Quiz 3 Solution

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1 QUIZ 3 Solution

1.1 Problem 1

1.2 (1.a)

The objective function is the sum of all decision variables mulitplied by their corresponding link travel times. In this shortest path problem the objective function Z is: $\min 6x_{12} + 2x_{13} + 10x_{14} + 5x_{24} + 4x_{25} + 7x_{32} + 3x_{35} + 8x_{36} + 4x_{47} + 3x_{54} + 10x_{57} + 3x_{67}$.

1.3 (2.a)

The decision variables are binary, i.e. they are either zero or one. Therefore, among all links exiting node (1), only one of them is on the shortest path and equals one and the others are zero. Thus, we have $x_{12} + x_{13} + x_{14} = 1$ as a constraint.

1.4 Problem 2

The conservation constraint explains that the sum of inflows must equal the sum of outflows. Therefore, for node (4) we have: $160 = x_{42} + x_{43} + x_{45} - x_{24} - x_{34} - x_{54}$.

1.5 Problem 3

In the transhipment problem, intermediate nodes are also allowed, in addition to nodes that are origins and nodes that are destinations. Thus, the correct answer includes all of them.

1.6 Problem 4

1.7 (4.1)

For points A, B, and E we cannot find any other point whose objective values are improved at all fronts. Therefore, they are noninferior.

1.8 (4.2)

Points D and C are performing worse compared to A and B points respectively, considering all four objective functions outcomes. Thus, D is dominated by A and C is dominated by B.

1.9 Problem 5

Consider four cases for objective functions:

min x_1 and min x_2: B and C are potential solutions

min x_1 and max x_2: A and B are potential solutions

max x_1 and min x_2: C and D are potential solutions

max x_1 and max x_2: Unbounded points are potential solutions

So all of the points are potential solutions.

1.10 Problem 6

1.11 (6.a)

According to the table, the value of z_1 for point B is the minimum objective function value. The corresponding zero slack variables are s_2 and s_3 which means that the second and third constraints are binding.

1.12 (6.b)

We conduct sensitivity analysis of the x_1 coefficient of the objective function. Allowable decrease of the x_1 coefficient is found when the slope of the objective function is equal to the slope of constraint (2). Thus $\frac{3}{-2} = \frac{a}{6}$ with $a = -9$ and allowable decrease is -9-4=-13 (we report the absolute value). Allowable increase of the x_1 coefficient is found when the slope of the objective function is equal to the slope of constraint (3). Thus $\frac{1}{1} = \frac{b}{6}$ with $b = 6$ and allowable decrease is 6-4=2. The unit change in the objective function value is synonymous to the shadow price for this objective function coefficient. Thus, shadow price= $\frac{Z_{new}-Z_{old}}{6-4} = \frac{6x_1^*+6x_2^*-4x_1^*-6x_2^*}{6-4} = \frac{2x_1^*}{2} = 3.20$

1.13 Problem 7

Definition: The **shadow price** for the i-th constraint of a linear program is the amount by which the optimal z-value of the objective function is improved, if the right hand side of the i-th constraint is increased by 1.

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