Programming Lab 2: The Pthreads Library

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Course: Parallel Computing

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Problem 2.1. The Trapezoidal Rule

The definite integral from a to b of a non-negative function $f(x)$ can be though of as the area bounded by the x-axis, the vertical lines $x = a$ and $x = b$, and the graph of the function $f(x)$. See Figure 1.

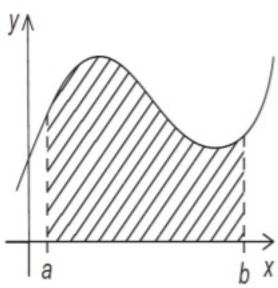

Figure 1: Definite Integral of a non-negative function

One approach to estimating this area of integral is to partition the region into **regular geometric shapes** and then add the areas of these shapes. In the trapezoidal rule, the regular geometric shapes are **trapezoids**;each trapezoid has its base on the x-axis, vertical sides and its top edge joining two points on the graph of $f(x)$. See Figure 2.

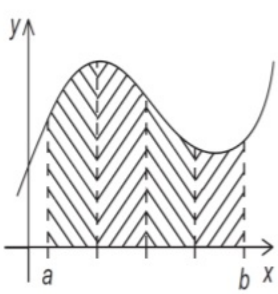

Figure 2: Trapezoids approximating definite integral

For our purposes, we will choose all the bases to have the same width. So if there are n trapezoids, the base of each will be $h = \frac{(b-1)}{n}$. The base of the leftmost trapezoid will be the interval $[a, a + h]$;the base of the next trapezoid will be $[a + h, a + 2h]$;the next, $[a + 2h, a + 3h]$; etc. In general, the base of the i^{th}

trapezoid will be $[a+ih, a+(i+1)h]$, $i = 0, ..., n-1$. In order to simplify notation, let x_i denote $a + ih$, $i = 0, ..., n-1$. Then, the length of the left side of the i^{th} trapezoid will be $f(x_i$, and its right side will be $f(x_{i+1})$. See Figure 3.

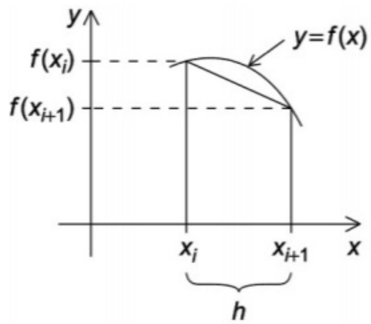

Figure 3: The i^{th} trapezoid

Thus, the area of the i^{th} trapezoid will be:

$$\frac{1}{2}h[f(x_i + f(x_{i+1})]$$

and the area of our entire approximation will be the sum of the areas of the trapezoids:

$$\begin{aligned}
&\frac{1}{2}h[f(x_0 + f(x_1)] + \frac{1}{2}h[f(x_1 + f(x_2)] + ... + \frac{1}{2}h[f(x_{n-1} + f(x_n)] \\
&= \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)] \\
&= [\frac{f(x_0)}{2} + \frac{f(x_n)}{2} + f(x_1) + f(x_2) + ...f(x_{n-1})]h
\end{aligned}$$

Implement a serial and a parallel program that estimate the definite integral $\int_{a=0}^{b=100} x^2 dx$ using the Pthreads library.

Problem 2.2. Complexity Analysis
1) Derive the running time (in terms of the number of trapezoids n) of the serial algorithm that you implemented in Problem 2.1.

2) Derive the running time of the parallel algorithm that you implemented in Problem 2.1. State your assumption(s) as necessary

Problem 2.3. Experimental Analysis
Measure the running times of the serial and the parallel program in Problem 2.1 for different values of n and different numbers of processors p. Plot the speedup curves. Also state the following parameters of the machine on which you carry out your experiment:

- the number of cores
- the number of threads per core (in case hyperthreading is enabled)

Avoid oversubscribing your machine.