#### Lecture 7

Environmental policy with pre-existing distortions

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## Roadmap

So far we have looked at single sector economies with:

- Pollution distortions
- Competitive markets
- Market power distortions

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So far we have looked at single sector economies with:

- Pollution distortions
- Competitive markets
- Market power distortions

Now we will learn about multi-sector economies

How does environmental policy spillover into these other sectors?

How does environmental policy interact with revenue-raising taxes (e.g. income taxes)?

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Households have a choice of either working or leisure time

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- There is a representative (single) firm
- There is a representative household

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This allows us to treat individual and aggregate behavior the same

1: The underlying critical assumption is that utility and profit functions take what's called a Gorman form.

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- *X* is consumption of the polluting good
- Z is consumption of the numeraire good (i.e. the relative good)
- N is the hours of leisure time
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where  $U_{XX}, U_{NN} < 0$  and  $U_{XX}U_{NN} - U_{NX}^2 > 0$  and the person is endowed with some amount of time T to allocate between work and leisure

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Household income is then:  $w \cdot (T - N)$ 

We can now write the households utility maximization problem as:

$$\max_{X,N,Z} U(X,Z,N,E) = U(X,N) + Z - D(E)$$

subject to: 
$$w \cdot (T - N) = Z + pX$$

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with FOCs:

$$U_X=p \qquad U_N=w$$

which implicitly define the demand function for consumption X(p,w) and the demand function for leisure N(p,w)

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We have two equations and two unknowns so we can solve to get:

$$rac{\partial N}{\partial p} = rac{-U_{XN}}{U_{XX}U_{NN} - U_{XN}^2} \qquad rac{\partial X}{\partial p} = rac{U_{NN}}{U_{XX}U_{NN} - U_{XN}^2}$$

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If X and N are substitutes,  $-U_{XN}>0$ , and leisure increases in the price of the consumption good

If they are complements,  $-U_{XN} < 0$ , and leisure decreases in the price of the consumption good

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The firm side of the economy will be the same as before: it produces X and emits E and for simplicity we will focus on the specific case:

$$\Pi = pX - C(X)$$
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- $\delta = 1$  so we can use E and X interchangeably
- $C'(X) > 0, C''(X) \ge 0$
- The polluting industry's demand for labor is small relative to the entire economy, i.e. wages are effectively fixed for the household

Now lets solve for the social optimum:

$$\max_{X} W = \underbrace{U(X,N) + w \cdot (T-N) - pX - D(X)}_{\text{Consumer Utility}} + \underbrace{pX - C(X)}_{\text{Firm profit}}$$

To focus on interactions with non-regulated industries, we assume the regulator cannot determine the allocation of leisure and labor

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One way you can think about this is as if the regulator imposes a quantity standard  $X^{st}$  and then a market price  $p^{st}$  arises which affects leisure demand

The FOC for the optimum is:

$$[U_X-D'(X)-C'(X)+[U_N-w]rac{\partial N}{\partial X}=0]$$

where the last term captures the households indirect leisure response to the regulator's policy choice

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Marginal abatement cost  $(U_X - C'(X))$  equals marginal damage (D'(X))!

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The consumer's utility maximization problem is:

$$\max_{X,Z,N} U = u(X,N) + Z - D(E)$$
subject to  $(1-m)w(T-N) = Z + pX$ 

Where the budget is scaled down by (1-m) reflecting the income tax

The FOCs are:

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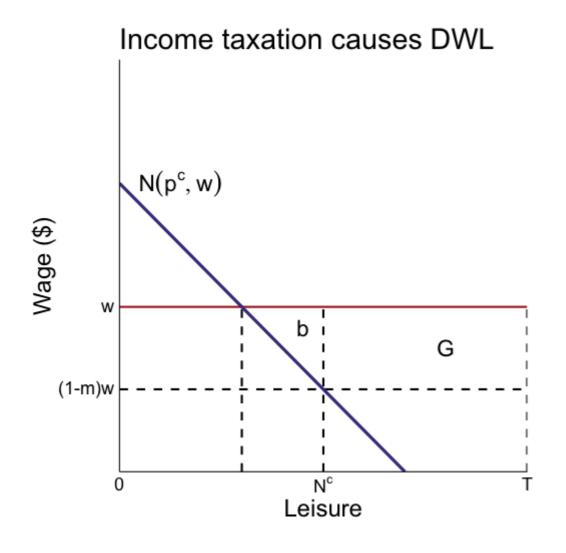
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The tax m makes the consumer act as if there is a subsidy mw on leisure

