Lenses, Folds, and Traversals

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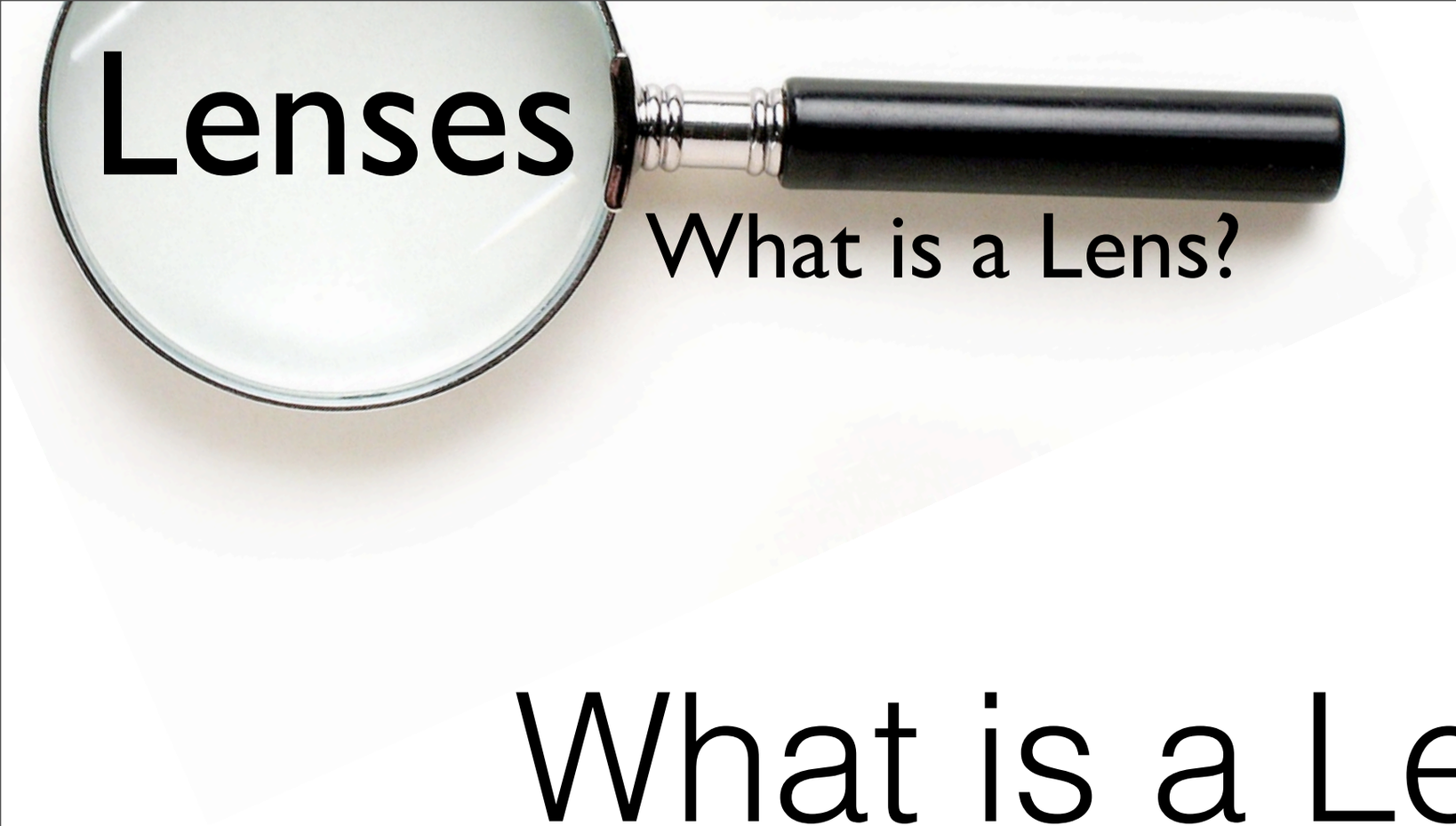
Lenses

Overview

- What is a Lens?
- “The Power is in the Dot”
  - Semantic Editor Combinators
  - Setters
  - Traversals
  - Folds
  - Lenses
  - Getters
  - Overloading Application
- Extras
  - Uniplate
  - Zippers



What is a Lens?

Lenses

What is a Lens?

Costate Comonad Coalgebra
is equivalent of Java's
member variable update
technology for Haskell

@PLT_Borat

Lenses

What is a Lens?

```
data Lens s a = Lens { set  :: s -> a -> s
                     , view :: s -> a
                     }

view :: Lens s a -> s -> a
set  :: Lens s a -> s -> a -> s
```

Laws:

```
1.) set l (view l s) s = s
2.) view l (set l s a) = a
3.) set l (set l s a) b = set l s b
```

Lenses

What is a Lens?

```
data Lens s a = Lens { set  :: s -> a -> s
                     , view :: s -> a
                     }
```

Lenses

What is a Lens?

```
data Lens s a = Lens { set  :: s -> a -> s
                     , view :: s -> a
                     }
```

Fusion (and data-lens).

```
data Lens s a = Lens (s -> (a -> s, a))
```

Lenses

What is a Lens?

```
data Lens s a = Lens { set  :: s -> a -> s
                     , view :: s -> a
                     }
```

