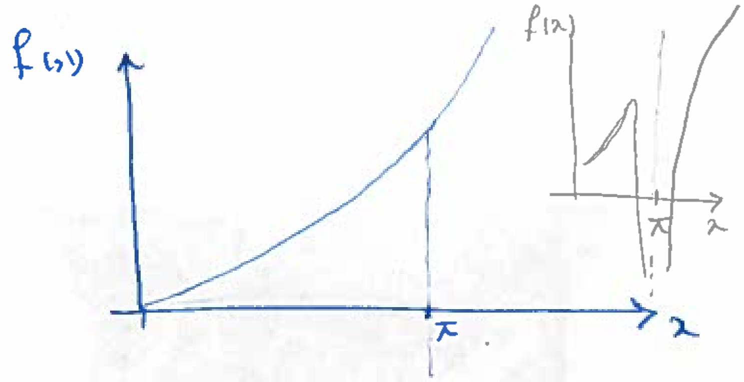
(a) Ehsan Kourkchi

HW02 Sep/02/15

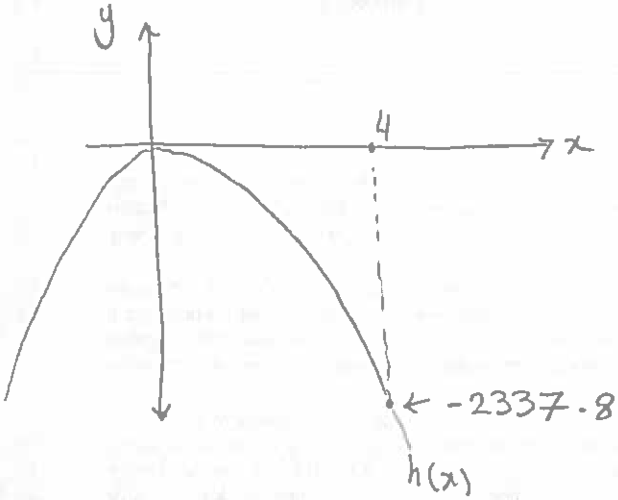
$$f(x) = 3\pi^4 x^2 + \ln\left((x-\pi)^2\right)$$

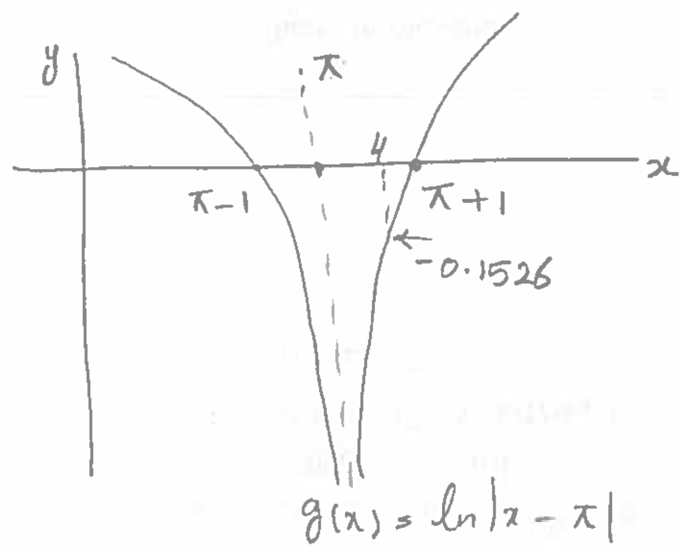


(b) proof that $f(x)$ has two roots in the range $\emptyset$ to 4 :

