocus of the local radius vs. center of



curvature equal to the zonal sagitta **t**, i.e. $h^2/2R$. Neglecting **t**, which is always much smaller than **R** leads to the exact expression for this term:

$$\Delta' = (\rho d)^4/2R^3$$

Since the longitudinal aberration proportional to the conic **K**, so is this geometric higher order term, thus can be written as $\Delta'=-K(\rho d)^4/2R^3$. This term is negligible except for very fast mirrors, for which the correct defocus is LA+Δ'. Obviously, this factor does not apply with moving source, when defocus is measured directly at the local radius.

Another possible correction factor is related to the surface (sagitta) equation, and could be called "**surface higher-order term**". From the geometry of ray originating at the intersection of local radius of curvature and axis and reflecting back onto itself (hence for moving source), it crosses axis at R-Kt from mirror vertex, **R** being the vertex radius of curvature and **t** the sagitta for given zonal height (for mirror oriented to left both **R** and **t** are negative, thus they add up for K<0). Thus the longitudinal differential between paraxial and zonal focus with moving source equals the zonal sagitta, given by $(\rho d)^2/2R$ for paraboloid. For fixed source, twice as much.

Thus, the higher-order longitudinal aberration terms for a paraboloid are zero. However, since the power series describing sagitta is:

$$t=(\rho d)^2/2R+(K+1)(\rho d)^4/8R^3+(K+1)^2(\rho d)^6/16R^5+...,$$

conics other than paraboloid have non-zero higher-order terms adding to the longitudinal defocus, with the defocus added given by:

$$\Delta'' = (K+1)(\rho d)^4/8R^3 + (K+1)^2(\rho d)^6/16R^5 + ...$$

It is very unlikely these terms would have any significance with even very fast/large mirrors for near-paraboloidal shape, but it could increase test accuracy with such mirrors if the second term is taken into account for figures that appreciably differ from parabolic. Since this is correction to where the local radius intersects the axis, these higher-order terms are also doubled for fixed source.

For very fast mirrors, the magnitude of both correction factors should be checked to determine if their magnitude would be sufficient to make the basic LA relation sufficiently different - as LA+Δ' (fixed source, for evaluating this term alone), LA+Δ" (moving source) or LA+Δ'+Δ" (fixed source, for evaluating both terms combined) - to affect measurement accuracy.

It should be noted that both, OSLO Edu and ATMOS do show non-zero secondary spherical aberration for paraboloid with source at the center of curvature, opposite in sign to the primary spherical. Since **R** is constant and surface equation **t** for parabola has no higher order terms, this appears to be a glitch. For 200mm f/2.5 paraboloid the secondary spherical is 1/67 of the primary, which would by somewhat less - probably closer to 1/100 - shorten the defocus for the outer zone, nearly offsetting the geometric higher-order correction. Since secondary spherical is in inverse proportion to the 5th power of F-number, already at f/3 it would be less than half as large, and roughly 50% smaller than the geometric higher-order term.

Foucault test aberrations

By establishing focus locations of annular zonal openings for non-spherical surfaces, the surface shape can be approximated with high level of accuracy. Common consensus seems to be that the general limit to a repeatable accuracy with the Foucault test is ~1/10 wave P-V on the wavefront, assuming needed testing skills. In practice, the accuracy limit vary with the type of deformation: it is to expect that it is higher for the rotationally symmetrical overall figure, where needed surface accuracy is only half that showing in the wavefront.

On the other hand, Foucault's accuracy is generally lower for zonal, local and rotationally asymmetrical figure errors, particularly astigmatism. It also becomes less reliable for relative apertures significantly larger than ~f/4.

Significant changes in the wavefront and ray geometry for the object at the center of curvature versus infinity, brings on quite a bit of change in surface aberrations. The P-V wavefront error of lower-order **spherical aberration** at the best focus, from Eq. 7 and 9, is given by:

$$W_s = -KD^4/256R^3 = KD/2048F^3,$$

with **K** being the mirror conic, and **F** being the mirror focal number for object at infinity. For prolate ellipsoids, paraboloid and hyperboloids, the negative sign determined by the conic indicates over-correction. Comparing it with Eq. 66 shows that paraboloid with object at the center of curvature exerts the same amount of spherical aberration as a comparable sphere for object at infinity, only of opposite sign.

However, since the effective focal ratio **F** has doubled, the geometric aberration has changed: as specified earlier, longitudinal spherical is larger by a factor of four vs. that for the corresponding sphere and object at infinity (L=D/32F for the diameter **D** in mm), while the transverse spherical is now doubled, keeping the same proportion to the Airy disc. The former changes in proportion to the square, and the latter in proportion to the cube of the zonal height.

The wavefront aberration, transverse and longitudinal aberrations change in proportion to the fourth, third and second power of the zonal height, respectively.

