

# Lecture 7

## Environmental policy with pre-existing distortions

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AEM 6510

# Roadmap

So far we have looked at single sector (partial equilibrium) economies with:

- Pollution distortions
- Competitive markets
- Market power distortions

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- Pollution distortions
- Competitive markets
- Market power distortions

Now we will learn about multi-sector (general equilibrium) economies

How does environmental policy spillover into these other sectors?

How does environmental policy interact with revenue-raising taxes (e.g. income taxes)?

# Environmental policy with leisure

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- There is a representative (single) firm
- There is a representative household

This allows us to treat individual and aggregate behavior the same

1: The underlying critical assumption is that utility and profit functions take what's called a Gorman form.

# Environmental policy with leisure

Define the following:

- $X$  is consumption of the polluting good
- $Z$  is consumption of the *numeraire* good (i.e. the relative good)
- $N$  is the hours of leisure time
- $E$  is aggregate emissions

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- $N$  is the hours of leisure time
- $E$  is aggregate emissions

Utility is then:

$$U(X, Z, N, E) = U(X, N) + Z - D(E)$$

where  $U_{XX}, U_{NN} < 0$  and  $U_{XX}U_{NN} - U_{NX}^2 > 0$  and the person is endowed with some amount of time  $T$  to allocate between work and leisure



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Household income is then:  $w \cdot (T - N)$

We can now write the households utility maximization problem as:

$$\max_{X, N, Z} U(X, Z, N, E) = U(X, N) + Z - D(E)$$

$$\text{subject to: } w \cdot (T - N) = Z + pX$$

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We can substitute the budget constraint in for  $Z$  to get an unconstrained problem:

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with FOCs:

$$U_X = p \quad U_N = w$$

which implicitly define the demand function for consumption  $X(p, w)$  and the demand function for leisure  $N(p, w)$

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We have two equations and two unknowns so we can solve to get:

$$\frac{\partial N}{\partial p} = \frac{-U_{XN}}{U_{XX}U_{NN} - U_{XN}^2} \quad \frac{\partial X}{\partial p} = \frac{U_{NN}}{U_{XX}U_{NN} - U_{XN}^2}$$

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If  $X$  and  $N$  are substitutes,  $-U_{XN} > 0$ , and leisure increases in the price of the consumption good

If they are complements,  $-U_{XN} < 0$ , and leisure decreases in the price of the consumption good

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The firm side of the economy will be the same as before: it produces  $X$  and emits  $E$  and for simplicity we will focus on the specific case:

$$\Pi = pX - C(X) \text{ where } E = \delta X$$

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We will also assume:

- $\delta = 1$  so we can use  $E$  and  $X$  interchangeably
- $C'(X) > 0, C''(X) \geq 0$
- The polluting industry's demand for labor is small relative to the entire economy, i.e. wages are effectively fixed for the household

# Environmental policy with leisure

Now lets solve for the social optimum:

$$\max_X W = \underbrace{U(X, N) + w \cdot (T - N) - pX - D(X)}_{\text{Consumer Utility}} + \underbrace{pX - C(X)}_{\text{Firm profit}}$$

To focus on interactions with non-regulated industries, we assume the regulator cannot determine leisure



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The consumer chooses  $N$  according to the FOC  $U_N(X^*, N) = w$  and then  $Z$  given the budget constraint  $Z = w(T - N) - pX^*$

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One way you can think about this is as if the regulator imposes a quantity standard  $X^*$  and then a market price  $p^*$  arises which affects leisure demand

# Environmental policy with leisure

The FOC for the optimum is:

$$U_X - D'(X) - C'(X) + [U_N - w] \frac{\partial N}{\partial X} = 0$$

where the last term captures the households **indirect** leisure response to the regulator's policy choice

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Given household utility maximization  $U_N - w = 0$  and the condition is then:

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Given household utility maximization  $U_N - w = 0$  and the condition is then:

$$U_X - C'(X) = D'(X)$$

Marginal abatement cost ( $U_X - C'(X)$ ) equals marginal damage ( $D'(X)$ ) !

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Now suppose the government needs to raise revenue with a labor income tax  $m$  in order to finance government services

It needs to finance a budget of size  $G$

The consumer's utility maximization problem is:

$$\begin{aligned} \max_{X,Z,N} U &= u(X, N) + Z - D(E) \\ \text{subject to } (1 - m)w(T - N) &= Z + pX \end{aligned}$$

