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SOAL QUIS 1

1. Tentukan; a. (A+B).D   b. (C-A)+B.A   c. (A+C)-(D-B)

   a. $A = \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$ b. $B = \begin{bmatrix} 5 & -1 \\ -4 & 0 \end{bmatrix}$ c. $C = \begin{bmatrix} 4 & 8 \\ -3 & -6 \end{bmatrix}$ d. $D = \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$

2. Tentukan matriks X jika :

   a. $X \begin{bmatrix} 4 & 5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 14 & 15 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$

   c. $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 28 \\ -14 \end{bmatrix}$ d. $X \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 14 & 5 \\ 10 & -2 \end{bmatrix}$

3. Carilah Determinan dan Invers matriks : a. $D = \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$ b. $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$

4. Tentukan determinan dan invers matriks berikut menggunakan metode Minor dan Kofaktor.

   $$A = \begin{bmatrix} 1 & 4 & 6 & 1 \\ 1 & 0 & 0 & 1 \\ 3 & 2 & 4 & 5 \\ 5 & 8 & 4 & 2 \end{bmatrix}$$

5. Selesaikan persamaan berikut dengan Eleminasi da Substitusi:
   $$x + y + 2z = 9$$
   $$2z + 4y - 3z = 1$$
   $$3z + 6y - 5z = 0$$

6. Selesaikan sistem persamaan berikut dengan cara Matriks:
   $$5x_1 + 2x_2 + 10x_3 + 16x_4 = 16$$
   $$3x_1 + x_2 - 2x_4 = 3x_1 + x_2 - 9x_3 - 19x_4 = -4x_1 + x_2 - 3x_4 = 5$$

Jawab

1

1a. $(A + B).D$

$A + B = \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 5 & -1 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 1 & 3 \end{bmatrix}$

$$(A+B).D = \begin{bmatrix} 4 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 4(10)+0(8) & 4(-6)+0(-5) \\ 1(10)+3(8) & 1(-6)+3(-5) \end{bmatrix}$$

$$\begin{bmatrix} 40 & -24 \\ 34 & -21 \end{bmatrix}$$

1b. $(C-A)+BA$

$$C-A = \begin{bmatrix} 4 & 8 \\ -3 & -6 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ -8 & -9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & -1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}$$

$$.= \begin{bmatrix} 5(-1)+(-1(5)) & 5(1)+(-1(3)) \\ -4(-1)+0(5) & -4(1)+0(3) \end{bmatrix}$$

$$.= \begin{bmatrix} -10 & 2 \\ 4 & -4 \end{bmatrix}$$

$$(C-A)+BA = \begin{bmatrix} 5 & 7 \\ -8 & -9 \end{bmatrix} \begin{bmatrix} -10 & 2 \\ 4 & -4 \end{bmatrix}$$

$$¿ \begin{bmatrix} -5 & 9 \\ -4 & -13 \end{bmatrix}$$

1c. $(A+C)-(D-B)$

$$A+C = \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ -3 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 9 \\ 2 & -3 \end{bmatrix}$$

$$D-B = \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ -4 & 0 \end{bmatrix}$$

$$.= \begin{bmatrix} 5 & -5 \\ 12 & -5 \end{bmatrix}$$

$$(A+C)-(D-B) = \begin{bmatrix} 3 & 9 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 5 & -5 \\ 12 & -5 \end{bmatrix}$$

$$.= \begin{bmatrix} -2 & 14 \\ -10 & 2 \end{bmatrix}$$

2

2a. $X \begin{bmatrix} 4 & 5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 14 & 15 \end{bmatrix}$

$XA = B$

$X = BA^{-1}$

Menentukan $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$X = \begin{bmatrix} 8 & 5 \\ 14 & 15 \end{bmatrix} \cdot \dfrac{1}{-10} \begin{bmatrix} 4 & 5 \\ 2 & 0 \end{bmatrix}$

$\begin{bmatrix} 8 & 5 \\ 14 & 15 \end{bmatrix} \begin{bmatrix} \dfrac{4}{-10} & \dfrac{5}{-10} \\ \dfrac{2}{-10} & \dfrac{0}{-10} \end{bmatrix}$

$\begin{bmatrix} -\dfrac{32}{10} - \dfrac{10}{10} & -\dfrac{40}{10} - \dfrac{0}{10} \\ -\dfrac{56}{10} - \dfrac{30}{10} & -\dfrac{70}{10} - \dfrac{0}{10} \end{bmatrix}$