Fusion (and data-lens).

```
data Lens s a = Lens (s -> (a -> s, a))

data Store s a = Store (s -> a) s

data Lens s a = Lens (s -> Store a s)
```

Lenses

What is a Lens?

```
newtype Lens s a = Lens (s -> Store a s)

data Store s a = Store (s -> a) s

instance Category Lens where
  id = Lens (Store id)
  Lens f . Lens g = Lens $ \r -> case g r of
    Store sr s -> case f s of
      Store ts t -> Store (sr . ts) t
```

The Power is in the Dot

Lenses

The Power is in the Dot

```
(.)          :: (a -> b) -> (c -> a) -> c -> b
```

Lenses

The Power is in the Dot

```
(.)          :: (a -> b) -> (c ->          a) -> c          -> b
```

Lenses

The Power is in the Dot

```
(.)        :: (a -> b) -> (c ->          a) -> c          -> b
(.).(.)    :: (a -> b) -> (c -> d ->     a) -> c -> d     -> b
```

Lenses

The Power is in the Dot

```
(.)         :: (a -> b) -> (c ->           a) -> c           -> b
(.).(.)     :: (a -> b) -> (c -> d ->      a) -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

Lenses

The Power is in the Dot

```
(.)         :: (a -> b) -> (c ->           a) -> c           -> b
(.).(.)     :: (a -> b) -> (c -> d ->      a) -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b

fmap           :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

Semantic Editor Combinators

Lenses

Semantic Editor Combinators

```
(.)         :: (a -> b) -> (c ->           a) -> c           -> b
(.).(.)     :: (a -> b) -> (c -> d ->      a) -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b

fmap           :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

These are sometimes known as **Semantic Editor Combinators**.

```
result = (.)

element = fmap

second = fmap

first f (a,b) = (f a, b)
```

Lenses

Semantic Editor Combinators

```
(.)         :: (a -> b) -> (c ->           a) -> c           -> b
(.).(.)     :: (a -> b) -> (c -> d ->      a) -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b

fmap           :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

These are sometimes known as **Semantic Editor Combinators**.

```
type SEC s t a b = (a -> b) -> s -> t

result :: SEC (e -> a) (e -> b) a b
result = (.)
element :: SEC [a] [b] a b
element = fmap
second :: SEC (c,a) (c,b) a b
second = fmap
first :: SEC (a,c) (b,c) a b
first f (a,b) = (f a, b)
```

Lenses

Semantic Editor Combinators

```
(.)         :: (a -> b) -> (c ->           a) -> c           -> b
(.).(.)     :: (a -> b) -> (c -> d ->      a) -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b

fmap           :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

These are sometimes known as **Semantic Editor Combinators**.

```
type SEC s t a b = (a -> b) -> s -> t

fmap :: Functor f => SEC (f a) (f b) a b
```

```
first :: SEC (a,c) (b,c) a b
first f (a,b) = (f a, b)
```

Setters

Lenses

Setters

```
(.)         :: (a -> b) -> (c ->           a) -> c           -> b
(.).(.)     :: (a -> b) -> (c -> d ->      a) -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b

fmap           :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
traverse                    :: (Traversable f, Applicative m)
  => (a -> m b) -> f a          -> m (f b)
traverse.traverse           :: (Traversable f, Traversable g, Applicative m)
  => (a -> m b) -> f (g a)      -> m (f (g b))
traverse.traverse.traverse :: (Traversable f, Traversable g, Traversable h, Applicative m)
  => (a -> m b) -> f (g (h a)) -> m (f (g (h b)))
```

Lenses

Setters

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
class (Functor f, Foldable f) => Traversable f where
  traverse :: Applicative m => (a -> m b) -> f a -> m (f b)
  ...
```

Lenses

Setters

```
fmap        :: Functor f     => (a -> b) -> f a -> f b
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

Lenses

Setters

```
fmap        :: Functor f     => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

Lenses

Setters

```
fmap        :: Functor f     => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)

newtype Identity a = Identity { runIdentity :: a }

instance Functor Identity where
  fmap f (Identity a) = Identity (f a)

instance Applicative Identity where
  pure = Identity
  Identity f <*> Identity x = Identity (f x)
```

Lenses

Setters

```
fmap           :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))

fmap        :: Functor f     => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)

newtype Identity a = Identity { runIdentity :: a }

instance Functor Identity where
  fmap f (Identity a) = Identity (f a)

instance Applicative Identity where
  pure = Identity
  Identity f <*> Identity x = Identity (f x)
```

Lenses

Setters

```
fmap            :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap       :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap  :: (Functor f, Functor g, Functor h)  => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)
```

```
over l f = runIdentity . l (Identity . f)
```

Lenses

Setters

```
fmap           :: Functor f                        => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)           => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)
```

```
over l f = runIdentity . l (Identity . f)

over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f
```

Lenses

Setters

```
fmap           :: Functor f                        => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)           => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)

over :: (a -> Identity b) -> s -> Identity t) -> (a -> b) -> s -> t
over l f = runIdentity . l (Identity . f)

over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f
```

