DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 473: Computational Finance Lab – IX 25/10/2021

1. Consider the following Black-Scholes PDE for European call:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta) S \dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\ V(S,t) = 0, \quad \text{for } S = 0, \\ V(S,t) = S - Ke^{-r(T-t)}, \quad \text{for } S \to \infty \\ \text{with suitable initial condition } V(S,0). \end{cases}$$

With the following transformation

$$\begin{cases} S = Ke^x, \quad t = T - \dfrac{2\tau}{\sigma^2}, \quad q := \dfrac{2r}{\sigma^2}, \quad q_\delta := \dfrac{2(r-\delta)}{\sigma^2}, \\ V(s,t) = V\left(Ke^x,\, T - \dfrac{2\tau}{\sigma^2}\right) =: v(x,\tau), \text{ and} \\ v(x,\tau) =: K \exp\left\{-\dfrac{1}{2}(q_\delta - 1)x - \left[\dfrac{1}{4}(q_\delta - 1)^2 + q\right]\tau\right\} y(x,\tau) \end{cases}$$

the above Black-Scholes PDE becomes the following 1-D heat conduction parabolic PDE:

$$\begin{cases} \dfrac{\partial y}{\partial \tau} = \dfrac{\partial^2 y}{\partial x^2},\ x\in\mathbb{R},\ \tau \geq 0, \\ y(x,0) = \max\left\{\exp(\frac{x}{2}(q_\delta + 1)) - \exp(\frac{x}{2}(q_\delta - 1)), 0\right\},\ x\in\mathbb{R}, \\ y(x,\tau) = 0, \text{ for } x \to -\infty, \\ y(x,\tau) = \exp\left(\frac{1}{2}(q_\delta+1)x + \frac{1}{4}(q_\delta+1)^2\tau\right) \text{ for } x\to\infty. \end{cases}$$

Solve the transformed PDE by the following finite element methods (FEMs):

(i) Piecewise-linear basis functions with the trapezoidal rule for the numerical quadratures and the Crank-Nicolson scheme.

(ii) Piecewise-linear basis functions with the Simpson's rule for the numerical quadratures and the Crank-Nicolson scheme.

The values of the parameters are $T = 1$, $K = 10$, $r = 0.06$, $\sigma = 0.3$ and $\delta = 0$.

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2. Consider the following Black-Scholes PDE for European put:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta) S \dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\ V(S,t) = Ke^{-r(T-t)} - S, \quad \text{for } S = 0, \\ V(S,t) = 0, \quad \text{for } S \to \infty \\ \text{with suitable initial condition } V(S,0). \end{cases}$$

Using the transformation given above the Black-Scholes PDE becomes the following problem:

$$\begin{cases} \dfrac{\partial y}{\partial \tau} = \dfrac{\partial^2 y}{\partial x^2},\ x \in \mathbb{R},\ \tau \geq 0, \\ y(x,0) = \max\left\{\exp(\frac{x}{2}(q_\delta - 1)) - \exp(\frac{x}{2}(q_\delta + 1)), 0\right\},\ x \in \mathbb{R}, \\ y(x,\tau) = \exp\left(\frac{1}{2}(q_\delta - 1)x + \frac{1}{4}(q_\delta - 1)^2\tau\right),\ \text{for } x \to -\infty, \\ y(x,\tau) = 0 \text{ for } x \to \infty, \end{cases}$$

Solve the transformed PDE by the following finite element methods (FEMs):

(i) Piecewise-linear basis functions with the trapezoidal rule for the numerical quadratures and the Crank-Nicolson scheme.

(ii) Piecewise-linear basis functions with the Simpson's rule for the numerical quadratures and the Crank-Nicolson scheme.

The values of the parameters are $T = 1$, $K = 10$, $r = 0.06$, $\sigma = 0.3$ and $\delta = 0$.