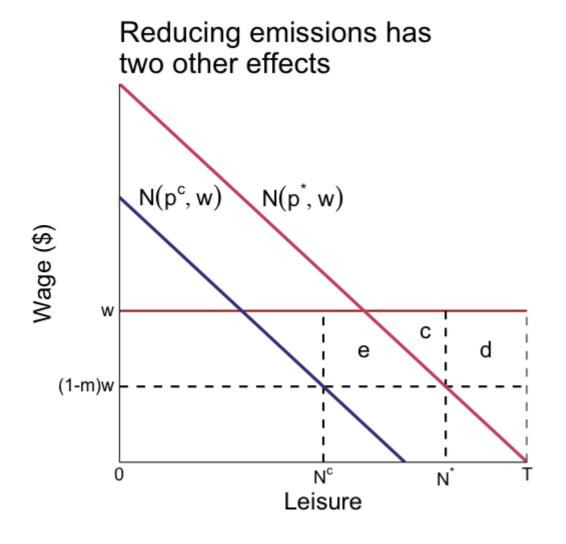


 $\boldsymbol{w}$  is the perfectly elastic demand for labor

 $N^c$  is how much leisure the consumer chooses, since (1-m)w < w this is too much and induces DWL equal to b

This is called excess burden

The tax raises revenues equal to G:  $mw imes (T-N^c)$ 

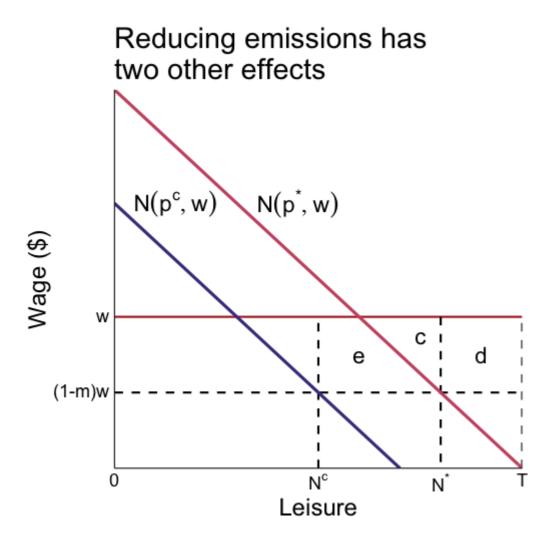


Suppose N and X are substitutes, and the regulator sets  $X=X^*$  where  $X^* \to MAC = MD$ 

This raises the price of X, shifts leisure demand to the right

New DWL is c, and government revenues are now only d

Change in DWL from  $X^c o X^*$  is indeterminant



Fixing the pollution externality had two effects:

- 1. Indeterminant effect on the distortion in the labor market
- 2. Reduced the amount of revenue the government raised through labor taxation

## Second-best environmental policy

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First let's consider the case where they can only raise revenue via a labor tax: this is non-revenue raising environmental policy

If we cannot raise revenue with the environmental policy, the regulator chooses X (and E) and the marginal tax rate m to maximize the sum of profit and utility, subject to the budget constraint

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The household consumes leisure according to the FOC:

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given the regulator chose  $X=ar{X}$ 

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The firm obtains profits:

$$\Pi=par{X}-C(ar{X})$$

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$$P(ar{X}) = u_X(ar{X},N)$$

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Let's do the comparative statics: differentiate the consumer's two FOCs with respect to  $\bar{X}$ 

$$u_{XX} \frac{\partial \bar{X}}{\partial \bar{X}} + u_{XN} \frac{\partial N}{\partial \bar{X}} = \frac{\partial p}{\partial \bar{X}}$$
 (X FOC)

$$u_{NX} rac{\partial ar{X}}{\partial ar{X}} + u_{NN} rac{\partial N}{\partial ar{X}} = 0$$
 (N FOC)

