Scrambling Time and Causal Structure of the Photon Sphere of a Schwarzschild Black Hole

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Abstract:

Recently, physicists have started applying quantum information theory to black holes. This led to the conjecture that black holes are the fastest scramblers of information, and that they scramble it in time order $M \log M$, where M is the mass of the black hole in natural units. As stated above, the conjecture is not completely defined, as there are several possible definitions of scrambling times. It appears that not all papers that refer to this conjecture interpret it the same way. We consider a definition of scrambling time stronger than the one given in the paper that first proposed this conjecture [Sekino and Susskind, *J. High Energy Phys.* **0810**:065 (2008)], and show that this stronger version of the conjecture appears to be incompatible with a number of other widely-believed and reasonable-sounding properties of black holes.

We argue that for the scrambling time of a black hole to be this fast, either relativity is violated or non-standard physics must be occurring outside the stretched event horizon of a black hole. More specifically, either information is being transferred faster than relativity would permit, the information is not carried by the Hawking radiation and thus must be carried by unknown physics, or the Hawking radiation carries much more information than standard thermodynamics would permit.

We analyze the situation from the viewpoint of an outside observer who never falls into the black hole. We assume that an outside observer never sees anything actually fall into the black hole. We also assume that from the viewpoint of this observer, the physics near a black hole are very much like the known laws of physics, except possibly in the stretched horizon, where physics at Planck-scale energies starts being relevant. For the physics near the horizon, all we assume is that the structure of space-time is roughly that predicted by general relativity.

Unlike Suskind's complementarity principle, we do not require that an outside observer's point of view be in any way compatible with that of an observer falling into the black hole. In fact, we do not consider the viewpoint of observers falling into the black hole at all. Thus, some of the solutions that have been proposed to evade the contradiction discovered in the AMPS paper [Almheiri et al., *J. High Energy Phys.* 2013:62 (2013)], in particular the assumption that black holes have firewalls just below the horizon that destroy any infalling information do not appear to address the problems our paper raises.

Naturally, in order to show this, we need to make some assumptions. Our first assumption is that, outside the stretched horizon, the laws of physics are well approximated by some quantum field theory. The second is that the causality structure of spacetime outside the horizon is dictated by the laws of general relativity. Third, we assume that the Hawking radiation carries the information that exits the black hole, as well as the information involved in scrambling the black hole. While we allow information to be stored by a different means in the stretched horizon, general relativity does not appear to permit fast scrambling unless this information leaves the stretched horizon. Finally, we assume that the usual laws of thermodynamics govern how much information can be carried by Hawking radiation.

Our argument considers the structure of the photon sphere from the point of view of an outsider who stays outside the black hole. The basic idea of our argument is to divide the photon sphere into cells, and use computer-science style arguments to show that it will take at least order M^2 time to transmit enough information from one side of the black hole to the other so as to maximally entangle the two sides.

1 Introduction

The black hole information paradox is the question of whether the dynamics of black hole evaporation is unitary, or whether information is lost when you throw it into a black hole. Classical general relativity says that anything that is thrown into a black hole can never come out, while quantum mechanics says that physical processes are reversible, so that information is never destroyed. In [3], Bekenstein proposed that black holes obey their own laws of thermodynamics, and that the entropy of a black hole was of order M^2 bits, where M is the mass of the black hole. In [5], Hawking continued the investigation of the thermodynamics of black holes and argued that radiation emerges from a black hole, and thus that the black hole will eventually evaporate after time order M^3 . While Hawking's argument does not appear to let information escape from a black hole, his argument is semi-classical, and thus can only be an approximation to the true physics. It is thus not clear at present whether information can escape from a black hole in a full theory of quantum gravity. Hawking radiation is very similar to Unruh radiation, the radiation that an accelerating observer in a vacuum state sees. While some treatments of black holes distinguish between Hawking and Unruh radiation, classifying the photons that escape the black hole as "Hawking radiation" and the virtual photons that remain inside the black hole as "Unruh radiation", we will not. We use the term "Hawking radiation" for both of these phenomena when they are in the vicinity of black holes.

