Automatic Dynamic Parallelotope Bundles for Reachability of Nonlinear Dynamical Systems

Edward Kim

A thesis submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Master of Science in the Department of Computer Science in the College of Arts and Science.

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Approved by:

Committee Member 1

Committee Member 2

Committee Member 3

Committee Member 4

Committee Member 5

Committee Member 6

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ABSTRACT

Edward Kim: Automatic Dynamic Parallelotope Bundles for Reachability of Nonlinear Dynamical Systems
(Under the direction of Parasara Sridhar Duggirala)

Reachable set computation is an important technique for the verification of safety properties of dynamical systems. In this thesis, we investigate reachable set computation for discrete nonlinear systems based on parallelotope bundles. The crux of the reachability algorithm relies on computing an upper and lower bound on the supremum and infimum respectively of a nonlinear function over a rectangular domain. We cover two ways of computing these bounds: one method utilizing Bernstein polynomials and the other relying on a non-linear optimization tool developed by NASA, Kodiak. We aim to improve the traditional parallelotope-based reachability method by removing the manual step of parallelotope template selection in order to make the procedure fully automatic. Furthermore, we show that adding templates dynamically during computations can improve accuracy. To this end, we investigate two techniques for generating the template directions. The first technique approximates the dynamics as a linear transformation and generates templates using this linear transformation. The second technique uses Principal Component Analysis (PCA) of sample trajectories for generating templates. We have implemented our approach in a Python-based tool called Kaa. The tool is modular and use two types of global optimization solvers, the first using Bernstein polynomials and the second using the aforementioned Kodiak library. Additionally, we leverage the natural parallelism of the reachability algorithm and parallelize the Kaa implementation. Finally, we demonstrate the improved accuracy of our approach on several standard nonlinear benchmark systems, including a high-dimensional COVID19 model proposed by the Indian Supermodel Committee.

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CHAPTER 1

Introduction

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