$$f(x,t) = \xi^t e^{ikx}$$

CFL: $\frac{v\,dt}{dx} \leq 1$

$$\frac{f(t+dt, x) - f(t-dt, x)}{2dt} = -v\,\frac{f(t, x+dx) - f(t, x-dx)}{2dx}$$

$$\frac{\xi^{t+dt} e^{ikx} - \xi^{t-dt} e^{ikx}}{dt} = -v\,\frac{\xi^t e^{ik(x+dx)} - \xi^t e^{ik(x-dx)}}{dx}$$

$$\xi^t e^{ikx}\,\frac{(\xi^{dt} - \xi^{-dt})}{dt} = -v\,\xi^t e^{ikx}\,\frac{(e^{ikdx} - e^{-ikdx})}{dx}$$

$$\xi^{dt} - \xi^{-dt} = -\frac{v\,dt}{dx}\,2i\sin(k\,dx)$$

solve

$$\xi^{dt} = \frac{1}{2}\left(-2i\,\frac{v\,dt}{dx}\sin(k\,dx) \pm 2\sqrt{1 - \left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2}\right)$$

take absolute squared

$$|\xi^{dt}|^2 = \left[\sqrt{\left(\sqrt{1 - \left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2}\right)^2 + \left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2}\,\right]^2$$

$$= 1 - \left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2 + \left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2$$

$$= \underline{1}$$

As the magnitude of the amplitude is constant, energy is conserved