Spherically-Symmetric Einstein-Geometric Proca Model

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1 Outline

The outline of the project is as follows:

1. We consider the Einstein-Proca system, that is, we consider the Einstein gravity with metric tensor $g_{\mu\nu}$ and a massive vector field Y_μ with mass m_Y. The system is described by the action

$$S[g,Y] = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G_N}R(g) - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - \frac{m_Y^2}{2}Y_\mu Y^\mu - g_Y Y_\mu J^\mu - \mathcal{L}_{rest}\right\} \quad (1)$$

where the curvature scalar $R(g) = g^{\mu\nu}R_{\mu\nu}\left({}^g\Gamma\right)$ is trace of the Ricci curvature tensor

$$R_{\mu\nu}({}^g\Gamma) = \partial_\alpha {}^g\Gamma^\alpha_{\mu\nu} - \partial_\nu {}^g\Gamma^\alpha_{\alpha\mu} + {}^g\Gamma^\beta_{\mu\nu}\,{}^g\Gamma^\alpha_{\alpha\beta} - {}^g\Gamma^\beta_{\mu\alpha}\,{}^g\Gamma^\alpha_{\nu\beta} \quad (2)$$

set by the Levi-Civita connection

$${}^g\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\rho}\left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right) \quad (3)$$

of the metric tensor $g_{\mu\nu}$. Apart from the curvature, in the action (1), $Y_{\mu\nu} = \nabla_\mu Y_\nu - \nabla_\nu Y_\mu$ is the field strength tensor of Y_μ, J_μ is the current which couples to the vector field Y_μ with charge g_Y, and $\mathcal{L}_{rest}$ collects all the fields and interactions (without containing Y_μ).

2. We now use the action (1) to construct the equations of motion for the metric and for the vector field:

$$R_{\mu\nu}\left({}^g\Gamma\right) - \frac{1}{2}R(g)g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \quad (4)$$

$$\nabla_\mu Y^{\mu\nu} - m_Y^2 Y^\nu = g_Y J^\nu \quad (5)$$

where the energy-momentum tensor is given by

$$T_{\mu\nu} = T^{(rest)}_{\mu\nu} + Y_{\mu\alpha}Y^\alpha_\nu - \frac{1}{4}Y_{\alpha\beta}Y^{\alpha\beta}g_{\mu\nu} - m_Y^2\left(Y_\mu Y_\nu - \frac{1}{2}Y_\alpha Y^\alpha g_{\mu\nu}\right) - 2g_Y\left(Y_\mu J_\nu - \frac{1}{2}Y_\alpha J^\alpha g_{\mu\nu}\right) \quad (6)$$

with

$$T^{(rest)}_{\mu\nu} = 2\frac{\delta\mathcal{L}_{rest}}{\delta g^{\mu\nu}} - \mathcal{L}_{rest} g_{\mu\nu} \tag{7}$$

being the energy-momentum tensor of the remaining interactions.

3. Now, the goal is to find a spherically-symmetric (r, θ, φ), static $(\partial g_{\mu\nu}/\partial t = 0)$ solution of the equations above, which is regular at the origin. To this end, it proves useful to follow the setup used in Ref. [1]:

$$\begin{aligned} g_{\mu\nu} &= \text{diag.}\left(-A(r)^2, B(r)^2, r^2, r^2\sin^2\theta\right)_{\mu\nu} && (8)\\ Y_\mu &= \psi(r)\delta^0_\mu && (9)\\ T^{(rest)}_{\mu\nu} &= (\rho+p)v_\mu v_\nu + p g_{\mu\nu} && (10)\\ J_\mu &= \rho v_\mu && (11)\end{aligned}$$

where $v_\mu = (-1, 0, 0, 0)_\mu$ is the velocity of the (ideal fluid) matter.

4. The goal is to find solutions of the equations of motion with nonzero matter density ρ and pressure p. (The static functions $A(r)$, $B(r)$ and $\phi(r)$ can be unambiguously related to the functions $f(r)$, $g(r)$ and $u(r)$ in [1].)

References

[1] Y. N. Obukhov and E. J. Vlachynsky, Annals Phys. **8**, 497-510 (1999) [arXiv:gr-qc/0004081 [gr-qc]].