Lecture 8

Theory of applied welfare analysis

Ivan Rudik
AEM 6510

Roadmap

- Review welfare theory
- Understand how the theory can be used to measure changes in welfare from changes in prices
- Understand different kinds of welfare measures, and when to use them

Establishing value

How do we establish monetary value?

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WTP and WTA are income-equivalents that link the starting and ending states to preferences

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WTP is how much the person is willing to give up to have the new price

WTA is how much the person needs to be given in lieu of the price decrease

WTP and WTA

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- If there's a lot of substitutes for the good, the price decrease isn't that valuable
- If there's few substitutes, the price decrease may be very valuable

The general model

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Let's start with a generalization of our consumer model

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- etc

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The consumer maximizes utility given some fixed level of q, vector of market prices $p = \{p_1, \ldots, p_J\}$, and income y:

$$\max_{z, x_1, \ldots, x_J} U(x_1, \ldots, x_J, z, q) + \lambda[y - z - \sum_{i=1}^{J} p_i x_i]$$

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This gives us the following FOCs:

$$U_{x_j} = \lambda p_j \quad j = 1, \dots, J$$

and

$$U_z = \lambda$$

The general model

With the FOCs we can solve for the *ordinary* demand functions $x_j(p, y, q)$, the Lagrange multiplier $\lambda(p, y, q)$, and z[1]

[1] Ordinary demand functions are the regular ones you've seen so far.

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Note we can directly estimate ordinary demand functions since they depend on observables p, y, q

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If we substitute x_j into U we get the **indirect utility function** $V(p, y, q)$

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Note that λ can be interpreted as the marginal utility of income

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We can also represent the consumer's behavior by the **dual** expenditure minimization problem:[1]

$$\min_{x_1,...,x_J,z} \sum_{i=1}^{J} p_i x_i + z + \mu[\bar{u} - U(x_1, \ldots, x_J, z, q)]$$

where $\bar{u}$ is a reference level of utility

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If we substitute the $h_j's$ into the minimization problem we get the expenditure function $E(p, \bar{u}, q)$ which is the minimum income required to achieve $\bar{u}$

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This links the solutions to utility max and cost min at the observed point of consumption by:

$$x_j(p, E(p, u^0, q), q) \equiv h_j(p, u^0, q) \quad \forall j$$

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$$\begin{aligned} \frac{\partial x_j}{\partial p_j} &= \frac{\partial h_j}{\partial p_j} - \frac{\partial x_j}{\partial y} \times \frac{\partial E_j}{p_j} \\ &= \frac{\partial h_j}{\partial p_j} - \frac{\partial x_j}{\partial y} \times x_j \end{aligned}$$

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The second equality comes from Shephard's Lemma: $h_j = \frac{\partial E_j}{\partial p_j}$ (envelope theorem) and the fact that $x_j(p, E(p, u^0, q), q) \equiv h_j(p, u^0, q)$

Ordinary and compensated demand

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What does this result show us?

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- The difference between compensated (h) and ordinary (x) demand is an **income gradient** $\frac{\partial x_j}{\partial y} \times x_j$
  - If there's no income effect $\frac{\partial x_j}{\partial y}$, then they are equivalent

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  - Moving along the ordinary demand curve confounds the pure price effect, and an implicit income effect (i.e. the substitution and income effects)

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This is important to understand the types of welfare measures we will be using

Price change welfare measures

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There are two concepts we can use to measure this effect, which just differ in reference point

Compensating variation

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CV is the income offset that gives you the pre-change utility back following the price change

Compensating variation

Given a change in price from p^0 to $p^1 < p^0$ the CV is:

$$V(p^0, y, q) = V(p^1, y - CV, q)$$

where V is the indirect (maximized) utility function

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Here $CV > 0$ since we are looking at a price decrease

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CV is the minimum someone is willing to accept to have a higher price

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Equivalent variation

The second concept is **equivalent variation** (EV)

> For a price decrease (increase) that provides a higher (lower) utility level, the EV is the payment (reduction) that moves the person to the new utility level, without the price change

EV uses the post-change level of utility as the reference, it's the income change that puts them at the post-change level of utility without the price change occuring

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Once we have CV or EV we have defensible measures for a consumer's value of an exogenous change in some variable

Two additional formulations

Before we continue, let's write down two additional expressions for CV and EV that will help us operationalize our theory:

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$$\begin{aligned} CV &= E(p^0, u^0, q) - E(p^1, u^0, q) \\ &= y - E(p^1, u^0, q) \end{aligned}$$

$$\begin{aligned} EV &= E(p^0, u^1, q) - E(p^1, u^1, q) \\ &= E(p^0, u^1, q) - y \end{aligned}$$

Where the second set of equalities come from the duality of the two problems: $E(\cdot)$ gives the expenditure (income) needed to achieve utility (u^0, u^1) given prices (p^0, p^1) in the utility maximization problem

From expenditure to demand

Now, by the fundamental theorem of calculus we have:

$$CV = E(p^0, u^0, q) - E(p^1, u^0, q) = \int_{p_j^1}^{p_j^0} \frac{\partial E(p, p_{-j}, u, q)}{\partial p_j} dp_j$$

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where p_{-j} is the set of prices without p_j

CV and EV as compensated demand

Remember that we found:

$$h_j(p, u, q) = \frac{\partial E(p, u, q)}{\partial p_j}, \quad j = 1, \dots, J$$

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What does this say about how we interpret CV and EV?

Value is area under the curve

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Key point: CV and EV are the area under the appropriate compensated demand curve, between two price levels

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One problem: we usually estimate *ordinary* demand curves because we don't observe u^0, u^1, we will get to this in a bit

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Let's look at how we can see WTP, WTA, CV, and EV graphically

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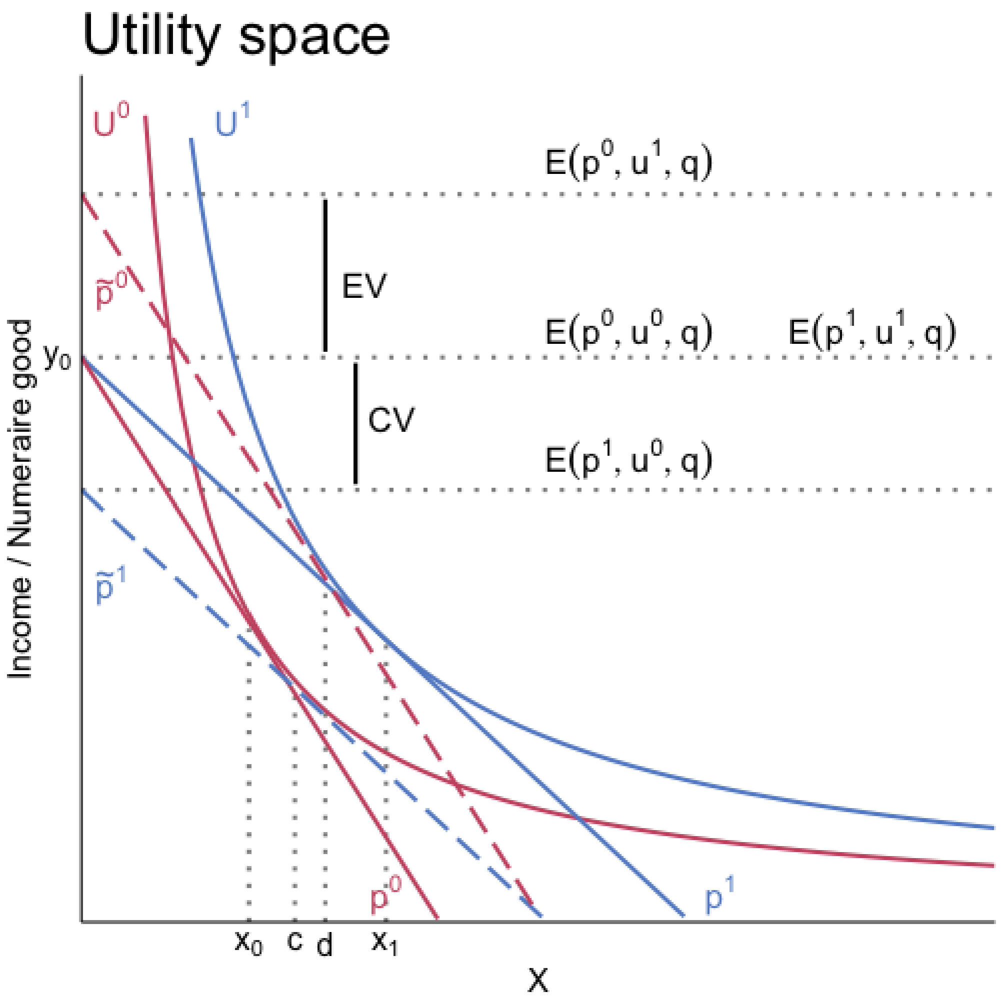
The book shows the intuition in indirect utility space if you're interested (Fig 14.1 Panel B)

CV and EV in utility space

The red solid budget line labeled p^0 is the budget constraint under price p^0

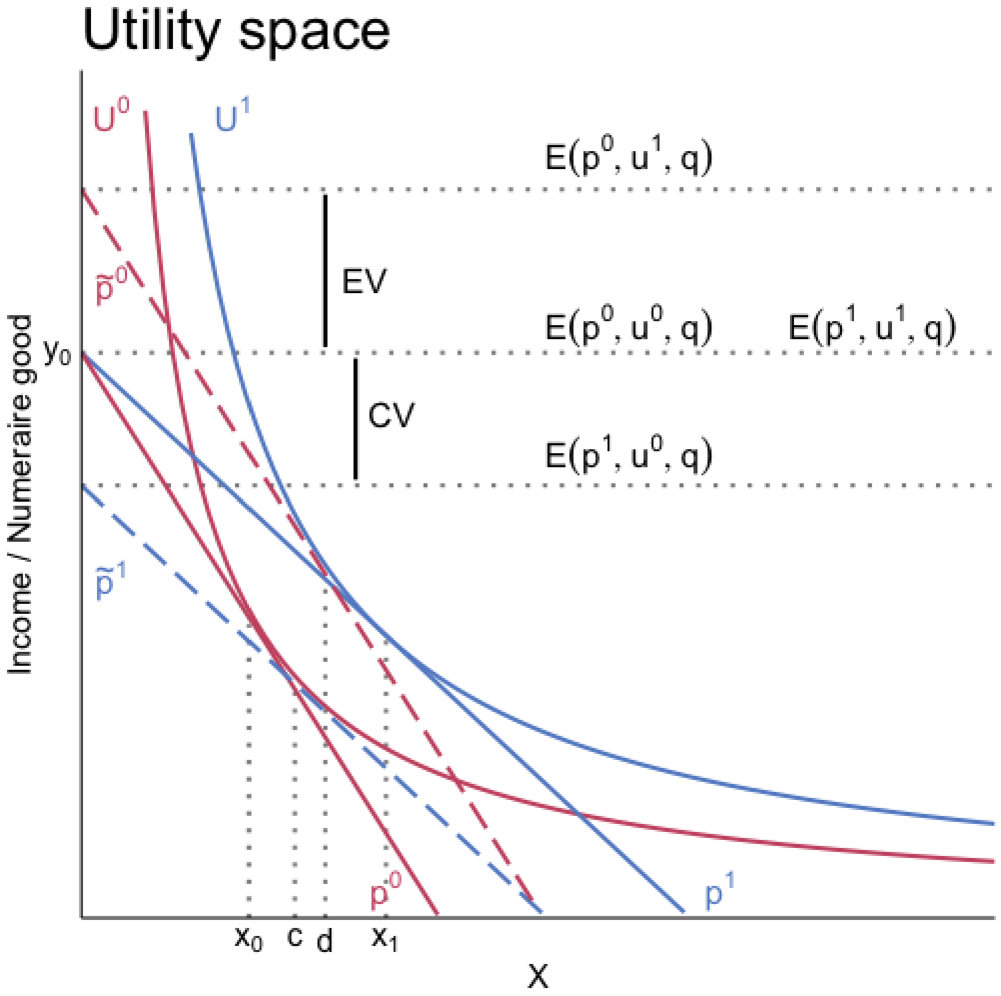
The blue solid budget line labeled p^1 is the budget constraint under price p^1

p_1 kicks out the budget constraint because $p_1 < p_0$



CV and EV in utility space

The consumer chooses consumption levels x^0 and x_1 to reach the highest indifference curves u^0 (red) and u^1 (blue)