 $\frac{\partial \bar{X}}{\partial \bar{X}} = 1$  so two equations, two unknowns;

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 (N FOC)

 $\frac{\partial X}{\partial \bar{X}} = 1$  so two equations, two unknowns; solving the system gives us:

$$egin{align} rac{\partial N}{\partial ar{X}} &= -rac{u_{XN}}{u_{NN}} \ rac{\partial p}{\partial ar{X}} &= rac{u_{XX}u_{NN} - u_{NN}^2}{u_{NN}} < 0 \ \end{pmatrix}$$

 $\operatorname{sign}(rac{\partial N}{\partial ar{X}})$  depends on whether X and N are complements or substitutes

Now that we know how the firm responds, return to the regulator's problem:

$$\max_{X,m} u(X,N) + Z - D(X) + pX - C(X) \quad ext{ s.t. } \quad wm(T-N) = G$$

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For convenience, we assume its returned to the consumer as a lump sum transfer so that:

$$Z=(1-m)w(T-N)-pX+G=(1-m)w(T-N)-pX+wm(T-N) \ \Rightarrow Z=w(T-N)-pX$$

Income is unchanged for a given level of N under the tax and transfer

The regulator's problem is then:

$$\max_{X,m} u(X,N) + \underbrace{w(T-N)}_Z - D(X) - C(X) + \lambda [wm(T-N) - G]$$

 $\lambda$  is called the marginal welfare cost of public funds

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What's the FOC for m?

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Whats the interpretation of the right hand side?

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Why?

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Why?

Higher m increases leisure demand  $\frac{\partial N}{\partial m}$ 

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Higher m increases leisure demand  $\frac{\partial N}{\partial m}$ 

This times the tax wedge mw, the gap between w and actual wage after taxes, gives us the change in excess burden (i.e. the DWL d in the graph)

$$\lambda = rac{wmrac{\partial N}{\partial m}}{w(T-N)-wmrac{\partial N}{\partial m}}$$

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First term is the increase in revenue on the inframarginal hours worked

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The denominator is:

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First term is the increase in revenue on the inframarginal hours worked

Second term is the decrease in revenue from reduced hours worked

• Similar to P(X) + P'(X)X for a monopolist

$$\lambda = rac{wmrac{\partial N}{\partial m}}{w(T-N)-wmrac{\partial N}{\partial m}}$$

Numerator and denominator combined give us:

The change in welfare from a change in m

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Numerator and denominator combined give us:

The change in welfare from a change in m over the change in tax revenue from a change in m

This is the change in welfare from a change in tax revenue!

Now consider the FOC for *X*:

$$[u_X-D'(X)-C'(X)+[u_N-w-\lambda wm]\,rac{\partial N}{\partial X}=0]$$

Now consider the FOC for *X*:

$$\left[u_X - D'(X) - C'(X) + \left[u_N - w - \lambda wm
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Recall that we know:

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So that we can substitute in the consumer leisure response:

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$$(1+\lambda)\left[-rac{\partial N}{\partial p}rac{\partial p}{\partial X}
ight]wm$$
 is new

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$$(1+\lambda)\left[-rac{\partial N}{\partial p}rac{\partial p}{\partial X}
ight]wm$$
 is new

What's the interpretation?

$$(1+\lambda)\left[-rac{\partial N}{\partial p}rac{\partial p}{\partial X}
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Suppose *N* and *X* are substitutes, what does this mean?

