P11. (a) $\frac{4!}{x(x-1)(x-2)(x-3)} = f(x) =$

$$\int dx\ f(x) = \int dx \left(\frac{-4}{x} + \frac{12}{x-1} - \frac{12}{x-2} + \frac{4}{x-3} \right) \quad (*)$$

$$\int \bigcirc\, d\bigcirc = \bigcirc^2 + c$$

$$\int \Delta^2\, d(\Delta^3) = \frac{\Delta^4}{4}$$

Schritt 1:

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2} + \frac{D}{x-3} = \frac{A(x-1)(x-2)(x-3) + Bx(x-2)(x-3) + Cx(x-1)(x-3) + Dx(x-1)(x-2)}{\cancel{x(x-1)(x-2)(x-3)}} =$$

$$24 = A(x-1)(x-2)(x-3) + B(x-2)(x-3)(x) + C(x-1)(x-3)x + Dx(x-1)(x-2)$$

$x=0 \Rightarrow 24 = -6A \Rightarrow \boxed{A=-4}$

$x=1 \Rightarrow 24 = +2B \Rightarrow \boxed{B=+12}$

$x=2 \Rightarrow 24 = -2C \Rightarrow \boxed{C=-12}$

$x=3 \Rightarrow 24 = 6D \Rightarrow \boxed{D=4}$

$$(*) = -4\int \frac{dx}{x} + 12\int \frac{dx}{x-1} - 12\int \frac{dx}{x-2} + 4\int \frac{dx}{x-3} = -4\ln|x| + 12\ln|x-1| - 12\ln(|x-2|) + 4\ln|x-3|$$

$$= +4\ln\left|\frac{x-3}{x}\right| + 12\ln\left|\frac{x-1}{x-2}\right|$$

11.(b) $\frac{1}{1+x+x^2+x^3} = \frac{1}{1+x+x^2(1+x)} = \frac{1}{(1+x)\underbrace{(1+x^2)}_{(x+i)(x-i)}} = f(x)$

$\frac{Bx}{1+x^2} + \frac{C}{1+x^2} \rightarrow \int \frac{f'(x)}{f(x)}dx$

$$\int f(x)\,dx = \frac{1}{2}\int\frac{dx}{x+1} - \frac{1}{2}\int\frac{x\,dx}{1+x^2} + \frac{1}{2}\int\frac{dx}{1+x^2}$$

$\frac{1}{|a|}$

$$= \frac{1}{2}\ln|x+1| - \frac{1}{2\cdot 2}\int\frac{d(1+x^2)}{1+x^2} + \frac{1}{2}\arctan(x)$$

$2x\,dx$

$$= \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x^2+1| + \frac{1}{2}\arctan(x)$$

Ansatz: $\frac{A}{x+1} + \frac{Bx+C}{1+x^2} = \frac{A(1+x^2) + \overbrace{Bx^2+(B+C)x+C}^{(Bx+C)(x+1)}}{(1+x)(x^2+1)}$

$\int\frac{d(x+1)}{1+x} = \ln|x+1|$

$\int\frac{dx}{x} = \ln x$

$x=-1 \Rightarrow A(2) = 1 \Rightarrow A = \frac{1}{2}$

$x = i \Rightarrow C - B + (B+C)i = 1 \Rightarrow C-B = 1 \wedge C+B = 0$

$x = -i \Rightarrow C - B - (B+C)i = 1 \quad C-B = 1 \wedge C+B = 0$

$\Rightarrow C = 1/2,\ B = -1/2$

$\frac{1}{3}\int\frac{x\,dx}{x^3}$

$\int\frac{x\,dx}{x^2} = \frac{1}{2}\int\frac{d(x^2)}{x^2} = \frac{1}{2}\ln(x^2)$

$2x\,dx$

P11.2 (a) $\int_0^\infty \frac{dx}{\sqrt{x}(1+x)}$

$x \to \infty \quad \wedge \quad x \to 0^+$

$x \to 0^+ \qquad \underline{1+x} \simeq 1 \Rightarrow \frac{1}{\sqrt{x}(1+x)} \simeq \frac{1}{\sqrt{x}}$

$\frac{1}{\sqrt{x}(1+x)} = O\left(\frac{1}{\sqrt{x}}\right) \quad (x \to 0^+)$

Sei $\varepsilon > 0$ beliebig klein $(\mathcal{U}(0))$

$$\int_0^\varepsilon \frac{dx}{\sqrt{x}} = 2\sqrt{\varepsilon}$$

$x \to 0^+$ Konv.

$$\int_L^\infty x^{-3/2}\,dx = 2\, x^{-1/2} \Big|_\infty^L$$

Sei $L > 0$ beliebig groß.

$x \to \infty \qquad 1+x \simeq x \qquad \frac{1}{\sqrt{x}(1+x)} \simeq \frac{1}{x^{3/2}}$

$$\int_L^\infty \frac{dx}{x^{3/2}} = -2\frac{1}{x^{1/2}}\Big|_L^\infty = \left[2\frac{1}{\sqrt{L}} - 0\right] = \frac{2}{\sqrt{L}} \in \mathbb{R}$$

Konvergiert!

$\sqrt{x} = z \Rightarrow dz = \frac{1}{2\sqrt{x}}\,dx$

$$\int_0^\infty \frac{dx}{\sqrt{x}(1+x)} = 2\int_{\sqrt{0}}^{\sqrt{\infty}} \frac{dz}{1+z^2} = 2\cdot \arctan(\sqrt{x})\Big|_0^\infty = 2\cdot\left[\frac{\pi}{2} - 0\right] = \pi$$

$\pi/2$

$-\pi/2$

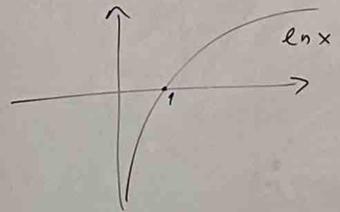
P11.2(b) $\int_0^\infty \frac{dx}{\sqrt{\cosh(x)-1}}$ $\qquad \cosh(x) = \frac{e^x+e^{-x}}{2}$

$x \to 0^+$ $\qquad$ Taylor'n um 0

$$\cosh(x) \stackrel{\wedge}{=} 1 + \frac{x^2}{2} + \frac{x^4}{4} + \dots \Rightarrow \cosh(x)-1 \simeq \frac{x^2}{2}$$

$$\frac{1}{\sqrt{\frac{x^2}{2}}} = \frac{\sqrt{2}}{x}$$

$$\sqrt{2}\int_0^\infty \frac{dx}{x} = \sqrt{2}\ln|x|\Big|_0^\infty$$



divergiert logarithmisch!

$x \to \infty$

$$= \frac{2\sqrt{2}}{e^{L/2}}$$