MAGNETIC MIRRORS, CHANNELS AND BOTTLES FOR COLD NEUTRONS

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It is well known that neutrons with spins oriented along a magnetic field are repelled from regions of strong field. This makes it possible to use magnetic mirrors and channels for obtaining focused beams of polarized neutrons. By surrounding an evacuated region by magnetic mirrors one can accomplish the confinement of cold neutrons. In the paper we consider conditions of adiabaticity necessary for maintaining the orientation of the spin relative to the field, we give some possible configurations for the magnetic field, and make estimates of the intensities of polarized beams and neutron densities in a magnetic bottle.

1. INTRODUCTION

THE energy of the magnetic moment of a neutron in a magnetic field is usually small compared to the kinetic energy of a thermal neutron. The magnetic moment of the neutron (− 1.913 nuclear magnetons) is equal to

$$\mu = -6.03\cdot 10^{-12}\ \text{ev/oe}. \quad (1)$$

The magnetic field in not too complex magnetic structures reaches values of the order of several thousands of oersteds. The corresponding energy is consequently of the order of $10^{-7} - 10^{-8}$ ev. Neutrons with such small energies can alter their motion drastically under the action of magnetic forces and, in particular with a suitable orientation of their spin, will be reflected from regions with high values of magnetic field. Neutrons with higher energy ϵ can be reflected from a magnetic mirror only for sufficiently small grazing angles

$$\sin\alpha \leqslant \sqrt{\mu H/\varepsilon}. \quad (2)$$

Despite the relatively small size of magnetic forces, there is a possibility of using a magnetic field in vacuum for obtaining beams of polarized neutrons. The choice of a suitable configuration of magnetic field gives a possibility for focussing beams and obtaining intensities somewhat greater than for the case of a magnetized cobalt mirror. Finally, when various conditions are satisfied, one can achieve an apparatus for confining super-cold neutrons which would make it possible to measure the lifetime of the neutron independently of an absolute calibration of the intensity of the neutron beam.

The magnetic energy of a neutron in an external field is equal to $\pm|\mu \mathrm{H}|$ for a neutron with spin projection $s_{\mathrm{H}} = \pm \frac{1}{2}$ on the direction of the magnetic field. The quantity s_{H} is an adiabatic invariant. This means that for sufficiently slow change in direction of the magnetic field the spin turns with the field and no depolarization results. The angular velocity of rotation of the field vector ν must be small compared with the frequency of precession of the magnetic moment in the field:

$$\nu \ll 2\mu H/\hbar. \quad (3)$$

When the adiabatic condition (3) is satisfied sufficiently closely, one can solve the problem of the motion of neutrons purely classically by treating $\pm|\mu \mathrm{H}(\mathbf{x})|$ as a potential energy for two types of particles (neutron with spin along the field, and neutron with spin opposite to the field), which do not change into one another. Before considering various applications, it is worthwhile to investigate in more detail cases of possible breakdown of the adiabatic condition and to estimate the probability of a flipping of the neutron spin in passing through such dangerous regions.

2. ADIABATIC CONDITION

A characteristic case of breakdown of adiabaticity is the passage of the neutron near a point where $\mathbf{H} = 0$, or a region of rapid rotation of the field. To investigate changes in orientation of the spin we may give the field as a function of time. Over a sufficiently small interval we may regard the field $\mathbf{H}$ as a linear function of time. Obviously then $|\mathbf{H}|$ reaches a minimum at that moment when $\mathbf{H} \perp \dot{\mathbf{H}}$. Choosing this point as the reference origin for the time, and choosing the x and z axes along $\mathbf{H}_{\min}$ and $\dot{\mathbf{H}}$, respectively, we obtain

$$H_x = \text{const}, \qquad H_y = 0, \qquad H_z = t\dot{H}, \qquad \dot{H} = \text{const}. \quad (4)$$

The spin-wave function $\psi = \binom{\varphi}{\chi}$ satisfies the equation

$$i\hbar\dot{\psi} + \mu H_x \sigma_x \psi + \mu t \dot{H} \sigma_z \psi = 0. \tag{5}$$

