Chapter 1

Real representation of complex numbers in dolfin

1.1 Helmholtz equation with Dirichlet and Neumann boundary conditions

Helmholtz equation

$$\boldsymbol{\nabla} \cdot \left(\bar{c}^2 \boldsymbol{\nabla} \hat{p}\right) + \omega^2 \hat{p} = 0 \tag{1.1}$$

with Dirichlet, $\hat{p} = 0$, and/or Neumann, $\partial \hat{p} / \partial n = 0$, boundary conditions. Discretization (finite element method)

$$-\int_\Omega \bar{c}^2 \boldsymbol{\nabla} \phi_k \cdot \boldsymbol{\nabla} \phi_j \, \mathrm{d}V \, \hat{p}_j + \omega^2 \int_\Omega \phi_k \phi_j \, \mathrm{d}V \, \hat{p}_j = 0 \tag{1.2}$$

ϕ_k is the trial function and ϕ_j is the test function. We use Lagrange polynomials of degree 1 or 2. In matrix form

$$\mathbf{A}\mathbf{p} + \omega^2 \mathbf{C}\mathbf{p} = 0 \tag{1.3}$$

$\mathbf{A}$ and $\mathbf{C}$ are real symmetric matrices. All the eigenvalues are real (and the eigenvectors are orthogonal). Let's ignore it! Let's suppose $\omega \in \mathbb{C}$ and $\mathbf{p} \in \mathbb{C}^N$.

$$\mathbf{A}(\mathbf{p}_r + i\mathbf{p}_i) + (\omega_r + i\omega_i)^2 \mathbf{C}(\mathbf{p}_r + i\mathbf{p}_i) = 0 \tag{1.4}$$

$$\mathbf{A}\mathbf{p}_r + \left(\omega_r^2 - \omega_i^2\right)\mathbf{C}\mathbf{p}_r - 2\omega_r\omega_i \mathbf{C}\mathbf{p}_i + i\left(\mathbf{A}\mathbf{p}_i + 2\omega_r\omega_i\mathbf{C}\mathbf{p}_r + \left(\omega_r^2 - \omega_i^2\right)\mathbf{C}\mathbf{p}_i\right) = 0 \tag{1.5}$$

We can write Eq. (1.5) in 'block matrix form'

$$\begin{pmatrix} \mathbf{A} & \\ & \mathbf{A} \end{pmatrix} \begin{pmatrix} \mathbf{p}_r \\ \mathbf{p}_i \end{pmatrix} + \begin{pmatrix} \omega_r & -\omega_i \\ \omega_i & \omega_r \end{pmatrix}^2 \begin{pmatrix} \mathbf{C} & \\ & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{p}_r \\ \mathbf{p}_i \end{pmatrix} = 0 \tag{1.6}$$

First line for the real part and second line for the imaginary part. This is how mass and stiffness matrices should be assembled in dolfin. If we pass them to the eigensolver, the generalized eigenvalue problem we actually solve is

$$\begin{pmatrix} \mathbf{A} & \\ & \mathbf{A} \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{pmatrix} + \lambda^2 \begin{pmatrix} \mathbf{C} & \\ & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{pmatrix} = 0 \tag{1.7}$$

We solve twice Eq. (1.3)...

Chapter 2

Helmholtz equation with unsteady heat release

$$\boldsymbol{\nabla} \cdot \left(\bar{c}^2(\boldsymbol{x})\boldsymbol{\nabla}\hat{p}(\boldsymbol{x})\right) + \omega^2\hat{p}(\boldsymbol{x}) = \frac{\dot{Q}_t}{U_b}\frac{\gamma - 1}{\bar{\rho}(\boldsymbol{x}_r)}v(\boldsymbol{x})F(\omega)\boldsymbol{\nabla}\hat{p}(\boldsymbol{x}_r) \cdot \boldsymbol{n}_r \tag{2.1}$$

All the dependences are explicit! $\dot{Q}_t$ is the total thermal power of the flame (or heat source), U_b is the average/bulk flow velocity, γ is the heat capacity ratio and $\bar{\rho}$ is the mean density. v is the heat release rate distribution, $F(\omega)$ is the flame transfer function, $\boldsymbol{x}_r$ is a point downstream of the flame and $\boldsymbol{n}_r$ is a unit vector. Some additional notes, $\int v \, \mathrm{d}V = 1$, and $F(\omega)$ is complex-valued.

TODO: Discretize the right-hand side and split into real and imaginary part.