

1 Significance Levels

When conducting hypothesis tests, significance levels are used to determine the probability of obtaining a test statistic as extreme or more extreme than the observed value, assuming the null hypothesis is true. Common significance levels include:

- $\alpha = 0.05$: The most commonly used significance level in scientific research.
- $\alpha = 0.01$: Used in cases where a higher level of confidence is desired.
- $\alpha = 0.001$: Used in cases where an even higher level of confidence is desired.

2 Inclusion of Systematic Uncertainties

Systematic uncertainties can arise from a variety of sources, including calibration errors, background estimation, and detector response. These uncertainties can be accounted for using different approaches:

2.1 Bayesian-Inspired Approach

A Bayesian-inspired approach involves treating the systematic uncertainties as nuisance parameters and marginalizing over them in the likelihood function. This approach can be computationally intensive but provides a full probability distribution for the parameter of interest.

2.2 Profile Likelihood Approach

The profile likelihood approach involves constructing a likelihood function that is marginalized over the nuisance parameters by maximizing the likelihood with respect to them. This approach can be computationally simpler than the Bayesian-inspired approach but does not provide a full probability distribution.

3 Combining Tests

When multiple tests are conducted, they can be combined using different methods:

3.1 Simple Combination

In a simple combination, the p-values from each test are combined using Fisher's method. This method assumes that the tests are independent and that the null hypothesis is true for each test.

3.2 Weighted Combination

In a weighted combination, the p-values from each test are combined using a weighted sum, where the weights are determined by the relative importance or reliability of each test.

4 Look-Elsewhere Effect

The look-elsewhere effect refers to the increased probability of observing a significant result by chance when multiple hypotheses are tested. This effect can be accounted for using methods such as the Bonferroni correction or the False Discovery Rate (FDR) method.

5 Inversion of Hypothesis Tests

Inversion of hypothesis tests involves testing the hypothesis that the observed data is consistent with a specified parameter value, rather than testing the hypothesis that the parameter value is equal to a specified value. This approach can provide a more informative and nuanced understanding of the data.

6 Bayesian Approach to Hypothesis Testing

The Bayesian approach to hypothesis testing involves assigning prior probabilities to the hypotheses and updating these probabilities based on the observed data using Bayes' theorem. This approach can provide a more intuitive and flexible approach to hypothesis testing, but requires specification of prior probabilities and can be computationally intensive.