Introducing the notation

$$\omega = \mu H_x / \hbar, \qquad a = \mu \dot{H} / \hbar, \tag{6}$$

we rewrite the equation in explicit expanded form

$$i\dot{\varphi} + \omega\chi + at\varphi = 0, \qquad i\dot{\chi} + \omega\varphi - at\chi = 0, \tag{7}$$

where ω is one-half the precession rate of the spin. Eliminating χ, we get

$$\ddot{\varphi} + (\omega^2 - ia + a^2 t^2)\varphi = 0 \tag{8}$$

The substitution

$$\varphi = e^{-z/2} u, \qquad z = -iat^2 \tag{9}$$

reduces this equation to the confluent hypergeometric equation

$$z\frac{d^2u}{dz^2} + \left(\frac{1}{2} - z\right)\frac{du}{dz} - \frac{\omega^2}{4ia} u = 0. \tag{10}$$

The two solutions of the confluent hypergeometric equation give the function $\varphi(t)$ which is regular for $t = 0$:

$$\varphi = e^{iat^2/2}\{c_1 F(\alpha, {}^1/_2, -iat^2) + c_2 i\omega t F(\alpha + {}^1/_2, {}^3/_2, -iat^2)\}, \tag{11}$$

where $\alpha = \omega^2/4ia$. Expressing the second amplitude in terms of these same constants we get

$$\chi = e^{iat^2/2}\{c_1 i\omega t F(\alpha + 1, {}^3/_2, -iat^2) + c_2 F(\alpha + {}^1/_2, {}^1/_2, -iat^2)\}. \tag{12}$$

In view of the identity $F(\alpha, \gamma, z) \equiv e^z F(\gamma - \alpha, \gamma, -z)$ these formulas are symmetric with respect to the interchange $\varphi \rightleftarrows \chi$, $z \to -z$. Using the asymptotic expressions for the confluent hypergeometric functions, it is easy to set up the expressions for the amplitudes, valid for $t \to \pm\infty$:

$$\varphi(t) = \sqrt{\pi} e^{-\pi\omega^2/8a}\left[\frac{c_1}{\Gamma({}^1/_2 - \alpha)} + \frac{i\omega}{2\sqrt{ia}}\frac{c_2}{\Gamma(1-\alpha)}\frac{t}{|t|}\right](at^2)^{-\alpha} e^{iat^2/2},$$

$$\chi(t) = \sqrt{\pi} e^{-\pi\omega^2/8a}\left[\frac{i\omega}{2\sqrt{-ia}}\frac{c_1}{\Gamma(1+\alpha)}\frac{t}{|t|} + \frac{c_2}{\Gamma({}^1/_2 + \alpha)}\right](at^2)^{\alpha} e^{-iat^2/2}. \tag{13}$$

Here t is assumed to be real, the factors to the right of the square brackets have a modulus equal to unity, and the normalization of the functions is the usual one:

$$|\psi|^2 = |\varphi|^2 + |\chi|^2 = |c_1|^2 + |c_2|^2. \tag{14}$$

In the asymptotic expressions only the leading terms have been kept.

It is now easy to obtain the probability for a reorientation of the spin. Taking for $t \to -\infty$,

$$|\varphi|^2 = 1, \quad |\chi|^2 = 0, \tag{15}$$

we find a solution for $t \to +\infty$,

$$|\varphi|^2 = e^{-\pi\omega^2/a}, \quad |\chi|^2 = 1 - e^{-\pi\omega^2/a}. \tag{16}$$

