AMSC661, Spring 2025 Maria Cameron

Homework 6. Due March 26.

Please upload a single pdf file on ELMS. Link your codes to your pdf (i.e., put your codes to Dropbox, GitHub, Google Drive, etc. and place links to them in your pdf file with your solutions.

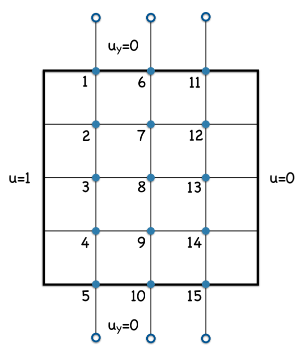
1. **(8 pts.) To be done on paper.** Consider Laplace's equation $u_{xx} + u_{yy} = 0$ in the given domain Ω with the given boundary conditions (BCs). Discretize the problem on the specified mesh using the 5-point stencil and write out an appropriate system of linear algebraic equations $Au = f$ for the numerical solution u as it is done in `elliptic.pdf`. Highlight the block structure of the matrix A.

   (a) $\Omega = [0,1] \times [0,1]$, the mesh and the BCs are shown in Fig. (a).

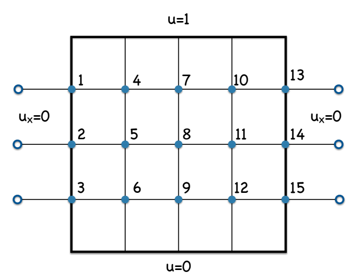
   (b) $\Omega = [0,1] \times [0,1]$, the mesh and the BCs are shown in Fig. (b).

   (c) $\Omega = [0,1] \times [0,1]$, the mesh and the BCs are shown in Fig. (c). (BCs are periodic in y and Dirichlet on the left and right boundaries.)

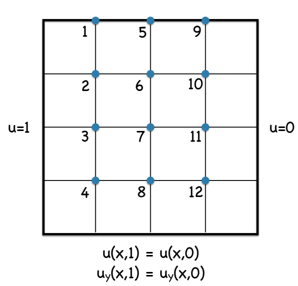
   (d) $\Omega = [0,1] \times [0,1]$, the mesh and the BC are shown in Fig. (d). (BCs are periodic in x and Dirichlet on thetop and bottom boundaries.)



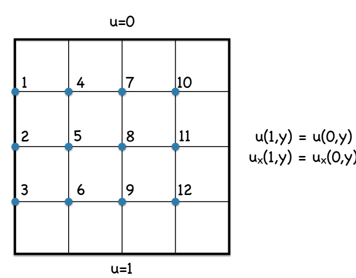
(a)



(b)

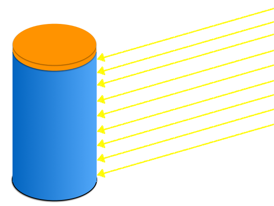


(c)



(d)

2. **(5 pts.) A programming task.** Suppose an empty cylindrical tin covered with an insulating top is standing on ice. The sun, located low above the horizon, is shining on one side of the tin as shown in the figure.



Assume that the height of the cylinder is 2 and the radius is 1. Take all physical constants such as the heat conductance and the intensity of the sunlight equal to 1. Then the stationary heat distribution satisfies the equation

$$u_{xx} + u_{yy} = \begin{cases} -\cos(x), & -\pi/2 \le x \le \pi/2, \\ 0, & \text{otherwise}, \end{cases}$$

$-\pi \le x \le \pi$, $0 \le y \le 2$.

   (a) Pick adequate boundary conditions and write down the boundary value problem (BVP).

   (b) Note that π and 2 are incommensurable, hence you need to pick different mesh steps in x and y directions. Write out the block structure and specify the blocks of the discrete differential operator for this problem.

   (c) Solve the BVP numerically and find the stationary temperature distribution. Plot a figure showing the distribution.

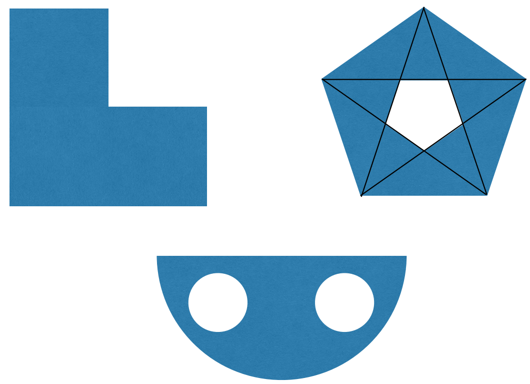
   Submit the formulation of the BVP, the figure with the stationary heat distribution, and a printout of your code.

3. **(5 pts)** *Triangular mesh generation using* `distmesh2d.m` *by P.-O. Persson.* Read [1], at least its first 12 pages (at least up to Section 6 "Mesh Generation in Higher Dimensions").

   If you prefer Matlab, download the distmesh package `distmesh.zip` available at http://persson.berkeley.edu/distmesh/.

   If you prefer Python, you can download my Python version of P.-O. Persson's code available at GitHub, user `mar1akc`, package `transition_path_theory_FEM_distmesh`, file `distmesh.py`.

   Mesh the shapes in the Figure below using distmesh2d.m following examples in [1].



You can pick arbitrary sizes as soon as topologies are preserved, and you can do uniform meshing.

4. **(5 pts)** Download Darren Engwirda's mesh generating package `MESH2D` from MathWorks file exchange. Read the code `tridemo`. Install the package on your computer by running `tridemo` with arguments from 0 to 10 (read the instruction). Use this package to triangulate the shapes from the previous code.

References

[1] P.-O. Persson, G. Strang, A Simple Mesh Generator in MATLAB. SIAM Review, Volume 46 (2), pp. 329-345, June 2004 (PDF)