DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 473: Computational Finance Labs – V and VI September 13, 2021

1. Consider the following American put option problem:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta) S \dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\ \text{with suitable initial and boundary and free boundary conditions.} \end{cases}$$

   (a) Solve the transformed PDE $y_\tau = y_{xx}$ of the above IBVP by using the Backward-Time and Central Space (BTCS) Scheme and the Crank-Nicolson finite difference scheme.

   (b) Plot $V(S,t)$ for $T=1$, $K=10$, $r=0.25$, $\sigma=0.6$, $\delta=0.2$, and the payoff.

   (c) Solve the problem by using $\delta\, x$ and $\delta\, \tau$, and $\delta\, x/2$ and $\delta\, \tau/2$ and calculate the error between these two numerical solution. Plot the error.

   (d) Also calculate the error mentioned above for different values of $\delta\, x/2$ and $\delta\, t/2$ and plot N *versus* the maximum absolute error.

2. Consider the following American call option problem:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta) S \dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\ \text{with suitable initial and boundary and free boundary conditions.} \end{cases}$$

   (a) Solve the transformed PDE $y_\tau = y_{xx}$ of the above IBVP by using the Backward-Time and Central Space (BTCS) Scheme and the Crank-Nicolson finite difference scheme.

   (b) Plot $V(S,t)$ for $T=1$, $K=10$, $r=0.06$, $\sigma=0.3$, $\delta=0.25$, and the payoff.

   (c) Solve the problem by using $\delta\, x$ and $\delta\, \tau$, and $\delta\, x/2$ and $\delta\, \tau/2$ and calculate the error between these two numerical solution. Plot the error.

   (d) Also calculate the error mentioned above for different values of $\delta\, x/2$ and $\delta\, t/2$ and plot N *versus* the maximum absolute error.