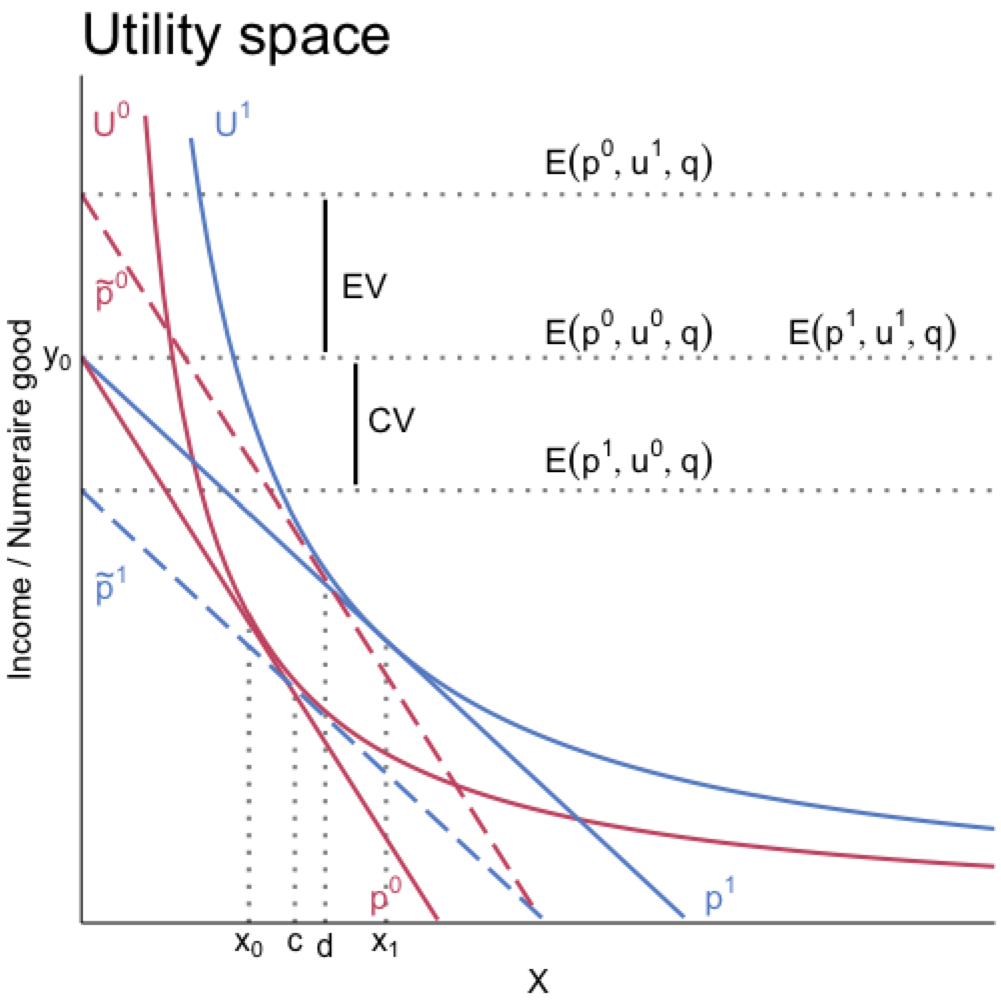


CV and EV in utility space

CV is given by the expenditure needed to reach u^0 given the new price

You can compute it by constructing a hypothetical budget line $\tilde{p}^1$ (blue dashed) from price p^1 but with reduced income so the consumer can only reach u^0

This change in income is CV



CV and EV in utility space

EV is given by the income needed to reach u^1 given the old price

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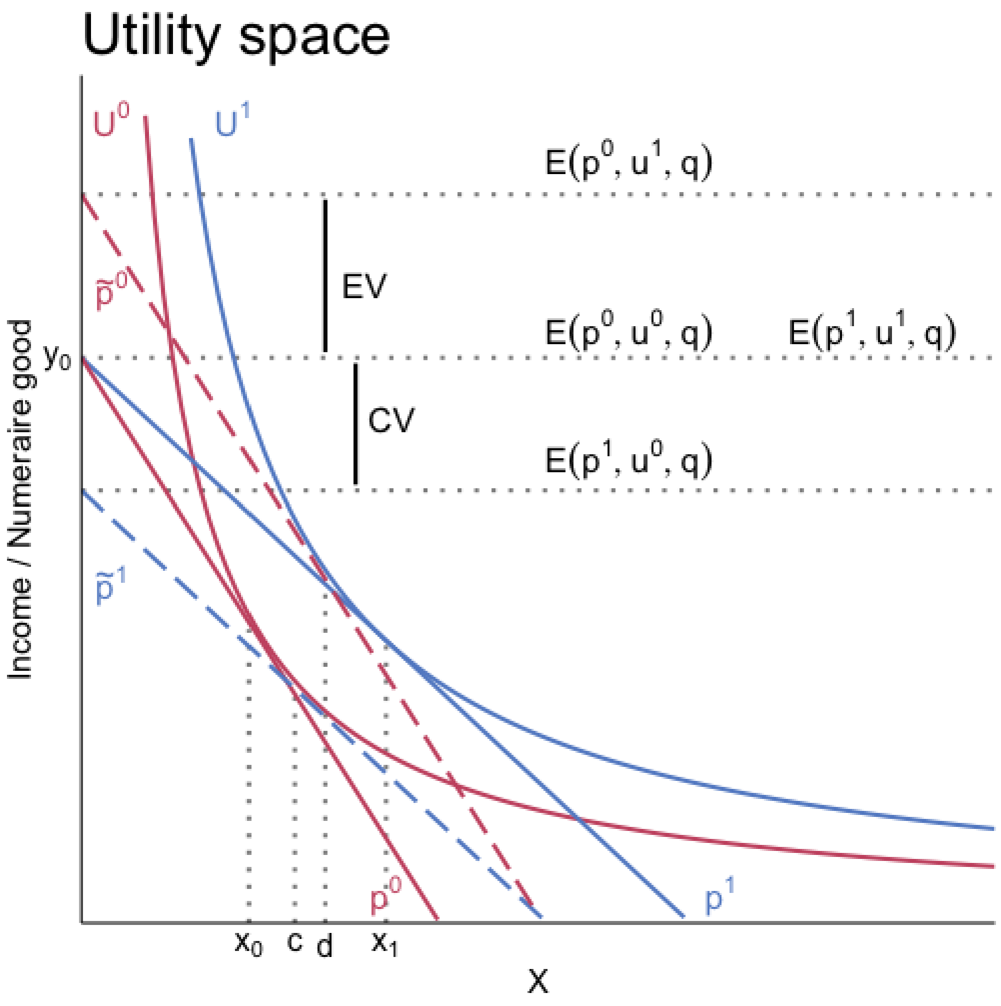
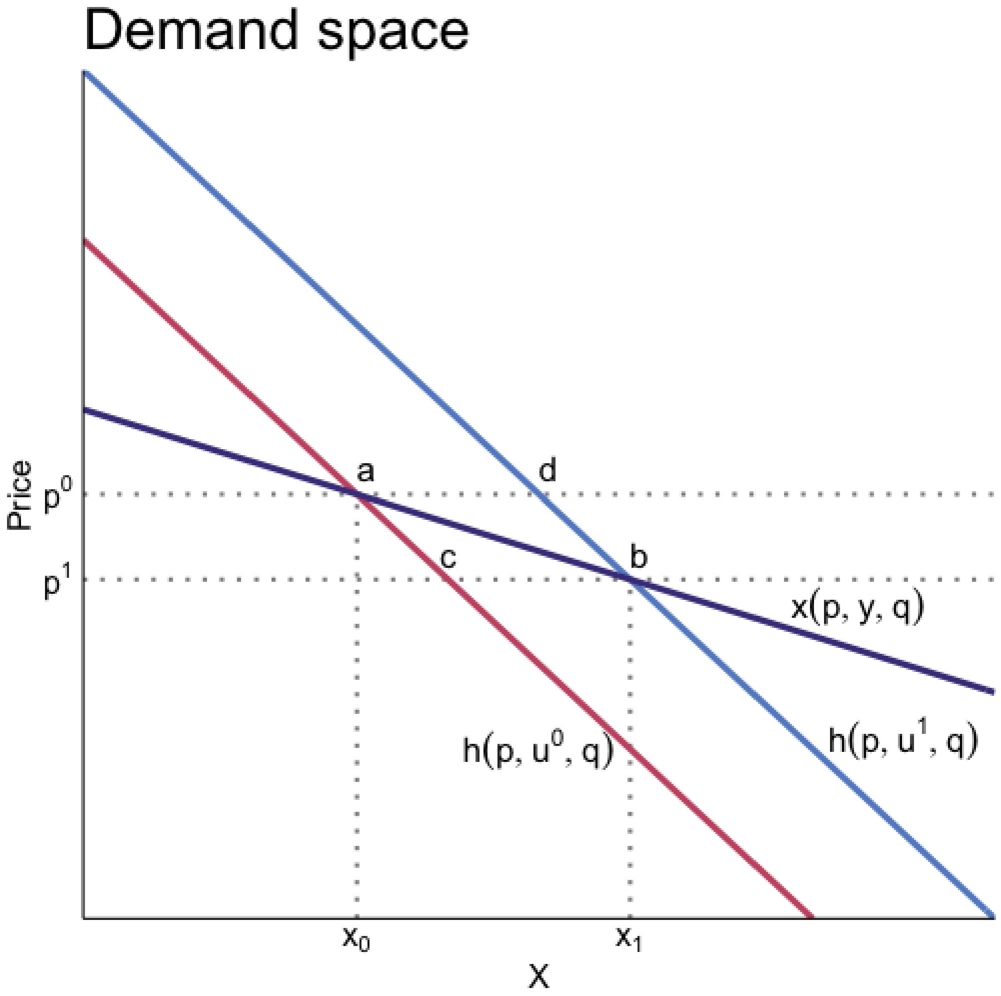
This change in income is EV



CV and EV in demand space

The price change traces out the ordinary demand curve (dark blue):

We are holding income y and environmental quality q fixed, so changes in price move us along $x(p, y, q)$

