Problem Set 3

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Efficient Algorithms

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Problem 3.1. Stack and Queue

1) How do you implement a queue with two stacks? Demonstrate your idea with pseudo-code.

2) Analyze the time complexity of the enqueue and dequeue operation of your two-stack queue.

Problem 3.2. Binary Heap

1) Verify whether the array $[15, 13, 9, 5, 12, 18, 7, 4, 0, 16, 2, 1]$ is a max heap. If not, show how to convert it to a max heap with $BuildMaxHeap$.

2) Compute the minimum number of elements of a heap of height h.

Hint: Use the fact that the maximum of elements of a heap of height h is $2^{h+1} - 1$. (We proved this in class.)

3) Show that a binary heap with n elements has height $\lfloor \log_2 n \rfloor$.

4) Show that the node with index $\lfloor \frac{n}{2} \rfloor$ is not a leaf in a binary heap tree with n elements.

Hint: Split your consideration into two cases: (Case 1) n is even and (Case 2) n is odd.

Reminder: In class, we proved that the elements in the subarray $A[\lfloor \frac{n}{2} \rfloor + 1, ..., n]$ are all leaves.

5) Based on your work in 4), compute the number of leaves in an n-element heap.

Problem 3.3. Priority Queue

1) Illustrate the operation of $Insert(A, 10)$ on the max heap $[100, 19, 36, 17, 3, 25, 1, 2, 7]$.

2) Show how to implement a queue using a min priority queue.