If we consider that the magnetic field has reversed its direction, then for complete adiabaticity we should expect the values $|\varphi|^2 = 0$, $|\chi|^2 = 1$ for $t \to +\infty$. The factor $e^{-\pi\omega^2/a}$ is equal to the probability of non-adiabatic re-orientation of the spin with respect to the field direction. Introducing the effective time for re-orientation of the field

$$\tau = \nu^{-1} = H_x/\dot{H} = \omega/a, \tag{17}$$

we can write the probability for re-orientation of the spin in the form

$$w = e^{-\pi\omega\tau}. \tag{18}$$

If the neutron undergoes a large number N_p of passages near danger points, the necessary condition for preserving adiabaticity is $N_p w \ll 1$. For the minimum value of the magnetic field we obtain the estimate

$$\pi\mu H_{min}^2 / \hbar |\dot{H}| > \ln N_p. \tag{19}$$

The important thing is that the probability of reorientation of the spin falls off exponentially with increase of the dimensionless factor $\omega\tau$. This makes it possible for each particular case to choose such a value of H_{min} that the non-adiabatic reorientation of the spin can be neglected.

3. MAGNETIC FIELD CONFIGURATION

Let us first consider a plane magnetic mirror. A period structure of conductors placed in the

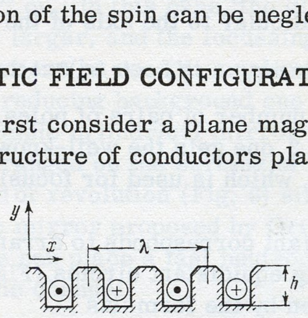

FIG. 1

grooves of an iron magnet support (Fig. 1) gives a magnetic field which, even at relatively small distances, is described very well by the formula

$$H_x = H_0 e^{-ky}\sin kx, \quad H_y = H_0 e^{-ky}\cos kx, \quad k = 2\pi/\lambda, \tag{20}$$

where λ is the period of the structure. For $H = |\mathbf{H}|$ we get

$$H = H_0 e^{-ky}. \tag{21}$$

Such a structure of magnetic field is convenient for obtaining a potential barrier near the walls

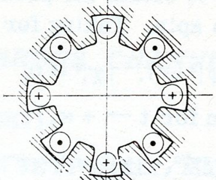

FIG. 2

and can serve as the prototype for constructing magnetic focussing channels and closed regions. We give some data for rough engineering computations. The power supply (in W/cm^2) for a good filling of the groove with copper is

$$P \approx H_0^2 h^{-1}, \qquad (22)$$

where H_0 is in kilooersted, and the depth of the groove h in centimeters. If the depth of the groove is approximately equal to the step in the structure, $h \sim \lambda$, the induction in the lower part of the groove will be approximately four times H_0. In this connection, the value $H_0 \approx 5{,}000$ oe should be regarded as a reasonable economic limit for the value of the mirror field. For smaller depth of groove, because of a somewhat increased power, one can achieve a field up to 7 – 8 koe.

For magnetic focussing channels (neutron pipes for cold neutrons) one can use different variants of arrangement of such structures on a cylindrical surface. Placing the poles along the generator of a cylinder (Fig. 2), we obtain a multipole lens (an octupole lens in the figure) with a field lying in the plane perpendicular to the axis of the lens:

$$H_r = (r/r_0)^{m-1} \sin m\varphi, \qquad H_\varphi = (r/r_0)^{m-1} \cos m\varphi, \qquad (23)$$

where m is the number of pairs of poles. In particular, for m = 3 one gets the well-known case of a sextupole lens, which is used for focusing molecular beams.

A second variant corresponds to arrangement of the poles along equidistant circles (Fig. 3). The field is then given by the formulas

$$H_r = I_0'(kr) \sin kz, \qquad H_z = I_0(kr) \cos kz, \qquad (24)$$

where I_0 is the Bessel function of imaginary argument.

Finally, a mixed variant is possible with an arrangement of the poles along a helix (Fig. 4).

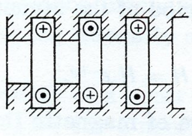

FIG. 3.

