Function f

$$f(x) = x^3 \tag{1}$$

$x \in [0, 1]$ and we have 4 elements in this range, so find $f'(0.33) =?$

Solution

Point	x	f(x)
0	0	0
1	0.25	0.01562
2	0.5	0.125
3	0.75	0.422
4	1	1

General formula for linear piecewise functions is;

$$\frac{y_1 - y_0}{x_1 - x_0} = m = f'(x) \tag{2}$$

where $f(x) = mx + k$

Line 01

$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{0.01562 - 0}{0.25 - 0} = 0.062 \tag{3}$$

Line 12

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0.125 - 0.01562}{0.50 - 0.25} = 0.437 \tag{4}$$

$$f(0.5) = 0.437(0.5) + k_{12} = 0.125 \qquad k_{12}\text{=-0.094} \tag{5}$$

Line 23

$$\frac{y_3 - y_2}{x_3 - x_2} = \frac{0.422 - 0.125}{0.75 - 0.50} = 1.188 \tag{6}$$

$$f(0.75) = 1.188(0.75) + k_{23} = 0.422 \qquad k_{23}\text{=-0.469} \tag{7}$$

Line 34

$$\frac{y_4 - y_3}{x_4 - x_3} = \frac{1 - 0.422}{1 - 0.75} = 2.312 \tag{8}$$

$$f(1) = 2.312(1) + k_{34} = 1 \qquad k_{34}\text{=-1.312} \tag{9}$$

$$f'(x) = \left\{ \begin{array}{ll} 0.062, & \text{for } 0 \leq x \leq 0.25 \\ 0.437, & \text{for } 0.25 < x \leq 0.50 \\ 1.188, & \text{for } 0.50 < x \leq 0.75 \\ 2.312, & \text{for } 0.75 < x \leq 1 \end{array} \right\}$$

Hence, $f'(0.33) = 0.437$