Simulation laboratory 3: Statistical analysis and bootstrapping

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4 March 2025

Overview

Objective:

- Understand the results of a discrete events simulation.
- Analyze the maximum queue length of a single road.

Implementation:

1. Calculation of mean and variance recursively
2. Stopping criteria for number of scenarios
3. Calculation of other statistics of distribution of maximum
4. Reliability of statistic: mean square error of simulation metrics via **bootstrap**

Statistical analysis

Jupyter notebook:

1. Implement your solution in the notebook **statistical_analysis_and_bootstrapping.ipynb**.
2. Import your solution from the previous lab.

TO DO:

1. Implement the function **moving_mean_var** for the recursive calculation of sample mean and variance.
2. Define a stopping criterion. Empirical consideration: choose a precision resulting in at least 100 simulation runs.
3. Plot sample mean and variance over the simulation runs and analyze the data.

Bootstrap mean square error

Calculate other statistics θ, e.g.:

- **95 percentile** of the maximum queue length

Calculate reliability of calculated statistics

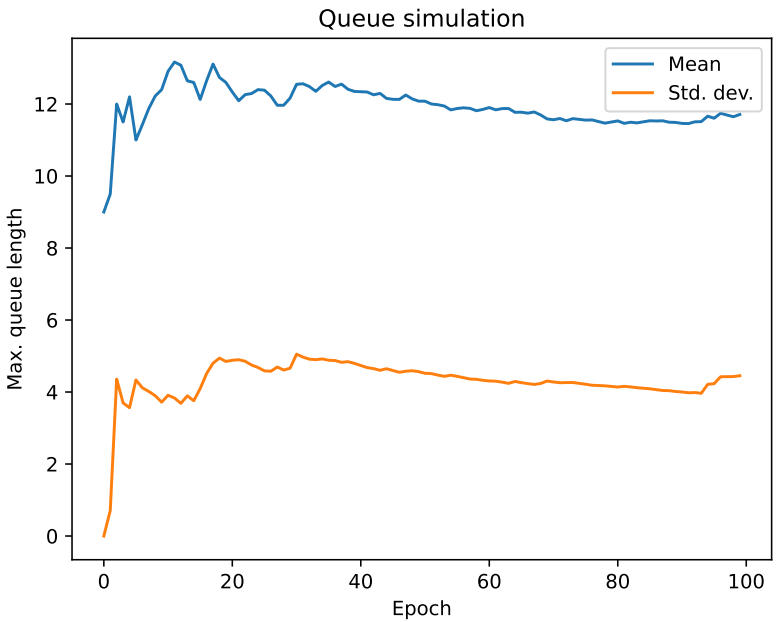
- Want to calculate MSE.
- **Problem**: do not know anything – distribution of estimated statistics, true value of statistic.
- **Solution**: bootstrap – approximate everything by empirical distribution

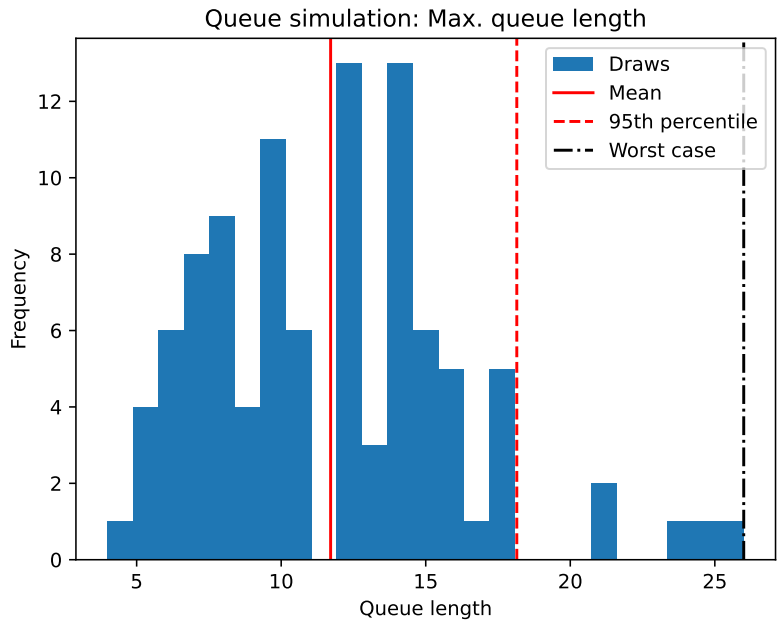
Bootstrap mean square error

TO DO:

- Implement the function **bootstrap** to calculate the bootstrap MSE of statistics of the maximum queue length distribution.

