COMP3121
Assignment 2
A17S1N2

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Question 1

a)

$$\text{let } A^n = \begin{pmatrix} F(n+1) & F(n) \\ F(n) & F(n-1) \end{pmatrix} \tag{1}$$

When $n = 1$

$$A^1 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

Assume $n = k$

$$A^k = \begin{pmatrix} F(k+1) & F(k) \\ F(k) & F(k-1) \end{pmatrix} \tag{3}$$

let $n = k + 1$

$$\begin{aligned} A^{k+1} &= AA^k & (4) \\ &= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F(k+1) & F(k) \\ F(k) & F(k-1) \end{pmatrix} & (5) \\ &= \begin{pmatrix} F(k+1)+F(k) & F(k)+F(k-1) \\ F(k+1) & F(k) \end{pmatrix} & (6) \\ &= \begin{pmatrix} F(k+2) & F(k+1) \\ F(k+1) & F(k) \end{pmatrix} & (7) \end{aligned}$$

Therefore the forumla is true by induction for all $n > 0$.

b) $F(n)$ can be found in $log_2(n)$ matrix multiplications using a recursive algorithm

```
matrix = ((1, 1), (1,0))
func(n):
    if n == 1:
        return matrix
    if n is even:
        return func(n/2)^2
    if n is odd:
        return func(n-1) * matrix
```

Question 2

a) First we calculate the Karastuba trick.

$$(a+b)(c+d) = ac + ad + bd + bc \tag{1}$$
$$ad + bd = (a+b)(c+d) - ac - bc \tag{2}$$

Then we substitute (2) into (4)

$$(a+ib)(c+id) = ac + adi + bdi + bc \tag{3}$$
$$= ac + i(ad + bd) + bc \tag{4}$$
$$= ac + i((a+b)(c+d) - ac - bc) + bc \tag{5}$$

b) First we calculate the Karastuba trick.

$$(a+b)^2 = a^2 + b^2 + 2ab \tag{1}$$
$$2ab = (a+b)^2 - a^2 - b^2 \tag{2}$$

Then we substitute (2) into (3)

$$(a+ib)^2 = a^2 + 2abi - b^2 \tag{3}$$
$$= a^2 + i((a+b)^2 - a^2 - b^2) - b^2 \tag{4}$$

c) By re-arranging by the laws of exponents:

$$(a+ib)^2(c+id)^2 = ((a+ib)(c+id))^2$$

thus from above, we then calculate the middle multiplication using 3 real number multiplications, and then we find the square as above using 2 more real number multiplications.

Question 3

Expand $P(x)$ and $Q(x)$ as follows.

$$P(x) = a_0 + x^{17}(a_{17} + a_{19}x^2 + a_{21}x^4 + a_{23}x^6)$$
$$Q(x) = b_0 + x^{17}(b_{17} + b_{19}x^2 + b_{21}x^4 + b_{23}x^6)$$

let $y = x^2$ so that

$$R_a(y) = a_{17} + a_{19}y + a_{21}y^2 + a_{23}y^3$$
$$R_b(y) = b_{17} + b_{19}y + b_{21}y^2 + b_{23}y^3$$

then

$$P(x)Q(x) = a_0b_0 + x^{17}(a_0R_b(x^2) + b_0R_a(x^2)) + x^{34}R_a(x^2)R_b(x^2)$$

through brute force, we then calculate

a_0b_0 to require 1 multiplication

$a_0R_b(x^2)$ and $b_0R_a(x^2)$ to require 4 multiplcations each

then to multiply $R_a(x^2)R_b(x^2)$ (of degree 3), we require $2(3) + 1 = 7$ multiplcations using the generalised Karatsuba method.
and so we get $1 + 4 + 4 + 7 = 16$ multiplications of large numbers.

Question 4

As $P(x)$ has all 15 roots of unity, and $x^{15} - 1$ and $P(x)$ are both of the same degree and are both monic then it follows that

$$P(x) = x^{15} - 1$$

Question 5

For any input $(a_0, a_1, a_2, ..., a_{2^n-1})$, we can describe a_i's new position by converting i to n binary places and finding the reversed sequence. e.g. $6 \to 110 \to 011 \to 3$

Question 6

let

$$f_m = \sum_{i+j=m} (j+1)q_jq_i \tag{1}$$
$$p_j = (j+1)q_j \tag{2}$$

then subsitute (2) into (1)

$$\sum_{i+j=m} p_jq_i = \vec{p} * \vec{q} \tag{3}$$

as (3) is a linear convolution, f_m can be computed in $O(nlogn)$

Question 7

a) Assuming the furtest distance from 1 corner to it's opposite corner, an inefficient algorithm would require (2+3)*2+(n-2)*4 such queries

b) text b goes
here

Question 8

text goes here