Lenses

Setters

```
fmap           :: Functor f                        => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)           => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)

over :: Setter s t a b -> (a -> b) -> s -> t
over l f = runIdentity . l (Identity . f)

over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f

type Setter s t a b = (a -> Identity b) -> s -> Identity t
```

Lenses

Setters

```
mapped                :: Functor f => Setter (f a) (f b) a b
mapped.mapped         :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)

over :: Setter s t a b -> (a -> b) -> s -> t
over l f = runIdentity . l (Identity . f)

over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f

type Setter s t a b = (a -> Identity b) -> s -> Identity t

mapped :: Functor f => Setter (f a) (f b) a b
mapped f = Identity . fmap (runIdentity . f)

over mapped f = runIdentity . mapped (Identity . f)
              = runIdentity . Identity . fmap (runIdentity . Identity . f)
              = fmap f
```

Lenses

Setters

```
mapped                :: Functor f => Setter (f a) (f b) a b
mapped.mapped         :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b

over :: Setter s t a b -> (a -> b) -> s -> t

over mapped :: Functor f => (a -> b) -> f a -> f b
over (mapped.mapped) :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
...

chars :: (Char -> Identity Char) -> Text -> Identity Text
chars f = fmap pack . mapped f . unpack

over chars :: (Char -> Char) -> Text -> Text
over (mapped.chars) :: Functor f => (Char -> Char) -> f Text -> f Text
over (traverse.chars) :: Traversable f => (Char -> Char) -> f Text -> f Text
```

Lenses

Setters

```
mapped                :: Functor f => Setter (f a) (f b) a b
mapped.mapped         :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b

over :: Setter s t a b -> (a -> b) -> s -> t

over mapped :: Functor f => (a -> b) -> f a -> f b
over (mapped.mapped) :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
...

chars :: (Char -> Identity Char) -> Text -> Identity Text
chars f = fmap pack . mapped f . unpack

over chars :: (Char -> Char) -> Text -> Text
over (mapped.chars) :: Functor f => (Char -> Char) -> f Text -> f Text
over (traverse.chars) :: Traversable f => (Char -> Char) -> f Text -> f Text
```

Lenses

Setters

```
mapped                :: Functor f => Setter (f a) (f b) a b
mapped.mapped         :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b

over :: Setter s t a b -> (a -> b) -> s -> t
```

Functor Laws:

1.) fmap id = id
2.) fmap f . fmap g = fmap (f . g)

Setter Laws for a legal Setter *l*.

1.) over *l* id = id
2.) over *l* f . over *l* g = over *l* (f . g)

```
both :: Setter (a,a) (b,b) a b
both f (a,b) = (,) <$> f a <*> f b

first :: Setter (a,c) (b,c) a b
first f (a,b) = (,b) <$> f a
```

Lenses

Setters (are like Functors)

```
type Simple f s a = f s s a a

sets :: ((a -> b) -> s -> t) -> Setter s t a b
mapped :: Functor f => Setter (f a) (f b) a b

over, mapOf, (%~) :: Setter s t a b -> (a -> b) -> s -> t
set, (.~) :: Setter s t a b -> b -> s -> t
(+~), (-~), (*~) :: Num c => Setter s t c c -> c -> s -> t
(//~) :: Fractional c => Setter s t c c -> c -> s -> t
(||~), (&&~) :: Setter s t Bool Bool -> Bool -> s -> t

assign :: MonadState s m => Setter s s a b -> b -> m ()
(.=) :: MonadState s m => Setter s s a b -> b -> m ()
(%=) :: MonadState s m => Setter s s a b -> (a -> b) -> m ()
(+=), (-=), (*=) :: (MonadState s m, Num a) => Simple Setter s a -> a -> m ()
(//=) :: (MonadState s m, Fractional a) => Simple Setter s a -> a -> m ()
(||=), (&&~) :: MonadState s m => Simple Setter s Bool -> Bool -> m ()
```

Traversals

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t

traverse           :: Traversable f => Traversal (f a) (f b) a b
traverse.traverse :: (Traversable f, Traversable g) => Traversal (f (g a)) (f (g b)) a b

over traverse f = runIdentity . traverse (Identity . f)
               = fmapDefault f
               = fmap f
```

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t

mapM :: (Traversable f, Monad m) => (a -> m b) -> f a -> m (f b)
mapM f = unwrapMonad . traverse (WrapMonad . f)

mapMOf :: Monad m => Traversal s t a b -> (a -> m b) -> s -> m t
mapMOf l f = unwrapMonad . l (WrapMonad . f)

traverse          :: Traversable f => Traversal (f a) (f b) a b
traverse.traverse :: (Traversable f, Traversable g) => Traversal (f (g a)) (f (g b)) a b

over traverse f = runIdentity . traverse (Identity . f)
               = fmapDefault f
               = fmap f

mapMOf traverse f = unwrapMonad . traverse (WrapMonad . f)
                  = mapM f
```

