

DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 473: Computational Finance Lab – VII

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1. The PDE modelling the *Asian option* for the *European arithmetic average strike call* (for  $S > 0$ ,  $A > 0$ ,  $0 \leq t \leq T$ ) is given by

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} + S \frac{\partial V}{\partial A} - rV = 0.$$

By using the transformation  $V(S, A, t) = \tilde{V}(S, R, t) = S \cdot H(R, t)$ , with  $R = A/S$ , transform the above PDE into the following form:

$$\left\{ \begin{array}{l} \frac{\partial H}{\partial t} + \frac{1}{2}\sigma^2 R^2 \frac{\partial^2 H}{\partial R^2} + (1 - rR) \frac{\partial H}{\partial R} = 0, \\ H = 0, \quad \text{for } R \rightarrow \infty, \\ \frac{\partial H}{\partial t} + \frac{\partial H}{\partial R} = 0, \quad \text{for } R = 0, \\ H(R_T, T) = \left(1 - \frac{R_T}{T}\right)^+ \end{array} \right.$$

- (a) Solve the above transformed PDE by the following schemes:
- (i) Forward-Euler for time & central difference for space (FTCS) scheme.
  - (ii) Backward-Euler for time & central difference for space (BTCS) scheme.
  - (iii) Crank-Nicolson finite difference scheme with a suitable second-order approximation for the left boundary condition.
- (b) Plot the solution surfaces for  $K = 100$ ,  $T = 0.2$ ,  $r = 0.05$ ,  $\sigma = 0.25$  at different time levels.
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