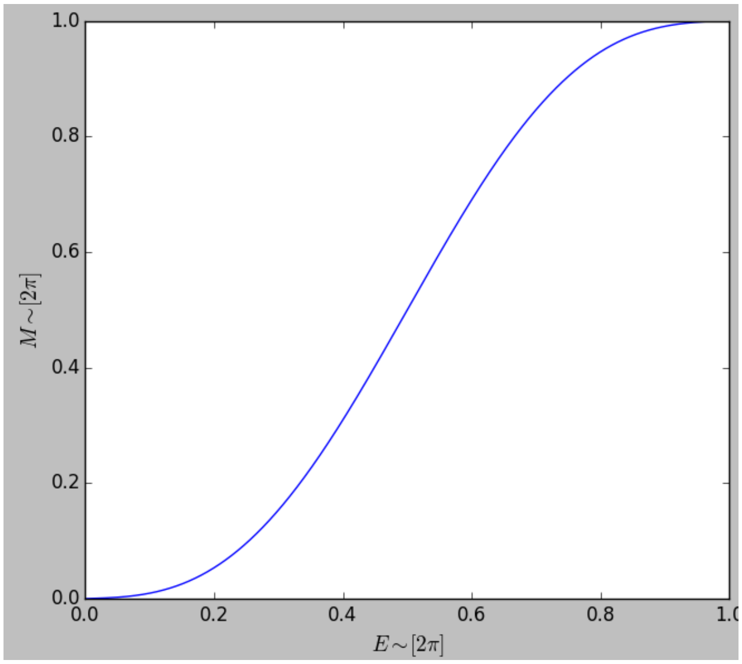
Ehsan Kourkchi …. AST735 --- HW07 --- Scientific Computation



How to numerically tackle the problem?

M = E – e sin E (1)

r = a (1 – e cos E) (2)

e = 0.9671 (3)

on the other hand: M = (2 pi / T) * t (4)

T is the period of the orbit and "t" is the elapsed time.

Since we want the time average, we start with t:

t → M using eq. 4

M → E using equations (1 and 3) … based on Newton method

E → (a/r) as a function of t … using eq. 2

Here, we really work with t/T, which starts from 0 and goes all the way up to 1.
dt = 1.E6 in our program, since later we realized that using this time interval we can get the precise numerical result which is the same as what we get analytically.

The other way is to increase the time revolution until we reach the certain accuracy we need and after that increasing the time resolution does not help that much. The other way is to sample the orbit more frequently when the planet/object is closer to the Sun. This way we can get more accuracy.

One way to test the code is to set $\varepsilon = 0$ to have a fully circular orbit, for which r = a and $< (a/r)^2 > = 1$ … The code passes this test easily.

Another way to test the code:

Using my python code, I know that I can get the solution for the time average of (a/r)^2 by 5 digits after the floating point.

The numerical result:

$<(r/a)^2> = 2.40292121316$

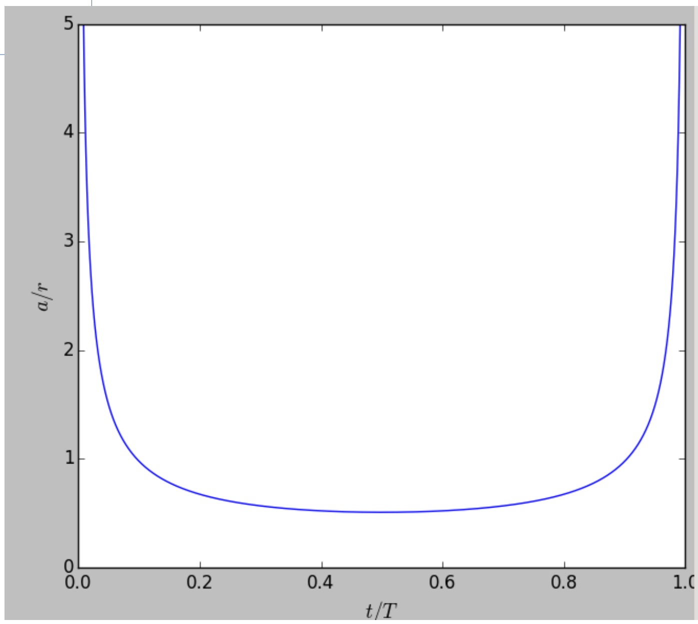
The actual analytical result:[1]

$$\left\langle \left(\frac{r}{a}\right)^2 \right\rangle_t = 1 + \tfrac{3}{2}\varepsilon^2$$

$$\left\langle \left(\frac{r}{a}\right)^2 \right\rangle_t \rightarrow (1+1.5*0.9671^{\wedge}2) = 2.402923615$$

We are required to get the time average of $(a/r)^2$ … The code has been change accordingly …

The final results: $< (a/r)^2 > = 3.93179129538$



As seen on the right, the diagram is symmetric, to calculate the time average we can just use the half if the time domain. Also we can use finer resolution near to t/T = 0 since the curve behaves more critically. Doing that:

$< (r/a)^2 > = 2.40292172607$

$< (a/r)^2 > = 3.93096389982$

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1 http://faculty.madisoncollege.edu/alehnen/kepler/kepler.htm