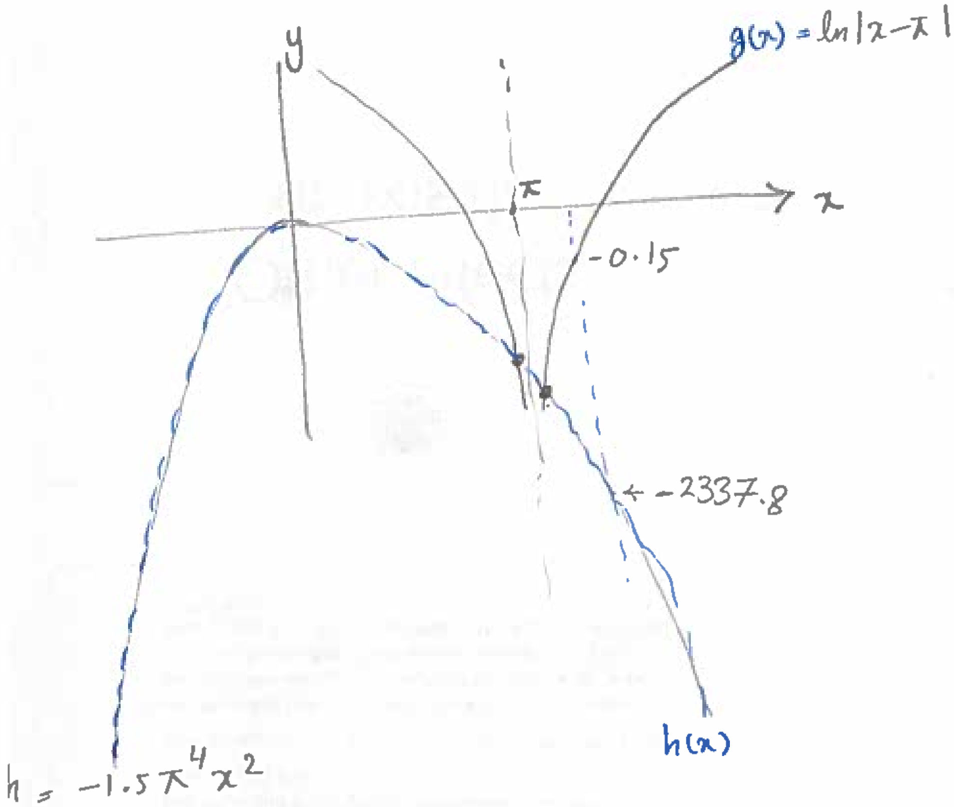
$$\begin{cases} f(x) = 3\pi^4 x^2 + 2\ln|x-\pi| \\ f(x) = 0 \quad \Rightarrow \quad \underbrace{\ln|x-\pi|}_{g(x)} = \underbrace{-1.5\pi^4 x^2}_{h(x)} \end{cases}$$

- Looking for x's for which $g(x) = h(x)$
- we plot $g(x)$ and $h(x)$:





Plotting $g(x)$ and $h(x)$ together :



- we know that two wings of $\ln|x-\pi|$ are asymtotically approaches to $-\infty$ as $x \to \pi^{\pm}$
- Since for $x=4$, $h(4)$ is much smaller than $g(4)$, $h(x)$ must have crossed $g(x)$ around $x=\pi$.

$$\Rightarrow \text{therefore} \begin{cases} h(x_1) = g(x_1) & \to \quad x_1 < \pi \\ h(x_2) = g(x_2) & \to \quad x_2 > \pi \end{cases}$$

x_1 and x_2 are roots of $f(x)$ $\begin{cases} f(x_1) = 0 \\ f(x_2) = 0 \end{cases}$

(C) Since $g(x)$ has two wings, f must have two roots very close to $x=\pi$. How to find both roots : ?

1) $\pi - 1 < x_1 < \pi$ → Searching the domain $[\pi-1, \pi)$ in a bisection algorithm.

2) $\pi < x_2 < \pi + 1$ → Doing the same for $(\pi, \pi+1]$ range.