Where the budget is scaled down by  $(1 - m)$  reflecting the income tax

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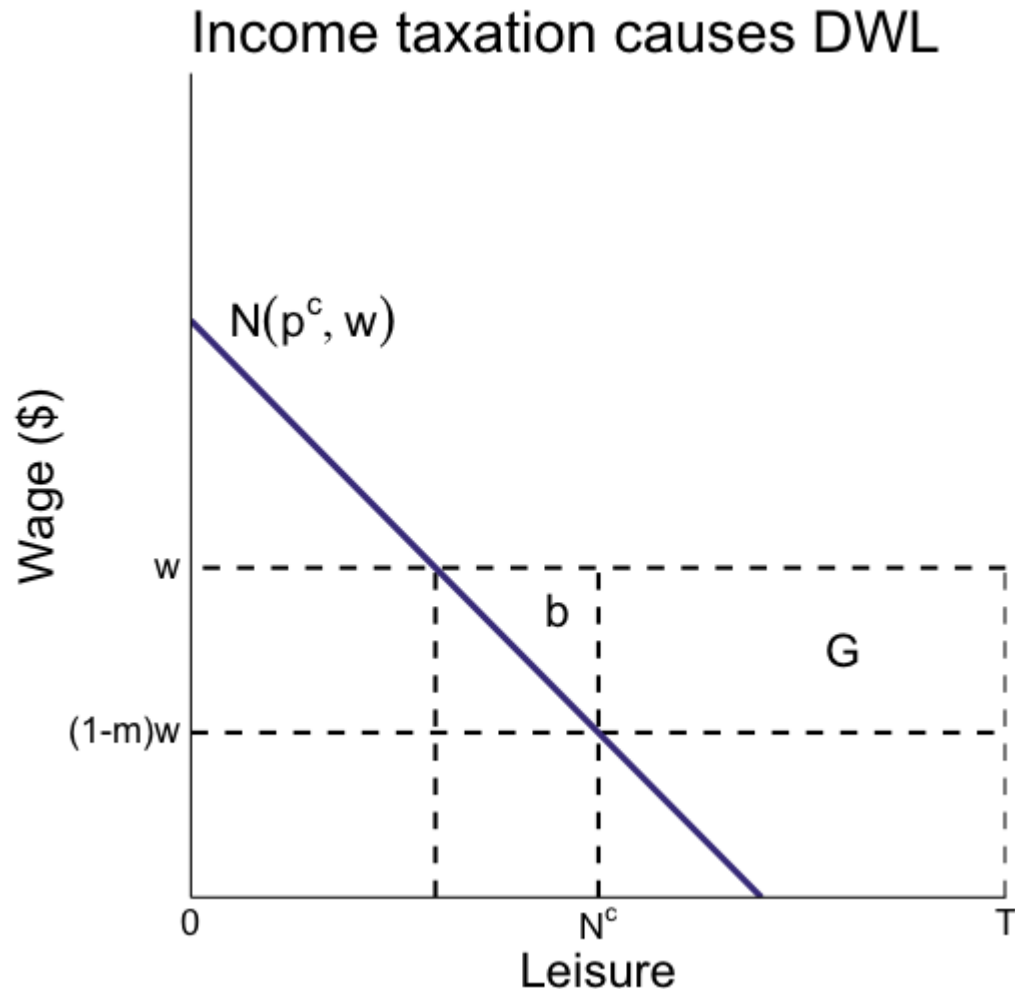
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The tax  $m$  makes the consumer act as if there is a subsidy  $mw$  on leisure

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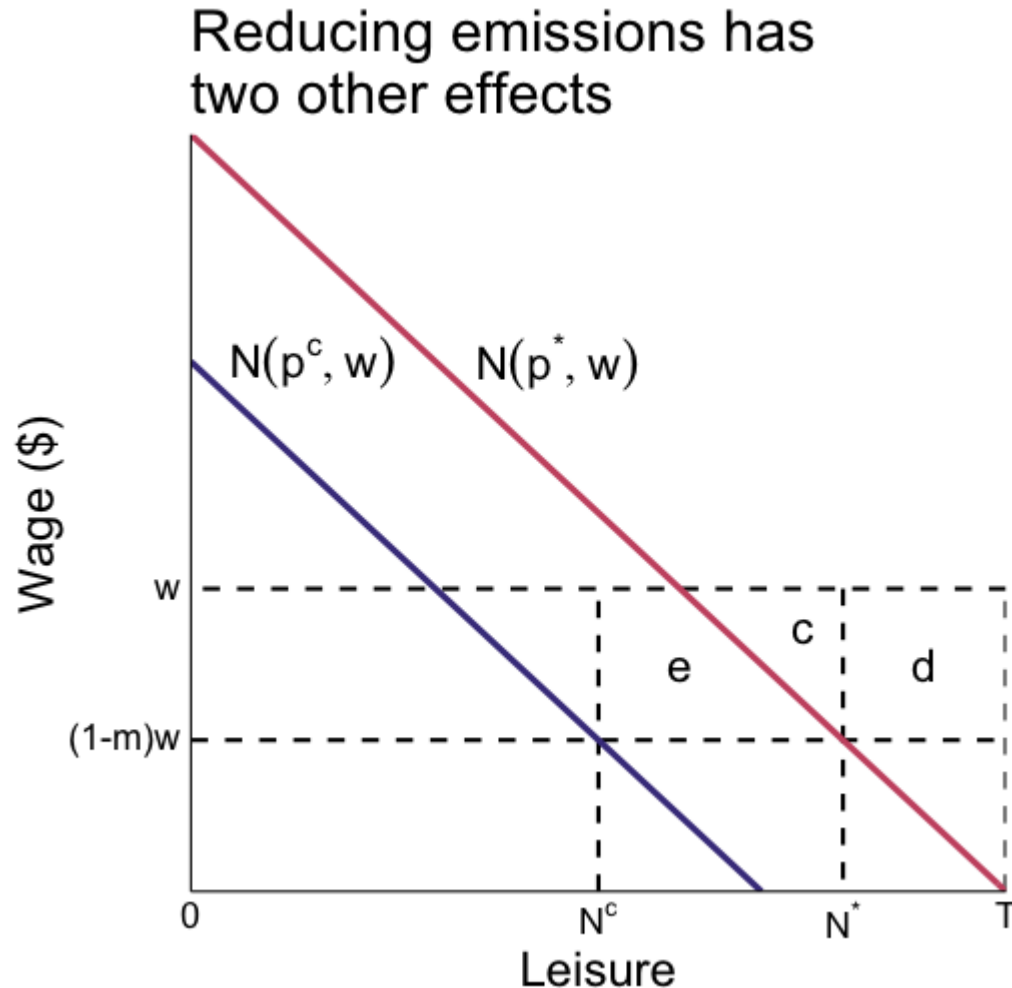


$N^c$  is how much leisure the consumer chooses, since  $(1 - m)w < w$  this is too much and induces DWL equal to  $b$

This is called **excess burden**

The tax raises revenues equal to  $G$ :  
 $mw \times (T - N^c)$

# Environmental policy with labor market distortions



Suppose  $N$  and  $X$  are substitutes,  
and the regulator sets  $X = X^*$   
where  $X^* \rightarrow MAC = MD$