Maldacena [11] discovered a correspondence between Anti de Sitter theories with gravity and conformal field theories without gravity (called AdS-CFT). Since conformal field theories are unitary, this implied that theories of quantum gravity should also be unitary.

In order to reconcile general relativity and quantum mechanics, Susskind [14] proposed a complementarity principle, where both an observer remaining outside the black hole and an observer falling in see things happening in accordance with the laws of physics, but where these two observers do not necessarily agree on exactly what happens. This complementarity principle has been seriously challened by an argument put forth in 2013 in a paper generally known as AMPS (from its authors' initials) [1]. More specifically, they use the monogamy of entanglement [4] to argue that the information inside a black hole cannot be entangled both with the information near the horizon and with the earlier Hawking radiation. They then argue that this means that information cannot escape from a black hole without producing a "firewall" across the horizon, where any infalling matter is destroyed.

Reasoning about black hole dynamics and complementarity, in 2007 Hayden and Preskill [6] gave arguments for why the scrambling time of a black hole had to be at least order $M \log M$, where M is the mass of the black hole in natural units. An informal definition of the scrambling time is how fast the information in a black hole gets "mixed up". This paper led to a more serious study of scrambling time. Several definitions of scrambling time have been proposed. These do not necessarily all yield the same quantity for the scrambling time [10]. These definitions will be discussed later in the paper.

Hayden and Preskill were reasoning about whether it was possible to use black holes to observe a violation of the no-cloning theorem. They assume that an observer, Bob, knows the exact state of a black hole. He then throws a quantum state into the black hole. Hayden and Preskill showed that if he waits for the scrambling time, Bob can use the Hawking radiation the black hole emits to recover the information he threw in. Bob finally jumps into the black hole in an attempt to catch up with the information he earlier threw in. If he can successfully do this, then he would have two copies of the quantum state, and thus would have effectively cloned a quantum state, a violation of the unitary principle of quantum mechanics (although nobody outside the black hole could verify that he has been successful at this).

What Hayden and Preskill showed was that as long as Bob has to wait a time of at least order $M \log M$, he can never catch up with the information thrown into the black hole. The fact that Hayden and Preskill needed about the scrambling time was thus that it was at least order $M \log M$, as this corresponded to the worst case of their analysis, where Bob comes the closest to catching up with the information he threw in.

A paper of Seskino and Susskind [12] followed up on the Hayden and Preskill paper. They gave a definition of scrambling time that seemed to be the minimum needed for the Hayden-Preskill argument to work, and made the conjecture that black holes were indeed fast scramblers, and mixed information up in order $M \log M$ time. Some support for this idea was presented in [13].

¹Natural units are chosen so that $G=c=\hbar=k_B=1$, where G is the gravitational constant, c is the speed of light, \hbar is Planck's constant, and k_B is Boltzmann's constant. All quantities in this paper will be given in natural units.

One argument given in [6, 12] for why the scrambling time should be order $M \log M$ was that if you add mass or electric charge to the black hole, it equalizes in a time of order $M \log M$. Thus, the time for information scrambling should also be $M \log M$. We believe this is a misleading argument. To see why, consider the analogy of putting a drop of dye into a pitcher of water. The water level will equalize in a matter of seconds, while it takes much longer for the dye to diffuse evenly throughout the water—certainly at least several minutes. This is because the process of equalizing the water level is driven by an energy difference, while the diffusion of the dye does not change the energy of the system. Similarly, distributing mass or electric charge uniformly around the black hole decreases its energy, while there is no energy decrease associated with spreading information around the black hole.