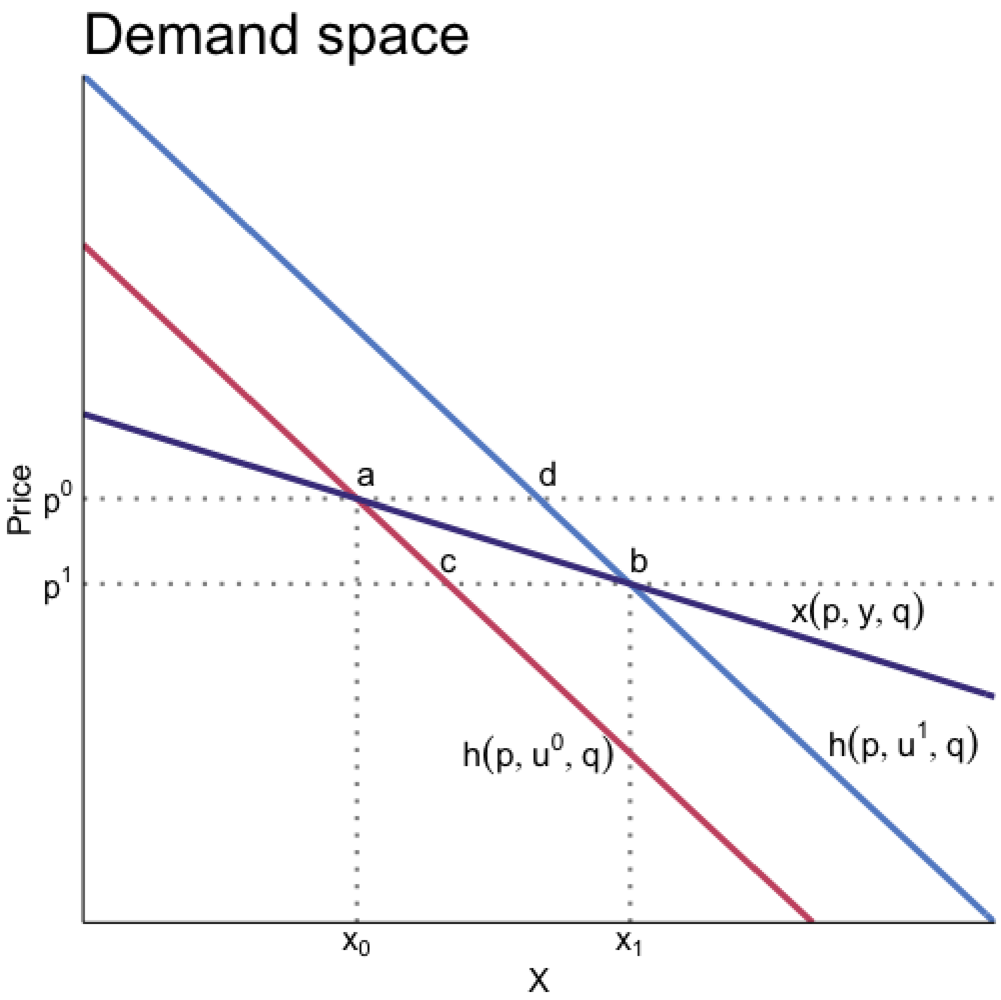


CV and EV in demand space

Utility is not held constant so we are moving across different compensated demand curves (red to light blue)

What traces out the compensated demand curves?

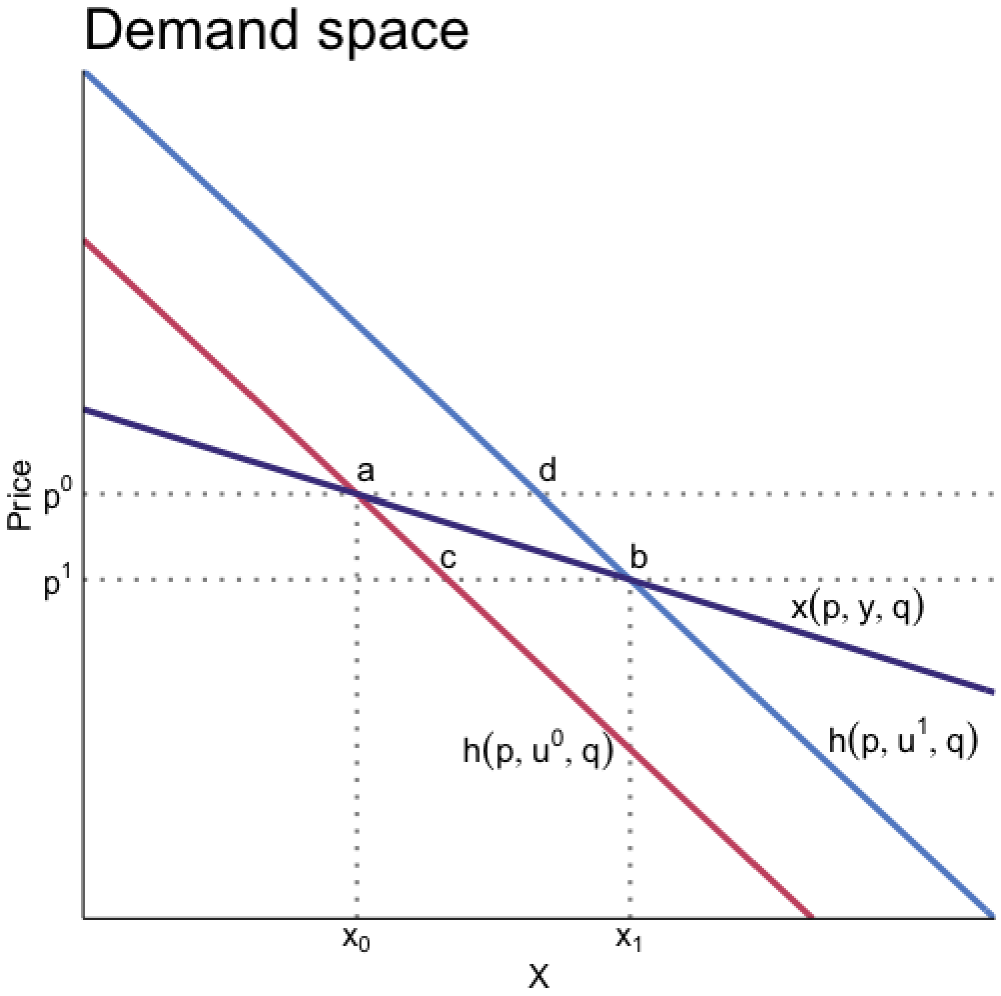
Changes in *budget constraints*



CV and EV in demand space

From the utility space example: we conceptualized moving from p^0 to $\tilde{p}^1$, a change in the budget constraint (price and also income) that kept utility constant, in order to recover CV

This change traces out $h(p, u^0, q)$



CV and EV in demand space

From the utility space example: we conceptualized moving from p^1 to $\tilde{p}^0$, a change in the budget constraint (price and also income) that kept utility constant, in order to recover EV

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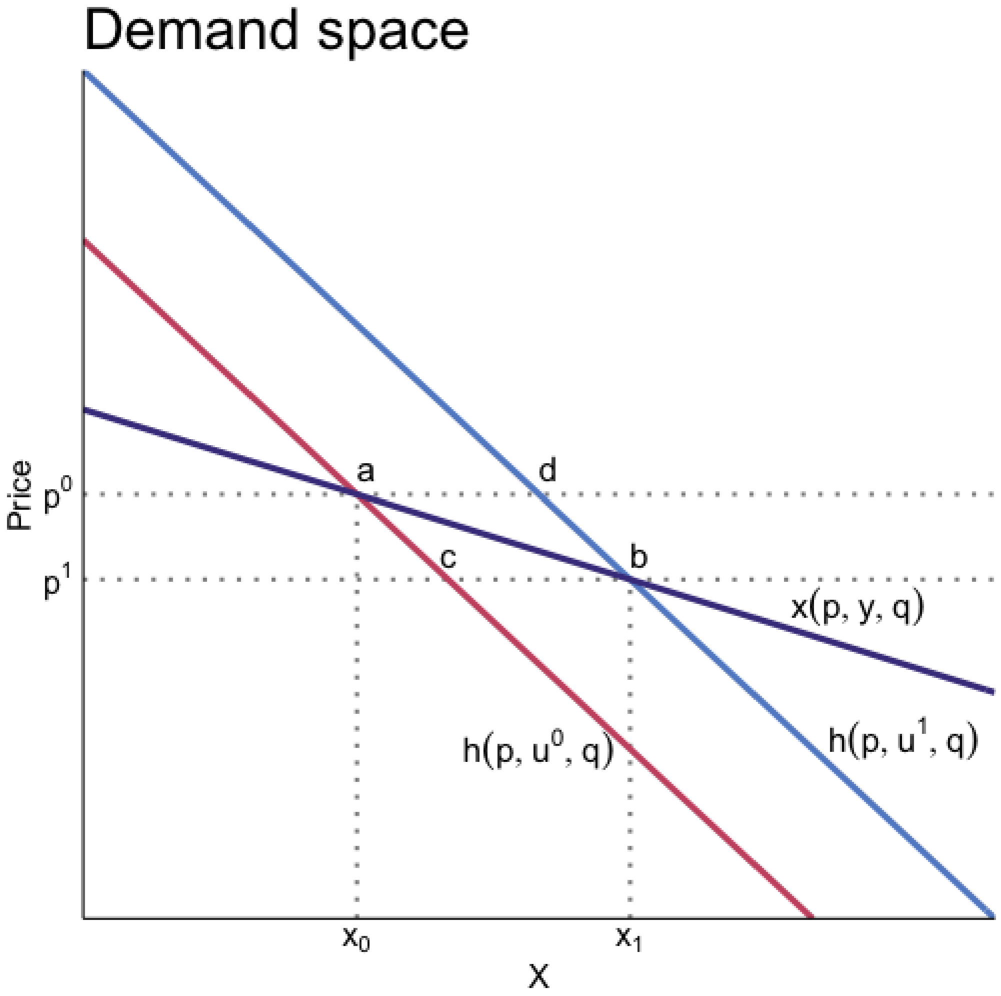
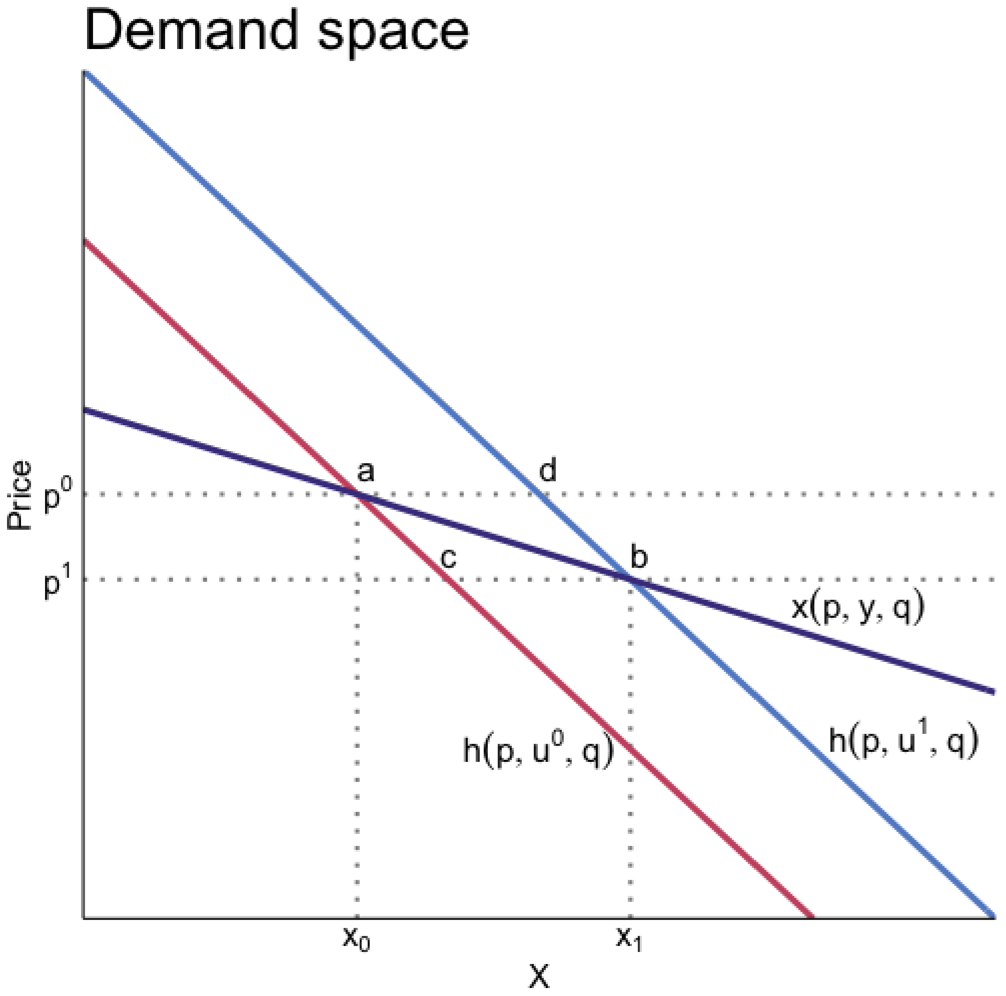