Substitutes means that MIE > 0

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This exacerbates the distortion caused by the income tax: the household was already undersupplying labor because of the income tax

Now the household undersupplies labor to an even greater extent

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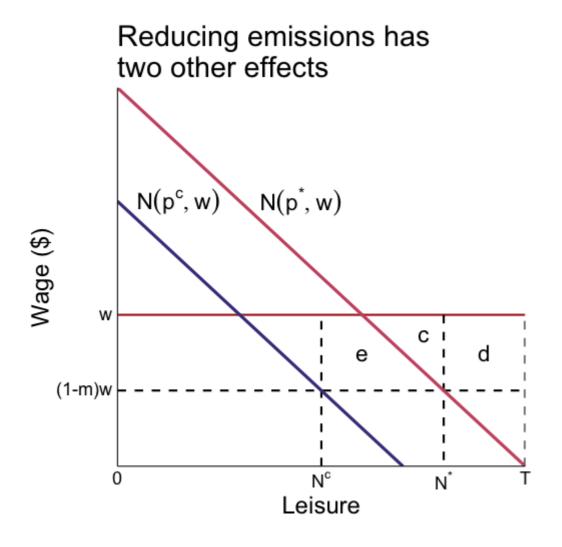
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Intuition?

Its less socially costly to reduce X because the household decreases N in response

This alleviates the distortion caused by the income tax: the household was undersupplying labor because of the income tax, but now reducing X increases labor supply, shrinking the labor market DWL



 $N^c o N^*$  when  $p^c o p^*$  because of a change in X

This is 
$$-\frac{\partial N}{\partial p} \frac{\partial p}{\partial X}$$

This reduces tax revenue by e+c which is just

$$egin{aligned} (N^*-N^c)(w-(1-m)w) \ &= \underbrace{(N^*-N^c)}_{pprox -rac{\partial N}{\partial p}rac{\partial p}{\partial X}} \end{aligned}$$

The marginal welfare cost of recovering the lost tax revenue (in order to maintain gov't revenues G) by raising m is  $\lambda$  giving us a total welfare cost of:

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But  $(N^* - N^c)mw$  also happens to be the increase in excess burden: its a direct welfare loss in addition to the loss from having to increase m

So the total welfare loss is:

$$(1+\lambda)(N^*-N^c)mw$$

The discrete version of MIE!

### Findings recap

If there's a government revenue constraint, and it can only be met with labor taxes then:

1. The marginal social cost of reducing X is higher if X and N are substitutes and lower if they are complements

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If there's a government revenue constraint, and it can only be met with labor taxes then:

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#### Findings recap

If there's a government revenue constraint, and it can only be met with labor taxes then:

- 1. The marginal social cost of reducing X is higher if X and N are substitutes and lower if they are complements
- 2. The optimal level of pollution is larger if they are substitutes, lower if they are complements
- 3. The absolute value of the difference in first and second-best pollution levels is larger if:
  - Demand for X is more inelastic
  - $\circ$  Elasticity of substitution between N and X is greater

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#### First define:

- $\varepsilon_x$  as the own price elasticity  $\frac{\partial X}{\partial p} \frac{p}{X}$
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and take advantage of the Slutsky symmetry condition  $\partial N/\partial p = \partial X/\partial w$ 

We can then use these to substitute into the MIE and get:

$$MIE = (1 + \lambda) \left[ -rac{\eta_{XN}}{arepsilon_X} 
ight] p rac{m}{1-m}$$

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The regulator's problem is thus to select two tax rates: m and au

For simplicity we still assume all tax revenues are returned lump sum to households

First derive household spending on the numeraire good:

$$Z=(1-m)w(T-N)-pX+G=w(T-N)-pX+ au X$$

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These are a function of the govt's choice of m and au

The household FOCs are:

$$u_X=p \qquad u_N=(1-m)w$$

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Next, as usual, differentiate the FOCs wrt  $\tau$ 

This gives us 3 equations and 3 unknown partial derivatives:

$$u_{XX} \frac{\partial X}{\partial \tau} + u_{XN} \frac{\partial N}{\partial \tau} = \frac{\partial p}{\partial \tau}$$
 (Household X FOC)

$$u_{XN} \frac{\partial X}{\partial au} + u_{NN} \frac{\partial N}{\partial au} = 0$$
 (N FOC)

$$C''(X)\frac{\partial X}{\partial \tau} = \frac{\partial p}{\partial \tau} - 1$$
 (Firm X FOC)

Substitute and solve...

