



Effect of Perforated Plates on the Acoustics of Annular Combustors

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This paper aims to show the influence of perforated plates on the acoustic modes in aeronautical gas turbine combustion chambers. The analytical model was implemented in a three-dimensional acoustic Helmholtz solver to account for the effect of perforated plates. First, an analytic test case is used to validate the coding in the acoustic solver. Then, a computation of the acoustic modes in an actual industrial chamber is conducted, taking into account the perforated liners. For both cases, a study of the influence of the bias flow speed is conducted. The acoustic energy budget is also used to evaluate the respective contributions of the perforated plates. In the case of the industrial chamber, some plates are proved to be more effective than the others, depending on the mode structure.

Nomenclature

A, B	=	$N \times N$ matrices
a	=	aperture radius, m
c	=	sound speed, $\text{m} \cdot \text{s}^{-1}$
d	=	aperture spacing, m
f	=	mode frequency, Hz
i	=	imaginary complex number
$\text{Im}()$	=	imaginary part
k	=	wave number, $\text{rad} \cdot \text{m}^{-1}$
K_a	=	Rayleigh conductivity, m^{-1}
N	=	number of discretization nodes
\mathbf{n}	=	unit normal vector
\mathbf{P}	=	acoustic pressure amplitude, N vector
\hat{p}	=	spatial pressure fluctuation
p_0	=	mean pressure, Pa
(r, θ, z)	=	polar coordinates
$\text{Re}()$	=	real part
S_t	=	Strouhal number; $\omega_r a / U$
t	=	time
U	=	bias flow speed, $\text{m} \cdot \text{s}^{-1}$
U_{md}	=	speed of maximal damping, $\text{m} \cdot \text{s}^{-1}$
$\hat{\mathbf{u}}$	=	spatial velocity fluctuation
Z	=	complex impedance
γ	=	polytropic coefficient
Ω	=	computational domain
$\delta\Omega$	=	boundary of computational domain
$\delta\Omega_{\text{MP}}$	=	boundary on which a perforated plate condition is applied
$\delta\Omega_P$	=	boundary on which a null acoustic pressure is applied
$\delta\Omega_W$	=	boundary on which an impermeable condition is applied
$\delta\Omega_Z$	=	boundary on which an impedance condition is applied
ν	=	cinematic viscosity, $\text{m}^2 \cdot \text{s}^{-1}$

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ρ_0	=	density, $\text{kg} \cdot \text{m}^{-3}$
ω	=	angular frequency, $\text{rad} \cdot \text{s}^{-1}$
ω_i	=	imaginary part of ω , $\text{rad} \cdot \text{s}^{-1}$
ω_r	=	real part of ω , $\text{rad} \cdot \text{s}^{-1}$
∇z	=	spatial gradient of z
$(\cdot)^*$	=	complex conjugate
$(\cdot)'$	=	derivative

Subscript

0	=	time index for averaged quantities
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Superscripts

1	=	fluctuating quantity
+	=	value upstream
-	=	value downstream

I. Introduction

TO CUT down pollutant emissions, industrial gas turbine combustion chambers are run in the lean premixed regime that, on the other hand, promotes combustion instabilities [1,2]. These instabilities result from a constructive coupling between the combustion process and the acoustic modes of the chamber. They might cause harmful damages to the combustion chamber, and therefore need to be accurately predicted.

Massive parallel computations and large-eddy simulations (LESs) now make it possible to represent the complex turbulent flow within combustion chambers [3–5]. Such LES solvers can predict instabilities in a reacting flow configuration [2,6], but their cost for full annular chambers remains prohibitive [7,8]. Moreover, these simulations cannot be repeated easily and cannot be used to optimize chamber designs to control azimuthal modes.

But in the cases where the flame response is known [9,10], acoustic Helmholtz solvers can prove to be sufficient to compute the thermoacoustic modes of a combustor. Helmholtz solvers reduce cost compared to solving the full Navier–Stokes equations [11–13]. To improve accuracy of these computations, the whole complex geometry of the combustor, including the combustion chamber, casing, swirlers, and dilution holes, needs to be taken into account [14,15].

