Bsp: $|q|<1$: $\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$

$\underbrace{\sum_{n=0}^{\infty} q^{n+1}}_{\frac{1}{1-q}-1} - \underbrace{q^0}_{=1}$

$x + \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = \pi$

Def: (Absolut konvergent): $\sum_{n=0}^{\infty} |a_n|$ konv.

$a_n \leq |a_n|$

Exkurs: Cauchyfolge: a_n

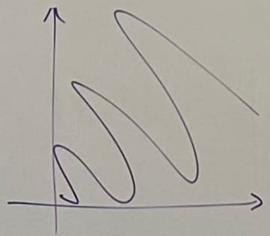
$\Rightarrow \forall \varepsilon > 0\ \exists N \in \mathbb{N} : |a_m - a_n| < \varepsilon\ ,\ \forall \underset{\in \mathbb{N}}{n,m} > N$

$\Downarrow$

Cauchyfolgen sind konv. in $\mathbb{R}^n$

Def: $\sum_{k=0}^{\infty} a_k\ ,\ \sum_{k=0}^{\infty} b_k$ abs. konv.

$\sum_{n=0}^{\infty} c_n \qquad c_n = \sum_{k=0}^{n} a_k\, b_{n-k}$



III. Potenzreihen

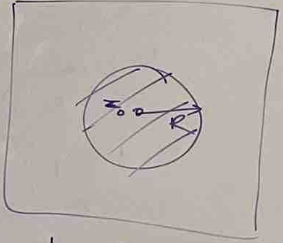
$$\sum_{n=0}^{\infty} a_n (z - z_0)^n$$

$\mathbb{C}$

„Entwicklungspkt."

$\mathbb{C} \ni a_n$ — „Koeff"

Konv. Rad.



$$\sum_{n=0}^{\infty} a_n \cdot 1^n$$

$|z_0 - s| < R \Longrightarrow$ Konv.

(1) Cauchy-Hadamard-Krit

$$R = \frac{1}{\limsup_{n\to\infty} \sqrt[n]{|a_n|}} \quad =: L$$

Wurzel Kriterium

- $L > 1$: div.
- $L < 1$: Konv.
- $L = 1$:

(2) Euler-Krit.

$$R = \frac{1}{\lim_{n\to\infty} \underbrace{\left|\frac{a_{n+1}}{a_n}\right|}_{=:s}}$$

, s muss existiern.

P5.1 (b) $\sum_{n=0}^{\infty} n \cdot q^n$ — (*) , $|q| < 1$

$$(*) = \sum_{n=0}^{\infty} (n+1) q^n - \sum_{n=0}^{\infty} q^n$$

$$= \left(\sum_{n=0}^{\infty} q^n\right)^2 - \frac{1}{1-q}$$

$$= \frac{1}{(1-q)^2} - \frac{1}{1-q}$$

$$= \frac{q}{(1-q)^2}$$

(c) $$\sum_{n=0}^{\infty} n \cdot \Big(\underbrace{\tfrac{1}{2}}_{\overset{!!}{q}}\Big)^n = \frac{n}{2^n} = n \cdot 2^{-n}$$

P5.3(b) $\sum_{n=0}^{\infty} \boxed{\frac{n!}{n^n}} \cdot z^n$

Euler: $S := \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left(\frac{\frac{(n+1)\cdot \cancel{n!}}{(n+1)^{n+1}}}{\frac{\cancel{n!}}{n^n}} \right).$

$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n =: e$

$R = \frac{1}{S} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$ (*)

$= e$

$\Rightarrow \lim_{n\to\infty} \sqrt[n]{|a_n|} = \frac{1}{R} = \lim_{n\to\infty} \frac{\sqrt[n]{n!}}{\frac{n}{e}} = 1$

$\boxed{\liminf = \limsup}$

(*)? $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$

$\ln(1+x) = \sum_{k=0}^{\infty} (-1)^{k+1} \frac{x^k}{k}$

$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

$a_n = \left(1+\frac{1}{n}\right)^n \Rightarrow \ln(a_n) = n \cdot \ln\left(1+\underbrace{\frac{1}{n}}_{=:x}\right)$

$\lim_{n\to\infty} \ln(a_n) = \lim_{n\to\infty} \left(1 - \frac{1}{2n} + \dots\right) = n \cdot \left(\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} - \dots\right)$

$= 1$

$= 1 - \frac{1}{2n} + \frac{1}{3n^2} - \dots$

$\ln(a_n) = 1$

$e^{\ln(a_n)} = e^1 \Rightarrow \ln(a_n) = 1$

$\log_e x = 1$ $\boxed{\lim_{n\to\infty} a_n = e}$

$\log_e e = 1$ $\boxed{x = e}$