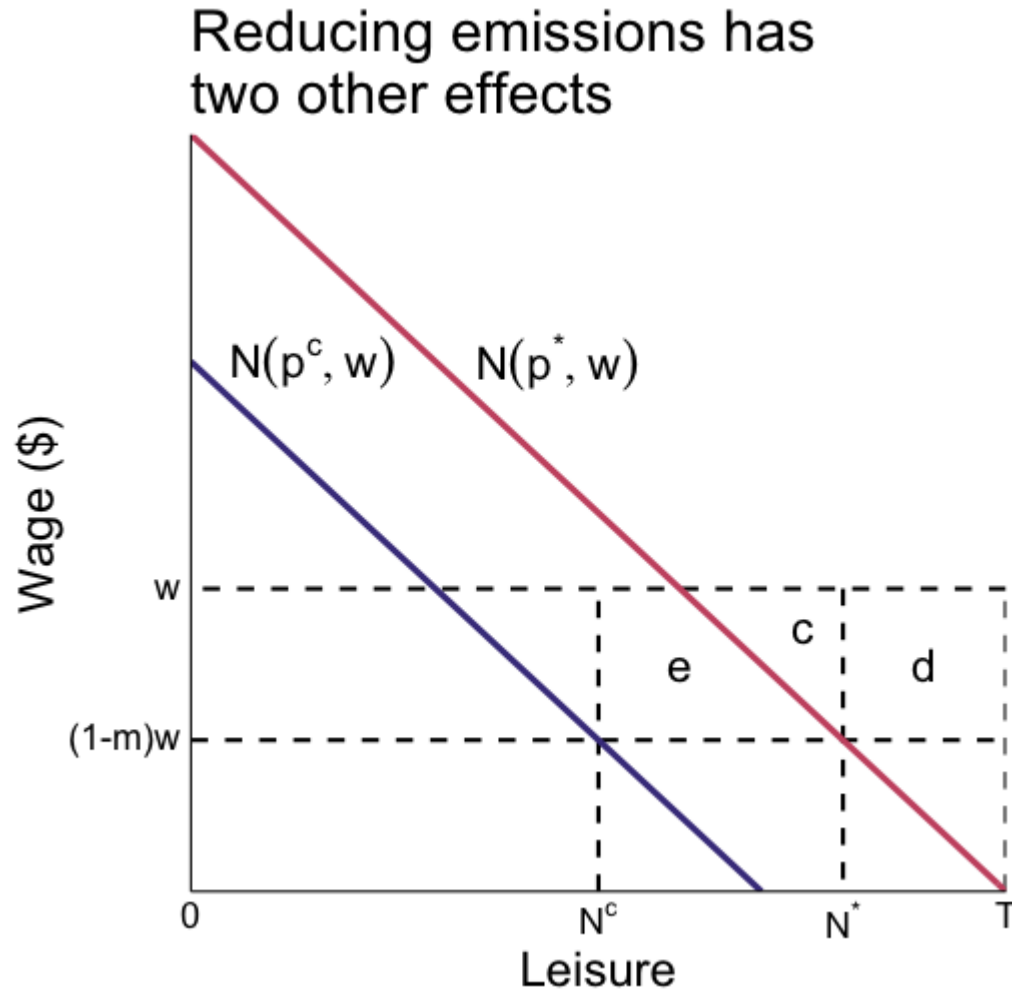
This raises the price of  $X$ , shifts  
leisure demand to the **right**

New DWL is  $c$ , and government revenues are now only  $d$

Change in DWL from  $X^c \rightarrow X^*$  is  
indeterminant



# Environmental policy with labor market distortions



Fixing the pollution externality had two effects:

1. Indeterminant effect on the distortion in the labor market
2. Reduced the amount of revenue the government raised through labor taxation

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First let's consider the case where they can only raise revenue via a labor tax: this is non-revenue raising environmental policy

# Second-best non-revenue raising environmental policy

If we cannot raise revenue with the environmental policy, the regulator chooses  $X$  (and  $E$ ) and the marginal tax rate  $m$  to maximize the sum of profit and utility, subject to the budget constraint

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The household consumes leisure according to the FOC:

$$U_N(\bar{X}, N) = (1 - m)w$$

given the regulator chose  $X = \bar{X}$

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given the regulator chose  $X = \bar{X}$

The firm obtains profits:

$$\Pi = p\bar{X} - C(\bar{X})$$

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The marginal value of the dirty good comes from the consumers inverse demand:

$$P(\bar{X}) = u_X(\bar{X}, N)$$

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Let's do the comparative statics: differentiate the consumer's two FOCs with respect to  $\bar{X}$

# Second-best non-revenue raising environmental policy

$$u_{XX} \frac{\partial \bar{X}}{\partial \bar{X}} + u_{XN} \frac{\partial N}{\partial \bar{X}} = \frac{\partial p}{\partial \bar{X}} \quad (\text{X FOC})$$

$$u_{NX} \frac{\partial \bar{X}}{\partial \bar{X}} + u_{NN} \frac{\partial N}{\partial \bar{X}} = 0 \quad (\text{N FOC})$$

$\frac{\partial \bar{X}}{\partial \bar{X}} = 1$  so two equations, two unknowns; solving the system gives us:

$$\frac{\partial N}{\partial \bar{X}} = - \frac{u_{XN}}{u_{NN}}$$

$$\frac{\partial p}{\partial \bar{X}} = \frac{u_{XX}u_{NN} - u_{NN}^2}{u_{NN}} < 0$$

$\text{sign}\left(\frac{\partial N}{\partial \bar{X}}\right)$  depends on whether  $X$  and  $N$  are complements or substitutes

# Second-best non-revenue raising environmental policy

Now that we know how the firm responds, return to the regulator's problem:

$$\max_{X,m} u(X, N) + Z - D(X) + pX - C(X) \text{ s.t. } wm(T - N) = G$$

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For convenience, we assume its returned to the consumer as a lump sum transfer so that:

$$\begin{aligned} Z &= (1 - m)w(T - N) - pX + G = (1 - m)w(T - N) - pX + wm(T - N) \\ &\Rightarrow Z = w(T - N) - pX \end{aligned}$$

Income is unchanged for a given level of  $N$  under the tax and transfer

# Second-best non-revenue raising environmental policy

The regulator's problem is then:

$$\max_{X,m} u(X, N) + w(T - N) - D(X) - C(X) + \lambda[wm(T - N) - G]$$

$\lambda$  is called the **marginal welfare cost of public funds**

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What's the FOC for  $m$ ?

