Some Results

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1 Corrections for PM

Although the draft of Principia Metaphysica has a remarkably high quality we were able to identify some minor issues and typographical errors during the formalization.

For example the follwing issue was noticed in the proof of the back implication of theorem (171). In the current draft of Principia Metaphysica the proof states:

It suffices to prove the contrapositive. So assume $[\lambda y\ y =_E x] = [\lambda y\ y =_E z]$. *Then by reasoning just given (inside the reductio),* $x =_E z$. [1, p. 477].

The back implication is $([\lambda y\ y =_E x] \neq [\lambda y\ y =_E z]) \rightarrow x \neq z$, though, and therefore the contrapositive $x = z \rightarrow [\lambda y\ y =_E x] = [\lambda y\ y =_E z]$. This can easily be proven by the substitution of identicals as demonstrated in our formalization. Nevertheless the proof in PM seems to have accidentally reversed the proof objective.

Another issue in the current draft was found in the proof of theorem (189.2). The proof states: *So by theorem (88), it follows that:* [1, p. 483]. The correct theorem to reference is (108), though. This kind of mistake can easily happen and can easily be missed in a review process. In the formalization in Isabelle on the other hand it immediately becomes apparent that the proof objective can not be solved by the stated theorem and the correct theorem can automatically be found.

2 Proof of a Meta-Conjecture

The Theory of Abstract Objects has a syntactic definition of possible worlds. Besides that it also has a semantic notion of possible worlds (i.e. a point in the Kripke semantics).

During a discussion between Zalta and Paleo the following meta-conjecture about possible worlds and their semantics in the Theory of Abstract Objects arose:

For every syntactic possible world "w", there exists a semantic point "p" which is the denotation of "w".

Using the formalization of the theory it was not only possible to immediately confirm the conjecture, but even to extend it to the more general statement that there exists a natural bijection between syntactic and semantic possible worlds (in fact the equivalence had already independently become apparent in an earlier formalization of a proof of the fundamental theorem of possible worlds in the meta-logic of the embedding).

Namely it is possible to show, that:

- For every syntactic possible world x, there exists a semantic possible world v, such that every proposition is semantically true in v, if and only if it is syntactically true in x:

  $\forall x.\ [\mathit{PossibleWorld}\ (x^P)\ \mathit{in}\ w] \longrightarrow (\exists v.\ \forall p.\ [p\ \mathit{in}\ v] = [x^P \models p\ \mathit{in}\ w])$

- For every semantic possible world v there exists a syntactic possible world x, such that every proposition is semantically true in v if and only if it is syntactically true in x:

  $\forall v.\ \exists x.\ [\mathit{PossibleWorld}\ (x^P)\ \mathit{in}\ w] \wedge (\forall p.\ [p\ \mathit{in}\ v] = [x^P \models p\ \mathit{in}\ w])$

References

[1] E. N. Zalta. Principia logico-metaphysica. http://mally.stanford.edu/principia.pdf. [Draft/Excerpt; accessed: October 28, 2016].