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MA 473: Computational Finance Lab – I,   August 9, 2021

1. Solve the following parabolic initial-boundary-value problem:

$$
\begin{cases}
\dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = \sin(2\pi\, x)\sin(4\pi\, t), & (x,t) \in (0,1) \times (0,1), \\
u(0,t) = u(1,t) = 0, & t \in (0,1], \\
u(x,0) = 0, & x \in (0,1),
\end{cases}
$$

by two-level finite difference schemes with the spatial step-size $h = 0.025$ and the time steps $k = 0.05$. Calculate the solution for three time levels.

2. Solve the following parabolic PDE by using two-level and three-level finite difference schemes:

$$
\begin{cases}
\dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0, & (x,t) \in (0,1) \times (0,T),\ T > 0 \\
u(0,t) = u(1,t) = 0, & t \in (0,T], \\
u(x,0) = f(x), & x \in (0,1),
\end{cases}
$$

for the following values of f:

(a) $f(x) = \sin(\pi\, x)$

(b) $f(x) = x\,(1 - x)$.

Here, the values of the step size in the x-direction is $\delta x = 0.025$, and in the t-direction is $\delta t = 0.0025$. Determine the solution for three time levels.

3. Solve the following parabolic problem:

$$
\begin{cases}
\dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0, & (x,t) \in (0,1) \times (0,t),\ t > 0, \\
u(x,0) = \cos(\pi\, x/2), & x \in (0,1), \\
u_x(0,t) = 0, & t \geq 0, \\
u(1,t) = 0, & t \geq 0,
\end{cases}
$$

by using the Neumann boundary condition approximation as given below:

$$
u_0^{n+1} = u_0^n + 2\frac{k}{h^2}(u_1^n - u_0^n).
$$

Use the spatial step-size $h = 0.10$, the time steps $k = 0.005$ and find the solution at $t = 0.1$.

4. Determine the numerical solution of the following one-dimensional parabolic IBVP:

$$
\begin{cases}
u_t - u_{xx} = 0, & (x,t) \in (0,1/2) \times (0,T),\ T > 0 \\
u(0,t) = 0,\quad u_x(1/2,t) = t^2, & \forall t \in (0,T] \\
u(x,0) = x(1 - x), & x \in (0,1/2),
\end{cases}
$$

by using the Crank-Nicolson scheme with $h = 0.3$ and $k = 0.05$ for three time levels.