# Second-best non-revenue raising environmental policy

The FOC for  $m$  is:

$$(u_N - w) \frac{\partial N}{\partial m} + \lambda \left[ w(T - N) - wm \frac{\partial N}{\partial m} \right] = 0$$

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Whats the interpretation of the right hand side?

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This times the **tax wedge**  $mw$ , the gap between  $w$  and actual wage after taxes, gives us the change in excess burden (i.e. the DWL  $d$  in the graph)

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First term is the increase in marginal revenue

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The denominator is:

The change in tax revenue from higher  $m$

First term is the increase in marginal revenue

Second term is the decrease in inframarginal revenue

- Similar to  $P(X) + P'(X)X$  for a monopolist

# Second-best non-revenue raising environmental policy

$$\lambda = \frac{wm \frac{\partial N}{\partial m}}{w(T - N) - wm \frac{\partial N}{\partial m}}$$

Numerator and denominator combined give us:

The change in welfare from a change in  $m$



# Second-best non-revenue raising environmental policy

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This is the change in welfare from a change in tax revenue!

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What's the interpretation?

# Second-best non-revenue raising environmental policy

$(1 + \lambda) \left[ -\frac{\partial N}{\partial p} \frac{\partial p}{\partial X} \right] w_m$  is called the **marginal interaction effect (MIE)**

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Suppose  $N$  and  $X$  are substitutes, what does this mean?



# Second-best non-revenue raising environmental policy

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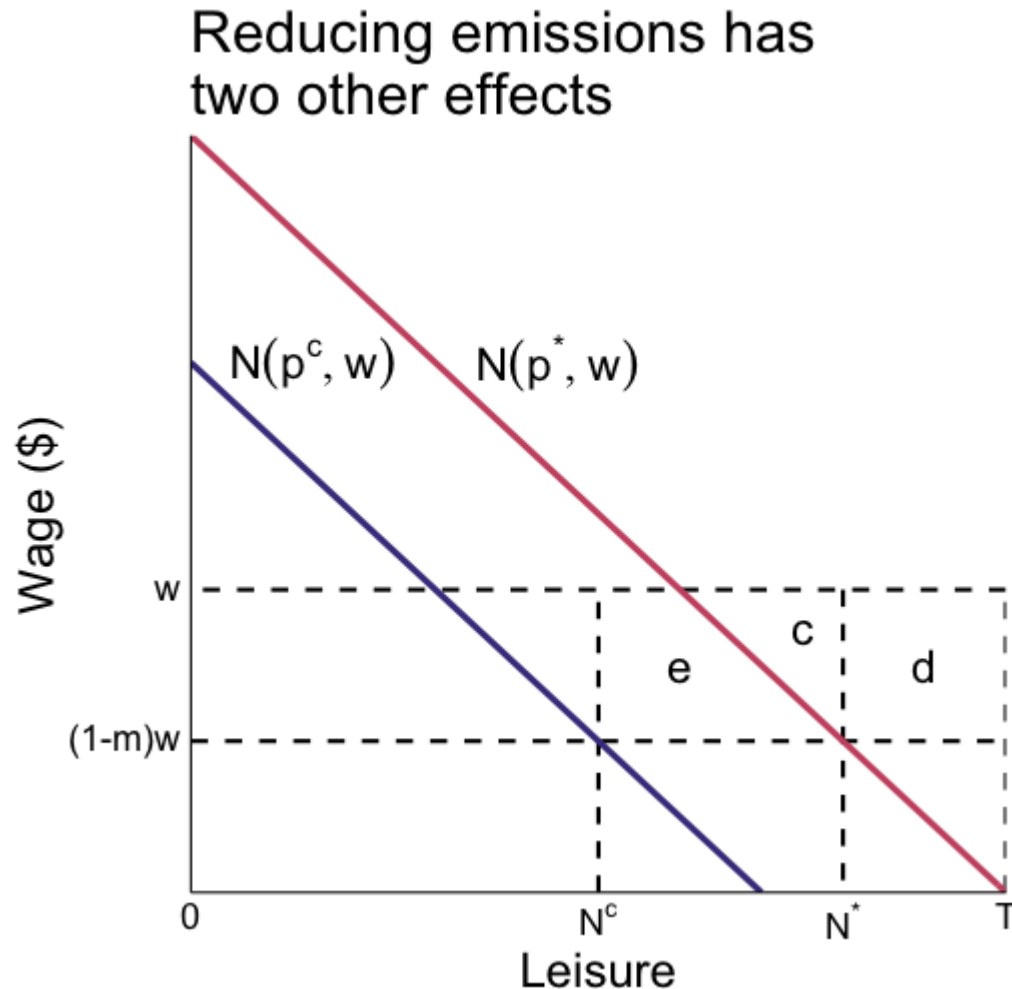
The marginal social cost of abatement ( $MAC + MIE$ ) has become **smaller**

Intuition?

Its less socially costly to reduce  $X$  because the household decreases  $N$  in response

This **alleviates** the distortion caused by the income tax: the household was undersupplying labor because of the income tax, but now reducing  $X$  increases labor supply

# Second-best non-revenue raising environmental policy



$N^c \rightarrow N^*$  when  $p^c \rightarrow p^*$  because of a change in  $X$

This is  $-\frac{\partial N}{\partial p} \frac{\partial p}{\partial X}$

This reduces tax revenue by  $e + c$  which is just

$$\begin{aligned} & (N^* - N^c)(w - (1 - m)w) \\ & = (N^* - N^c)mw \end{aligned}$$

# Second-best non-revenue raising environmental policy

The marginal welfare cost of recouping the lost tax revenue by raising  $m$  is  $\lambda$  giving us a total welfare cost of:

$$\lambda(N^* - N^c)mw$$

But  $(N^* - N^c)mw$  is also the excess burden: its a **direct welfare loss!**

So the total welfare loss is:

$$(1 + \lambda)(N^* - N^c)mw$$

The discrete version of MIE!