CV and EV in demand space

We never observe $\tilde{p}^0$ or $\tilde{p}^1$, they're just hypothetical

This illustrates how we do not directly observe compensated demand curves even though they are how we compute CV and EV



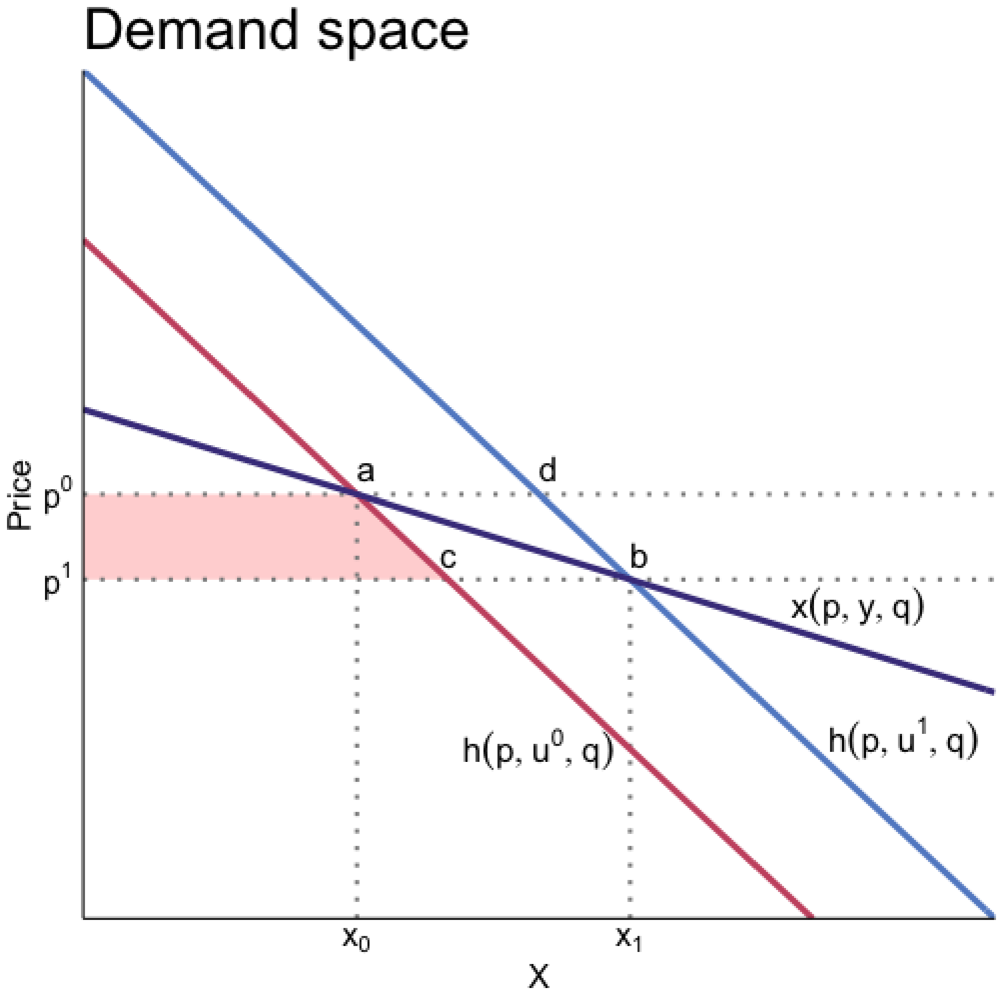
CV and EV in demand space

What is CV and EV on the graph?

CV is (p^0, a, c, p^1)

It is the area under the original compensated demand curve

And note that area under is flipped because price is on the y axis for the inverse demand curves we plot

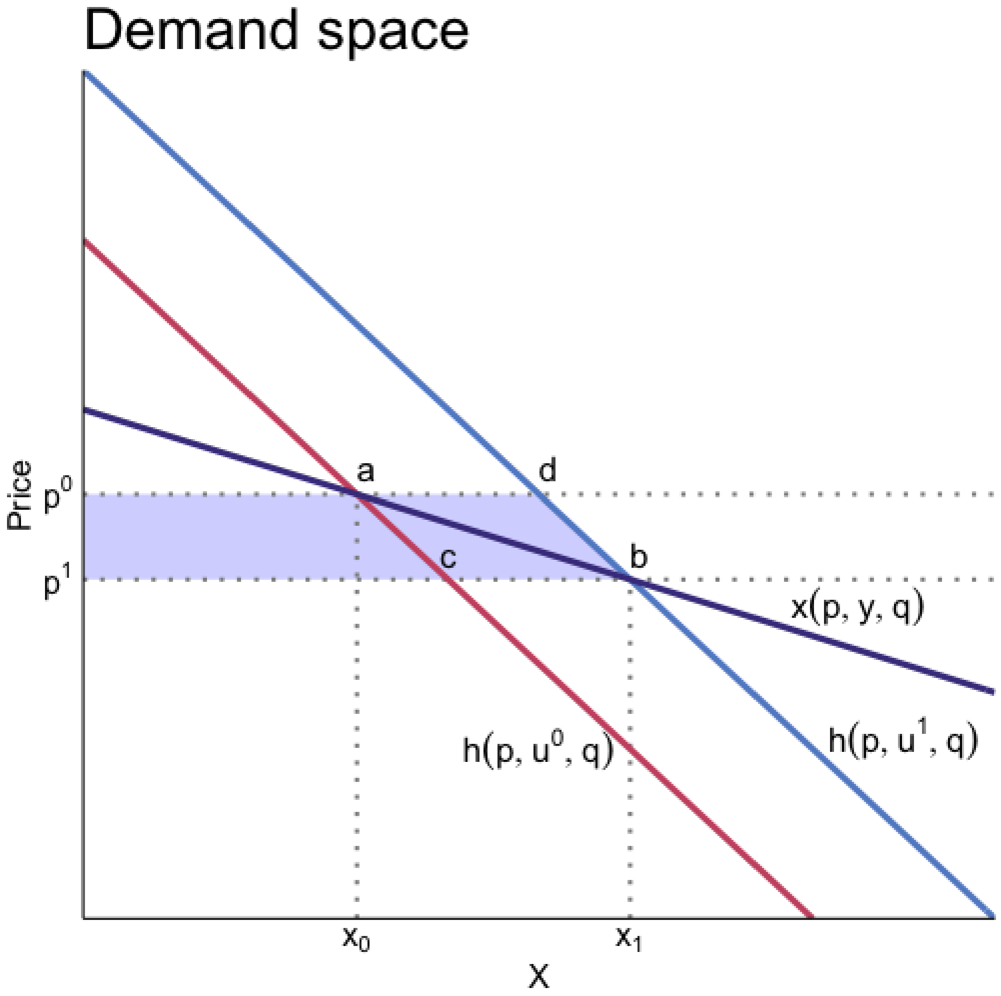


CV and EV in demand space

What is CV and EV on the graph?

EV is (p^0, d, b, p^1)

It is the area under the new compensated demand curve



Toward computing CV and EV

We saw that we can compute CV and EV using compensated demand curves, so we can link these valuation concepts to behaviorial function for the good

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Our problem again is that we do not observe compensated demand curves, but ordinary demand curves

Often times economists will use **consumer surplus (CS)** in place of CV or EV

Consumer surplus

CS is effectively the ordinary demand version of EV and CV

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$$CS = \int_{p_j^0}^{p_j^1} x_j(p, y, q) dp_j$$

Since this is based on ordinary demand, we can compute it easily if we have an estimate of consumer demand

CS in demand space

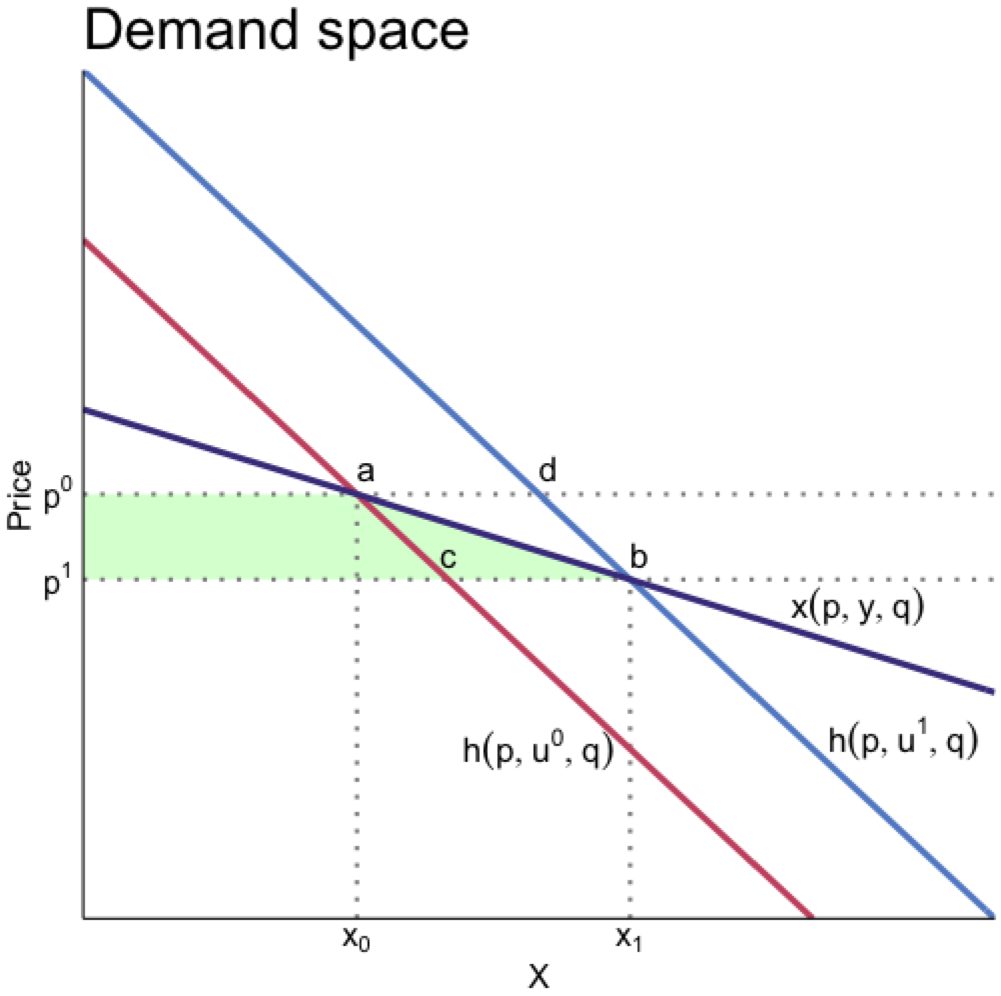
What is CS on the graph

CS is (p^0, a, b, p^1)

It is the area under the ordinary demand curve

What is this measuring?