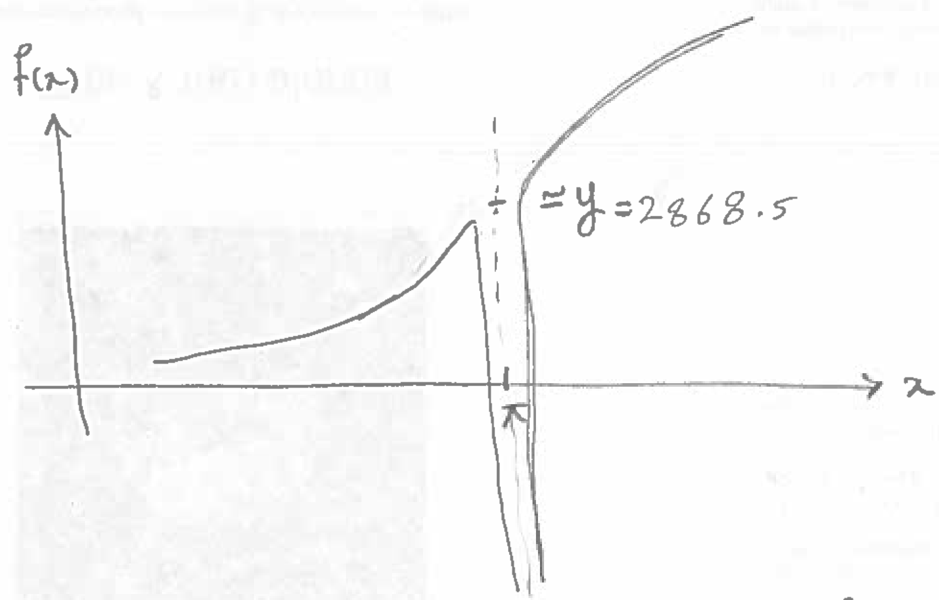
* Since $\ln((x-\pi)^2)$ is indefinite for $x=\pi$, we can just make an estimation of the roots of $f(x)$:

$$\begin{cases} \text{we know that } x_1 \text{ and } x_2 \text{ are very close to } \pi: \\ \Rightarrow f(x) = 3\pi^2(\pi)^2 + \ln((x-\pi)^2) = 0 \end{cases}$$

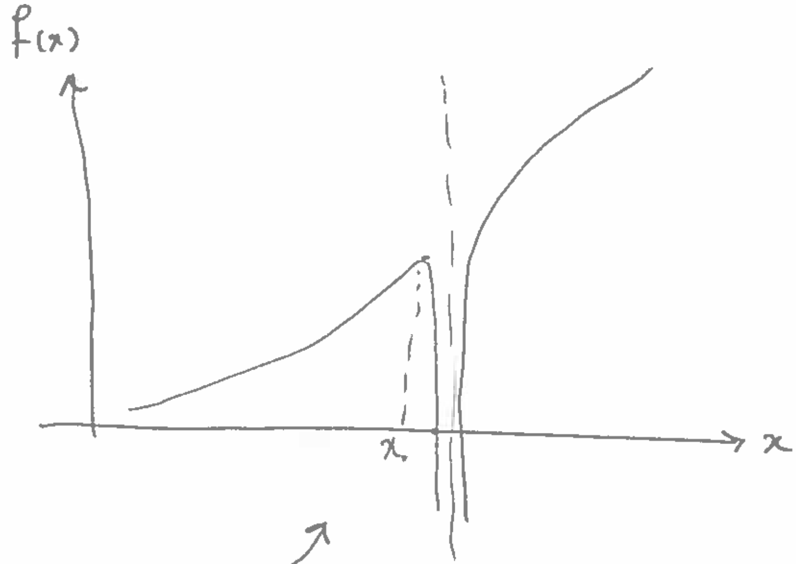
$$\Rightarrow \begin{cases} -3\pi^6 = \ln((x-\pi)^2) \\ x_1 \doteq \pi - \sqrt{\exp(-3\pi^6)} \;;\; \boxed{x_2 \simeq \sqrt{\exp(-3\pi^6)} + \pi} \end{cases}$$

$$\sqrt{\exp(-3\pi^6)} = 5.14 \times 10^{-627} \checkmark$$

d)



At $x=\pi$, $f(\pi)$ is indefinite. On both sides of $x=\pi$, it goes to $-\infty$ asymptotically



local maximum $\begin{cases} x_0 = \dfrac{3\pi^3 + \sqrt{9\pi^6 - 24}}{6\pi^2} \cong 3.1394 \\ f'(x) = 0 \\ x < \pi \end{cases} \Rightarrow -3\pi^4 x^2 + 3\pi^5 x - 2 = 0$

* Both plot are exagerated around $x = \pi$.

* $f(x)$ changes sign very close to π.

roots $\simeq \pi \pm O(10^{-627})$