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
```

Traversable Laws:

1.) traverse pure = pure
2.) Compose . fmap (traverse f) . traverse g ≡ traverse (Compose . fmap f . g)

Laws for a valid Traversal l:

1.) l pure = pure
2.) Compose . fmap (l f) . l g = l (Compose . fmap f . g)

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t

traverse :: Traversable f => Traversal (f a) (f b) a b

both :: Traversal (a,a) (b,b) a b
both f (a,b) = (,) <$> f a <*> f b

traverseLeft :: Traversal (Either a c) (Either b c) a b
traverseLeft f (Left a) = Left <$> f a
traverseLeft f (Right c) = pure (Right c)

traverseRight :: Traversal (Either c a) (Either c b) a b
traverseRight f (Left c) = pure (Left c)
traverseRight f (Right a) = Right <$> f a

traverseLeft.both :: Traversal (Either (a,a) c) (Either (b,b) c) a b

>>> over (traverseLeft.both) (+1) $ Left (2,3)
Left (3,4)

(.~) = over
traverseLeft.both .~ (+1) $ Left (2,3)
traverseLeft.both +~ 1 $ Left (2,3)
```

Lenses

Traversals (are like Traversables)

```
type LensLike f s t a b = (a -> f b) -> s -> f t
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t

traverse :: Traversable t => Traversal (t a) (t b) a b
ignored :: Traversal s s a b
traverseLeft :: Traversal (Either a c) (Either b c) a b
traverseRight :: Traversal (Either c a) (Either c b) a b
both :: Traversal (a, a) (b, b) a b

traverseOf :: LensLike f s t a b -> (a -> f b) -> s -> f t
forOf :: LensLike f s t a b -> s -> (a -> f b) -> f t
mapMOf :: LensLike (WrappedMonad m) s t a b -> (a -> m b) -> s -> m t
forMOf :: LensLike (WrappedMonad m) a b c d -> s -> (a -> m b) -> m t
```

Folds

Lenses

Folds

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Foldable f where
  foldMap :: Monoid m => (a -> m) -> f a -> m
  ...

class (Functor f, Foldable f) => Traversable f where
  traverse :: Applicative m => (a -> m b) -> f a -> m (f b)
  ...
```

Lenses

Folds

```
(.)         :: (a -> b) -> (c -> a)          -> c           -> b
(.).(.)     :: (a -> b) -> (c -> d -> a)     -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b

fmap           :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap      :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))

foldMap                 :: (Foldable f, Monoid m)
  => (a -> m) -> f a         -> m
foldMap.foldMap         :: (Foldable f, Foldable g, Monoid m)
  => (a -> m) -> f (g a)     -> m
foldMap.foldMap.foldMap :: (Foldable f, Foldable g, Foldable h, Monoid m)
  => (a -> m) -> f (g (h a)) -> m
```

Lenses

Folds

```
(.)          :: (a -> b) -> (c -> a)           -> c           -> b
(.).(.)      :: (a -> b) -> (c -> d -> a)      -> c -> d      -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b

fmap            :: Functor f                          => (a -> b) -> f a         -> f b
fmap.fmap       :: (Functor f, Functor g)             => (a -> b) -> f (g a)     -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))

foldMap                  :: (Foldable f, Monoid m)
  => (a -> m) -> f a         -> m
foldMap.foldMap          :: (Foldable f, Foldable g, Monoid m)
  => (a -> m) -> f (g a)     -> m
foldMap.foldMap.foldMap :: (Foldable f, Foldable g, Foldable h, Monoid m)
  => (a -> m) -> f (g (h a)) -> m

traverse                    :: (Traversable f, Applicative m)
  => (a -> m b) -> f a         -> m (f b)
traverse.traverse           :: (Traversable f, Traversable g, Applicative m)
  => (a -> m b) -> f (g a)     -> m (f (g b))
traverse.traverse.traverse :: (Traversable f, Traversable g, Traversable h, Applicative m)
  => (a -> m b) -> f (g (h a)) -> m (f (g (h b)))
```

Lenses

Folds

```
fmap        :: Functor f     => (a -> b) -> f a -> f b

foldMap        :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m

traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

Lenses

Folds

```
fmap        :: Functor f     => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b

foldMap        :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m

traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

Lenses

Folds

```
fmap        :: Functor f     => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)

foldMap        :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m
foldMapDefault f = getConst . traverse (Const . f)

traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)

newtype Const m a = Const { getConst :: m }

instance Functor (Const m) where
  fmap _ (Const m) = Const m

instance Monoid m => Applicative (Const m) where
  pure _ = Const mempty
  Const m <*> Const n = Const (m <> n)
```

Lenses

Folds

```
foldMap        :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m
foldMapDefault f = getConst . traverse (Const . f)

foldMapOf l f = getConst . l (Const . f)

folded :: Foldable f => Fold (f a) a
folded.folded :: (Foldable f, Foldable g) => Fold (f (g a)) a
folded.folded.folded :: (Foldable f, Foldable g, Foldable h) => Foldable (f (g (h a))) a

folded f = Const . foldMap (getConst . f)

foldMapOf folded f = getConst . Const . foldMap (getConst . Const . f)
                   = foldMap f

view l = getConst . l Const

view folded = getConst . Const . foldMap (getConst . Const)
            = foldMap id
            = fold
```

```
type Fold s a = forall m. (a -> Const m a) -> s -> Const m s
```

Lenses

Folds