How does it relate to WTP and WTA?



What is CS

In general CS has no WTP/WTA interpretation since utility is not held fixed for movements along an ordinary demand curve (look at last figure, we jumped compensated demand curves!)

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Let's see if CS is something else that can be useful

Roy's identity

First we need to derive a central result in economics, **Roy's Identity**

$$x_j = -\frac{\partial V/\partial p_j}{\partial V/\partial y}$$

Roy's identity relates ordinary demand to the indirect utility function $V(p, y)$

Roy's identity

The derivation is pretty simple

Plug the expenditure function into V at $\bar{u}$:

$$V(p, e(p, \bar{u})) = \bar{u}$$

Roy's identity

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Plug the expenditure function into V at $\bar{u}$:

$$V(p, e(p, \bar{u})) = \bar{u}$$

Differentiate both sides with respect to p_j:

$$\frac{\partial V}{\partial p_j} + \frac{\partial V}{\partial y}\frac{\partial e}{\partial p_j} = 0$$

and recall that $\frac{\partial e}{\partial p_j} = h_j(p, \bar{u}) = x_j(p, e(p, \bar{u}))$

Roy's identity

We then get:

$$\frac{\partial V}{\partial p_j} + \frac{\partial V}{\partial y} x_j = 0$$

and finally

$$x_j = -\frac{\partial V / \partial p_j}{\partial V / \partial y}$$

It's kind of like an MRS, the demand for good x_i is the income increase required to compensate for a change in the price of good i

What is CS?

Now plug this expression for x_i into our definition of CS to get:

$$CS = \int_{p_j^0}^{p_j^1} x_j(p,y,q)dp_j = \int_{p_j^0}^{p_j^1} -\frac{V_{p_j}(p,y,q)}{V_y(p,y,q)}dp_j = \int_{p_j^0}^{p_j^1} -\frac{V_{p_j}(p,y,q)}{\lambda(p,y,q)}dp_j$$

where $V_{p_j}(p,y,q) = \partial v/\partial p_j$ and $V_y(p,y,q) = \partial v/\partial y$, and λ is the marginal utility of income

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Now lets look at our first result

Assume that $\lambda(p,y,q)$ is not a function of p_j: $\partial\lambda/\partial p_j = 0$

What is CS?

We can re-write CS as:

$$CS = \frac{1}{\lambda(p,y,q)} \int_{p_j^0}^{p_j^1} -V_{p_j}(p,y,q)dp_j = [V(p^0,y,q) - V(p^1,y,q)] \frac{1}{\lambda(p,y,q)}$$

If the marginal utility of income is constant with respect to price, **CS is a money-metric reflection of the change in utility!**

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- Change in utility: $V(p^0,y,q) - V(p^1,y,q)$
- Translated into dollar terms by: $\frac{1}{\lambda(p,y,q)}$

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- Translated into dollar terms by: $\frac{1}{\lambda(p,y,q)}$

CS is the change in money implied by a change in utility when $\partial\lambda/\partial p_j = 0$

What is CS?

$\partial\lambda/\partial p_j = 0$ is generally **not** going to be true

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$\partial\lambda/\partial p_j = 0$ is generally **not** going to be true

Pg 399-400 in the book show how assuming $\partial\lambda/\partial p_j = 0$ implies that the **income elasticity of demand** must be equal for all goods whose prices may change in the analysis:

$$\frac{\partial x_j}{\partial y}\frac{y}{x_j} = \frac{\partial x_k}{\partial y}\frac{y}{x_k} \quad \forall j, k$$

CS as an approximation

CS doesn't generally recover the money-equivalent change in utility

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Willig (1976) shows that CS is a first-order approximation if the income elasticity is small or the change in CS is small relative to the budget

CS and measurement

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To see this let's first look at two identities in consumer economics

1. From before: the observed demand level solves the utility maximization and expenditure minimization problems
2. $\bar{u} = V(p, E(p, \bar{u}, q), q)$, the indirect utility given an income y equal to the expenditure to achieve $\bar{u}$ is equal to $\bar{u}$

CS and measurement

Differentiate $\bar{u} = V(p, E(p, \bar{u}, q), q)$ with respect to p_j:

$$\frac{\partial V}{\partial p_j} + \frac{\partial V}{\partial y}\frac{\partial E}{\partial p_j} = 0$$

which gives us that:

$$\frac{\partial E(p, \bar{u}, q)}{\partial p_j} = -\frac{\partial V}{\partial p_j}\bigg/\frac{\partial V}{\partial y} = x_j(p, y, q)$$

where the second equality is **Roy's identity**

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where the second equality is **Roy's identity**

This relates income (equal to expenditures at the optimum) and price

CS and measurement

$$\frac{\partial E(p,\bar{u},q)}{\partial p_j} = \frac{\partial y(p)}{\partial p_j} = x_j(p,y,q)$$

Suppose we:

1. Parameterized x_j with some functional form
2. Estimated the parameters of x_j using real world data

We can then solve $\frac{\partial y(p)}{\partial p_j} = x_j(p,y,q)$ for y to get: $y[p_j, k(p_{-j},q)]$

k is a constant of integration

CS and measurement

If $k(p_{-j}, q)$ is held fixed, then $y[p_j, k(p_{-j}, q)]$ is a quasi-expenditure function

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Since utility is ordinal we only care about comparisons, not levels, so we can set $u^0 = k(p_{-j}, q)$ so that

$$y[p_j, k(p_{-j}, q)] = y[p_j, u^0] = \hat{E}(p_j, u^0)$$

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We can compute CV for a change in p_j easily using this quasi-expenditure function, but we need to assume prices of other goods and environmental quality are **fixed**

Quantity change welfare measures

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Quantity change welfare measures

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How is this analysis different and similar to our analysis of price changes?

First let's define CV and EV in terms of environmental quantity changes

Here we will be thinking about increasing q from q^0 to q^1

CV and EV: indirect utility

Compensating variation CV is given by:

$$V(p, y, q^0) = V(p, y - CV, q^1)$$

and equivalent variation EV is given by:

$$V(p, y, q^1) = V(p, y + CV, q^0)$$

CV and EV: indirect utility

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$$V(p, y, q^0) = V(p, y - CV, q^1)$$

and equivalent variation EV is given by:

$$V(p, y, q^1) = V(p, y + CV, q^0)$$

CV is the WTP to have the environmental improvement $q^0 \to q^1$

EV is the WTA to forgo the environmental improvement $q^0 \to q^1$

WTP vs WTA with quantity changes

Unlike with price changes the choice of EV or CV matters conceptually

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One implies the individual has property rights to the improvement (EV), and one implies they do not have property rights (CV)

WTP vs WTA with quantity changes

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One implies the individual has property rights to the improvement (EV), and one implies they do not have property rights (CV)

This may matter in practice because WTP and WTA can diverge due to budget constraints:[1]

- You can only pay as much as your budget but you can accept any positive amount

There are also other behavioral reasons, but we won't touch on them here. See Sec 14.4 for details on the budget argument.

CV and EV: expenditure function

We can also define CV and EV with the expenditure function:

$$\begin{aligned} CV =& E(p, u^0, q^0) - E(p, u^0, q^1) \\ =& y - E(p, u^0, q^1) \end{aligned}$$

$$\begin{aligned} EV =& E(p, u^1, q^0) - E(p, u^1, q^1) \\ =& E(p, u^1, q^0) - y \end{aligned}$$

CV and EV: demand curves

We can also compute quantity change CV and EV with demand curves like we did for price changes

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Note that since q is fixed from the individual's perspective, we will need to look at inverse demand curves (the usual kind on the graphs we draw)

CV and EV: demand curves

We can also compute quantity change CV and EV with demand curves like we did for price changes

Note that since q is fixed from the individual's perspective, we will need to look at inverse demand curves (the usual kind on the graphs we draw)

The compensated inverse demand is given by:

$$\Pi^q(p,u,q) = -\frac{\partial E(p,u,q)}{\partial q}$$

which is the marginal willingness to pay for q: it's the change in income that holds utility constant given a marginal increase in q

CV and EV: demand curves

CV and EV are then the area under the MWTP/inverse demand curves:

$$
\begin{aligned}
CV =& \int_{q^0}^{q^1} \pi^q(p, u^0, q) dq \\
=& \int_{q^0}^{q^1} -\frac{\partial E(p, u^0, q)}{\partial q} dq \\
=& E(p, u^0, q^0) - E(p, u^0, q^1)
\end{aligned}
$$

$$
\begin{aligned}
EV =& \int_{q^0}^{q^1} \pi^q(p, u^1, q) dq \\
=& \int_{q^0}^{q^1} -\frac{\partial E(p, u^1, q)}{\partial q} dq \\
=& E(p, u^1, q^0) - E(p, u^1, q^1)
\end{aligned}
$$