Now solve for how the endogenous variables change in au

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$$rac{\partial X}{\partial au} = rac{u_{NN}}{H} < 0$$

$$\frac{\partial N}{\partial au} = \frac{-u_{XN}}{H} \lessgtr 0$$

$$rac{\partial p}{\partial au} = rac{u_{XX}u_{NN} - u_{XN}^2}{H} > 0$$

where 
$$H=u_{XX}u_{NN}-u_{XN}^2-C^{\prime\prime}(X)u_{NN}>0$$

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The regulator wants to maximize social welfare given the budget constraint:

$$\max_{m, au} \underbrace{U(X,N) + Z - D(X)}_{ ext{household utility}} + \underbrace{pX - C(X) - au X}_{ ext{firm profit}}$$
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Substitute in for Z from household spending:

$$Z = w(T-N) - pX + au X$$

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$$\left[u_X - C'(X) - D'(X)\right] \frac{\partial X}{\partial au} + \left[\underbrace{u_N - w}_{-wm} - \lambda wm\right] \frac{\partial N}{\partial au} + \lambda \left[X + au \frac{\partial X}{\partial au}\right] = 0$$

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Since the tax is per unit, we have that:  $\frac{\partial N}{\partial \tau} / \frac{\partial X}{\partial \tau} = \frac{\partial N}{\partial p} / \frac{\partial X}{\partial p}$ , MIE is similar in revenue and non-revenue raising contexts

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MRE changes the marginal social cost of X because changes in  $\tau$  affect how much revenue we need to raise with distorting labor taxation

Let's get some intuition at the corner case of  $\tau=0$ 

What's the sign of MRE?

$$MRE( au=0)$$
:  $\lambda\left[x/rac{\partial X}{\partial au}
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- $\rightarrow$  this reduces the distortionary tax in the labor market
- → this reduces welfare losses in the labor market
- $\rightarrow$  this reduction in welfare losses reduces the marginal social cost of reducing X, decreasing the optimal level of X

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$$MRE \equiv \lambda \left[ au + X \Big/ rac{\partial X}{\partial au} 
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ight] = \lambda \left[ au + p/arepsilon_X 
ight] = \lambda au \left[ 1 + 1/arepsilon_X^ au 
ight]$$

where  $\varepsilon_X < 0$  is the elasticity of demand for the dirty good and  $\varepsilon_X^{\tau}$  is the elasticity with respect to the tax

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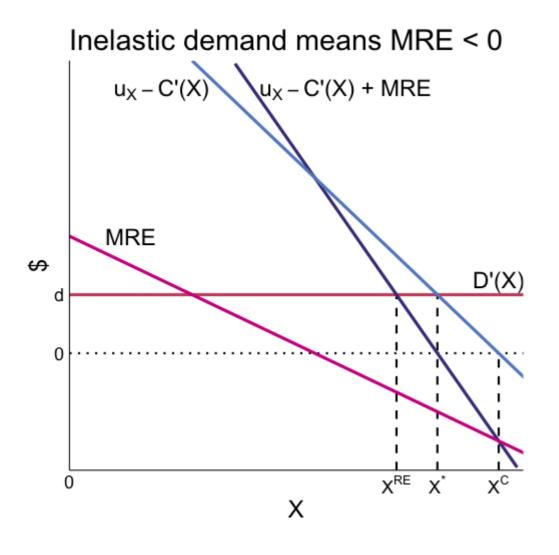
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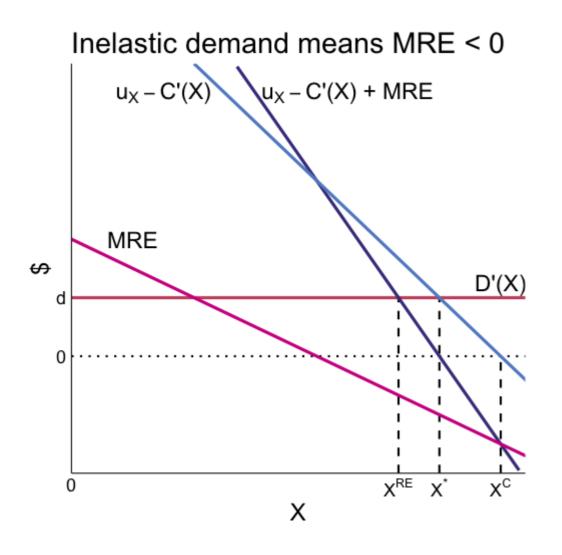
Why?