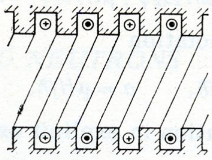

FIG. 4

The field in this case has a somewhat more complicated structure:

$$H_r = I_m'(kr) \cos(m\varphi - kz),$$
$$H_\varphi = -(m/kr) I_m(kr) \sin(m\varphi - kz),$$
$$H_z = I_m(kr) \sin(m\varphi - kz). \qquad (25)$$

Each of these variants has its advantages, and no one of them is universal. The variant with the multipole lens (Fig. 2) is relatively simple in construction. The danger associated with breakdown of adiabaticity at the field nodes can be easily eliminated if we place along the cylindrical surface a small auxiliary winding giving a weak constant longitudinal field H_z. Some construction difficulties can arise because of the necessity of changing the diameter of the pipe along its length and for bends in the channel.

The variant with disc-shaped poles (Fig. 3) does not have these disadvantages, since the magnetic circuit can consist of individual discs with their own windings. By smoothly varying the diameter and number of ampere turns in the lenses, one can achieve a widening of the channel and even a change in the period of the structure. By placing the disc lenses at small angles to one another, one can change the direction of the channel. We must regard as a defect of this variant a certain difficulty in satisfactorily eliminating possible breakdowns of adiabaticity at the field nodes: in this case the field vanishes not on nodal lines, but at isolated nodal points on the axis. The probability of passage through a node is small, but nevertheless it is not equal to zero. At the nodal points, all three Cartesian components of the field have a finite longitudinal derivative, so that the addition of a small constant field can only shift the nodal point, but not eliminate it. When nodal lines were present in the case of a plane field, it was sufficient to add a small component of field along the nodal line.

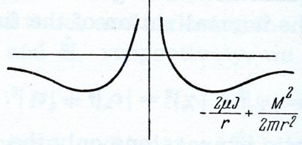

FIG. 5

A channel with disc lenses can be used apparently only in those cases where the total number of passages in the neighborhood of nodal points is small: for example, for single passage of neutrons through a relatively short channel.

The helical structure of the channel for m = 1 (two-pole helix) gives a magnetic field which has no nodal lines or nodal points. Therefore, the magnetic channel shown in Fig. 4 can be used without any auxiliary windings. Construction difficulties arise for turns of the channel and changes in cross section. The transition to another cross section does not require changing the ampere turns in the winding, as in the case of multipole lenses, but one need only preserve sufficiently well the constancy of the width of the pole and the groove. However, the shape of the poles and windings in these transitions will be quite complex.

All the configurations enumerated for magnetic mirrors and channels are computed on the basis of repulsion of neutrons with spin parallel to the field from regions with high field value. One can, of course, also use the attraction of neutrons of opposite polarization. However, neutrons with anti-parallel spin will necessarily be attracted to the magnetic poles or conductors carrying current and will be lost.

An exception might be the following lone case: a rectangular conductor with field $H = 2J/r$. In the neighborhood of the conductor, for neutrons with anti-parallel spin, there will be an attraction with potential $-2\mu J/r$. In combination with the centrifugal potential $M^2/2mr^2$ this gives a certain region of stable helical motion (Fig. 5) for neutrons which have a suitable value of their initial angular momentum $M < 2\sqrt{m\mu r_0 J}$ around the axis of the conductor. Here it is essential that the region accessible for motion of the neutrons be doubly connected, and any supports of the conductor will result in a partial loss in beam.

4. POLARIZED BEAMS

One possible application of mirrors with periodic magnetic structure may be the obtaining of beams of polarized neutrons. It is easy to see that the plane mirrors of the type shown in Fig. 1 do not have any advantages compared to the cobalt mirrors used at present, since the maximum grazing angle for a cobalt mirror is 2 — 3 times greater than for the period structure and correspondingly one has a higher intensity for the same size of mirror.