```
foldMapOf l f = getConst . l (Const . f)
anyOf l f = getAny . foldMapOf l (Any . f)
sumOf l = getSum . foldMap Sum

>>> sumOf both (10,20)
30

>>> sumOf (traverse.both) [(10,20),(30,40)]
100
```

More Than a Traversal

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t

type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s


view l = getConst . l Const


foldMapOf l f = getConst . l (Const . f)


toListOf l = getConst . l (\c -> Const [c])


_1 f (a,b) = (,b) <$> f a

>>> view _1 (10,20)
10

_1 :: Traversal (a,c) (b,c) a b
>>> view _1 (10,20)
No instance of Monoid for a
No instance of Num for a
```

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t
type Lens s t a b      = forall f. Functor f     => (a -> f b)        -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s
type Getting r s t a b =                            (a -> Const r b)  -> s -> Const r t

view :: Getting a s t a b -> s -> a
view l = getConst . l Const

foldMapOf :: Getting m s t a b -> (a -> m) -> s -> m
foldMapOf l f = getConst . l (Const . f)

toListOf :: Getting [a] s t a b -> s -> [a]
toListOf l = getConst . l (\a -> Const [a])

_1 :: Lens (a,c) (b,c) a b
_1 f (a,b) = (,b) <$> f a

>>> view _1 (10,20)
10
```

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t
type Lens s t a b      = forall f. Functor f     => (a -> f b)        -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s
type Getting r s t a b =                            (a -> Const r b)  -> s -> Const r t

view :: Getting a s t a b -> s -> a
view l = getConst . l Const

set :: Setter s t a b -> b -> s -> t
set l d = runIdentity . l (Identity . const d)

lens :: (s -> a) -> (b -> s -> t) -> Lens s t a b
lens sa bst afb s = (`bst` s) <$> afb (sa s)
```

A Lens updates a *single* target. A Traversal can update (and read from) *many*.

A lens is just a valid traversal that targets a single element by only using the Functor out of whatever instance it is supplied.

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t
type Lens s t a b      = forall f. Functor f     => (a -> f b)        -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s
type Getting r s t a b =                            (a -> Const r b)  -> s -> Const r t

_1 :: Lens (a,c) (b,c) a b
_1 f (a,b) = (,b) <$> f a

_2 :: Lens (c,a) (c,b) a b
_2 f (a,b) = (a,) <$> f b

at :: Ord k => k -> Simple Lens (Map k a) a
at

traverse._1 :: Applicative m => (a -> m b) -> [(a,c)] -> m [(a,c)]

>>> over (traverse._1) length $ [("hello","San"),("Francisco",":)")]
[(5,"San"),(9,":)"]

view _1 (10,20)
10
```

Lenses

Folds (are like Foldables)

```
type Getting r s t a b = (a -> Const r b) -> s -> Const r t

(^?) :: s -> Getting (First a) s t a b -> Maybe a
(^..) :: s -> Getting [a] s t a b -> [a]
folding :: (Foldable f, Applicative g, Gettable g) => (s -> f a) -> LensLike g s t a b
folded :: Foldable f => Fold (f a) a

unfolded :: (s -> Maybe (a, s)) -> Fold s a
iterated :: (a -> a) -> Fold a a
backwards :: LensLike (Backwards f) s t a b -> LensLike f s t a b
repeated :: Fold a a
replicated :: Int -> Fold a a
takingWhile :: (Gettable f, Applicative f)
            => (a -> Bool) -> Getting (Endo (f s)) s s a a -> LensLike f s s a a

foldMapOf :: Getting r s t a b -> (a -> r) -> s -> r
foldOf :: Getting a s t a b -> s -> a
foldrOf :: Getting (Endo r) s t a b -> (a -> r -> r) -> r -> s -> r
toListOf :: Getting [a] s t a b -> s -> [a]
anyOf :: Getting Any s t a b -> (a -> Bool) -> s -> Bool
traverseOf_ :: Functor f => Getting (Traversed f) s t a b -> (a -> f r) -> s -> f ()
```

Getters

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t
type Lens s t a b      = forall f. Functor f     => (a -> f b)        -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s
type Getting r s t a b =                            (a -> Const r b)  -> s -> Const r t
type Getter s a        = forall r.                  (a -> Const r a)  -> s -> Const r s

to :: (s -> a) -> Getter s a
-- to :: (s -> a) -> (a -> Const r a) -> s -> Const r s
```

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t
type Lens s t a b      = forall f. Functor f     => (a -> f b)        -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s
type Getting r s t a b =                            (a -> Const r b)  -> s -> Const r t
type Getter s a        = forall r.                  (a -> Const r a)  -> s -> Const r s

to :: (s -> a) -> Getter s a
-- to :: (s -> a) -> (a -> Const r a) -> s -> Const r s

to_ :: (s -> a) -> (a -> r) -> s -> r
to_ f g = g . f
```

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t
type Lens s t a b      = forall f. Functor f     => (a -> f b)        -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s
type Getting r s t a b =                            (a -> Const r b)  -> s -> Const r t
type Getter s a        = forall r.                  (a -> Const r a)  -> s -> Const r s

to :: (s -> a) -> Getter s a
to f g = Const . getConst . g . f

view :: Getting a s t a b -> s -> a
view l = getConst . l Const

view (to f) = getConst . to f Const
            = getConst . Const . getConst . Const . f
            = f
```