# Findings recap

If there's a government revenue constraint, and it can only be met with labor taxes then:

1. The marginal social cost of reducing  $X$  is higher than the first-best if  $X$  and  $N$  are substitutes and lower if they are complements

# Findings recap

If there's a government revenue constraint, and it can only be met with labor taxes then:

1. The marginal social cost of reducing  $X$  is higher than the first-best if  $X$  and  $N$  are substitutes and lower if they are complements
2. The optimal level of pollution is larger if they are substitutes, lower if they are complements

# Findings recap

If there's a government revenue constraint, and it can only be met with labor taxes then:

1. The marginal social cost of reducing  $X$  is higher than the first-best if  $X$  and  $N$  are substitutes and lower if they are complements
2. The optimal level of pollution is larger if they are substitutes, lower if they are complements
3. The absolute value of the difference in first and second-best pollution levels is larger if:
  - Labor supply is more elastic
  - Demand for  $X$  is more inelastic
  - Elasticity of substitution between  $N$  and  $X$  is greater

# Second-best non-revenue raising environmental policy

We didn't actually show the last part yet



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First define:

- $\varepsilon_x$  as the own price elasticity  $\frac{\partial X}{\partial p} \frac{p}{X}$
- $\eta_{XN}$  as the elasticity of substitution between  $X$  and  $N$ :  $\frac{\partial X}{\partial w} \frac{(1-m)w}{X}$

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and take advantage of the **Slutsky symmetry condition**  $\partial N / \partial p = \partial X / \partial w$

We can then use these to substitute into the MIE and get:

$$MIE = (1 + \lambda) \left[ -\frac{\eta_{XN}}{\varepsilon_X} \right] p \frac{m}{1 - m}$$

# Second-best non-revenue raising environmental policy

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Still need to show labor supply part

# Second-best non-revenue raising environmental policy

Define the elasticity of labor supply **at the after-tax wage** as:

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And recognize that:

$$\frac{\partial N(p, (1 - m)w)}{\partial m} = -w \frac{\partial N(p, w)}{\partial w}$$

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Combining these two gives:

$$\frac{\partial N}{\partial m} m = \varepsilon_L L m / (1-m)$$



# Second-best non-revenue raising environmental policy

Finally, put:

$$\frac{\partial N}{\partial m} m = \varepsilon_L L m / (1 - m)$$

in

$$\lambda = \frac{wm \frac{\partial N}{\partial m}}{w(T - N) - wm \frac{\partial N}{\partial m}}$$

To get the welfare cost of public funds in terms of fundamental economic parameters

# Second-best non-revenue raising environmental policy

We get:

$$\lambda = \frac{\varepsilon_L m / (1 - m)}{1 - \varepsilon_L m / (1 - m)}$$

Side note: If labor is more elastic  $\varepsilon_L$  is larger, the numerator is larger, denominator is smaller  $\rightarrow$  MIE is bigger

If labor supply is perfectly inelastic (vertical), there is no welfare cost of public funds!

# Second-best non-revenue raising environmental policy

Finally use:

$$\lambda = \frac{\varepsilon_L m / (1 - m)}{1 - \varepsilon_L m / (1 - m)}$$

to get:

$$\frac{m}{(1 - m)} = \frac{\lambda}{(1 + \lambda)\varepsilon_L}$$

and then substitute into our MIE expression:

$$MIE = (1 + \lambda) \left[ -\frac{\eta_{XN}}{\varepsilon_X} \right] p \frac{m}{1 - m}$$

# Second-best non-revenue raising environmental policy

Finally we will get:

$$MIE = \lambda \frac{-\eta_{XN}}{\varepsilon_X} \frac{p}{\varepsilon_L}$$

and substitute in our expression for  $\lambda$ :

$$\lambda = \frac{\varepsilon_L m / (1 - m)}{1 - \varepsilon_L m / (1 - m)}$$

To get:

$$MIE = p \frac{m / (1 - m)}{1 - \varepsilon_L m / (1 - m)} \frac{-\eta_{XN}}{\varepsilon_X}$$

# Second-best non-revenue raising environmental policy

$$MIE = p \frac{m/(1-m)}{1 - \varepsilon_L m/(1-m)} \frac{-\eta_{XN}}{\varepsilon_X}$$

MIE is bigger and the absolute value of the difference in first and second-best pollution levels is larger if:

1. Labor supply is more elastic:  $\varepsilon_L$  bigger
2. Demand for  $X$  is more inelastic:  $\varepsilon_X$  smaller
3. Elasticity of substitution between  $N$  and  $X$  is greater:  $\eta_{XN}$  bigger in magnitude

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In our model the government has a revenue requirement:

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For simplicity we still assume tax revenues are returned lump sum to households

# Revenue raising environmental policy

First derive household spending on the numeraire good:

$$Z = (1 - m)w(T - N) - pX + G = w(T - N) - pX + \tau X$$

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These are a function of the govt's choice of  $m$  and  $\tau$

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The household FOCs are:

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Next, as usual, differentiate the FOCs wrt  $\tau$

# Revenue raising environmental policy

This gives us 3 equations and 3 unknown partial derivatives:

$$u_{XX} \frac{\partial X}{\partial \tau} + u_{XN} \frac{\partial N}{\partial \tau} = \frac{\partial p}{\partial \tau} \quad (\text{Household X FOC})$$

$$u_{XN} \frac{\partial X}{\partial \tau} + u_{NN} \frac{\partial N}{\partial \tau} = 0 \quad (\text{N FOC})$$

$$C''(X) \frac{\partial X}{\partial \tau} = \frac{\partial p}{\partial \tau} - 1 \quad (\text{Firm X FOC})$$



