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Subject ____

Date: __/__/__

MON TUE WED THR FRI SAT SUN

Class Work-1 (ADA)

AIM: Matrix Multiplication by
(a) Divide & Conquer
(b) Strassen's Method

(A) Divide & Conquer

Method: For two 4X4 matrices A and B, divide each into 2X2 blocks:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

The product is:

$C_1 = A_{11} B_{11} + A_{12} B_{21}$, $C_{12} = A_{11} B_{12} + A_{12} B_{22}$

$C_{21} = A_{21} B_{11} + A_{22} B_{21}$, $C_{22} = A_{21} B_{12} + A_{22} B_{22}$

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

split into 2X2 submatrices

$$A_{11} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \qquad A_{12} = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix} \qquad A_{21} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}$$

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$A_{22} = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$ $B_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Multiply each:

- $A_{11} B_{11} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$
- $A_{12} B_{21} = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}$

$C_{11} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 12 & 14 \end{bmatrix}$

- $A_{11} B_{12} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$
- $A_{12} B_{22} \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}$

$C_{12} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 12 & 14 \end{bmatrix}$

- $A_{21} B_{11} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}$
- $A_{22} B_{21} = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$

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$$C_{21} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix} + \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix} = \begin{bmatrix} 20 & 22 \\ 28 & 30 \end{bmatrix}$$

- $$A_{21} B_{12} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}$$

- $$A_{22} B_{22} = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$$

- $$C_{22} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix} + \cancel{20}\ \cancel{12} \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix} = \begin{bmatrix} 20 & 22 \\ 28 & 30 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 6 & 4 & 6 \\ 12 & 14 & 12 & 14 \\ 20 & 22 & 20 & 22 \\ 28 & 30 & 28 & 30 \end{bmatrix}$$

(B) Strassen's Method:

Method: Strassen reduces the number of multiplications from 8 to 7

For 2×2 submatrices:

$$M_1 = (A_{11} + A_{22})(B_{11} + B_{22})$$
$$M_2 = (A_{21} + A_{22})(B_{11})$$
$$M_3 = (A_{11})(B_{12} - B_{22})$$
$$M_4 = (A_{22})(B_{21} - B_{11})$$

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$M_5 = (A_{11} + A_{12})(B_{22})$

$M_6 = (A_{21} - A_{11})(B_{11} + B_{12})$

$M_7 = (A_{12} - A_{22})(B_{21} + B_{22})$

Then:

$C_{11} = M_1 + M_4 - M_5 + M_7$

$C_{12} = M_3 + M_5$

$C_{21} = M_2 + M_4$

$C_{22} = M_1 - M_2 + M_3 + M_6$

Example: $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$M_1 = (A_{11} + A_{22})(B_{11} + B_{22})$

$= \begin{bmatrix} 1+11 & 2+12 \\ 5+15 & 6+16 \end{bmatrix} \begin{bmatrix} 1+1 & 0+0 \\ 0+0 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 12 & 14 \\ 20 & 22 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 24 & 28 \\ 40 & 44 \end{bmatrix}$

$M_{22} = (A_{21} + A_{22})(B_{11}) = \begin{bmatrix} 9+11 & 10+12 \\ 13+15 & 14+16 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 22 \\ 28 & 30 \end{bmatrix}$

$M_3 = (A_{11})(B_{12} - B_{22}) = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

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$$M_4 = (A_{22})(B_{21} - B_{11}) = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_5 = (A_{11} + A_{12})(B_{22}) = \begin{bmatrix} 1+3 & 2+4 \\ 5+7 & 6+8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 12 & 14 \end{bmatrix}$$

$$M_6 = (A_{21} - A_{11})(B_{11} + B_{12}) = \begin{bmatrix} 9-1 & 10-2 \\ 13-5 & 14-6 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$M_7 = (A_{12} - A_{22})(B_{21} + B_{22}) = \begin{bmatrix} 3-11 & 4-12 \\ 7-15 & 8-16 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -8 \\ -8 & -8 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -16 & -16 \\ -16 & -16 \end{bmatrix}$$

$$C_{11} = M_1 - M_4 - M_5 + M_7 = \begin{bmatrix} 29 & 28 \\ 40 & 44 \end{bmatrix} + 0 - \begin{bmatrix} 4 & 6 \\ 12 & 14 \end{bmatrix} + \begin{bmatrix} -16 & -16 \\ -16 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 \\ 12 & 14 \end{bmatrix}$$

$$C_{12} = M_3 + M_5 = 0 + \begin{bmatrix} 4 & 6 \\ 12 & 14 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 12 & 14 \end{bmatrix}$$

$$C_{21} = M_2 + M_4 = \begin{bmatrix} 20 & 22 \\ 28 & 30 \end{bmatrix} + 0 = \begin{bmatrix} 20 & 22 \\ 28 & 30 \end{bmatrix}$$

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$$C_{22} = M_1 - M_2 + M_3 + M_6$$

$$= \begin{bmatrix} 24 & 28 \\ 40 & 44 \end{bmatrix} - \begin{bmatrix} 20 & 22 \\ 28 & 30 \end{bmatrix} + 0 + \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 22 \\ 28 & 30 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 6 & 4 & 6 \\ 12 & 14 & 12 & 14 \\ 20 & 22 & 20 & 22 \\ 28 & 30 & 28 & 30 \end{bmatrix}$$

Conclusion:

Both divide & conquer and Strassen's Method gives the same result for matrix multiplicaton. Divide & conquer is simple but needs 8 multiplicatons while Strassen takes 7, making ~~out~~ it more efficient for large matrices.

Divide & Conquer Time Complexity $= O(n^3)$

Strassen's Method Time Complexity $= O(n^{2.81})$