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)        -> s -> f t
type Lens s t a b      = forall f. Functor f     => (a -> f b)        -> s -> f t
type Setter s t a b    =                            (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m      => (a -> Const m a)  -> s -> Const m s
type Getting r s t a b =                            (a -> Const r b)  -> s -> Const r t
type Getter s a        = forall r.                  (a -> Const r a)  -> s -> Const r s

to :: (s -> a) -> Getter s a
to f g = Const . getConst . g . f

view :: Getting a s t a b -> s -> a
view l = getConst . l Const

view (to f) = getConst . to f Const
            = getConst . Const . getConst . Const . f
            = f
```

Lenses

Getters (are like functions)

```
class Functor f => Gettable f where
  coerce :: f a -> f b

type Getter a c = forall f. Gettable f => (c -> f c) -> a -> f a
newtype Accessor r a = Accessor { runAccessor :: r }

type Getting r a b c d = (c -> Accessor r d) -> a -> Accessor r b

to :: (s -> a) -> Getter s a
to f g = coerce . g . f

(^.) :: s -> Getting a s t a b -> a
view, (^$) :: Getting a s t a b -> s -> a

use :: MonadState s m => Getting a s t a b -> m a
```

Lenses

Lenses (Both Getter and Traversal)

```
type Lens s t a b = forall f. Functor f => (a -> f b) -> s -> f t

lens     :: (s -> a) -> (s -> b -> t) -> Lens s t a b
resultAt :: Eq e => e -> Simple Lens (e -> a) a
chosen   :: Lens (Either a a) (Either b b) a b

(<+~), (<-~), (<*~) :: Num a => LensLike ((,) a) s t a a -> a -> s -> (a, t)
(<//~)              :: Fractional a => LensLike ((,) a) s t a a -> a -> s -> (a, t)
(<||~), (<&&~)      :: LensLike ((,) Bool) s t Bool Bool -> Bool -> s -> (Bool, t)
(%%~)               :: LensLike f s t a b -> (a -> f b) -> s -> f t

(<+=),(<-=),(<*=) :: (MonadState s m, Num a) => SimpleLensLike ((,) a) s a -> a -> m a
(<//=)            :: (MonadState s m, Fractional a) => SimpleLensLike ((,) a) s a -> a -> m a
(<||=),(<&&=)     :: MonadState s m => SimpleLensLike ((,) Bool) s Bool -> Bool -> m Bool
(%%=)             :: MonadState s m => LensLike ((,) r) s s a b -> (a -> (r, b)) -> m r
```

Isomorphisms

Lenses

Fun Overloading

```
class Category k => Isomorphic k where
  isomorphic :: (a -> b) -> (b -> a) -> k a b

instance Isomorphic (->) where
  isomorphic = const

data Isomorphism a b = Isomorphism (a -> b) (b -> a)

instance Category Isomorphism where
  id = Isomorphism id id
  Isomorphism bc cb . Isomorphism ab ba = Isomorphism (bc . ab) (ba . cb)