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$$\frac{\partial X}{\partial \tau} = \frac{u_{NN}}{H} < 0$$

$$\frac{\partial N}{\partial \tau} = \frac{-u_{XN}}{H} \lessgtr 0$$

$$\frac{\partial p}{\partial \tau} = \frac{u_{XX}u_{NN} - u_{XN}^2}{H} > 0$$

where  $H = u_{XX}u_{NN} - u_{XN}^2 - C''(X)u_{NN} > 0$

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Substitute in for  $Z$  from household spending:

$$Z = w(T - N) - pX + \tau X$$

And look at the  $\tau$  FOC

# Revenue raising environmental policy

$$\left[ u_X - C'(X) - D'(X) \right] \frac{\partial X}{\partial \tau} + \lambda \left[ X + \tau \frac{\partial X}{\partial \tau} \right] + \left[ \underbrace{u_N - w}_{-wm} - \lambda wm \right] \frac{\partial N}{\partial \tau} = 0$$

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Just follow the same steps as we did with the non-revenue raising case and divide by  $\frac{\partial X}{\partial \tau}$  to get:

$$\underbrace{u_x - C'(X)}_{MAC} + \underbrace{(1 + \lambda)wm \left[ -\frac{\partial N}{\partial \tau} / \frac{\partial X}{\partial \tau} \right]}_{MIE} + \underbrace{\lambda \left[ \tau + x / \frac{\partial X}{\partial \tau} \right]}_{MRE} = D'(X)$$

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Since the tax is per unit, we have that:  $\frac{\partial N}{\partial \tau} / \frac{\partial X}{\partial \tau} = \frac{\partial N}{\partial p} / \frac{\partial X}{\partial p}$ , MIE is similar in revenue and non-revenue raising contexts



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Let's get some intuition at the corner case of  $\tau = 0$

What's the sign of  $MRE$ ?

# Revenue raising environmental policy

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- this reduces the distortionary tax in the labor market
- this reduces welfare losses in the labor market
- this reduction in welfare losses reduces the marginal social cost of reducing  $X$

# Revenue raising environmental policy

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We can get some intuition by making a substitution:

$$MRE \equiv \lambda \left[ \tau + x / \frac{\partial X}{\partial \tau} \right] = \lambda \left[ \tau + x / \frac{\partial X}{\partial p} \right] = \lambda [\tau + p / \varepsilon_X]$$

where  $\varepsilon_X < 0$  is the elasticity of demand for the dirty good

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# Revenue raising environmental policy

$$MRE \equiv \lambda [\tau + p/\varepsilon_X]$$

MRE is negative and increases total abatement if:

- demand for dirty good is sufficiently inelastic
- the price of the dirty good is sufficiently larger than the emission tax

# Revenue raising environmental policy

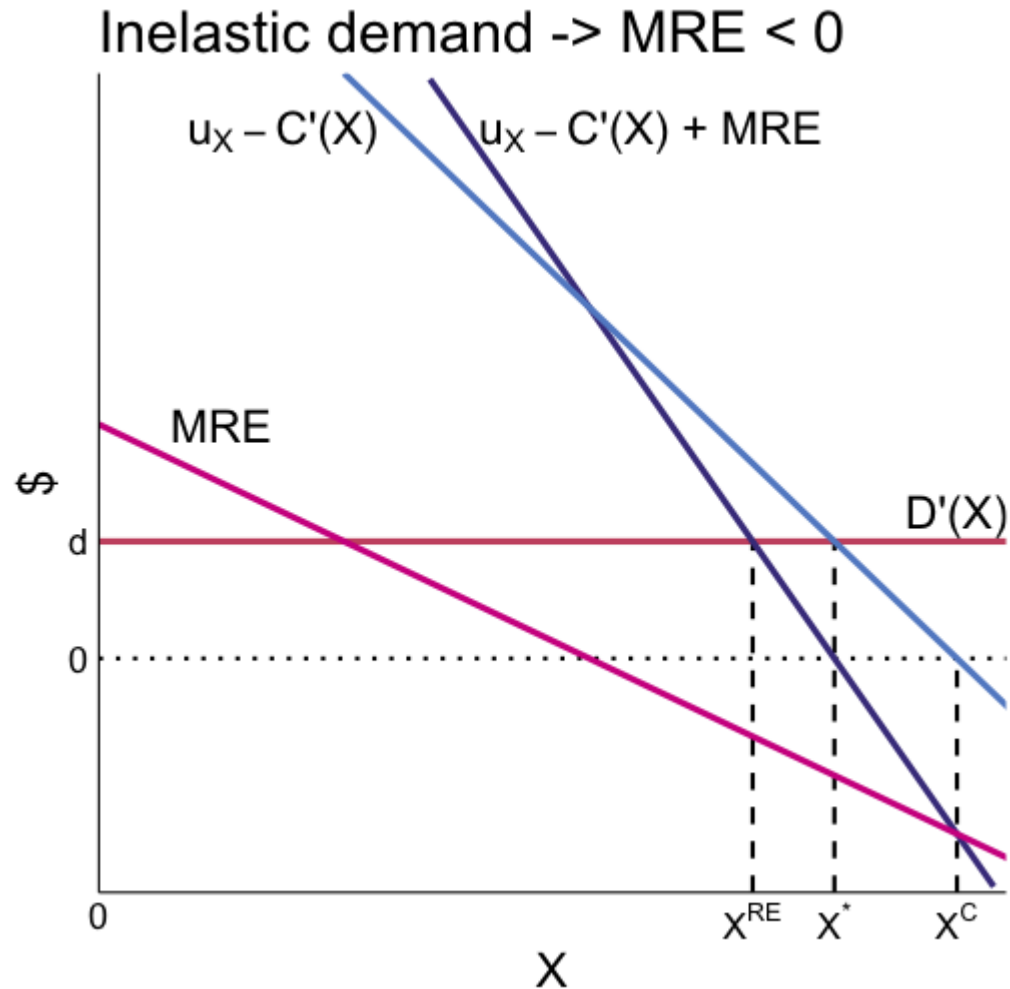
$$MRE \equiv \lambda [\tau + p/\varepsilon_X]$$

MRE is negative and increases total abatement if:

- demand for dirty good is sufficiently inelastic
- the price of the dirty good is sufficiently larger than the emission tax

Why?