In the case of magnetic channels of cylindrical shape, certain advantages appear, associated with

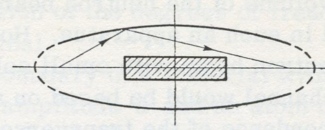

FIG. 6

focussing of the beam. The beam intensity in a long magnetic channel is equal to

$$N = \frac{n\overline{v}}{2} \frac{\pi\alpha^2}{4\pi} S, \tag{26}$$

where $\overline{nv}$ is the flux of thermal neutrons at the wall of the reactor, α is an angle given by formula (2), S is the cross section area of the channel. The factor associated with solid angle is smaller than in the case of the cobalt mirror. However, the beam cross section in this case may be considerably greater. As a result, by using a sufficiently large magnetic channel with periodic structure, one can apparently obtain in practice a beam intensity several times (3 — 5) greater than with a good cobalt mirror. Of course, one should keep in mind that the magnetic channel is an extremely complicated gadget compared with a cobalt mirror. Thus, for example, for neutrons with energy $\sim 3 \times 10^{-2}$ ev emerging from the reactor, the limiting angle for a field of 5000 oe is $\alpha = 10^{-3}$. The length of channel must be no less than $2r/\alpha$, i.e. several tens of meters even for a beam only a few centimeters in diameter.

It is possible that the use of magnetic channels will turn out to be useful for obtaining beams of cold polarized neutrons with energies of the order of $10^{-4} - 10^{-3}$ ev. In this case, the glancing angle is somewhat larger, and the focussing properties of the channel can be used at smaller sizes. For purposes of reducing background one can use in place of a simple cylindrical mirror a periodic structure arranged on the surface of a highly prolate ellipsoid of revolution (Fig. 6) similar to the electrostatic mirror proposed by Grigor'ev.[1] In this case one can place a fast neutron shield along the axis of the channel.

5. CONFINEMENT OF COLD NEUTRONS

By applying the principle of reflection of neutrons from regions with high values of magnetic field, one can realize confinement of neutrons in vacuum by two methods: by bending the magnetically focusing channel into a ring, or making a simple, singly-connected region surrounded by a strong magnetic field. In the first case, there is a possibility for keeping neutrons with energies considerably exceeding the height of the magnetic potential barrier, and even to increase considerably the phase volume of the neutron beam which can be captured in such an apparatus. However, motion of the neutron beam with small solid angle in such a ring channel would be based on an approximate independence of the transverse and longitudinal degrees of freedom, i.e., on the existence of approximate integrals of the motion. Over a long period of time even slight inaccuracies in the magnetic field configuration would lead to exchange of energy between these degrees of freedom and to loss of the neutrons as a result of their going beyond the admissible boundaries in solid angle.

In this connection, the estimates of possible neutron densities in "magnetic bottle" which follow refer to such a filling of the phase volume for which the moving of neutrons away from the walls is associated not with approximate integrals of the motion, but only with their total energy in the magnetic and gravitational fields, which for a field constant in time is an exact integral of the motion:

$$p^2/2m + |\mu H| + mgz = \text{const.} \tag{27}$$

Here at each point inside the magnetic bottle motion of the neutrons is permitted in any direction, but the velocities are extremely small - of the order of 2 — 3 m/sec. Complicated regions similar to a torus have no advantages in this case, and magnetic channels are conveniently used only for initial filling of the volume with neutrons. Such a method for confining cold neutrons is very similar to that proposed by Zel'dovich[2] for preserving cold neutrons in vacuum by means of a potential barrier at the surface of a material with a positive coherent scattering amplitude.

If we take the magnetic channel from the reactor wall to some vacuum region surrounded by a magnetic field of periodic structure, it can be filled with super-cold neutrons with energy less than the value of the potential barrier $\mu H \approx 3 \times 10^{-8}$ ev (for H = 5000 oe). Moreover, one can carry out an isolation of the magnetic channel from the volume intended for storage by using a magnetic plug. After this the magnetic field in the channel can be switched off, or one can even disconnect the channel mechanically to eliminate background.

The time during which cold neutrons will remain in a magnetic bottle should be determined only by the half-life if the experimental arrangement is properly done. The number of neutrons remaining after a definite time of storage can be measured by a counter which during the storage time is either switched off the bottle by a magnetic field, or simply fed in and removed from the volume of the bottle.