$\begin{bmatrix} -\dfrac{21}{5} & -4 \\ -\dfrac{43}{5} & -7 \end{bmatrix}$

2b. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$

$AX = B$

$X = A^{-1} B$

$X = \dfrac{1}{-2} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$

$\begin{bmatrix} -\dfrac{1}{2} & -\dfrac{2}{2} \\ -\dfrac{3}{2} & -\dfrac{4}{2} \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} -\dfrac{1}{2} & -1 \\ -\dfrac{3}{2} & -2 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -2+(-2) & -\dfrac{3}{2}+1 \\ -6+(-4) & -\dfrac{9}{2}+2 \end{bmatrix}$$

$$. = \begin{bmatrix} -4 & -\dfrac{1}{2} \\ -10 & -\dfrac{5}{2} \end{bmatrix}$$

2c. $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 28 \\ -14 \end{bmatrix}$

$AX = B$
$X = A^{-1}B$

$$X = \frac{1}{14}\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 28 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{3}{14} & -\dfrac{2}{14} \\ \dfrac{1}{14} & \dfrac{4}{14} \end{bmatrix}\begin{bmatrix} 28 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{84}{14}+\dfrac{28}{14} \\ \dfrac{28}{14}-\dfrac{56}{14} \end{bmatrix}$$

$$\begin{bmatrix} 28 \\ -2 \end{bmatrix}$$

2d. $X \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 14 & 5 \\ 10 & -2 \end{bmatrix}$

$X = BA^{-1}$
$$X = \begin{bmatrix} 8 & 2 \\ 14 & 5 \\ 10 & -2 \end{bmatrix}\frac{1}{6}\begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix}$$

$$. = \begin{bmatrix} 8 & 2 \\ 14 & 5 \\ 10 & -2 \end{bmatrix} \begin{bmatrix} \dfrac{2}{6} & -\dfrac{1}{6} \\ \dfrac{4}{6} & \dfrac{1}{6} \end{bmatrix}$$

$$. = \begin{bmatrix} 8\left(\dfrac{2}{6}\right)+2\left(\dfrac{4}{6}\right) & 8\left(-\dfrac{1}{6}\right)+2\left(\dfrac{1}{6}\right) \\ 14\left(\dfrac{2}{6}\right)+5\left(\dfrac{4}{6}\right) & 14\left(-\dfrac{1}{6}\right)+5\left(\dfrac{1}{6}\right) \\ 10\left(\dfrac{2}{6}\right)+-2\left(\dfrac{4}{6}\right) & 10\left(-\dfrac{1}{6}\right)+-2\left(\dfrac{1}{6}\right) \end{bmatrix}$$

$$. = \begin{bmatrix} 4 & -1 \\ 8 & -\dfrac{3}{2} \\ 2 & -2 \end{bmatrix}$$

3

3a. $D = \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$

$Det\, D = \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$
$\quad\quad . = 10(-5)-(-6(8))$
$\quad\quad . = -50+48$
$\quad\quad . = -2$

$D^{-1} = -\dfrac{1}{2}\begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$

$\quad . = \begin{bmatrix} -5 & 3 \\ -4 & -\dfrac{5}{2} \end{bmatrix}$

3b. $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$

Metode minor kofaktor

$Det\, A = a_{11}.C_{11}+a_{12}.C_{12}+a_{13}.C_{13}$
$\quad . = 2.(-1)^{1+1}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}+1.(-1)^{1+2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}+(-1).-1^{1+3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$
$\quad . = 2.(1-(-2))-1.(1-1)+(-1(-2-1))$
$\quad . = 2(3)-1(0)+(-1(-3))$

$$.= 6 - 0 + 3$$
$$.= 9$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$.= \begin{bmatrix} \dfrac{2}{9} & \dfrac{1}{9} & -\dfrac{1}{9} \\ \dfrac{1}{9} & \dfrac{1}{9} & \dfrac{1}{9} \\ \dfrac{1}{9} & -\dfrac{2}{9} & \dfrac{1}{9} \end{bmatrix}$$

4. $A = \begin{bmatrix} 1 & 4 & 6 & 1 \\ 1 & 0 & 0 & 1 \\ 3 & 2 & 4 & 5 \\ 5 & 8 & 4 & 2 \end{bmatrix}$