Another argument for why the scrambling time should be order $M \log M$ arises from the AdS-CFT correspondence. Shenkar and Stanford [13] compute time-ordered correlation functions in the CFT side of the correspondence, and show that the time scale of the decay of these correlations is order $M \log M$. They conclude that the scrambling time on the CFT side of the correspondence is order $M \log M$, and thus, that the scrambling time in the AdS side of the correspondence should also be order $M \log M$. While we agree that it seems that the out-of-time-order correlations probably do decay with time scale order $M \log M$, we do not see why the timescale for the decay of time-ordered correlation functions should be the same as the scrambling time, especially for the stronger definitions of scrambling time that have been proposed.

In this paper, we try to assume that physics near a black hole behaves as much as possible like established physics. We assume that the black hole dynamics are those seen by an outside observer, and that nothing actually ever reaches the event horizon. We assume that the structure of the space-time outside the black hole is well-approximated by the laws of general relativity. We assume further that the black hole dynamics is unitary; that outside the stretched horizon, all information the black hole is carried by the Hawking radiation; and that the thermodynamics of Hawking radiation is given by the standard formulas for the thermodynamics of thermal radiation. Other than the laws of general relativity, we try to make no assumption on the physics in the stretched horizon, where the energy scales are the Planck energy.

Using a definition of scrambling time stronger than Sekino and Susskind, but comparable to definitions considered in several other papers that discuss the scrambling conjecture, we show that under these assumptions, for black holes to scramble information faster than order M^2 , they must be able to transmit information from one part of the black hole to another faster than the speed of light, a violation of causality in relativity theory.

Let us note that, under the assumptions of relativity, information cannot be transmitted quickly in the stretched event horizon of the black hole. Suppose we constrain the information to move around the black hole at a height h or less above the horizon, i.e., at radius at most 2M+h. The line element for Schwartschild coordinates is

$$ds^{2} = -\left(1 - \frac{2M}{r}\right)dt^{2} + \left(1 - \frac{2M}{r}\right)^{-1}dr^{2} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}).$$

If we set $ds^2 \leq 0$, to specify a lightlike or timelike trajectory, then we see that

$$r^2(d\theta^2 + \sin^2\theta d\phi^2) \le \left(\frac{h}{2M+h}\right) dt^2,$$

showing that in time t, we can move a distance of at most $t\sqrt{h/(2M)}$ radially. The stretched horizon is of order the Planck distance above the event horizon, which corresponds to h=1/M. Thus, to transmit information that stays within the stretched horizon from one side of a black hole to another, relativity says that we need time order M^2 . One of the assumptions we make in this paper is that this bound holds.

2 Scrambling Time

Let us consider two hemispheres of the black hole, which we will call the north and south hemisphere. In a Haar-random pure state, these two hemispheres will have nearly maximal entanglement; that is, since a black hole has order M^2 bits of entropy, they will have order M^2 bits of entanglement. It seems likely that there is some minimum entanglement between these two hemispheres in all low-energy states, but this should be governed by the boundary, and thus be on the order of M bits, much less than the maximal entanglement. Assume that the two hemispheres start out in a random nearly unentangled state. We ask how long it will take the state to evolve to a nearly maximally entangled state. This is our definition of scrambling time.

One might object that it is hard to initialize the black hole in a nearly unentangled state, i.e., a state having much less than M^2 entanglement. To address this objection, let us note that if such states exist, one can ask the question of how long it will take these states to evolve into nearly maximally entangled states; one does not need a plausible physical mechanism for producing these states. We see no reason why these states should not exist.

There are many other possible definitions of scrambling time. One is simply the time scale it takes for out-of-time-order correlations to decay. This is the definition used in [13, 15]. Another is the time it takes to reach a maximally entangled state, starting not with a product state of two nearly equal halves, but with a tensor product of a pure state on one qubit with a Harr-random pure state on the remaining qubits. This last definition is close to Sekino and Susskind's [12]. We do not see any reason why the scrambling time should not be of order $M \log M$ for these definitions. Sekino and Susskind chose this definition so that it would be close to what would is needed to be able to recover the information in the Gedanken experiment of Hayden and Preskill [6]; however, we don't know of any proofs that this definition of scrambling is sufficient to let one recover the information in order $M \log M$ time. The paper [9] defines the scrambling time in a way much closer to the one we use in this paper: the time it takes to evolve from a tensor product of n qubits each in a pure state to a state that is nearly maximally entangled on subsystems of size κn for some constant κ . The paper [18] assumes that the quantum system is in a Haar-random state after the scrambling time, also a stronger criterion than our definition.