To estimate the density of cold neutrons in the bottle we can use the formula

$$n = (2n_\Theta / 3\sqrt{\pi})\,(\mu H/\Theta)^{3/2}, \tag{28}$$

where n is the neutron density in the bottle, n_Θ is the density of thermal neutrons at the boundary of the reactor, Θ is the neutron temperature. Here it is assumed that the cold part of the neutrons at the surface of the reactor is described by the usual thermal spectrum. Actually, the number of cold neutrons will be somewhat smaller because of absorption. The correction factor is approximately equal to

$$\sigma_0/(\sigma_0 + \sigma_a), \tag{29}$$

where σ_a is the absorption cross section, σ_0 is the cross section for the process of energy exchange of the neutron with the moderator. In crystalline bodies, the cross section for inelastic scattering (collision with phonons) increases for cold neutrons proportional to v^{-1}, as does the absorption cross section. Because of this, there is no reason to expect any essential reduction in the factor (29) for super-cold neutrons. For good moderators (graphite, beryllium) it is close to unity. Heavy water in both liquid and solid state also satisfies the condition $\sigma_a \ll \sigma_0$ for extremely cold neutrons, and can be used as a source in such an experiment. The feasibility of using ice made from ordinary water and other cooling agents of solid hydrogenous materials requires more detailed investigation, since the absorption cross section of hydrogen is comparatively large, while the experimental data concerning inelastic cross sections for supercooled neutrons are not known in sufficient detail. Butterworth, Egelstaff, et al.[3] have used liquid hydrogen inside a reactor as moderator and obtained an increase in yield of cold neutrons compared with the beam from a graphite scatterer by a factor of 20 — 25.

The interaction of cold neutrons with vibrational degrees of freedom for chemically bound hydrogen in solids at low temperatures is relatively small.[4,5] The cross section for exchange of energy at low temperatures is of the same order as the absorption cross section, which limits the efficiency of cold coolants of hydrogen-containing moderator. It is possible that, despite the technical difficulties of working with liquid hydrogen inside a reactor, this method for obtaining an intense beam of cold neutrons is the most efficient.

For a reactor with a thermal neutron density of 10^8 cm^{-3}, the estimate (28) gives a value of the density in the magnetic bottle of the order of 10^4 m^{-3}. The use of cryogenic techniques inside a reactor may increase this quantity by at least an order of magnitude.

In connection with the small value of the magnetic potential barrier, it is necessary to consider various secondary phenomena which may affect the collection and storage of cold neutrons in a magnetic bottle. Among such effects are: the influence of gravitation; the influence of a surface potential barrier on the boundary of the body, which serves as a source of cold neutrons, and also in possible thin shells along the path of the beam; heating and absorption of neutrons in collisions with residual gas and heating associated with mechanical vibrations of the magnetic bottle and noise in the magnetic field.

The force of gravity produces a potential energy varying with height approximately by 10^{-9} ev/cm. Consequently, neutrons with an energy of 3×10^{-8} ev can rise above the surface of the mirror to a height no greater than 30 cm. In this connection, the magnetic bottle need not necessarily be closed, but can be constructed in the form of a cup or a plate with the heavy neutron gas poured into it. The gravitational potential has an essential influence on the possible arrangement of the magnetic bottle near the reactor. Obviously, one can not place the storage region for cold neutrons lower than the point where the magnetic channel meets the reactor. In this case, even the slowest neutrons would be accelerated by the gravitational field, and the potential barrier of the magnetic field could not retain them. If the magnetic channel is arranged horizontally between the reactor and the storage region, the gravitational field will have practically no effect on the process of filling the magnetic bottle with neutrons.

There is also the possibility of arranging the magnetic bottle above the reactor and connecting it with the neutron source by a vertical channel. In this case, those neutrons will reach the storage region which when they were below had a velocity sufficient for overcoming the gravitational barrier. For a sufficiently exact arrangement of the magnetic channel, the degrees of freedom corresponding to the vertical and horizontal motion in it will be separated. In motion along the channel, the vertical component will change in accordance with the increase in gravitational potential, while the distribution of horizontal velocities will not depend on height. In the lower region, neutrons which are able to reach the bottle will move within a comparatively small solid angle. In the upper portion of the channel there will be all directions of motion. Since we are considering a single passage through the channel, the effect of inaccuracies on the separation of the degrees of freedom will not be too critical.

