Evaluating heuristics

Admissibility

A search is *admissible* if it is guaranteed to find a minimal path to a solution if such a path exists.

Breadth first is admissible. Why?

Depth first is not. Why not?

What about *depth first with iterative deepening?*

We are interested in the class of *admissible heuristic (informed) strategies.*

An Evaluation Function

- If *n* is a node in a search space, g(n) is the depth at which n was found, and h(n) is the heuristic *estimate* from n to the goal, then f(n) is an estimate of the total cost from the start state, via n, to the goal state.

  In Best First Search, f(n) is used to order the Open list (low to high).

$$f(n) = g(n) + h(n)$$

The optimal evaluation function

If g*(n) is the cost of the *shortest* path to n, and h*(n) is the *actual* cost of the shortest path from n to the goal, then f*(n) is the actual cost of the optimal path through n:

$$f^*(n) = g^*(n) + h^*(n)$$

Best-first-search using f* is admissible, but *oracles* such as f* don't really exist – but we would like f to be a close estimate of f*.

Algorithm A*

If best-first-search is used with an evaluation function in which h(n) is less than or equal to h*(n), the resulting algorithm is called A*.

All A* algorithms are admissible.

[see text for proof]

Note that BFS is A*, with h(n) = 0.

Examples

Consider some heuristics for the 8 puzzle:

1. # tiles out of place
2. Sum of manhattan distances to place for all tiles
3. 3

Are these admissible? Why or why not?

Monotonicity (local admissibility, aka "consistency" in the text)

Recall that A* does not require g(n)=g*(n).

A heuristic is *monotone* if, for all states n_i and n_j where n_j is a descendant of n_i:

1. $h(n_i) - h(n_j) <= cost(n_i,n_j)$ where $cost(n_i,n_j)$ is the actual number of moves from n_i to n_j; and
2. The heuristic evaluation of the goal is zero, i.e. h(Goal)=0.

Any monotonic heuristic is admissible. Why?

Informedness

For two A* heuristics h1 and h2, if h1(n)<=h2(n) for all states n and h1(m)<h2(m) for some state m, then heuristic h2 is said to be *more informed* than h1.

Of the two 8-square heuristics, which is more informed? Why?

Summary of Informed Search

- Informed search orders the OPEN list according to a estimate f(n) of the distance to a goal (lower is better).
- We like our heuristics to be as informed as possible, while still being admissible (so that they find the shortest path, if there is one).
- Cost tradeoff: If a heuristic makes search 10x more efficient (in nodes searched), but takes 20x as long to evaluate each node, it is not worth it!