3 The Cell Structure

The photon sphere of a Schwarzschild black hole is defined as everything inside the smallest possible circular orbit but inside the horizon. For a Schwarzschild black hole, which has radius 2M, the smallest possible circular orbit has radius 3M. What we do is first divide the photon sphere the black hole into cells, with the property that, as seen from an observer far from the black hole, information can be transmitted from one part of a cell to any other part of the same cell in time order M. We then calculate that the Hawking radiation within each cell only can carry a constant number of bits. Thus, we can model the causality structure of the black hole as a distributed network of processors, where each processor only contains a constant number of bits, and takes time order M (as measured by an observer far from the black hole) to communicate with its neighbors.

We show that this distributed network cannot transmit information from one side of the black hole to the other quickly enough to be a fast scrambler.

In order to give bounds on the flow of information in the photon sphere, we divide the photon sphere into cells. The cells (from the point of view of an accelerating observer hovering within a cell) should be roughly constant diameter in all directions. And from the point of view of an outside observer far from the black hole, each round trip from a face of the cell to the opposite face and back should take time order M.

Recall that the line element of the Schwarzschild coordinates is

$$ds^{2} = -\left(1 - \frac{2M}{r}\right)dt^{2} + \left(1 - \frac{2M}{r}\right)^{-1}dr^{2} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}).$$

Let h be the height above the horizon, i.e. h=r-2M. For an observer who stays at the same r,Ω in Schwarzschild coordinates, we have

$$d\tau = \sqrt{1 - \frac{2M}{r}}dt = \sqrt{\frac{h}{r}}dt.$$

In the photon sphere, $2M \le r \le 3M$, so

$$\sqrt{\frac{h}{r}} \approx \sqrt{\frac{h}{M}}.$$

Thus, the diameter of these cells in Schwarzschild coordinates in the direction parallel to the horizon should be order $M\sqrt{h/M}$. Because radial distance is lengthened by an additional factor of order $\sqrt{h/M}$, the vertical diameter of these cells in Schwarzschild coordinates order is M(h/M). Thus, the cells have diameter order \sqrt{Mh} in the direction parallel to the horizon, and diameter order h in the vertical direction. For a two-dimensional representation of what these cells look like, see Figure 1.

It follows that the number of cells that cross a great circle at radius 2M+h is of order

$$\frac{2M+h}{\sqrt{Mh}} \approx \sqrt{M/h},$$

and the number of cells that cross a line from the photon sphere to a point at 2M+h is order

$$\log(M/h)$$
.

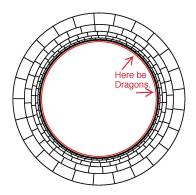


Figure 1: The black hole cell structure in Schwarzschild coordinates, depicted in two dimensions. Note that the aspect ratio of the cells appears larger as you approach the horizon. This is an artifact of the Schwarzschild coordinates; while the cells do grow smaller as you approach the horizon, an observer hovering near the horizon would see these cells as having an aspect ratio of near unity.

Let us now calculate the entropy, i.e., the number of bits contained in the Hawking radiation in each cell. The entropy of black-body radiation in a cell of volume V is

$$S = \frac{4\pi^2}{45} VT^3,$$

and so is proportional to VT^3 , where T is the temperature. The side length of each of these cells, as seen from a stationary observer, is of order \sqrt{Mh} , so the volume is of order $M^{3/2}h^{3/2}$.