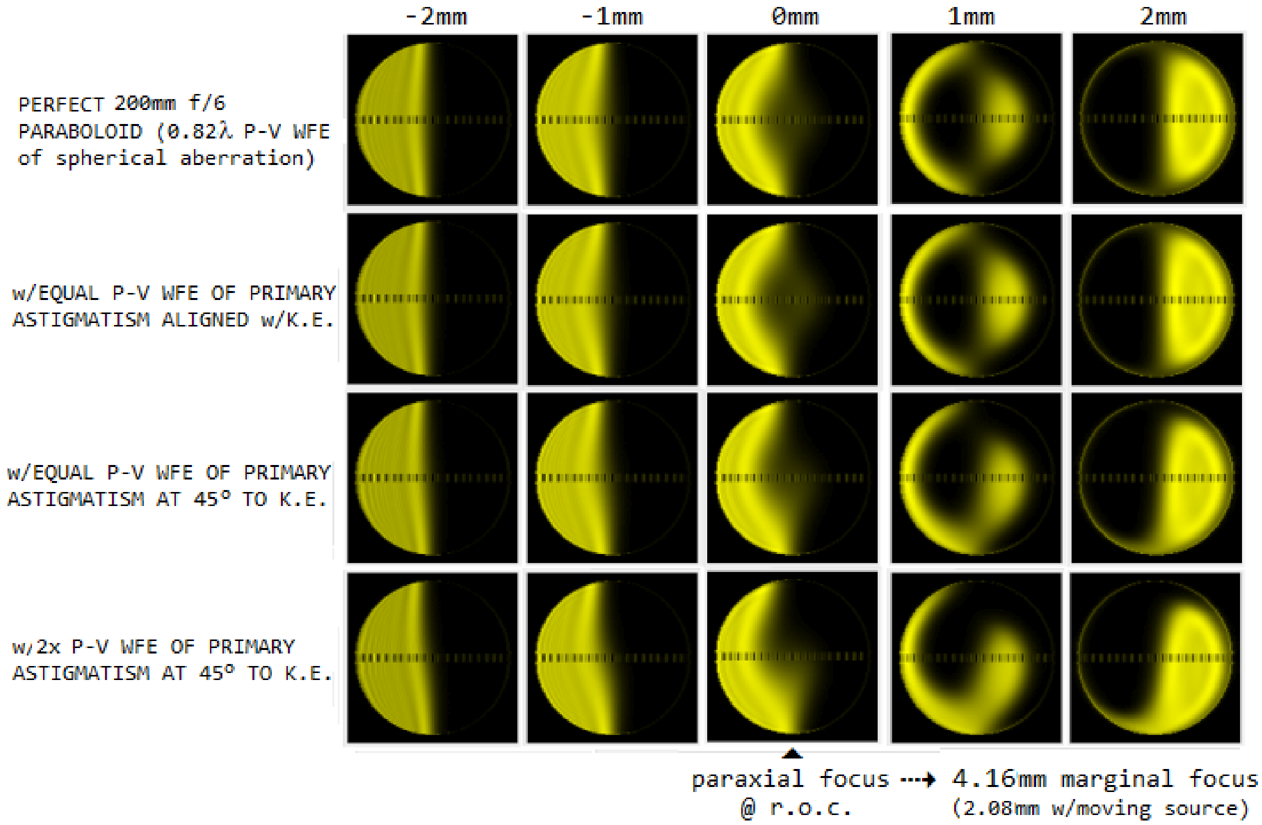
For the stop at the surface, coma with object at the center of curvature is cancelled, regardless of the conic. The wavefront error of astigmatism, on the other hand, doesn't change with object distance, remaining as given by Eq. 71.1. It quickly increases with angular separation between the source and mirror focus, which needs to be kept at a minimum in order to preserve best possible focus quality. Since this separation is the incidence angle (with respect to the axis passing through mirror center) doubled, the value corresponding to off-axis height **h** in the equation is s/2, **s** being the source-to-focus separation. Due to doubled mirror-to-image distance, the effective angle is yet another two times smaller, corresponding to s/4 off axis height at the infinity focus. Thus, replacing **h** with s/4 gives the RMS wavefront error of astigmatism induced as $\omega=s^2/627DF^3$, for the aperture diameter **D** in mm (**s** is in mm as well) and infinity focal ratio **F** (in units of 550nm wavelength, $\omega=2.9s^2/DF^3$; as little as 20mm source-to-focus separation with 200mm ï¿½/4 mirror induces 0.09 wave RMS error of lower-order astigmatism). For **s** and **D** in inches, $\omega=74s^2/DF^3$.

Unlike the wavefront aberration, the geometric aberrations of astigmatism does change due to the doubled effective focal ratio **F** for mirror with object at its center of curvature, with the longitudinal aberrations larger by a factor of four, and the transverse aberration nominally doubled. The wavefront error changes in proportion to the square of the zonal height, transverse aberration with the zonal height, and the longitudinal is, as expected, constant.

