Automatic Dynamic Parallelotope Bundles for Reachability of Nonlinear Dynamical Systems

Edward Kim

A thesis submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Master of Science in the Department of Computer Science in the College of Arts and Science.

Chapel Hill 2022

Approved by:

Committee Member 1

Committee Member 2

Committee Member 3

Committee Member 4

Committee Member 5

Committee Member 6

©2022 Edward im ALL RIGHTS RESERVED

ABSTRACT

Edward Kim: Automatic Dynamic Parallelotope Bundles for Reachability of Nonlinear Dynamical Systems

(Under the direction of Parasara Sridhar Duggirala)

Reachable set computation is an important technique for the verification of safety properties of dynamical systems. In this thesis, we investigate reachable set computation for discrete nonlinear systems based on parallelotope bundles. The crux of the reachability algorithm relies on computing an upper and lower bound on the supremum and infimum respectively of a nonlinear function over a rectangular domain. We cover two ways of computing these bounds: one method utilizing Bernstein polynomials and the other relying on a non-linear optimization tool developed by NASA, Kodiak. We aim to improve the traditional parallelotope-based reachability method by removing the manual step of parallelotope template selection in order to make the procedure fully automatic. Furthermore, we show that adding templates dynamically during computations can improve accuracy. To this end, we investigate two techniques for generating the template directions. The first technique approximates the dynamics as a linear transformation and generates templates using this linear transformation. The second technique uses Principal Component Analysis (PCA) of sample trajectories for generating templates. We have implemented our approach in a Python-based tool called Kaa. The tool is modular and use two types of global optimization solvers, the first using Bernstein polynomials and the second using the aforementioned Kodiak library. Additionally, we leverage the natural parallelism of the reachability algorithm and parallelize the Kaa implementation. Finally, we demonstrate the improved accuracy of our approach on several standard nonlinear benchmark systems, including a high-dimensional COVID19 model proposed by the Indian Supermodel Committee.

ACKNOWLEDGEMENTS

I would like to thank the UNC Computer Science community for supporting my work and providing me a warm community for pursuing this research. In particular, I would like to thank my advisor, Parasara Sridhar Duggirala, for the opporunity to perform research and participate in his group. Bineet Ghosh, Manish Goyal, Abel Karimi, and Meghan Stuart were all great colleagues and I enjoyed my time exchanging ideas, proofreading drafts, and socializing over group lunches. I would also like to thank Stanley Bak for helping me improve the implementation of Kaa and providing valuable feedback during this research. Juan Garcia made a wonderful collegue and an equally wonderful friend. The nights we raided Peabody and Phillips hall was certainly a memorable time. Finally, I would like to thank my parents and my family for their ardent, un-ending support.

TABLE OF CONTENTS

LIST OF TABLES	vi
LIST OF FIGURES	vii
1 Introduction	1
1.1 A Very Long Section Title that Should be Split Across Two Lines in the Table of Contents	2
BIBLIOGRAPHY	3

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

Introduction

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer

1.1 A Very Long Section Title that Should be Split Across Two Lines in the Table of Contents

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Nam liber tempor cum soluta nobis eleifend option congue nihil imperdiet doming id quod mazim placerat facer

BIBLIOGRAPHY