Disproving Asymptotic Claims

Think of this as a game between you and me.

Me: I'll give you c and n_0.

You: OK, I'll find (at least) one value for n that works with c an n_0 from you.

1) $$n \notin O(\sqrt{n})$$

<u>Sol^n:</u> Pick arbitrary c and n_0 for which we can find n that works:

$n > c\sqrt{n}$ and $n \geq n_0$

$n^2 > c^2 n$ and $n \geq n_0$

$n^2 - c^2 n > 0$ and $n \geq n_0$

$n(n-c^2) > 0$ and $n \geq n_0$

We take only non-negative values for n,

$n > c^2$ and $n \geq n_0$

$n \geq \lceil c^2 \rceil + 1$ and $n \geq n_0$

Pick $n = \max(\lceil c^2 \rceil + 1, n_0)$ $\blacksquare$

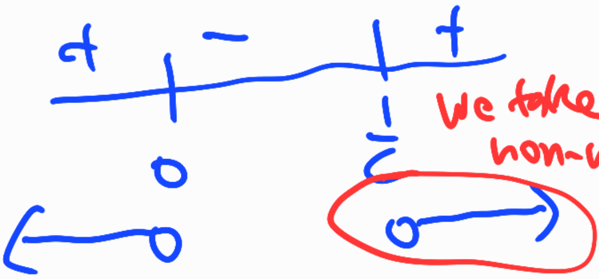
2) $n^2 \notin \Omega(n^3)$

Sol^n: Pick arbitrary c and n_0 for which we can find n that works:

$$n^2 < c \cdot n^3 \quad \text{and} \quad n \geq n_0$$

$$cn^3 - n^2 > 0 \quad \text{and} \quad n \geq n_0$$

$$n^2(cn-1) > 0 \quad \text{and} \quad n \geq n_0$$



We take only non-negative values for n

$$n > \frac{1}{c} \quad \text{and} \quad n \geq n_0$$

$$n \geq \left\lceil \frac{1}{c} \right\rceil + 1 \quad \text{and} \quad n \geq n_0$$

Pick $n = \max\left(\left\lceil \frac{1}{c} \right\rceil + 1, n_0 \right)$ $\blacksquare$

3) $n^2+1 \notin O(2n)$

<u>Sol</u>

Sol^n: Pick arbitrary c and n_0 for which we can find n that works:

$n^2+1 > c \cdot 2n$ and $n \geq n_0$

$n^2-2cn+1 > 0$ "

$(n-c)^2 - c^2+1 > 0$ "

$(n-c)^2 > c^2-1$ "

Case $c^2-1 \geq 0 \rightarrow c \in (-\infty,-1] \cup [1,\infty)$

$n-c > \sqrt{c^2-1}$

$n > c+\sqrt{c^2-1}$ "

$n \geq \lceil c+\sqrt{c^2-1} \rceil + 1$ and $n \geq n_0$

Pick $n = \max(\lceil c+\sqrt{c^2-1} \rceil + 1, n_0)$ $\blacksquare$

Case $c^2-1 < 0 \rightarrow c \in (-1,1)$

Since $(n-c)^2 \geq 0$,

$n \in (-\infty, \infty)$ $n \geq n_0$

Pick $n = n_0$. $\blacksquare$

4) $n \notin O(1)$

Soln: Pick arbitrary c and n_0 for which we can find n that works:

$$n > c \cdot 1 = c \quad \text{and} \quad n \geq n_0$$
$$n \geq \lceil c \rceil + 1 \quad \text{and} \quad n \geq n_0$$

Pick $n = \max(\lceil c \rceil + 1, n_0)$ ■

5) $n \notin \Omega(n \log n)$

Soln: Pick arbitrary c and n_0 for which we can find n that works:

$$n < c \cdot n \log n \quad \text{and} \quad n \geq n_0$$
$$1 < c \log n \quad \text{and} \quad n \geq n_0$$
$$\frac{1}{c} < \log n \quad \text{and} \quad n \geq n_0$$
$$\log 2^{\frac{1}{c}} < \log n \quad \text{and} \quad n \geq n_0$$
$$n > 2^{\frac{1}{c}} \quad \text{and} \quad n \geq n_0$$
$$n \geq \lceil 2^{\frac{1}{c}} \rceil + 1 \quad \text{and} \quad n \geq n_0$$

Pick: $n = \max\left(\lceil 2^{\frac{1}{\varepsilon}} \rceil + 1, n_0\right)$ ■