Simulation laboratory 2: Discrete events simulation

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Goals

Discrete events simulation:

- Understand how to simulate events
- Apply the Poisson process
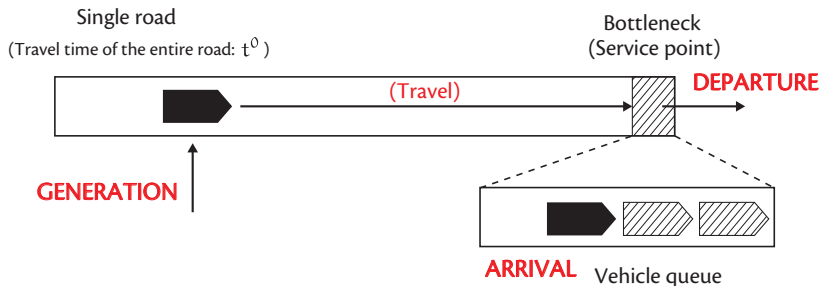- Simulate a time-varying queue

Implementation:

- Simulate vehicle queue on a single-lane road with a bottleneck

Problem definition: Vehicle queue on a single road

Single road
(Travel time of the entire road: t^0)

Bottleneck
(Service point)

(Travel)

DEPARTURE

GENERATION

ARRIVAL Vehicle queue

Events:

1. Vehicle **GENERATION**
2. Vehicle **ARRIVAL** at the queue
3. Vehicle **DEPARTURE** from the queue

Vehicle **GENERATION**

Single road
(Travel time of the entire road: t^0)

Bottleneck
(Service point)

**GENERATION**

Vehicle queue
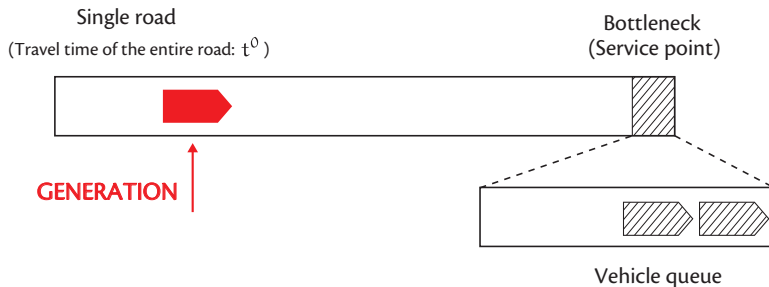
- New vehicle enters at time t^g, following a Poisson process with rate λ.
- Create **GENERATION** event at time $t_{i+1}^g = t_i^g + \text{Exp}(\lambda)$.
- Vehicle entry point is uniformly distributed over the road (imagine vehicles pulling out of their driveways).

Vehicle **ARRIVAL** at queue

Single road
(Travel time of the entire road: t^0)

Bottleneck
(Service point)

(Travel)

ARRIVAL Vehicle queue
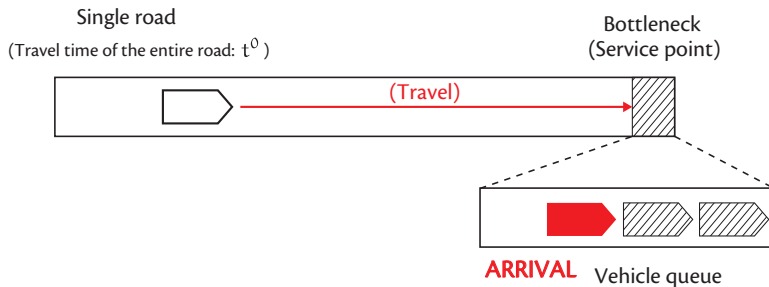
- Travel time on the entire road is t^0, thus vehicle arrives downstream of the queue $tt \sim U(0, t^0)$ after entering (see the previous slide).
- Create **ARRIVAL** event at time $t_i^a = t_i^g + tt$.
- Increase queue by 1, i.e., $q := q + 1$

Vehicle **DEPARTURE** from queue

Single road

(Travel time of the entire road: t^0)

Bottleneck
(Service point)

DEPARTURE
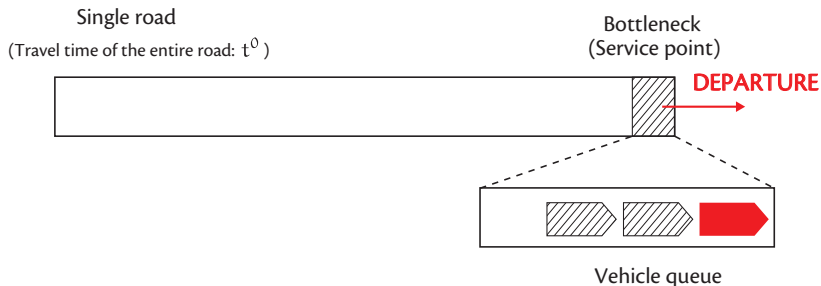
Vehicle queue

- Service time for a vehicle at bottleneck is $t^s \sim \text{Exp}(\mu)$.
- Create **DEPARTURE** event at time $t^d_{i+1} = t^d_i + t^s$.
- Reduce queue by 1, i.e., $q := q - 1$
- Attention at special cases, e.g., only one vehicle in the queue.

Simulation

State variables:

- t: Time
- q: Number of vehicle in the queue

Parameters (scenario):

- T: The duration of vehicle generation
- t^0: Travel time of the entire road
- λ: Rate for vehicle generation
- μ: Rate for service time (road capacity)

Simulation

Events:

- List of future events sorted in chronological order.
- Initialization of the simulation: first event (Generation).
- Process the next event:
    1. Update the variables.
    2. Collect statistics.
    3. Generate and add new events to the list.
    4. Remove the processed event from the list.
    5. Finish the simulation if the list is empty, go to next event otherwise.

Event triggers event

Event	Triggered event	Queue
Sim. Start	Generation, Sim. End	
Generation	Generation (if $t < T$), Arrival	
Arrival	Departure (if $q = 1$)	$q = q + 1$
Departure	Departure (if $q > 0$)	$q = q - 1$
Sim. End		

1 Queue simulation

2 Exercise

3 My results

Exercise

Jupyter notebooks:

1. **classes_examples.ipynb** illustrates object-oriented programming features of Python.
2. **discrete_events_simulation_template.ipynb** for implementation of queue simulation.

To do:

- Implement the simulation procedure in function "simulate".
- Collect and analyse simulation statistics.

Hints:

- Use classes to represent different types of events.
- Use the provided Scenario class.

Queue length over time - random runs