Santa's Workshop Tour 2019 MIP formulation

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Definition of variables

$$x_{f,d} = \begin{cases} 1, & \text{if family } f \text{ is assigned to day } d \\ 0, & \text{otherwise, for } f = 0, \ldots, 4999, \ d = 0, \ldots, 99. \end{cases}$$

$$y_{d,i,j} = \begin{cases} 1, & \text{if on day } d \text{ there are } i \text{ and on day } d{+}1 \text{ there are } j \text{ people on the workshop} \\ 0, & \text{otherwise, for } d = 0, \ldots, 99 \text{ and } i = 125, \ldots, 300 \text{ and } j = 125, \ldots, 300. \end{cases}$$

n_d is the number of people attending the workshop on day d

Parameters

$c_{f,d}$ is the preference cost associated with $x_{f,d}$

p_f is the number of people in family f

$a_{d,i,j}$ is the accounting penalty associated with $y_{d,i,j}$

$$a_{d,i,j} = \left(\frac{i - 125}{400}\right) \cdot i^{\left(\frac{1}{2} + \frac{|i-j|}{50}\right)}$$

Model

$$\begin{aligned}
\text{minimize} \quad & \sum_{f=0}^{4999} \sum_{d=0}^{99} c_{f,d}\, x_{f,d} + \sum_{d=0}^{99} \sum_{i=125}^{300} \sum_{j=125}^{300} a_{d,i,j}\, y_{d,i,j} \\
\text{subject to} \quad & \sum_{d=0}^{99} x_{f,d} = 1 && f = 0, \ldots, 4999 && (1) \\
& \sum_{f=0}^{4999} p_f\, x_{f,d} = n_d && d = 0, \ldots, 99 && (2) \\
& \sum_{i=125}^{300} \sum_{j=125}^{300} i\, y_{d,i,j} = n_d && d = 0, \ldots, 99 && (3) \\
& \sum_{i=125}^{300} \sum_{j=125}^{300} y_{d,i,j} = 1 && d = 0, \ldots, 99 && (4) \\
& \sum_{i=125}^{300} y_{d,i,j} = \sum_{k=125}^{300} y_{d+1,j,k} && d = 0, \ldots, 98, \ j = 125, \ldots, 300 && (5) \\
& 125 \leq n_d \leq 300 && d = 0, \ldots, 99 && (6) \\
& x_{f,d}, \ y_{d,i,j} \in \{0, 1\}
\end{aligned}$$

Explanations

Constraints (1) assure that every family f is assigned to a single day.

Constraints (2) and (3) are two different ways of computing n_d. The variables n_d will be the link between the $x_{f,d}$ and the $y_{d,i,j}$ variables.

Constraints (4) assure that, for each day, a single variable $y_{d,i,j}$ is equal to 1.

Constraints (5): $y_{d,i,j} = 1$ means that on day d there are i people and on day $d+1$ there are j people on the workshop. These constraints force $y_{d+1,j,k}$ to be equal to 1 for some (unique) k.

Constraints (6) are the bounds for variables n_d.