What is the temperature? The temperature seen by a near-horizon observer (which still holds within a constant factor for h < M) is

$$T \approx \frac{1}{4\pi\sqrt{2Mh}}$$

Thus, the number of bits in each cell is proportional to

$$VT^3 = O(1)$$

and so approximately constant. This radiation must consist of virtual particles, since it does not contribute to the mass of the black hole. However, virtual particles can have observable effects, and thus presumably can carry information.

Conventional wisdom says that the continuous nature of space and time breaks down at the Planck scale. The value of h which gives Planck-scale cells is order 1/M. If we stop the process at h=1/M, there are order M^2 cells, a constant fraction of them just above the horizon of the black hole. The exact constant factor on the M^2 depends on exactly where we stop dividing the cells.

Black hole thermodynamics predicts that the entropy of a black hole is A/4, where A is the surface area. So if we assume that each cell contains a constant number of bits

of Hawking radiation, then for an outside observer, the entropy encoded in the Hawking radiation is sufficient to account for the black hole's entropy. This explanation for the entropy has been previously proposed [7, 17]. Is the entropy really encoded in the Hawking radiation this way? While we may have to wait until there is a microscopic theory of quantum gravity to answer this question definitively, it seems consistent with our current knowledge of physics.

Recall Landauer's precept "information is physical." In order for the black hole's quantum state to be scrambled, information must be carried from one part of the black hole to another. And if we accept Landauer's precept, it must be carried by some physical process. Given our assumption that outside the stretched horizon, the physics agreed more or less with known physics, it appears that outside the stretched horizon, the only possible carrier of information is the Hawking radiation. This assumption does permit that some information could be stored by unknown physics within the stretched horizon, but relativity does not allow information within the stretched horizon to be transmitted quickly enough for the scrambling time to be order $M \log M$.

4 Transmitting Information in the Photon Sphere

We now switch to a more computer-science mode of reasoning. Suppose we have a network of these cells, where each cell contains order 1 bits, and we wish to send order M^2 bits from one side of the black hole to the other. We divide time into time steps of size M, as measured by an outside observer. Each cell can communicate with its neighbor in one time step.

More specifically, suppose we have two hemispheres of the black hole, one near the North pole and one near the South pole, each of which is nearly pure. Each of these hemispheres contains half of the black hole's suface, so each contains order M^2 bits. To obtain a maximally entangled state, we need to send order M^2 information from one of these regions to the other.

Let us consider what paths we could send this information along. The quickest path (as measured by an outside observer) between two points in the stretched horizon is the shortest null geodesic between them (see Figure 2). There is a path that might be easier to think about that is not much longer: go straight up at the speed of light until you reach a level where the two points are covered by adjacent cells, travel at this height until you are directly above the second point, and then go back down. If the two points in the stretched horizon are separated by an angle θ , then this second path will take time order $\log(M\theta)$, and this will thus apply to the null geodesic as well.

So one possibility is that we send the information up to the outer region of the photon sphere, send it along the outside of this sphere, and then send it back down to near the horizon once it reaches the other side. The length of these paths is order $\log M$, which is fast enough for the scrambling time to be order $M \log M$. However, there are only a constant number of cells on the outside of the photon sphere. These cells form a bottleneck, since each of these paths must go through one of these cells. Each of these cells on the outer boundary can only contain a constant number of qubits per time step, so if we are going to send M^2 qubits along paths involving these cells, we need to take order M^2 time steps, resulting in total time order M^3 .

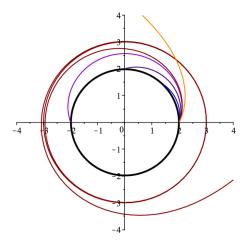


Figure 2: Trajectories of photons from a point on the horizon of a black hole. The red trajectory makes a complete orbit at R=3M before it escapes to infinity.

Another possibility is that we send the information while keeping it close the horizon of the black hole. There are order M disjoint paths from one of these regions to the other along the horizon, but each of these paths has length order M. It would thus take order M time steps to send all the information along these paths, making order M^2 time altogether. In fact, we will show that this is the best we can do.