instance Isomorphic Isomorphism where
  isomorphic = Isomorphism

type a <-> b = forall k. Isomorphic k => k a b

from :: Isomorphism a b -> b <-> a
from (Isomorphism a b) = isomorphic b a
```

Lenses

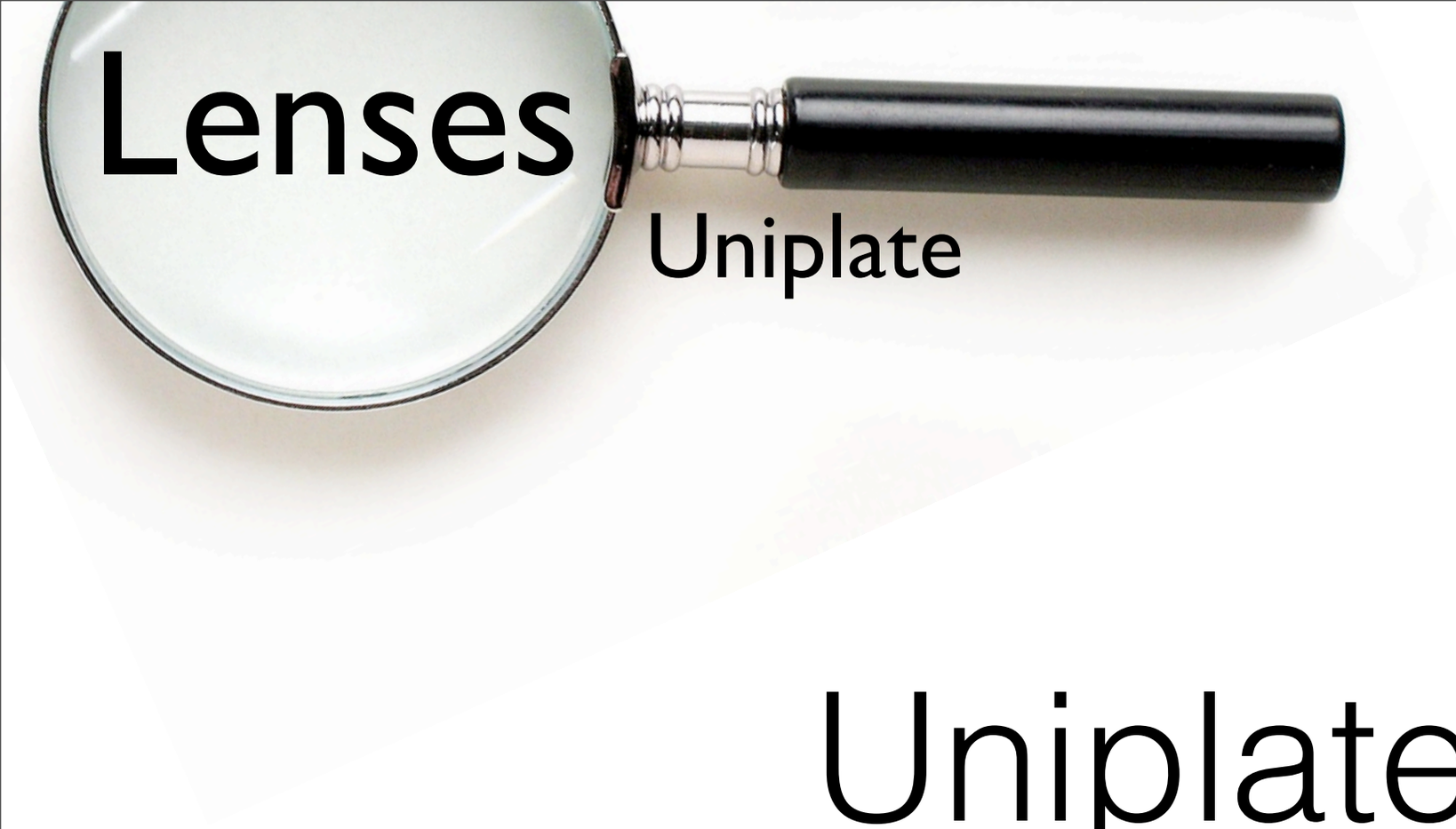
Fun Overloading

```
class Category k => Isomorphic k where
  isomorphic :: (a -> b) -> (b -> a) -> k a b

inc :: Num a => a -> a <-> a
inc = isomorphic (+1) (subtract 1)

>>> inc 4
5

>>> from inc 5
4
```

Lenses

Fun Overloading

```
type Traversal s t a b = forall f. Applicative f                => (a -> f b) -> s -> f t
type Lens s t a b      = forall f. Functor f                    => (a -> f b) -> s -> f t
type Iso s t a b       = forall k f. (Isomorphic k, Functor f) => k (a -> f b) (s -> f t)

iso :: (a -> b) -> (b -> a) -> Simple Iso a b
iso ab ba = isos ab ba ab ba

isos :: (a -> c) -> (c -> a) -> (b -> d) -> (d -> b) -> Iso a b c d
isos ac ca bd db = isomorphic (\cfd a -> db <$> cfd (ac a))
                              (\afb c -> bd <$> afb (ca c))

packed :: Simple Iso String Text
packed = iso pack unpack

text :: Simple Traversal Text Char
text = from packed . traverse

>>> anyOf (both.text) (=='c') ("chello"^.packed,"world"^.packed)
True
```



Uniplate

Lenses

Uniplate

```
class Plated a where
  plate :: Simple Traversal a a
  plate = ignored

instance Plated (Tree a) where
  plate f (Node a as) = Node a <$> traverse f as

template :: (Data a, Typeable b) => Simple Traversal a b
uniplate :: Data a => Simple Traversal a a
biplate :: (Data a, Typeable b) => Simple Traversal a b

children :: Plated a => a -> [a]
children = toListOf plate

rewriteOf :: Simple Setter a a -> (a -> Maybe a) -> a -> a
rewriteOf l f = go where go = transformOf l (\x -> maybe x go (f x))

contextsOf     :: SimpleLensLike (Bazaar a a) a a -> a -> [Context a a a]
holesOf        :: LensLike (Bazaar c c) s t c c -> s -> [Context c c t]
paraOf         :: Getting [a] a b a b -> (a -> [r] -> r) -> a -> r
partsOf        :: LensLike (Bazaar c c) s t c c -> Lens s t [c] [c]
unsafePartsOf :: LensLike (Bazaar c d) s t c d -> Lens s t [c] [d]
```

Zippers

Lenses

Zippers

```
zipper ("hello","world")
  % down _1
  % fromWithin traverse
  % focus .~ 'J'
  % rightmost
  % focus .~ 'y'
  % rezip
("Jelly","world")
```

```
zipper :: a -> Top :> a
up :: (a :> b :> c) -> a :> b
down :: Simple Lens b c -> (a :> b) -> a :> b :> c
within :: Simple Traversal b c -> (a :> b) -> Maybe (a :> b :> c)
fromWithin :: Simple Traversal b c -> (a :> b) -> a :> b :> c

left, right  :: (a :> b) -> Maybe (a :> b)
lefts, rights :: Int -> (h :> a) -> Maybe (h :> a)
leftmost, rightmost :: (a :> b) -> a :> b

save :: (a :> b) -> Tape (a :> b)
restore :: Tape (h :> a) -> Zipped h a -> Maybe (h :> a)
```

Lenses

References

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- T. van Laarhoven. Talk on Lenses http://twanvl.nl

Lenses

Questions

Any
Questions

?

Extra Slides

Lenses

The Power is in the Dot