Metode Minor kofaktor

$$M_{11} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 4 & 5 \\ 8 & 4 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 2 & 4 \\ 8 & 4 \end{bmatrix} = (0 \times 4 \times 2) + (0 \times 5 \times 8) + (1 \times 2 \times 4) - (1 \times 4 \times 8) - (0 \times 5 \times 4) - (0 \times 2 \times 2)$$
$$¿ 0 + 0 + 8 - 32 - 0 - 0$$
$$¿ -24$$

$$C_{11} = (-1)^{1+1} \times M_{11}$$
$$.= -24$$

$$M_{21} = \begin{bmatrix} 4 & 6 & 1 \\ 2 & 4 & 5 \\ 8 & 4 & 2 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 2 & 4 \\ 8 & 4 \end{bmatrix} = (4 \times 4 \times 2) + (6 \times 5 \times 8) + (1 \times 2 \times 4) - (1 \times 4 \times 8) - (4 \times 5 \times 4) - (6 \times 2 \times 2)$$
$$¿ 32 + 240 + 8 - 32 - 80 - 24$$
$$¿ 144$$

$$C_{21} = (-1)^{1+2} \times M_{21}$$
$$.= -144$$

$$M_{31} = \begin{bmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ 8 & 4 & 2 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 0 & 0 \\ 8 & 4 \end{bmatrix} = (4 \times 0 \times 2) + (6 \times 1 \times 8) + (1 \times 0 \times 4) - (1 \times 0 \times 8) - (4 \times 1 \times 4) - (6 \times 0 \times 2)$$
$$¿ 0 + 48 + 0 - 0 - 16 - 0$$
$$¿ 32$$

$$C_{31} = (-1)^{1+3} \times M_{31}$$

$$. = 32$$

$$M_{41} = \begin{bmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 0 & 0 \\ 2 & 4 \end{bmatrix} = (4 \times 0 \times 5) + (6 \times 1 \times 2) + (1 \times 0 \times 4) - (1 \times 0 \times 2) - (4 \times 1 \times 4) - (6 \times 0 \times 5)$$

$$. = 0 + 12 + 0 - 0 - 16 - 0$$
$$. = -4$$

$$C_{41} = (-1)^{1+4} \times M_{41}$$
$$. = 4$$

$$Det\, A = (a_{11} \times C_{11}) + (a_{21} \times C_{21}) + (a_{31} \times C_{31}) + (a_{41} \times C_{41})$$
$$. = (1 \times -24) + (1 \times -144) + (3 \times 32) + (5 \times 4)$$
$$. = -24 - 144 + 96 + 20$$
$$. = -52$$

Invers Matriks

$$A^{-1} = \frac{1}{-52} \begin{bmatrix} 1 & 4 & 6 & 1 \\ 1 & 0 & 0 & 1 \\ 3 & 2 & 4 & 5 \\ 5 & 8 & 4 & 2 \end{bmatrix}$$

$$. = \begin{bmatrix} -\dfrac{1}{52} & -\dfrac{4}{52} & -\dfrac{6}{52} & -\dfrac{1}{52} \\ -\dfrac{1}{52} & 0 & 0 & -\dfrac{1}{52} \\ -\dfrac{3}{52} & -\dfrac{2}{52} & -\dfrac{4}{52} & -\dfrac{5}{52} \\ -\dfrac{5}{52} & -\dfrac{8}{52} & -\dfrac{4}{52} & -\dfrac{2}{52} \end{bmatrix}$$

5. $x + y + 2z = 9$ persamaan (1)
$2x + 4y - 3z = 1$ persamaan (2)
$3x + 6y - 5z = 0$ persamaan (3)

Hitung persamaan (1) *dan* (2), *metode eliminasi*
$$x + y + 2z = 9 \|\times 2\| 2x + 2y + 4z = 18$$
$$2x + 4y - 3z = 1 \|\times 1\| \underline{2x + 4y - 3z = 1} \quad -$$
$$-2y + 7z = 17 \qquad (4)$$

Hitung persamaan (1) *dan* (3), *metode eliminasi*
$$x + y + 2x = 9 [\times 3] 3x + 3y + 6z = 27$$
$$3x + 6y - 5z = 0 [\times 1] 3 \underline{x + 6y - 5z = 0} \quad -$$

$$-3y+11z=27 \qquad (5)$$

Hitung persamaan (4) *dan* (5), *metode eliminasi*

$-2y+7z=17 \left[\times 3\right] -6y+21z=51$

$-3y+11z=27 \left[\times 2\right] \underline{-6y+22z=54}$

$$\qquad\qquad\qquad z=3 \qquad (6)$$

Substitusi persamaan (6) *ke* (4)

$-2y+7(3)=17$

$\quad -2y+21=17$

$\qquad -2y=4$

$\qquad\quad y=-2 \qquad\qquad\qquad (7)$

substitusikan persamaan (6) *dan* (7) *ke persamaan* (1)

$\qquad x+y+2z=9$

$x+(-2)+2(3)=9$

$\qquad x-2+6=9$

$\qquad\quad x+4=9$

$\qquad\qquad x=9-4$

$\qquad\qquad x=5$

6. $5x_1+2x_2+10x_3+16x_4=16$

$\quad 3x_1+x_2-2x_4=3x_1+x_2-9x_3-19x_4=-4x_1+x_2-3x_4=5$