Recall that we are assuming that any non-standard physics is confined to the stretched horizon. Thus, any information that is outside the stretched horizon must be carried by Hawking radiation. (Presumably, gravitons could also carry information from one region of the black hole to another, but since gravitons are massless particles, their thermodynamics should be similar to the thermodynamics of Hawking radiation.)

Suppose we cut the black hole in half by a plane through its center. The intersection of the cell structure with this plane will look something like that in Figure 1a. There are order M cells in this cut. We can see this by observing that the inner layer has order M cells, and the number of cells in each layer forms a geometric series. If we assume that each cell can only send a constant number of bits to its neighbors during each time step, then each cell can process a constant number of bits per time step. Since the time steps each take time order M, and we need order M of them, it will take order M^2 time to pass order M^2 bits from one side of this cut to the other. The scrambling time must be at least this large. Thus, with our assumptions, a lower bound for the scrambling time is order M^2 . Note that the same argument shows that to get Q bits of quantum information from one hemisphere of the black hole to the other, we need time order QM.

5 Hayden and Preskill Revisited

We have seen that to get Q bits of quantum information from one hemisphere of the black hole to the other, we need to take time order QM. Recall that in Hayden and Preskill's paper, Alice tosses her diary into the black hole. Let us assume her diary weighs 0.02 mg—one Planck mass (rather a small mass for a diary, but we assume that Alice writes small). How many bits does the hole receive? Ignoring the gravitational potential energy contained in the diary, if we add 1 Planck mass to the hole, then the mass of the black hole increases from M to M+1, and the number of bits goes from M^2 to M^2+2M+1 . We have thus increased the number of bits in the black hole by 2M. We would need to wait order M^2 time for all these bits to be spread evenly through the black hole, and possibly this is the minimum amount of time it takes for these bits to come out in the Hawking radiation.

Suppose we take one photon of light and send it into the hole, and encode a qubit in it by arranging for it to be in some specific polarization. This doesn't improve things much. The photon has energy α in Planck units. Thus, when we add it to the black hole, the information content of the black hole increases by $2\alpha M$ bits, and it takes order αM^2 time for the black hole to scramble, which is still proportional to M^2 , even if α is very small (around 10^{-25} for visible light). We do not seem to be saved by the fact that Bob knows everything about the photon except the polarization, because we might need to wait for all the extra information we've added to be evenly distributed through it, and not just the unknown polarization. It is possible that the one unknown qubit of polarization scrambles quickly despite the fact that we've added a lot of known information, but this would require some justification.

To add one just bit to the black hole, we do not see any better way of doing it than sending a photon of energy 1/(2M), which will have wavelength comparable to the radius of the black hole. The fact that we can recover a photon of wavelength comparable to the radius in time order $M\log M$ seems much less remarkable than recovering the polarization of a visible light photon; it seems as though a photon of radius 2M may essentially already be spread throughout the black hole as soon as it enters it.

6 Discussion and Speculations

The arguments in this paper appear to indicate that at least one of the commonly held beliefs about black holes in the following list is incorrect:

- 1. Black hole evolution is unitary.
- 2. The causal structure in the neighborhood of a black hole is that predicted by general relativity.
- 3. The scrambling time of a black hole is order $M \log M$.
- 4. Outside of the stretched horizon, any information in a black hole is contained in the Hawking radiation. This includes the information leaving the black hole, and the information that scrambles it.

5. The amount of information that Hawking radiation contains is the amount predicted by thermodynamics.

It is not clear whether this argument can be extended to narrow down the above list of assumptions which might possibly be incorrect.