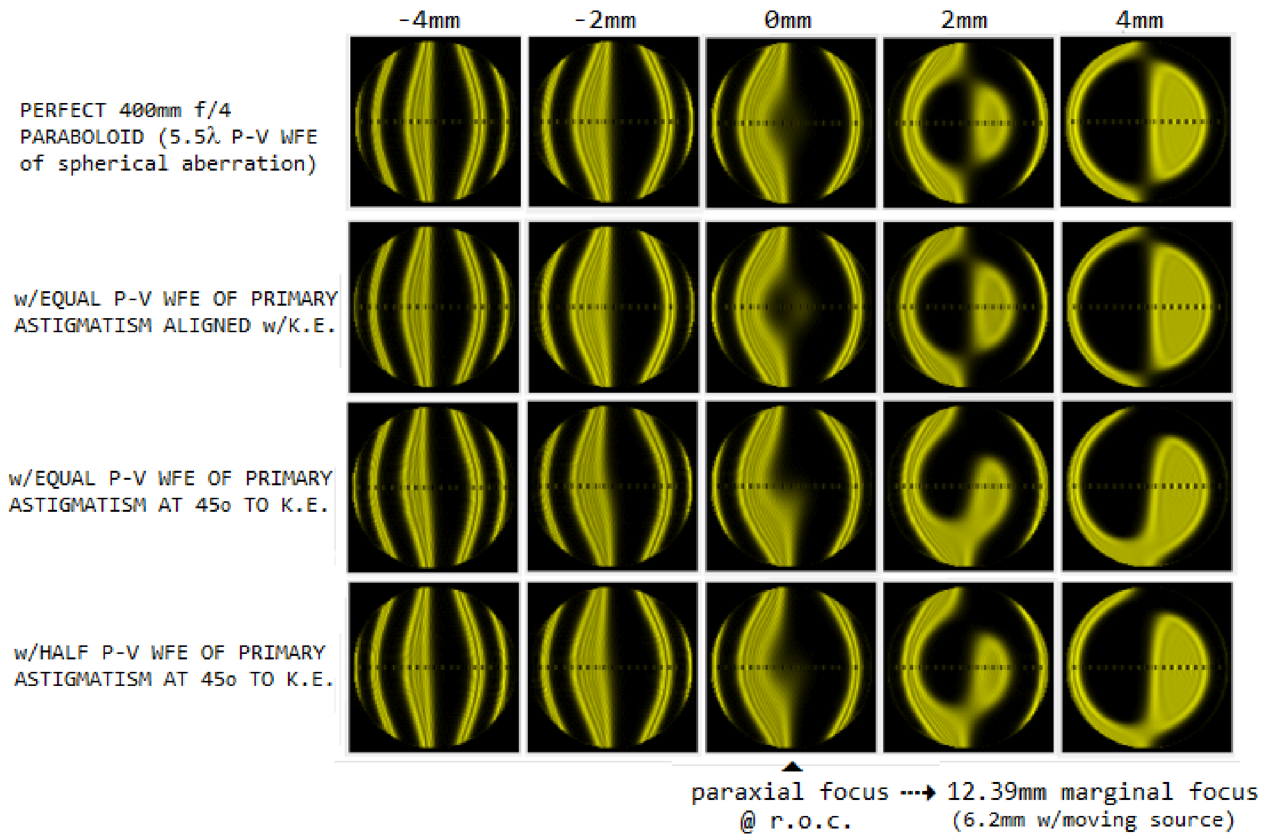
Contrary to the common belief, mirror astigmatism can be detected with the Foucault setup (as well as Ronchi), even when quite low. Both, geometric and diffraction analysis (Astigmatism under the Foucault test, Linfoot, click on "print this article" for PDF) predict that astigmatism produces uneven intensity distribution along one of the two perpendicular (or nearly so, for less symmetrical forms) astigmatic axes, which has one side brighter, and the other darker than the rest of illuminated surface. Depending on its orientation in the setup, and the point of interception, this illumination asymmetry results on more or less obvious apparent rotation of the shadow (in the Ronchi, also depending on the orientation, it will cause either gradual change in line width, similar to the effect of spherical aberration - when the grating orientation coincides with that of astigmatic axes - or S-like line deformation when astigmatic axes are at 45 angle vs. grating). John Sherman's spot test web pages describe some practical approaches for detecting astigmatism in the Foucault or Ronchi setup, along with graphic illustrations. Quantifying astigmatism with either of the two tests is, however, more difficult.

ASTIGMATISM IN THE FOUCAULT TEST

Foucault test is known as insufficient at detecting astigmatic deformation of the test surface, but it does have the capability to detect relatively large errors, under certain conditions. Following two examples should illustrate more specifically its limitations in this respect. The first is a smaller, medium fast mirror, with which detection limit falls generally above the level of spherical aberration of the mirror at the r.o.c. The sensitivity is the lowest when astigmatic axis is aligned with the KE, and highest when it is at a 45-degree angle.



The other one is a large, fast mirror, with which the detection limit is, roughly, at half its spherical aberration at r.o.c. However, since it is several times higher than in the first example, the detection threshold is actually about twice larger astigmatic deformation. If compared to the Ronchi test, it is significantly less sensitive in this respect.



The astigmatism in these simulations is of saddle-like shape, which corresponds to the wavefront error at the best focus location. Actual surfaces commonly have cylindrical deformation, but the P-V error remains unchanged, only the RMS error is larger by a factor of $\sqrt{1.5}$.

◄ 4.8.1. Testing optical quality ▌ 4.8.3. Ronchi test ►

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