Problem Set 8

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Efficient Algorithms

Problem 1. BFS and DFS

Consider the following undirected graph G.

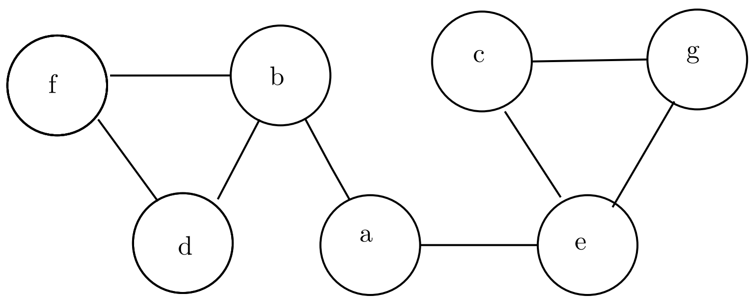

Figure 1: Undirected Graph $G = (V, E)$

(1) Use BFS to traverse the graph G in Figure 1 in the alphabetical order starting from vertex a. Construct a BFS search tree and identify the frontier set at each level.

(2) Use DFS to traverse the graph G in Figure 1 in the alphabetical order starting from vertex a. Identify the type of each edge of G.

Problem 2. Applications of BFS and DFS

(1) How can you detect a cycle in an **undirected** graph with BFS?

(2) Given an **unweighted** graph $G = (V, E)$ and a designated root vertex r, explain how to compute a shortest path from r to every other nodes in V in linear time in the number of vertices $|V|$ and edges $|E|$. Assume that a shortest path from a vertex i to a vertex j in an **unweighted** graph is defined as a path with the minimum number of edges.

(3) Based on your idea in (2), compute a shortest path from a to every other vertex $v \in V$ of the graph in Figure 1.

Problem 3. Dijkstra's Algorithm

(1) Solve the following shortest path problem, with vertex a as the source.

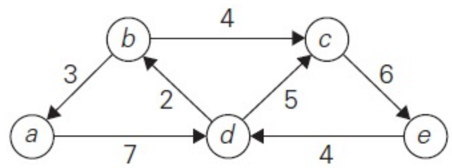

Figure 2: Solve the single-source shortest path with a as the source.

(2) How do you apply Dijkstra's algorithm on a **directed** graph $G = (V, E, w)$ to find a shortest path from every vertex $v \in V$ to a given destination vertex $t \in V$ in the G?

(3) Apply the algorithm you proposed in (2) to the graph in Figure 2.

(4) Analyze the running time of your algorithm in (2) in terms of $|V|$ and $|E|$.

(5) How do you apply Dijkstra's algorithm on a **undirected** graph $G = (V, E, w)$ to find a shortest path from every vertex $v \in V$ to a given destination vertex $t \in V$ in the G?

(6) Analyze the running time of your algorithm in (5) in terms of $|V|$ and $|E|$.

Problem 4. Bellman-Ford

(1) Apply the Bellman-Ford algorithm to the graph in Figure 2 with vertex a as the source vertex. Show your work in each pass. Determine whether $d[v] = \delta(s, v)$ before $|V| - 1$ passes for all $v \in V$.

(2) Given a directed graph $G = (V, E)$, suppose that G contains negative-weight cycles. Modify the Bellman-Ford algorithm so that it sets $v.d = -\infty$ for all vertices v for which there is a **negative-weight cycle** on some path from the source s to v.

(3) How do you detect if a directed graph $G = (V, E)$ has a **negative-weight cycle** using the Bellman-Ford algorithm? Analyze the running time of your solution in terms of $|V|$ and E.

Problem 5. Floyd-Warshall and Johnson's algorithm

$$\begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

Figure 3: Apply the Floyd-Warshall algorithm to the following weight matrix.

(1) Apply the Floyd-Warshall algorithm to the graph represented by the weight matrix in Figure 3 to find shortest paths amoung all pairs of vertices $u, v \in V$. Give the matrix $D^{(k)}$ in each step k.

(2) Apply the Floyd-Warshall algorithm to the graph represented by the weight matrix in Figure 3 to find the transitive closure of G. Give the matrix $T^{(k)}$ in each step k.

(3) How can you detect the presence of negative-weight cycles in a directed graph using the output matrix of the Floyd-Warshall algorithm?

(4) Apply Johnson's algorithm to the graph in Figure 4. Show the values of h and $\hat{w}$.

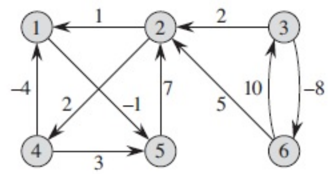

Figure 4: Apply Johnson's algorithm to the graph.