Chapter 1

The Second Chapter

1.1 Matrix Representation of Complex Matrices

The equation system;

$$A\boldsymbol{P} + \omega^2 \boldsymbol{P} = 0 \tag{1.1}$$

where A is complex matrix and B is real matrix. Since $\boldsymbol{P}$ is a complex eigenfrequency, $\boldsymbol{P} = p_r + ip_i$. It can be written $\boldsymbol{P}$ in matrix form;

$$\boldsymbol{P} = \begin{bmatrix} p_r \\ p_i \end{bmatrix} \tag{1.2}$$

The block-matrix form can be written as;

$$\begin{bmatrix} A_r & -A_i \\ A_i & A_r \end{bmatrix} \begin{bmatrix} p_r \\ p_i \end{bmatrix} + \omega^2 \begin{bmatrix} C & \\ & C \end{bmatrix} \begin{bmatrix} p_r \\ p_i \end{bmatrix} = 0 \tag{1.3a}$$

$$\begin{bmatrix} A_r & -A_i \\ A_i & A_r \end{bmatrix} \begin{bmatrix} p_r \\ p_i \end{bmatrix} + \begin{bmatrix} w_r^2 - w_i^2 & -2w_r w_i \\ 2w_r w_i & w_r^2 - w_i^2 \end{bmatrix} \begin{bmatrix} C & \\ & C \end{bmatrix} \begin{bmatrix} p_r \\ p_i \end{bmatrix} = 0 \tag{1.3b}$$

The equation system in 1.3a can be split into real and imaginary parts. The real part is;

$$A_r p_r - A_i p_i + (w_r^2 - w_i^2) C p_r - 2 w_r w_i C p_i = 0 \tag{1.4}$$

The imaginary part is;

$$A_i p_r - A_r p_i + 2 w_r w_i C p_r + (w_r^2 - w_i^2) C p_i = 0 \tag{1.5}$$