# Demand for dirty good is sufficiently inelastic:

Suppose 
$$rac{\partial N}{\partial p}=0$$
 so  $MIE=0$ ,  $C'(X)=c,$   $D'(X)=d$ 

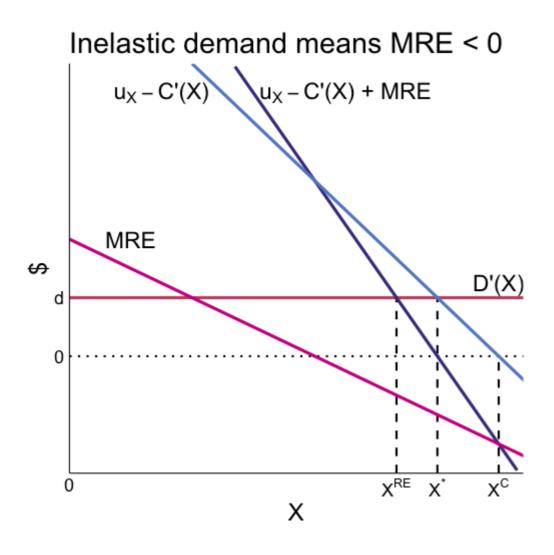
Inelastic demand lets us raise more revenue from a small change in the tax



Inelastic demand lets us raise more revenue from a small change in the tax

This reduces the marginal social cost of reducing X

Optimal X with revenue-raising is lower than without:  $X^{RE} < X^*$ 



We can also see that if D'(X) was very large, making au larger, we would be where MRE>0

Is there a prospect for a double dividend?

There is a weak double dividend if welfare is always greater when revenue raised via environmental taxation is used to reduced distortionary taxation rather than refunded lump sum

This is always true

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There is a **strong double dividend** if the emission tax should always be set above the MAC=MD level, resulting in greater pollution reductions and more revenue raised

• This may or may not be true

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Price of X rises from  $\tau$ , demand for leisure goes down, labor goes up

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Let's look at this pathway in more detail

Again, assume C'(X) = c, this gives us that:

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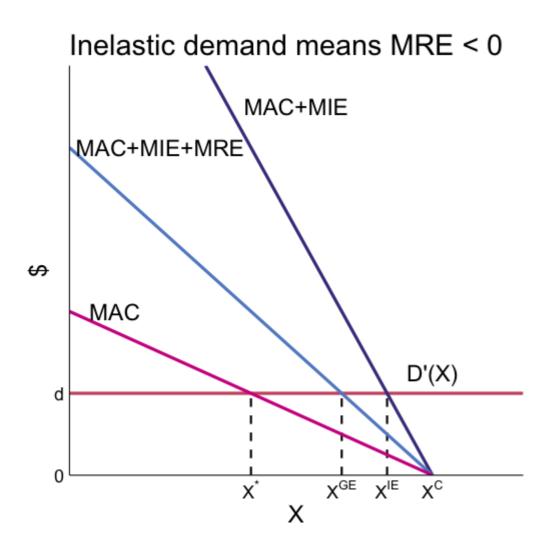
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Suppose N and X are average substitutes which means  $\eta_{XN}=\varepsilon_L$ , then:

$$MIE = \lambda \left( -rac{p}{arepsilon_X} 
ight) < \lambda \left( rac{p}{arepsilon_X} + au 
ight) = MRE$$

⇒ we shouldn't expect a strong double dividend



Even though there isn't a double dividend, MIE and MRE still matter for the optimal second-best pollution level

Optimal pollution  $X^{GE}$  is larger than first-best  $X^*$ , but less than the level without revenue recycling  $X^{IE}$ 

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What about freely allocated permits or command and control?

This would lead to the same *environmental* outcome, but not achieve the the welfare maximizing outcome

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Setting  $X^{GE} < X^c$  raises the price of X, increases leisure, and reduces revenues via the interaction effect

Without revenue from permits or taxes, the optimal pollution level is higher