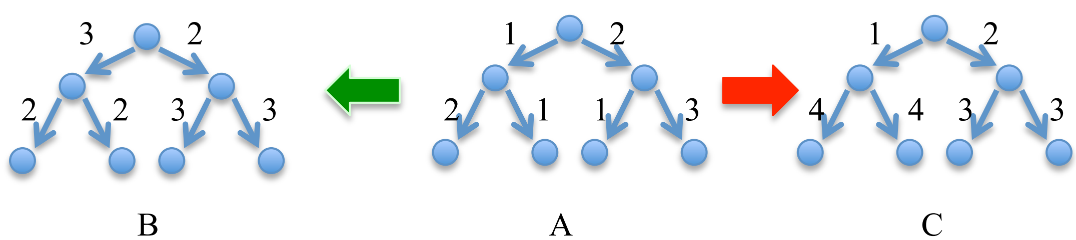
Assignment 3
Due date: Monday, April 24, 2017 at Noon

1. (Timing Problem in VLSI chips) Consider a complete binary tree with $n = 2^k$ leaves. Each edge has an associate positive number that we call the length of this edge (see figure below). The distance from the root to a leaf is the sum of the lengths of all edges from the root to this leaf. The root sends a clock signal and the signal propagates along the edges and reaches the leaf in time proportional to the distance from the root to this leaf. Design an algorithm which increases the lengths of some of the edges in the tree in a way that ensures that the signal reaches all the leafs at the same time while the total sum of the lengths of all edges is minimal. (For example, on the picture below if the tree A is transformed into trees B and C all leaves of B and C are on the distance 5 from the root and thus receive the clock signal in the same time, but the sum of lengths of edges in C is 17 while sum of lengths in B is only 15.)

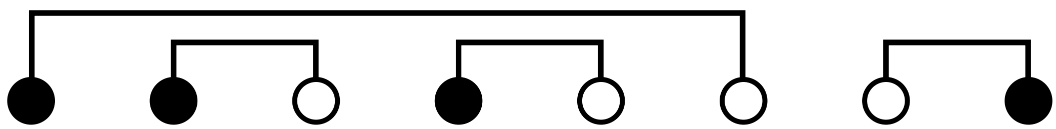


2. You are running a small manufacturing shop with plenty of workers but with a single milling machine. You have to produce n items; item i requires m_i machining time first and then p_i polishing time by hand. The machine can mill only one object at a time, but your workers can be polishing in parallel as many objects as you wish. You have to determine the order in which the objects should be machined so that the whole production is finished as quickly as possible. Prove that your solution is optimal.

3. Alice wants to throw a party and is deciding whom to call. She has n people to choose from, and she has made up a list of which pairs of these people know each other. She wants to pick as many people as possible, subject to two constraints: at the party, each person should have at least five other people whom they know and at least five other people whom they do not know. Give an efficient algorithm that takes as input the list of n people and the list of all pairs who know each other and outputs a subset of these n people which satisfies the constraints and which has the largest number of invitees. Argue that your algorithm indeed produces a subset with the largest possible number of invitees.

4. Assume that you are given n white and n black dots, lying on a line, equally spaced. The dots appear in any order of black and white, see the example picture below. We

need an algorithm which connects each black dot with a (different) white dot, so that the total length of wires used to form such connected pairs is minimal. The length of wire used to connect two dots is equal to their distance along the line.



(a) Someone has proposed the following algorithm: start by connecting the closest pair of a black and a white dot. Repeat. Give an example where such an algorithm fails to produce an optimal solution.

(b) Design an algorithm which produces an optimal solution.

5. Assume you are given n tasks each of which takes the same, unit amount of time to complete. Each task has an integer deadline and penalty associated with it which you pay if you do not complete the task in time. Design an algorithm that schedules the tasks so that the total penalty you have to pay is as small as possible.

6. You have to write a very long paper. You compiled a sequence of books in the order you will need them, some of them multiple times. Such a sequence might look something like this:

$$B_1, B_2, B_1, B_3, B_4, B_5, B_2, B_6, B_4, B_1, B_7, \ldots$$

Unfortunately, the library lets you keep at most 10 books at home at any moment, so every now and then you have to make a trip to the library to exchange books. On each trip you can exchange any number of books (of course, between 1 and all of 10 books you can keep at home). Design an algorithm which decides which books to exchange on each library trip so that the total number of trips which you will have to make to the library is as small as possible.