

COMP3121
Assignment 2
A17S1N2

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Question 1

a)

$$\text{let } A^n = \begin{pmatrix} F(n+1) & F(n) \\ F(n) & F(n-1) \end{pmatrix} \quad (1)$$

When $n = 1$

$$A^1 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad (2)$$

Assume $n = k$

$$A^k = \begin{pmatrix} F(k+1) & F(k) \\ F(k) & F(k-1) \end{pmatrix} \quad (3)$$

let $n = k + 1$

$$A^{k+1} = AA^k \quad (4)$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F(k+1) & F(k) \\ F(k) & F(k-1) \end{pmatrix} \quad (5)$$

$$= \begin{pmatrix} F(k+1) + F(k) & F(k) + F(k-1) \\ F(k+1) & F(k) \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} F(k+2) & F(k+1) \\ F(k+1) & F(k) \end{pmatrix} \quad (7)$$

Therefore the formula is true by induction for all $n > 0$.

b) $F(n)$ can be found in $\log_2(n)$ matrix multiplications using a recursive algorithm

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matrix = ((1, 1), (1, 0))
func(n):
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if n == 1:
    return matrix
if n is even:
    return func(n/2)^2
if n is odd:
    return func(n-1) * matrix

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Question 2

a) First we calculate the Karastuba trick.

$$(a + b)(c + d) = ac + ad + bd + bc \quad (1)$$

$$ad + bd = (a + b)(c + d) - ac - bc \quad (2)$$

Then we substitute (2) into (4)

$$(a + ib)(c + id) = ac + adi + bdi + bc \quad (3)$$

$$= ac + i(ad + bd) + bc \quad (4)$$

$$= ac + i((a + b)(c + d) - ac - bc) + bc \quad (5)$$

b) First we calculate the Karastuba trick.

$$(a + b)^2 = a^2 + b^2 + 2ab \quad (1)$$

$$2ab = (a + b)^2 - a^2 - b^2 \quad (2)$$

Then we substitute (2) into (3)

$$(a + ib)^2 = a^2 + 2abi - b^2 \quad (3)$$

$$= a^2 + i((a + b)^2 - a^2 - b^2) - b^2 \quad (4)$$

c) By re-arranging by the laws of exponents:

$$(a + ib)^2(c + id)^2 = ((a + ib)(c + id))^2$$

thus from above, we then calculate the middle multiplication using 3 real number multiplications, and then we find the square as above using 2 more real number multiplications.

Question 3

Question 4

As $P(x)$ has all 15 roots of unity, and $x^{15} - 1$ and $P(x)$ are both of the same degree and are both monic then it follows that

$$P(x) = x^{15} - 1$$

Question 5

For any input $(a_0, a_1, a_2, \dots, a_{2^n-1})$, we can describe a_n 's new position by converting n to binary and finding the reversed sequence. e.g. $6 \rightarrow 110 \rightarrow 011 \rightarrow 3$

Question 6

text goes here

Question 7

- a) Assuming the furthest distance from 1 corner to it's opposite corner, an inefficient algorithm would require $(2+3)*2+(n-2)*4$ such queries
- b) text b goes
here

Question 8

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