# Revenue raising environmental policy

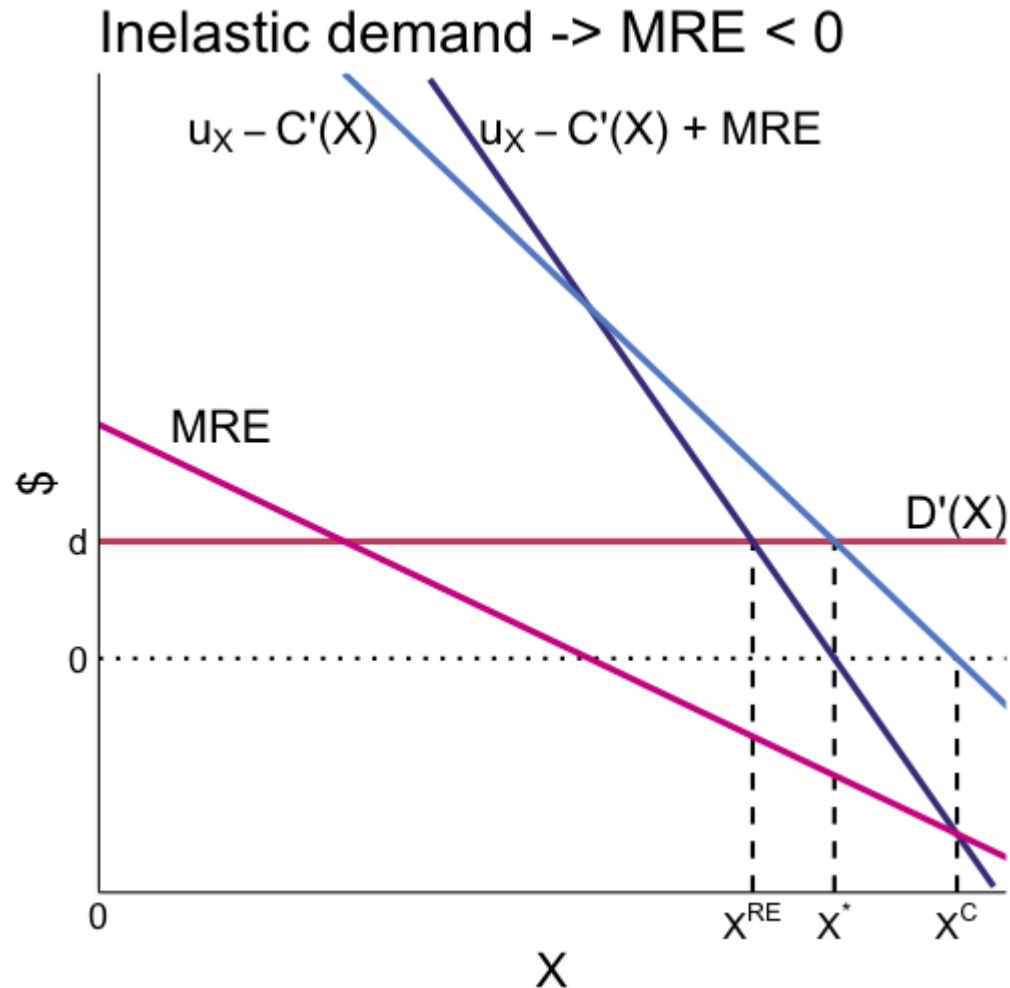


**Demand for dirty good is sufficiently inelastic:**

Suppose  $\frac{\partial N}{\partial p} = 0$  so  $MIE = 0$ ,  
 $C'(X) = c$ ,  $D'(X) = d$