```
fmap        :: Functor f     => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)

foldMap        :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m
foldMapDefault f = getConst . traverse (Const . f)

traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)

newtype Const m a = Const m

instance Functor (Const m) where
  fmap _ (Const m) = Const m

instance Monoid m => Applicative (Const m) where
  pure _ = Const mempty
  Const m <*> Const n = Const (m <> n)
```

Lenses

Isomorphism-Based Lenses

```
data a ↔ b
  = Iso (a -> b) (b -> a)

newtype Lens a b =
  forall c. Lens (a ↔ (c,b))
```

Lenses

Isomorphism-Based Lenses

```
data a ↔ b
  = Iso (a -> b) (b -> a)

newtype Lens a b =
  forall c. Lens (a ↔ (c,b))

(%%~) :: Functor f =>
         Lens a b -> (b -> f b) -> a -> f a
Lens (Iso to fro) f a = case to a of
  (c, b) -> fro . ((,) c) <$> f b
```

Lenses

Isomorphism-Based Lenses

```
data a ↔ b
  = Iso (a -> b) (b -> a)

newtype Lens a b =
  forall c. Lens (a ↔ (c, b))

(%%~) :: Functor f =>
         Lens a b -> (b -> f b) -> a -> f a
Lens (Iso to fro) f a = case to a of
  (c, b) -> fro . ((,) c) <$> f b
```

Lenses

van Laarhoven Lenses

```
type Lens a b = forall f.
  Functor f => (b -> f b) -> a -> f a

(%%~) :: Lens a b -> (b -> f b) -> a -> f a
(%%~) = id
```

Lenses

van Laarhoven Lenses

```
type Lens a b = forall f.
  Functor f => (b -> f b) -> a -> f a

(%%~) :: Lens a b -> (b -> f b) -> a -> f a
(%%~) = id

(.~) :: Lens a b -> b -> a -> a
l .~ b = runIdentity . l (Identity . const b)

(^.) :: a -> Lens a b -> b
a ^. l = getConst (l Const a)
```

Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
  Functor f => (c -> f d) -> a -> f b

(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
(%%~) = id

(.~) :: Lens a b c d -> d -> a -> b
l .~ b = runIdentity . l (Identity . const b)

(^.) :: a -> Lens a b c d -> c
a ^. l = getConst (l Const a)
```

Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
  Functor f => LensLike f a b c d

type LensLike f a b c d =
  (c -> f d) -> a -> f b

(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
(%%~) = id

(.~) :: Lens a b c d -> d -> a -> b
l .~ b = runIdentity . l (Identity . const b)

(^.) :: a -> Lens a b c d -> c
a ^. l = getConst (l Const a)
```

Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
  Functor f => LensLike f a b c d

type LensLike f a b c d =
  (c -> f d) -> a -> f b

(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
(%%~) = id

(.~) :: Lens a b c d -> d -> a -> b
l .~ b = runIdentity . l (Identity . const b)

(^.) :: a -> Lens a b c d -> c
a ^. l = getConst (l Const a)
```

Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
  Functor f => LensLike f a b c d

type LensLike f a b c d =
  (c -> f d) -> a -> f b

(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
(%%~) = id

(.~) :: LensLike Identity a b c d -> d -> a -> b
l .~ b = runIdentity . l (Identity . const b)

(^.) :: a -> LensLike (Const c) a b c d -> c
a ^. l = getConst (l Const a)
```

Lenses

Indexed Lenses, Traversals, etc.

```
class Indexed i k where
  index :: ((i -> a) -> b) -> k a b

type Indexable i a b = forall k. Indexed i k => k a b

instance Indexed i (->) where
  index f = f . const

newtype Index i a b = Index { withIndex :: (i -> a) -> b }

instance i ~ j => Indexed i (Index j) where
  index = Index

indexed :: Indexed Int k => LensLike (Indexing f) a b c d) -> k (c -> f d) (a -> f b)
```

Lenses

Indexed Lenses, Traversals, etc.

```
type Overloaded k f a b c d = k (c -> f d) (a -> f b)

-- turn a normal traversal or setter into an indexed traversal or setter.
indexed :: Indexed Int k => LensLike (Indexing f) a b c d) -> k (c -> f d) (a -> f b)
class At k m | m -> k where
  at :: k -> SimpleIndexedLens k (m v) (Maybe v)

traverseAt :: At k m => k -> SimpleIndexedTraversal k (m v) v
value :: (k -> Bool) -> SimpleIndexedTraversal k (k, v) v
iwhereOf :: (Indexed i k, Applicative f) =>
  Overloaded (Index i) f a b c c -> (i -> Bool) -> Overloaded k f a b c c

... and there are similarly a ton of combinators for these, too...
```

Lenses
Overview

Overview

What are Lenses?

The Power is in the Dot

Families of Generalized van Laarhoven Lenses

General Purpose Combinators

Indexed Traversals (and Isomorphisms)

Uniplate

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