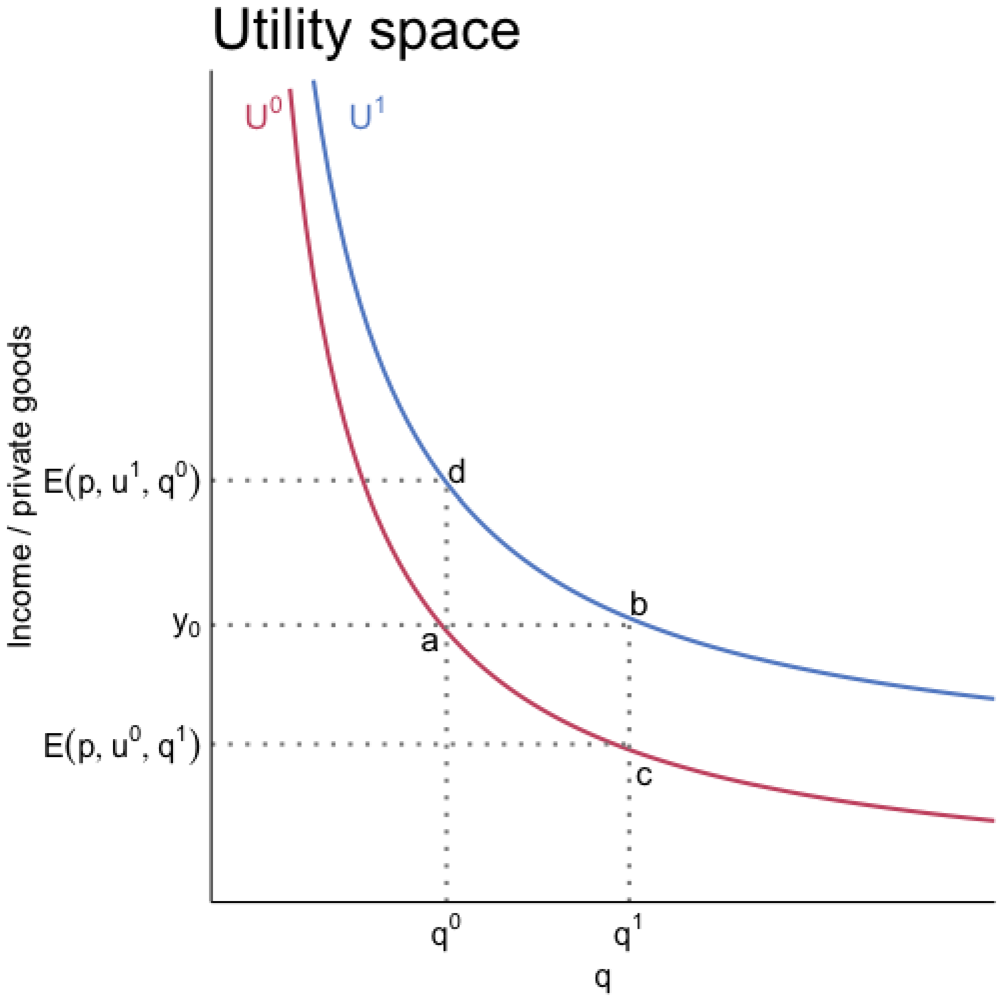
CV and EV in utility space

Y axis is income/spending on all private goods

X axis is quantity of the environmental good

We start at a where we are at u^0 and q^0

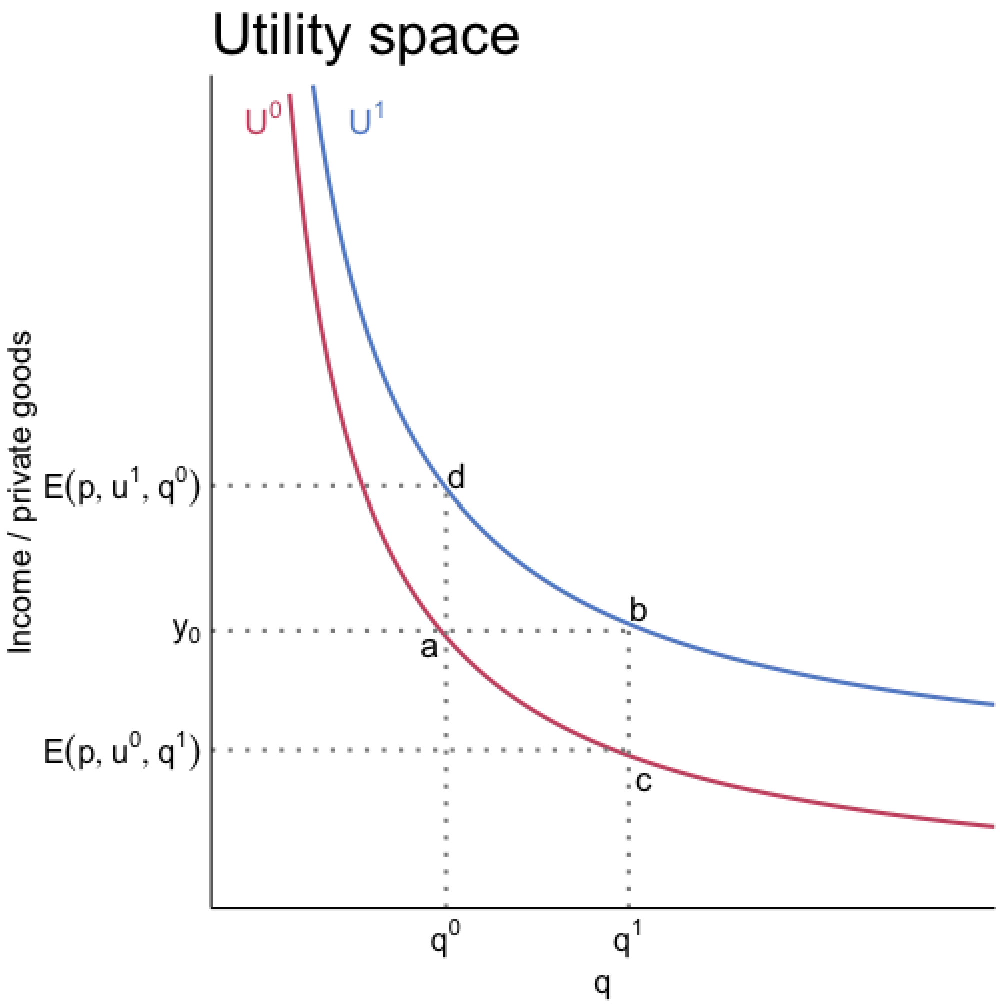
(Skipping drawing inverse demand curves)



CV and EV in utility space

When $q^0 \to q^1$ we move to point b and $u^0 \to u^1$

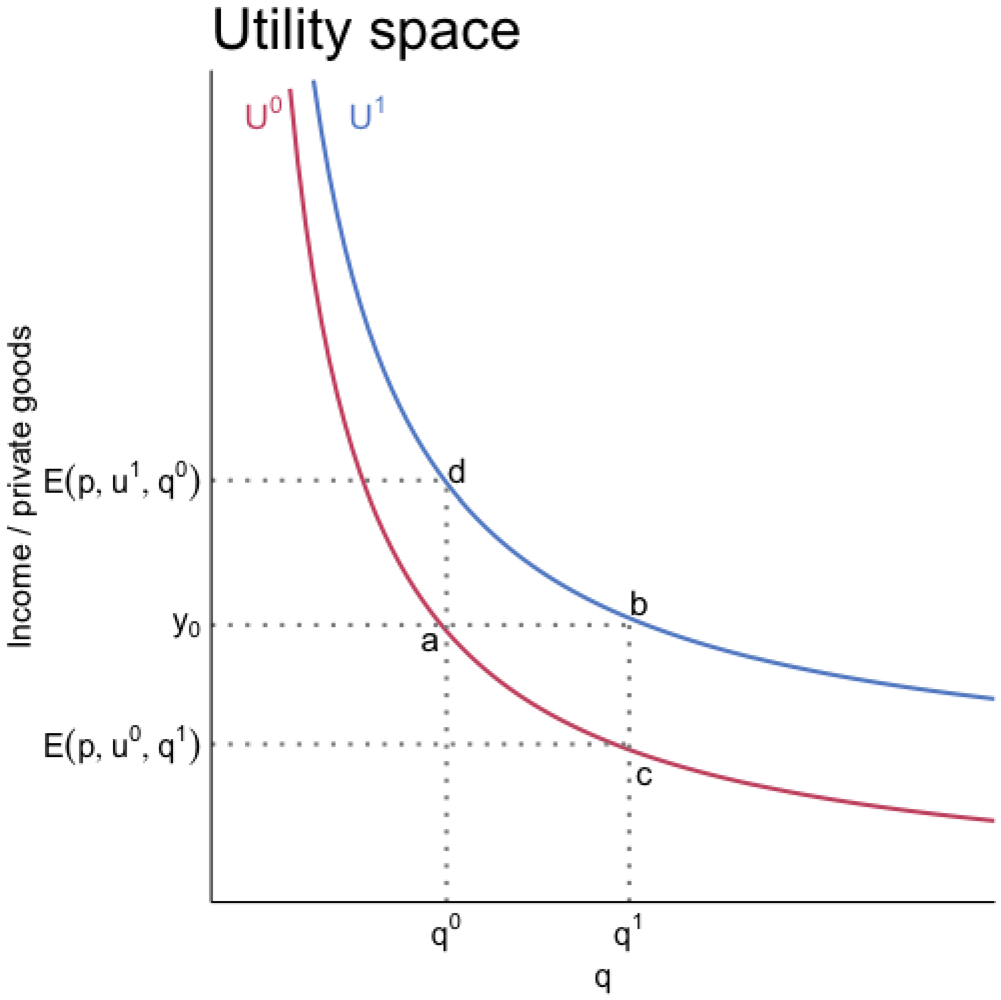
Income/expenditures is held constant because q has no price so we just move horizontally



CV and EV in utility space

CV is the change in income needed to go from $u^0 \rightarrow u^1$ at q^1:
$y_0 - E(p, u^0, q^1)$

EV is the change in income needed to go from $u^0 \rightarrow u^1$ at q^0:
$E(p, u^1, q^0) - y_0$

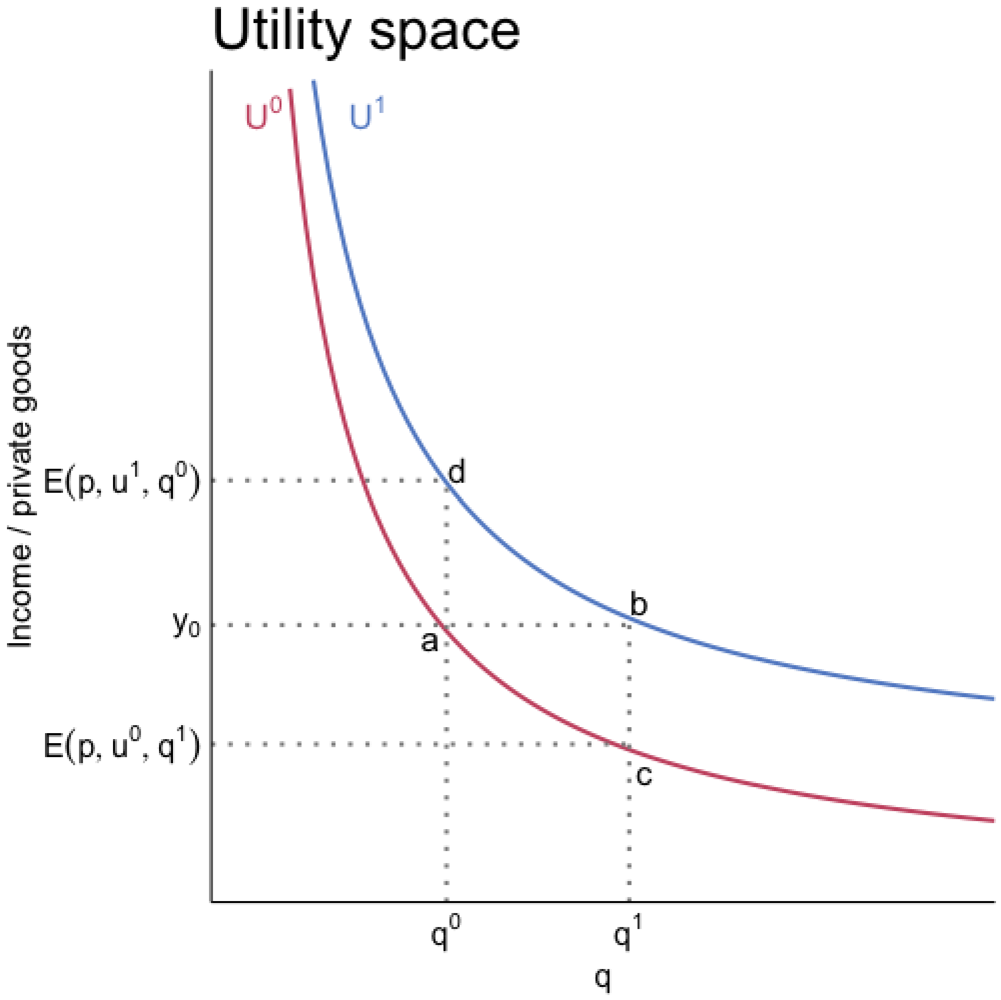


CV and EV in utility space

Tracing out demand curves is a little trickier here since q has no price

Here's how to think about it:

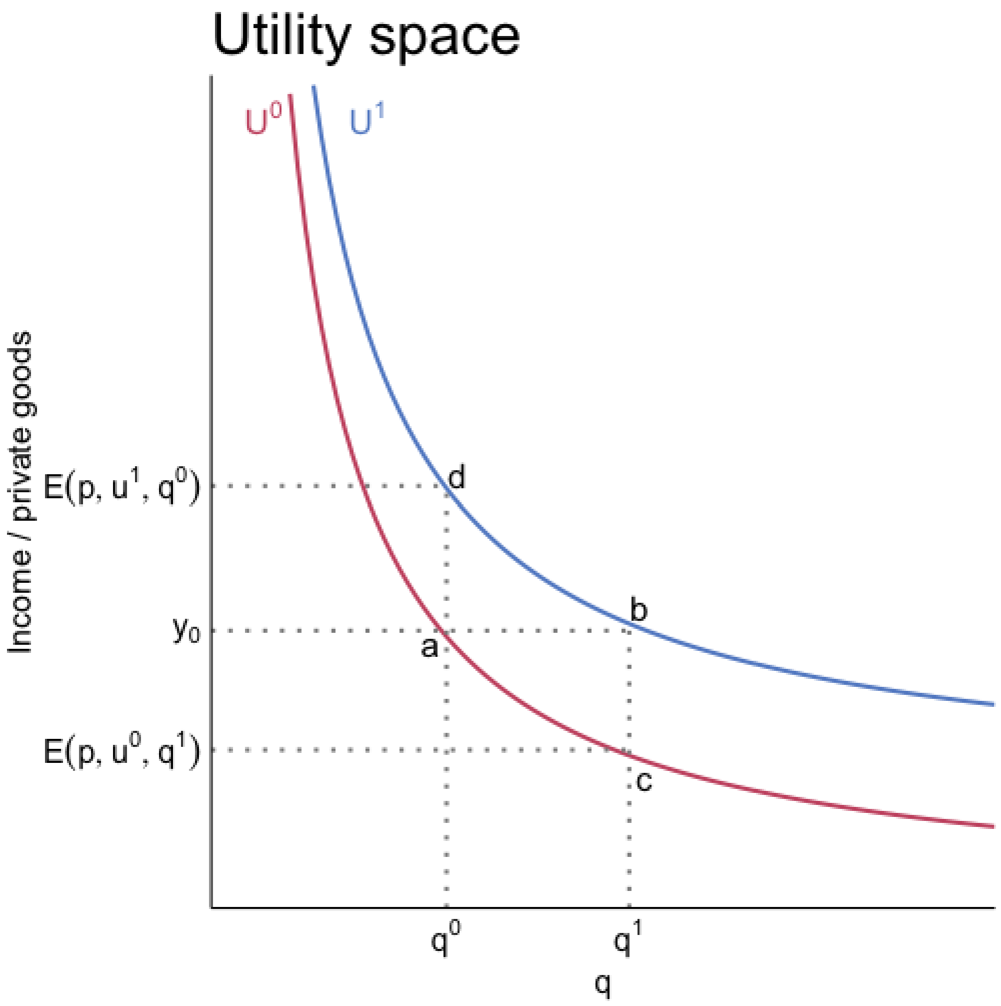
- Suppose q was traded in a market at some virtual price π
- The person's virtual income to **compensate** them and keep their private good spending to be y_0 is: $\tilde{y} = y_0 + \pi \tilde{q}$



CV and EV in utility space

$$\tilde{y} = y_0 + \pi\tilde{q}$$

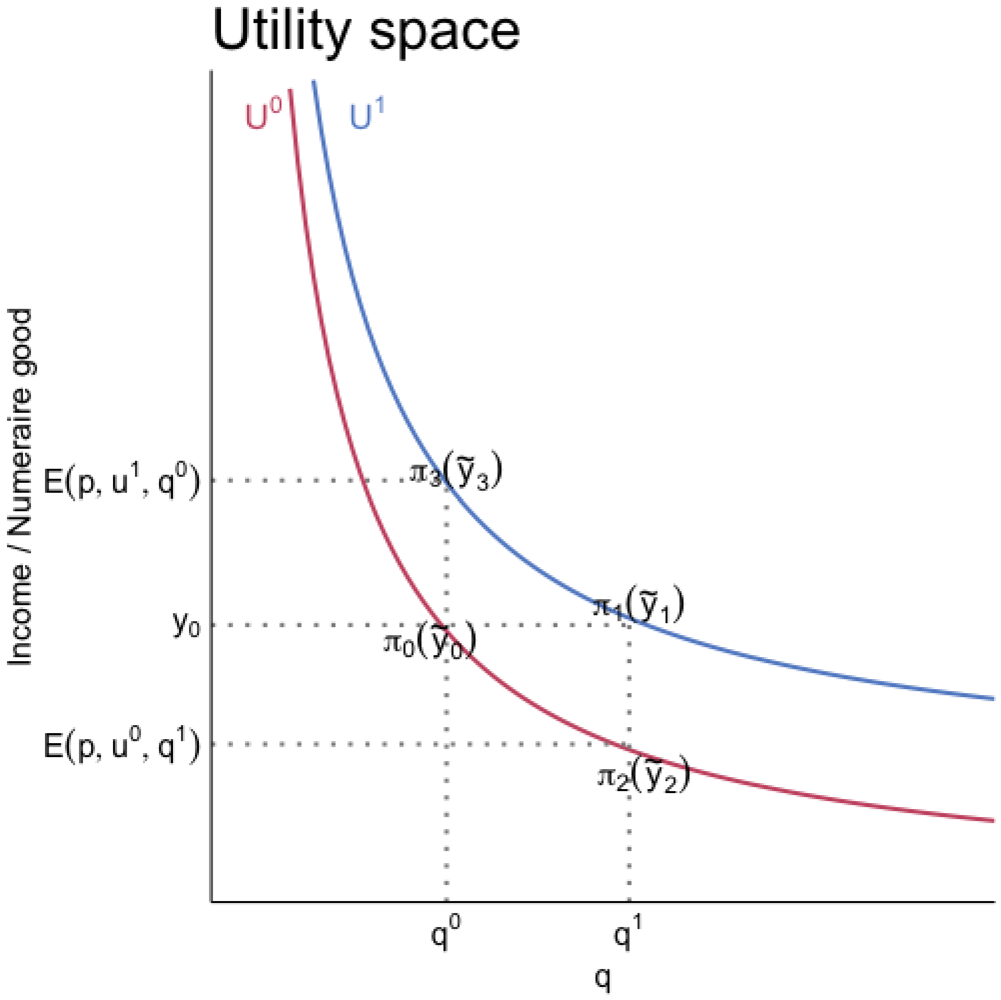
Given some income $\tilde{y}$ the consumer "buys" $\tilde{q}$ units such that the budget constraint is tangent to an indifference curve (like usual)



CV and EV in utility space

Let $\pi_0(\tilde{y}_0), \pi_1(\tilde{y}_1), \pi_2(\tilde{y}_2), \pi_3(\tilde{y}_3)$ be the virtual price/income combinations tangent at points a, b, c, d

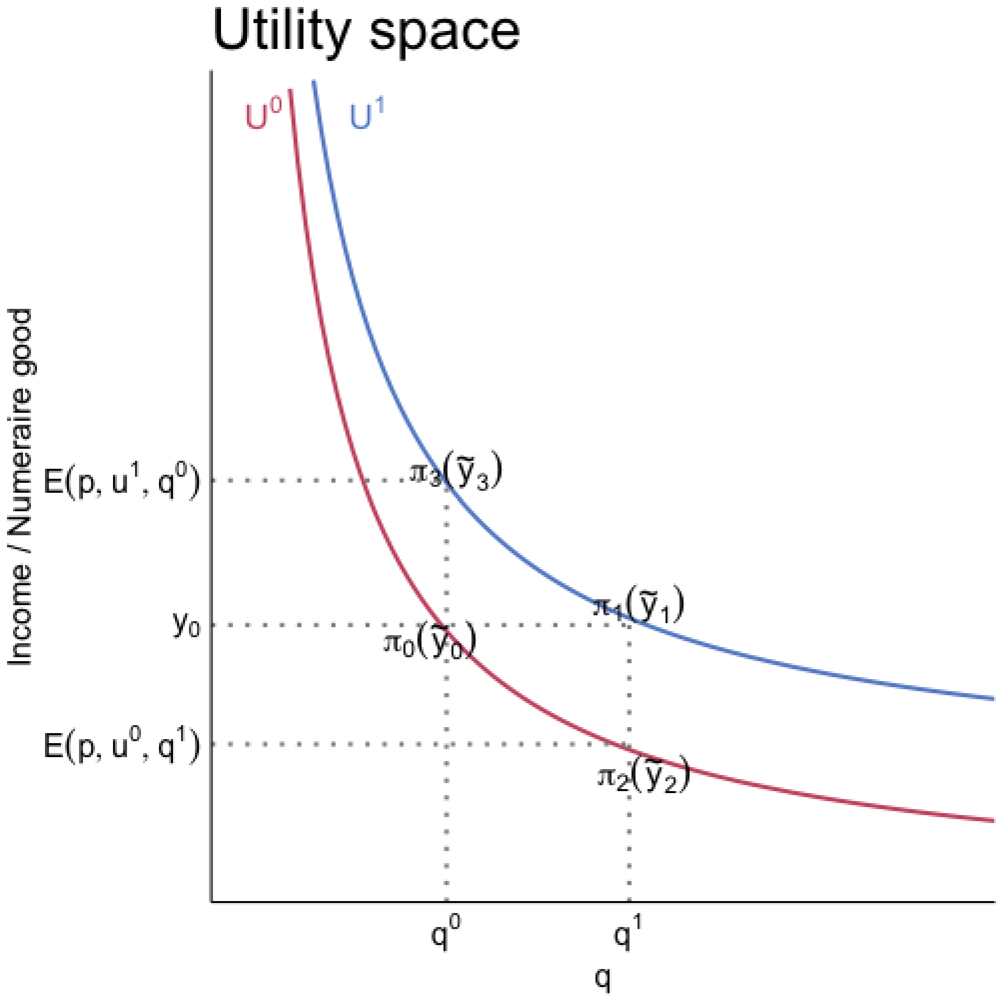
We can use these to trace out our compensated inverse demand curves by moving along the same indifference curve to different levels of q



