

Accounting for Unsteady Heat Release

The Helmholtz equation with flame term;

$$\nabla \cdot (\bar{c}^2(\mathbf{x}) \nabla \hat{p}(\mathbf{x})) + \omega^2 \hat{p}(\mathbf{x}) = \frac{\dot{Q}}{U_b} \frac{\gamma - 1}{\rho(\bar{\mathbf{x}}_r)} \nu(\mathbf{x}) F(w) \nabla \hat{p}(\mathbf{x}_r) \cdot \mathbf{n}_r \quad (1)$$

The right-hand side of the equation (1) can be multiplied with test function $\{\phi_k\}_{K=1}^N$. Then trial function \hat{p}_h can be substituted with \hat{p} . Next, integration over the domain ϕ gives;

$$(\gamma - 1) \int_{\phi} \phi_k \frac{\dot{Q}}{U_b} \frac{\nu(\mathbf{x}) F(w)}{\rho(\bar{\mathbf{x}}_r)} \nabla \sum_{j=1} \hat{p}_j \phi_j \cdot \mathbf{n}_r d\mathbf{x} \quad k = 1, 2, 3, \dots N \quad (2)$$

Therefore we can take the summation out of the integral and if \hat{p}_j is constant;

$$(\gamma - 1) \sum_{j=1} \left[\phi_k \frac{\dot{Q}}{U_b} \frac{F(w)}{\rho(\bar{\mathbf{x}}_r)} \nabla \phi_j(\mathbf{x}_r) \cdot \mathbf{n}_r \int_{\phi} \nu(\mathbf{x}) d\mathbf{x} \right] \hat{p}_j \quad k = 1, 2, 3, \dots N \quad (3)$$

Since $\omega = \omega_r + i\omega_i$, Flame Transfer Function(FTF) can be introduced as

$$F(\omega) = K e^{i\omega\tau} = K e^{i(\omega_r + i\omega_i)\tau} = K \frac{\cos(\omega_r\tau) + i\sin(\omega_r\tau)}{e^{\omega_i\tau}} \quad (4)$$

where K is a constant, and τ is a time delay between initial perturbation and the heat release perturbation. Hence, inserting equation (4) into (3) gives;

$$(\gamma - 1) \sum_{j=1} \left[\phi_k \frac{\dot{Q}}{U_b} \frac{K \cos(\omega_r\tau) + i\sin(\omega_r\tau)}{\rho(\bar{\mathbf{x}}_r) e^{\omega_i\tau}} \nabla \phi_j(\mathbf{x}_r) \cdot \mathbf{n}_r \int_{\phi} \nu(\mathbf{x}) d\mathbf{x} \right] \hat{p}_j \quad k = 1, 2, 3, \dots N \quad (5)$$

Matrix D can be defined for representing equation (5) in block-form. Real part of the equation (5) is

$$(\gamma - 1) \sum_{j=1} \left[\phi_k \frac{\dot{Q}}{U_b} \frac{K \cos(\omega_r\tau)}{\rho(\bar{\mathbf{x}}_r) e^{\omega_i\tau}} \nabla \phi_j(\mathbf{x}_r) \cdot \mathbf{n}_r \int_{\phi} \nu(\mathbf{x}) d\mathbf{x} \right] \hat{p}_j \quad k = 1, 2, 3, \dots N \quad (6)$$

Imaginary part yields as

$$(\gamma-1) \sum_{j=1} \left[\phi_k \frac{\dot{Q}}{U_b} \frac{K \sin(\omega_r \tau)}{\rho(\mathbf{x}_r) e^{\omega_i \tau}} \nabla \phi_j(\mathbf{x}_r) \cdot \mathbf{n}_r \int_{\phi} \nu(\mathbf{x}) d\mathbf{x} \right] \hat{p}_j \quad k = 1, 2, 3, \dots, N \quad (7)$$

Matrix D

$$D = \begin{bmatrix} D_r & -D_i \\ D_i & D_r \end{bmatrix} \quad (8)$$