Inelastic demand lets us raise more revenue from a small change in the tax

# Revenue raising environmental policy

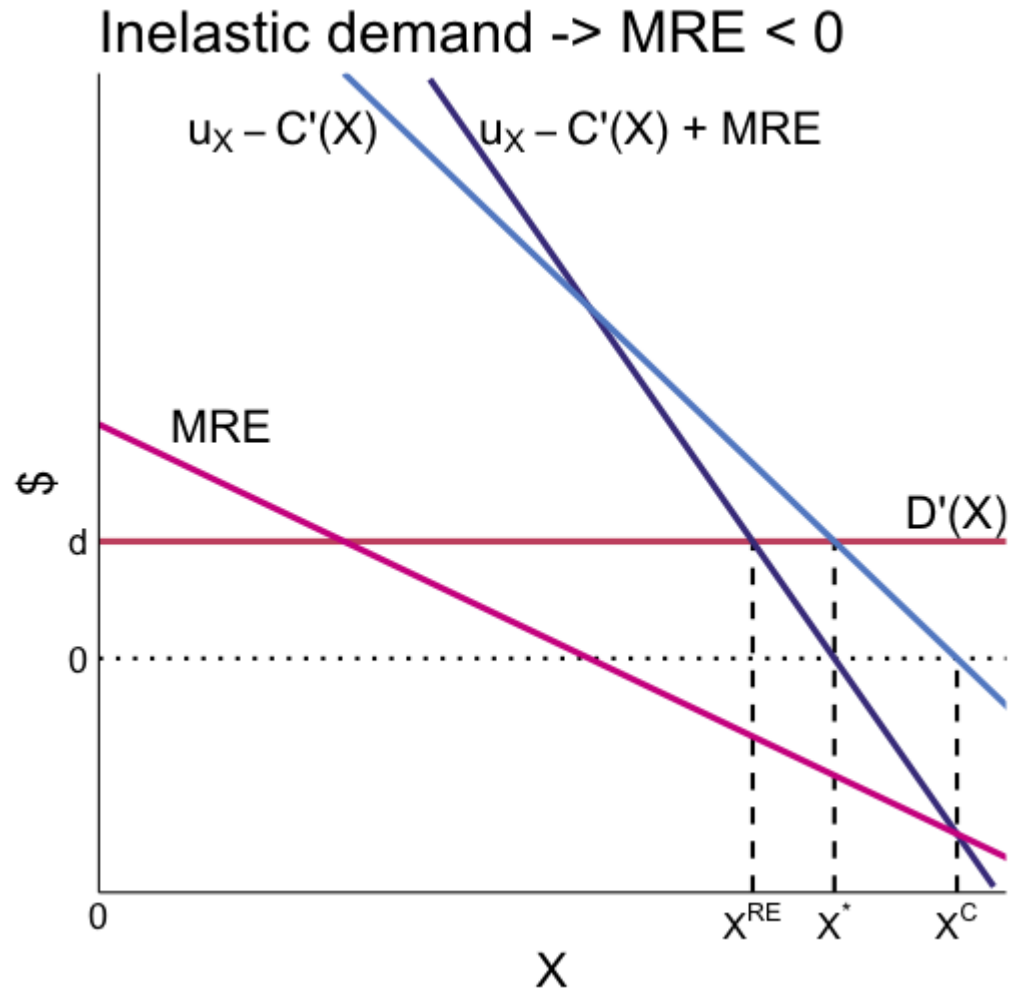


Inelastic demand lets us raise more revenue from a small change in the tax

This reduces the marginal social cost of reducing  $X$

Optimal  $X$  with revenue-raising is lower than without:  $X^{RE} < X^*$

# Revenue raising environmental policy



We can also see that if  $D'(X)$  was very large, making  $\tau$  larger, we would be where  $MRE > 0$

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There is a **weak double dividend** if welfare is always greater when revenue raised via environmental taxation is used to reduced distortionary taxation rather than refunded lump sum

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There is a **strong double dividend** if the emission tax should always be set above the  $MAC = MD$  level, resulting in greater pollution reductions and more revenue raised

- This may or may not be true

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Price of  $X$  rises from  $\tau$ , demand for leisure goes down, labor goes up

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Let's look at this pathway in more detail

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Again, assume  $C'(X) = c$ , this gives us that:

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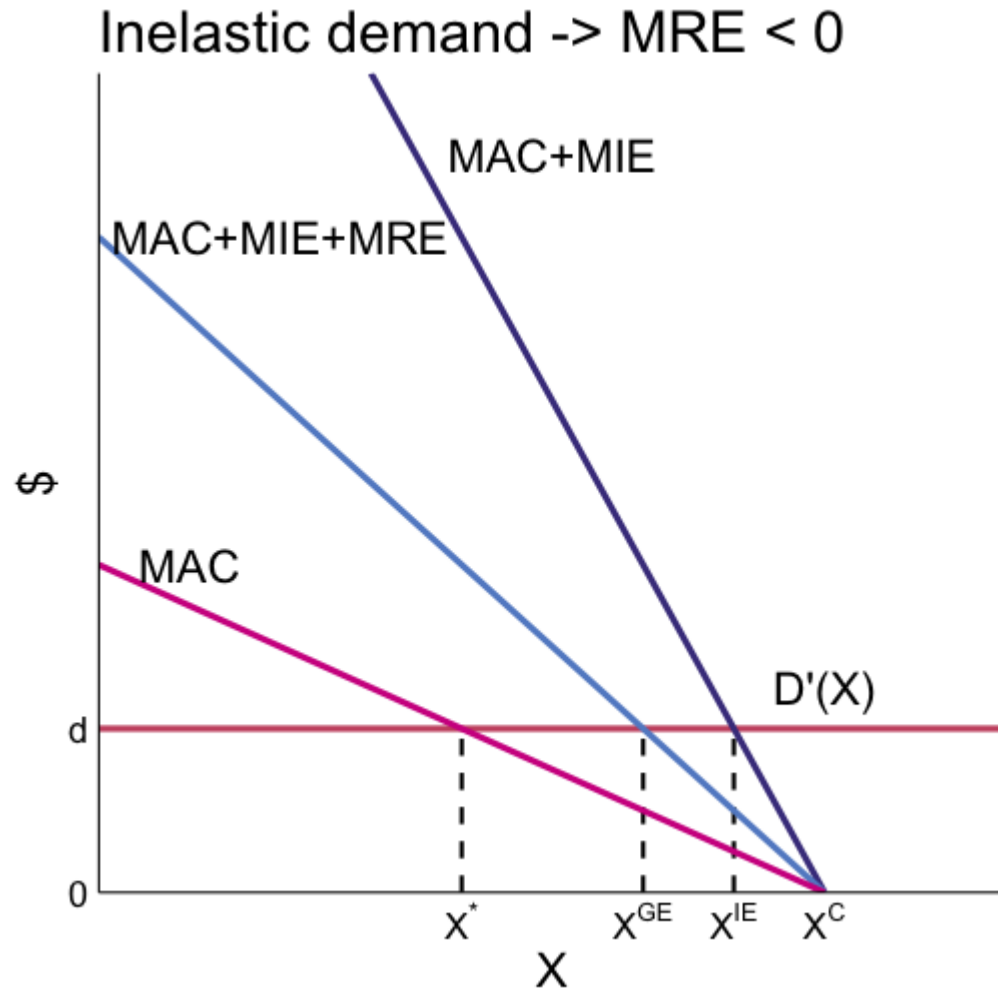
$$MIE = \lambda \left( -\frac{\eta_{XN}}{\varepsilon_X} \right) \frac{p}{\varepsilon_L}$$
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Suppose N and X are *average substitutes* which means  $\eta_{XN} = \varepsilon_L$ , then:

$$MIE = \lambda \left( -\frac{p}{\varepsilon_X} \right) < \lambda \left( \frac{p}{\varepsilon_X} + \tau \right) = MRE$$

$\Rightarrow$  we shouldn't expect a strong double dividend

# Revenue raising environmental policy



Even though there isn't a double dividend, MIE and MRE **still matter** for the optimal second-best pollution level

Optimal pollution  $X^{GE}$  is larger than first-best  $X^*$ , but less than the level without revenue recycling  $X^{IE}$

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What about freely allocated permits or command and control?

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Without revenue from permits or taxes, the optimal pollution level is higher