At the boundary of a material with a coherent scattering amplitude a and an atom density N, there is a potential barrier

$$u = 2\pi\hbar^2 Na/m, \tag{30}$$

where m is the neutron mass. For most light materials this potential is of the order of 10^{-7} ev. Obviously, in the neutron spectrum emerging from a material with positive scattering amplitude there will be very few neutrons with energy below the height of the barrier, since such neutrons can be produced only in the immediate surface layer of the material. (They must have negative kinetic energy inside the body.) If along the path of the neutrons from the source, located at the surface of the reactor, there is no change in the velocity of the neutrons until they reach the bottle region, then one can use as a source of cold neutrons with energy below 10^{-7} ev only materials with negative scattering amplitude. Of the light elements only hydrogen has a negative coherent scattering amplitude.

By making use of the gravitational potential one can, as was described above, transform the neutron spectrum leaving the reactor and obtain neutrons with kinetic energy of the order 10^{-8} ev even from a source with positive scattering amplitude. So, for example, in the case of a graphite source, which has a positive potential $\sim 1.5 \times 10^{-7}$ ev, one can obtain very cold neutrons with a length of vertical magnetic channel of the order of 1.5 m and more.

The repulsive action of nuclei with positive scattering amplitude eliminates the use of even the very thinnest films of such materials in cold neutron counters. For these purposes one can use only materials with negative scattering amplitude.

Let us consider the estimates of heating of neutrons associated with vibrations of the walls of the bottle. The number of collisions of the neutron with the walls, N_{coll}, during the time of observation t is equal to

$$N_{coll} \approx vt/l, \tag{31}$$

where l is the linear dimension of the bottle, v is the velocity of the neutrons. The change in kinetic energy of the neutron is given by

$$2m^{-1}\Delta\varepsilon = \Delta\overline{v^2} = 2N_{coll}\overline{v_w^2}, \tag{32}$$

where v_W is the effective velocity of the reflecting wall which is made up of mechanical vibrations and the fluctuations in the magnetic field

$$v_{ст} = v_{vib} + k\dot{H}/H. \tag{33}$$

The important vibrations are those with periods at least as great as the collision time of the neutron with the wall

$$\tau_{\text{coll}} \approx (kv)^{-1}, \tag{34}$$

i.e., in practice noise and vibrations with frequencies no higher than several hundred cycles per second. These estimates lead to requirements on mechanical vibrations which are easily fulfilled. The admissible modulation depth for the magnetic field at standard frequencies is found to be of the order of 10^{-4}, which is completely achievable technically if we consider the filtering action of the magnetic system itself. In fact, the depth of modulation of the field is $R/L\Omega$ times less than the depth of modulation of the voltage. Even for a time constant L/R of the order of 0.3 sec and a modulation depth for the voltage of 3×10^3, which is usual for rectifying equipment, the depth of modulation of a field of frequency 50 cps will be of the order of 3×10^{-5}.

Field fluctuations with higher frequencies $\Omega > \tau_{\text{coll}}^{-1}$ will also give some contribution to the heating of the neutrons, but they are easily filtered out by the conducting walls of the vacuum chamber and can in practice be completely eliminated.

These estimates show that the losses of cold neutrons due to heating upon reflection from the walls can be made sufficiently small. An elementary computation shows that losses of neutrons because of heating and absorption in collisions with residual gas is small compared with the natural decay even for pressures of the order of 10^{-5} mm Hg. Thus apparently there is nothing to prevent a direct determination of the neutron lifetime by using magnetic bottles, except for the relatively small density $10^4 - 10^5$ m^{-3} and the difficulties associated with background in the neighborhood of the reactor.

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