One question that could be asked is whether we can learn any more about black holes by considering the cell structure given by this paper. We believe that one thing the cell structure seems to indicate is that we should not think of black hole dynamics as taking place solely on the horizon, as in the membrane picture [16]. In the cell structure, there are order $\log M$ layers in the photon sphere, and the outer layers seem to play a crucial role, even though they do not contain very many quantum bits. Without these layers it would be difficult or impossible for an unequal distribution of charge on the surface of a black hole to equalize quickly without a violation of relativity, as the information about the amount of charge (or mass) added to the black hole could not propagate from one side of the hole to the other in time $M \log M$. With the cell structure, it is possible to equalize the charge (or mass), as we do not need to get much information from one side to the other; the charge and mass are scalar quantities, so to communicate how much charge there is, one needs only to communicate order $\log M$ bits, which is possible in order $M \log M$ time using the cell structure.

Similarly, if one believes that all the information is sitting on the horizon, then in order for the out-of-time order correlations to decay in order $M \log M$ time, either one has to accept non-local causality, or one has to realize that information can be carried on short paths, through the outer layers of the photon sphere (in our case, by virtual photons of Hawking radiation).

Further, we believe that even near the horizon, something can be learned about black holes by imagining the dynamics in three dimensions. Suppose we try to extend the cell structure inwards beyond the Planck scale. What happens? The cells at a Planck distance from the horizon are (as seen from an outside observer) at Planck temperature. If we consider possible layers closer to the horizon, a naive application of the formula would say that the temperature should increase. But what happens when you try to add energy to a system at Planck temperature? Paradoxically, the temperature decreases. A system at Planck temperature contains lots of Planck-size black holes, which are constantly forming and evaporating (or nearly forming and then evaporating, if you assume that black holes can never actually form in finite time). If you add energy to these, you obtain larger-than-Planck-size black holes, which have lower temperature. These will start absorbing mass, and increasing in size, until the ambient temperature of the space outside them is the same as the temperature of a black hole. This means that in a static universe of constant volume at thermal equilibrium², there is at most one black hole, and the amount of mass contained in it is a constant fraction of the mass of the universe. This is because the black hole will absorb radiation until the ambient temperature outside of the black hole decreases faster than the temperature of the black hole. This can only happen if the black hole contains much of the mass of the universe. (If the volume of the universe is too large compared to the mass it contains, the black hole will evaporate completely.)

²It is not clear that such a thing is allowed by the laws of physics, as general relativity may only allow static universes with the aid of a cosmological constant, and these may be unstable

Thus, below the Planck-scale layer, we expect to find large black holes. And indeed, it seems quite likely that below the layer of Planck-scale black holes just above the horizon, we indeed find the surface of a single black hole—namely, the actual black hole.

For an eternal black hole which stays at the same mass, the black hole horizon should be in thermodynamical equilibrium. Thus, if it is constantly absorbing Planck-scale near-black-holes, one expects it to be constantly emitting them as well. (Of course, these Planck-scale black holes may never completely form from an outside observer's point of view.) One might thus expect space-time at the stretched horizon, as inferred by an outside observer, to be very irregular. Confirmation of this and more details of this phenomenon may require a theory of quantum gravity.

We have identified one way in which information might be communicated from one side of the black hole to the order in time less than order M^2 . However, this is a fairly far-fetched speculation that we believe is ruled out by several considerations. If the Planck-scale black holes in the atmosphere are not just black holes, but also wormholes, then in a mature black hole, we might expect to find wormholes connecting one side of the black hole to the other. Information falling on one side could then be propagated to the other side quickly. There are numerous problems with this proposal. We believe it is very unlikely these wormholes could last long enough to travel very far from where they are formed before they evaporate. Further, there is a large gradient in the time dilation constant near the horizon; unless there is a mechanism for preventing it [8], space-like separated wormhole mouths would turn into time-like separated wormhole mouths, and information falling into them might be unavailable for long periods of time, or might even emerge before it fell in. This would give rise to causality violation, and it is difficult to construct a consistent theory of physics with causality violation. Finally, unless these wormholes last long enough that each of them can transmit much more than a constant number of bits, arguments similar to those in our paper show that we these wormholes cannot be moved around the stretched horizon quickly enough to enable fast scrambling.

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