Statistical indexes

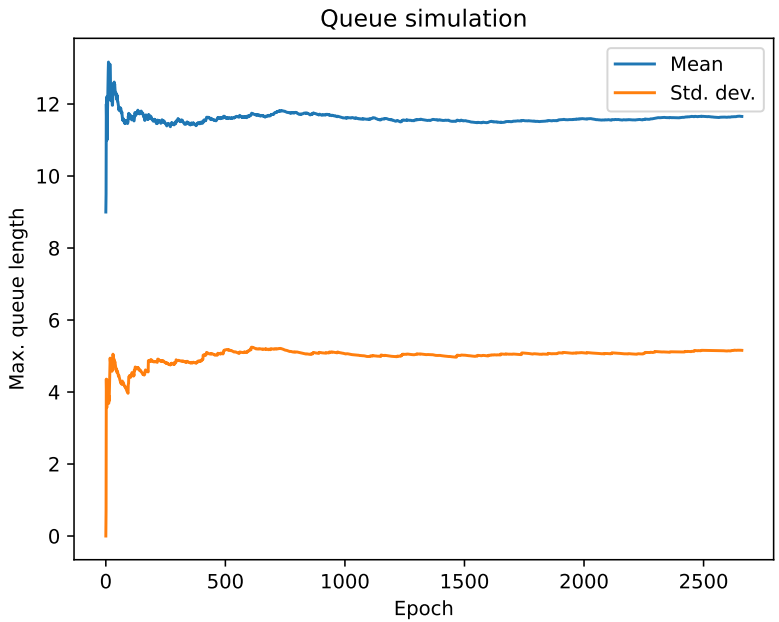


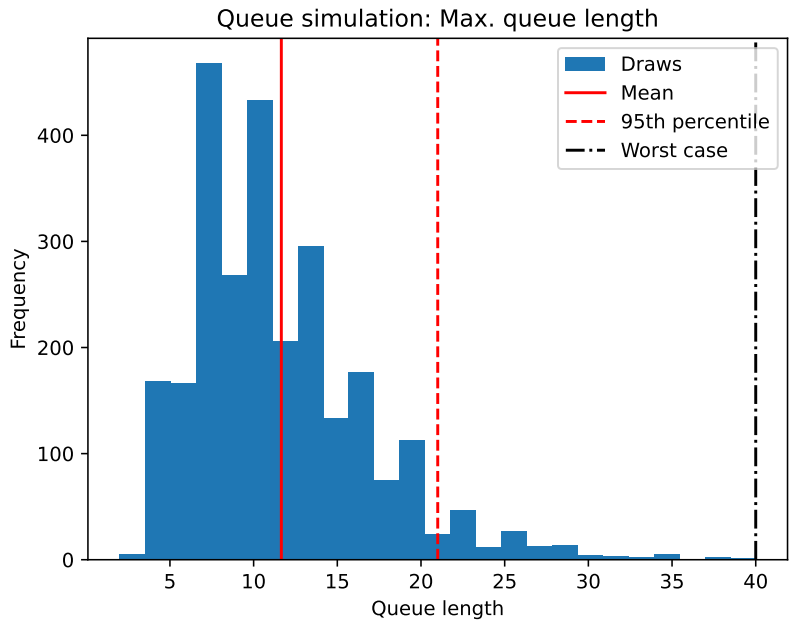


Statistics of maximum queue length (stops when $\sigma/\sqrt{n} < 0.5$):

- Mean = 11.7 (MSE = 0.196, BootstrapMSE = 0.201)
- 95 percentile = 18.2 (BootstrapMSE = 6.17)
- Worst = 26.0 (BootstrapMSE = 2.07)

Statistical indexes





Statistics of maximum queue length (stops when $\sigma/\sqrt{n} < 0.1$):

- Mean = 11.7 (MSE = 0.00999, BootstrapMSE = 0.00966)
- 95 percentile = 21.0 (BootstrapMSE = 0.455)
- Worst = 40.0 (BootstrapMSE = 5.41)