DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 473: Computational Finance Lab – 4 August 30, 2021

1. Consider the following European call option problem:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2(S)S^2\dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad 0 \le S < \infty,\ t \le T, \\ V(S,t) = V_T(S), \quad 0 \le S < \infty. \end{cases}$$

With the following transformation

$$\begin{cases} \xi = \dfrac{S}{S+q}, \\ \tau = T - t, \\ V(S,T) = (S+q)\overline{V}(\xi,\tau). \end{cases}$$

the above European call option problem becomes the following parabolic PDE:

$$\begin{cases} \dfrac{\partial \overline{V}}{\partial \tau} = \dfrac{1}{2}\overline{\sigma}^2(\xi)\xi^2(1-\xi^2)\dfrac{\partial^2 \overline{V}}{\partial \xi^2} + (r-\delta)\xi(1-\xi)\dfrac{\partial \overline{V}}{\partial \xi} - [r(1-\xi)+\delta\xi]\overline{V},\ 0 \le \xi \le 1,\ 0 \le \tau \le T, \\ \overline{V}(\xi,0) = \max(2\xi-1,0),\ 0 \le \xi \le 1, \\ \overline{V}(0,\tau) = \overline{V}(0,0)e^{-r\tau}, \qquad \overline{V}(1,\tau) = \overline{V}(1,0)e^{-\delta\tau},\ 0 \le \tau \le T. \end{cases}$$

Solve the transformed PDE by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.

(ii) Backward-Euler for time & central difference for space (BTCS) scheme.

(iii) Crank-Nicolson finite difference scheme

The values of the parameters are $T = 1,\ r = 0.04,\ \sigma = 0.25,\ \delta = 0.1$.

To solve the system of linear algebraic equations arising from the implicit schemes use the iterative methods (Jacobi method, Gauß-Seidel method and SOR method).

Use conjugate gradient method to solve the system.

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2. Consider the following European put option problem:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2(S)S^2\dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad 0 \le S < \infty,\ t \le T, \\ V(S,t) = V_T(S), \quad 0 \le S < \infty. \end{cases}$$

With the following transformation

$$\begin{cases} \xi = \dfrac{S}{S+q}, \\ \tau = T - t, \\ V(S,T) = (S+q)\overline{V}(\xi,\tau). \end{cases}$$

the above European put option problem becomes the following parabolic PDE:

$$\begin{cases} \dfrac{\partial \overline{V}}{\partial \tau} = \dfrac{1}{2}\overline{\sigma}^2(\xi)\xi^2(1-\xi^2)\dfrac{\partial^2 \overline{V}}{\partial \xi^2} + (r-\delta)\xi(1-\xi)\dfrac{\partial \overline{V}}{\partial \xi} - [r(1-\xi)+\delta\xi]\overline{V},\ 0 \le \xi \le 1,\ 0 \le \tau \le T, \\ \overline{V}(\xi,0) = \max(1-2\xi,0),\ 0 \le \xi \le 1, \\ \overline{V}(0,\tau) = \overline{V}(0,0)e^{-r\tau}, \qquad \overline{V}(1,\tau) = \overline{V}(1,0)e^{-\delta\tau},\ 0 \le \tau \le T. \end{cases}$$

Solve the transformed PDE by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.

(ii) Backward-Euler for time & central difference for space (BTCS) scheme.

(iii) Crank-Nicolson finite difference scheme

The values of the parameters are $T = 1,\ r = 0.04,\ \sigma = 0.25,\ \delta = 0.1$.

To solve the system of linear algebraic equations arising from the implicit schemes use the iterative methods (Jacobi method, Gauß-Seidel method and SOR method).

Use conjugate gradient method to solve the system.