CV and EV in utility space

The virtual price change along an indifference curve trace out the compensated inverse demands:

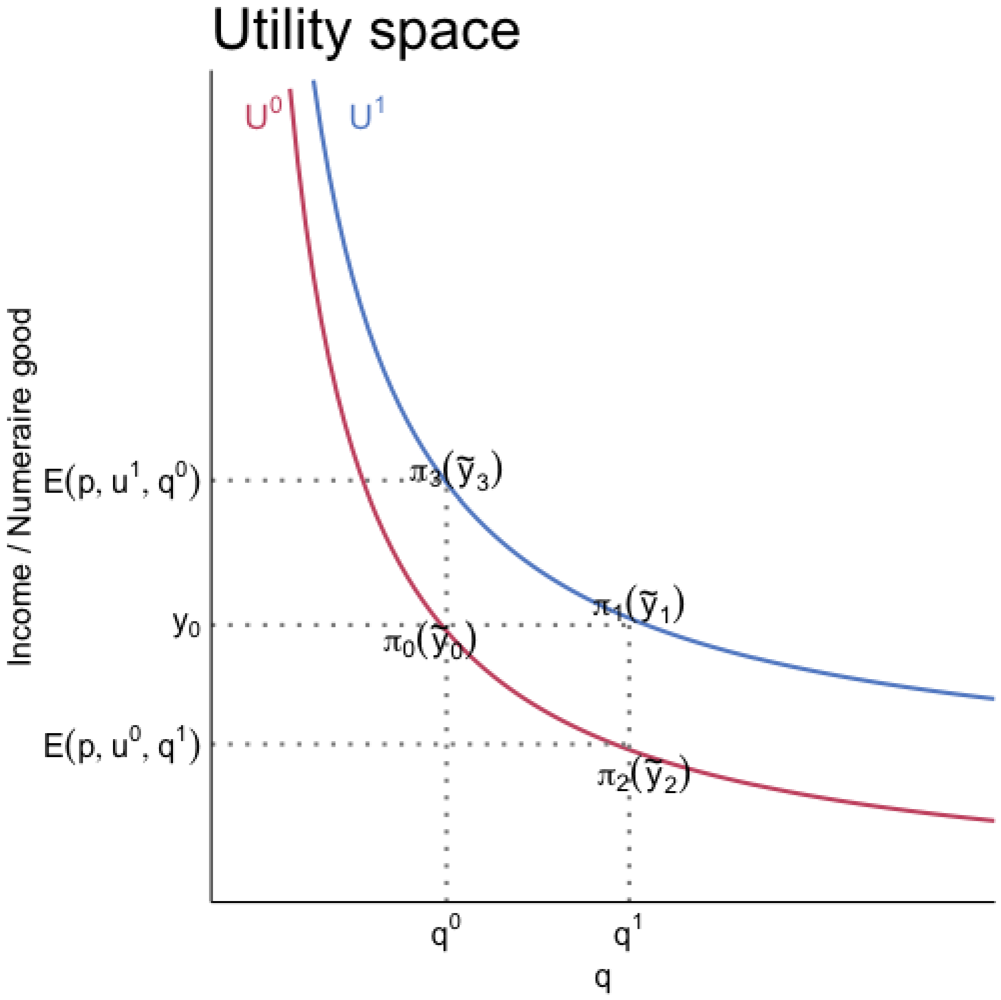
- $\pi_0(\tilde{y}_0)$ and $\pi_2(\tilde{y}_2)$ trace out $\pi^q(p, u^0, q)$
- $\pi_3(\tilde{y}_3)$ and $\pi_1(\tilde{y}_1)$ trace out $\pi^q(p, u^1, q)$



CV and EV in utility space

The virtual price change from q^0 to q^1 holding income fixed traces out the ordinary inverse demand curve

- $\pi_0(\tilde{y}_0)$ and $\pi_1(\tilde{y}_1)$ trace out $\theta^q(p, y, q)$



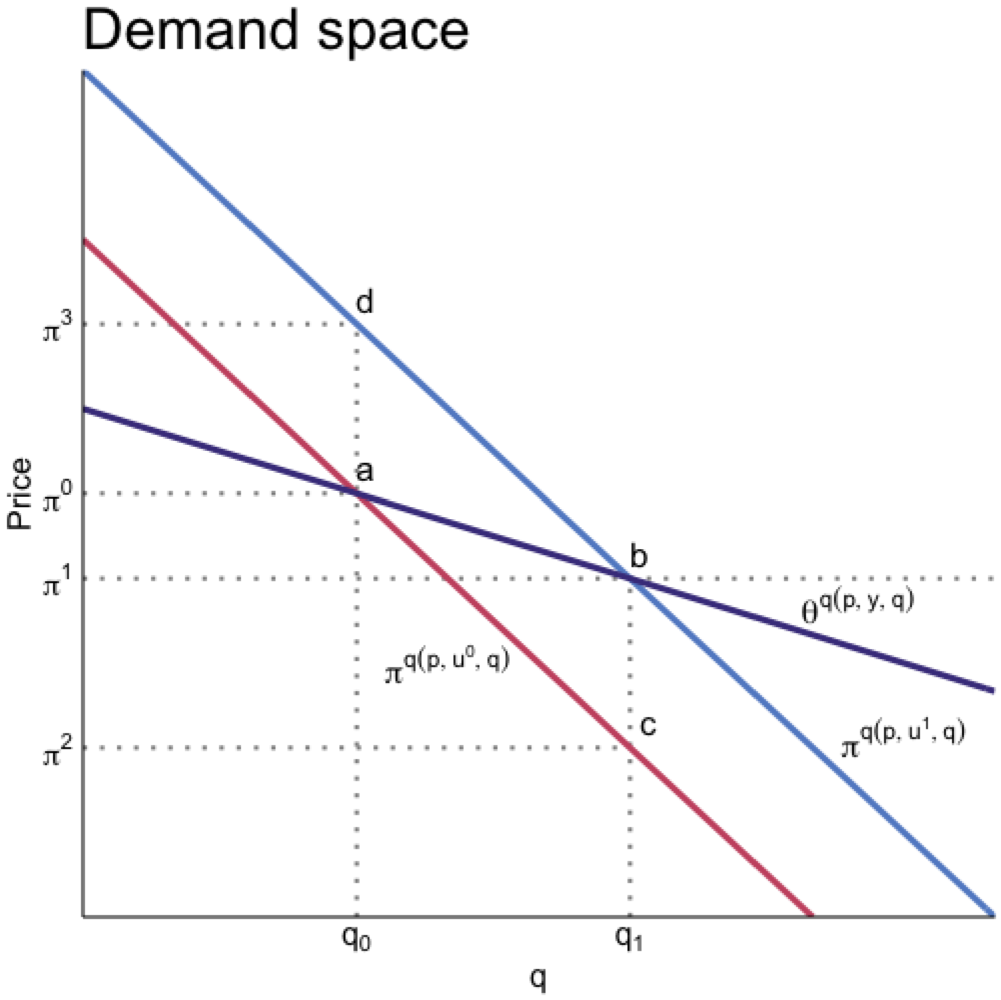
CV and EV in demand space

Similar to before

CV is given by the area q_0, a, c, q_1

EV is given by the area q_0, d, b, q_1

CS is given by the area q_0, a, b, q_1



Compensated demand and virtual prices

Now we have started to get the intuition for why it's called compensated demand

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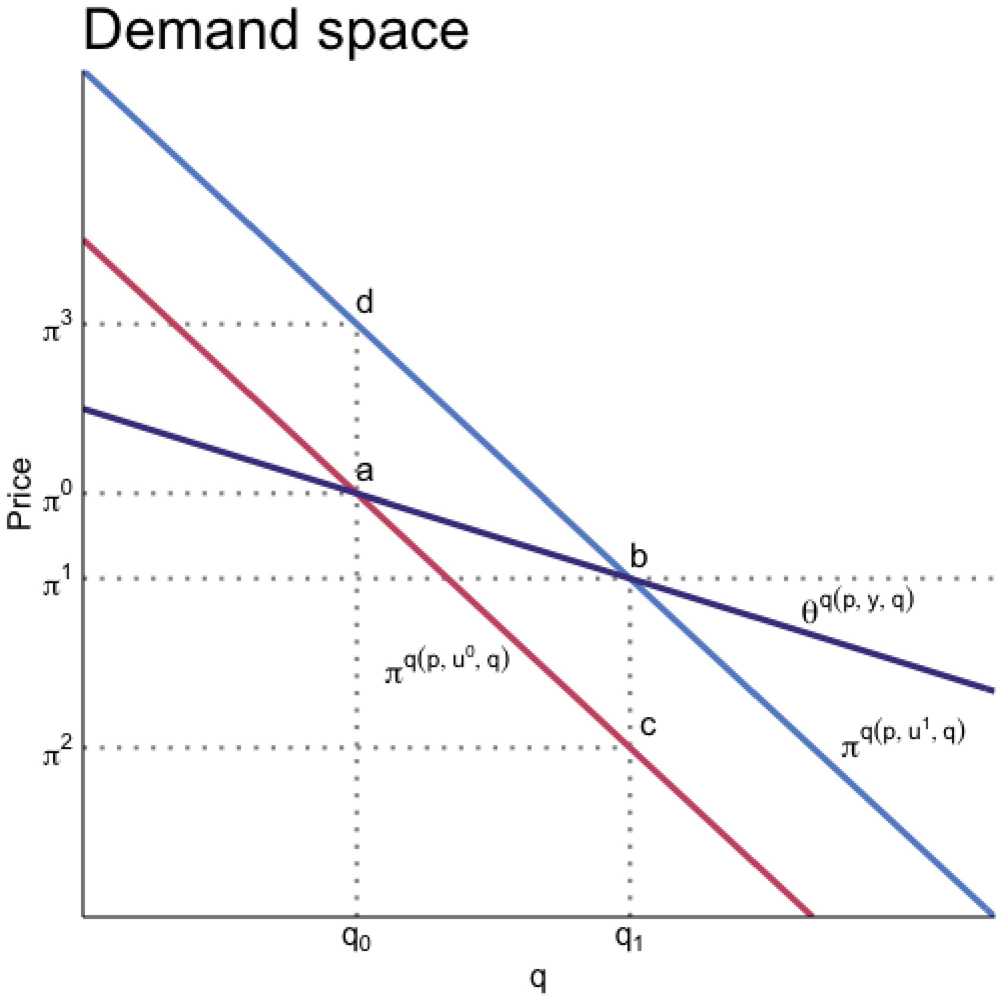
We are directly **compensating** the person's income to maintain constant utility



The fundamental challenge of measurement

Recall with price changes we were able to value them by relating them to (quasi-)expenditures:

$$\frac{\partial E(p,\bar{u},q)}{\partial p_j} = \frac{\partial V}{\partial p_j} \Bigg/ \frac{\partial V}{\partial y} = x_j(p,y,q)$$

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If we can obtain the ordinary inverse demand curve for q then we can calculate welfare measures!

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q isn't traded in markets

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We solve this challenge by studying market goods that capitalize the value of environmental goods