AMSC661, Spring 2025 Maria Cameron

**Homework 1. Due Feb. 12**

**Please upload a single pdf file on ELMS. Link your codes to your pdf (i.e., put your codes to dropbox, github, google drive, etc. and place links to them in your pdf file with your solutions.**

1. **(3 pts)** Solve Exercise 1 from my Lecture notes `ODEsolvers.pdf` (see Section 1.4). Note, $y(t)$ is a scalar function.

2. **(3 pts)** Solve Exercise 2 from my Lecture notes `ODEsolvers.pdf` (see Section 1.4). Note, $y(t)$ is a scalar function.

3. **(3 pts)** Solve Exercise 5 from my Lecture notes `ODEsolvers.pdf` (see Section 3.4).

4. **(3 pts)** Show that the implicit midpoint rule (a symplectic method)

$$k = f\left(t_n + \frac{h}{2}, u_n + \frac{h}{2}k\right), \quad u_{n+1} = u_n + hk. \tag{1}$$

   is consistent (at least) of order 2.

5. **(8pts)**

   (a) Use the method of undetermined coefficients to determine $a_0$, $a_1$, $b_0$, and $b_1$ that make the linear two-step explicit method consistent of as high order as possible:

$$u_{n+1} - a_0 u_n - a_1 u_{n-1} = h(b_0 f_n + b_1 f_{n-1}). \tag{2}$$

   (b) Show that this method is unstable by applying it to the ODE $y' = 0$ and mimicking the technique shown in Section 3.5.

   (c) Apply this method to the 2D gravity problem with a unit-circle solution:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} = \begin{bmatrix} u \\ v \\ -\frac{x}{x^2+y^2} \\ -\frac{y}{x^2+y^2} \end{bmatrix}, \quad \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3}$$

   Integrate the numerical solution for two periods. Show that the solution blows up by reporting the norm of the solution at time $4\pi$ for $h = 2\pi/N$ with $N = 20$, 40, 80. Also, plot the $x$ and $y$ components of the solution in the $xy$ plane.

   (d) Apply the midpoint rule with the Forward Euler predictor to the 2D gravity problem with a unit-circle solution. Use the time interval $8\pi$, set $h = 2\pi/N$ with $N = 20$, 40, 80 and plot the numerical solutions.