The walls of recent combustion chambers in aeronautical gas turbines are generally perforated with submillimeter holes for cooling purposes. These holes allow fresh air flowing in the casing (outside the combustion chamber) to enter the combustion chamber

due to the pressure difference across the walls. The jets issuing from the holes coalesce to form a protecting cooling film that isolates the wall from the hot gases flowing in the chamber. This technique is called full-coverage film cooling (FCFC) [16].

In addition to modifying the dynamical and thermal characteristics of the flow near the walls, FCFC may have an impact on the acoustics of the chamber. Indeed, multiperforated plates (MPs) are known to have a damping effect on incident acoustic waves [17–21]. Acoustic waves force the shear layer between the jet and the main flow to break down and form vortex rings. This mechanism efficiently converts part of the acoustic energy into vortical energy [22]. Vortices are in turn dissipated by viscosity without substantial sound production [23]. The damping effect is generally enhanced by the presence of a bias flow through the plate [23–25], as encountered in FCFC. It strongly depends on the parameters of this bias flow [24], on the geometrical characteristics of the plate (porosity or shape of the hole) [26,27], and on the amplitude of the perturbation [28,29].

To perform predictive simulations of combustion instabilities, one should then either account for the presence of multiperforated walls in the chamber or show that their effect can be neglected. To the authors' knowledge, no study in the literature has properly addressed this question. Indeed, accounting for FCFC in simulations is a difficult task: due to the tiny diameter of the apertures, meshing the holes in numerical computations is clearly impossible. It is thus necessary to use models to represent the macroscopic effect of perforated walls on both near-wall flow structure and acoustics. This issue is encountered in computational fluid dynamics calculations [30–32] but also when resolving the Helmholtz equation to determine the acoustic modes of a chamber.

Analytical models have been developed and can be used to model the acoustic behavior of a MP. Howe proposed a model to represent the acoustic damping due to the vortex shedding in perforated plates submitted to a bias flow [23]. Howe's model [23] provides the acoustic impedance of a MP, and is therefore well adapted to Helmholtz solvers. To account for the plate thickness, which is neglected in Howe's model, Jing and Sun [33] proposed a modified version of Howe's model [23]. However, the use of the modified model does not provide a significant difference on the configurations studied here. Melling's model [21] also provides the impedance of a MP, considering the damping is due to the shear layers inside the orifices. Efforts have also been made to account for the presence of the grazing flow [34,35], but no analytical model has yet been developed, and thus cannot be implemented numerically.

In the present study, we will show how such models can be integrated in a general-purpose Helmholtz solver to account for the presence of perforated walls in aeronautical combustion chambers. The resulting tool will then be applied to a helicopter industrial chamber. The derivation of the Helmholtz equation under low Mach assumption is recalled in Sec. II.A, and the Helmholtz solver is described in Sec. II.B. A short overview of the available analytical models for MP will be given in Sec. II.C, and their numerical implementation is explained in Sec. II.D. Section III is dedicated to the verification of the implementation of the model derived by Howe [23] in an academic test case consisting of two coaxial cylinders separated by a perforated plate. A study of the influence of the parameters is also conducted in this section. Section IV shows how an energy budget can be computed from the results of the Helmholtz solver. Computations and energy budgets are performed on the longitudinal modes of a real industrial chamber in Sec. V. Finally, a computation on the whole chamber allows us to obtain the azimuthal modes of the industrial chamber in Sec. VI.

II. Incorporating Multiperforated Plate Models into a Helmholtz Solver

A. Derivation of the Helmholtz Equation Under Low Mach Assumption

Under the hypothesis of a nonviscous low-Mach-number mean flow and linear acoustics, the wave equation can be written in a reactive media as follows [2]:

$$\frac{1}{\gamma p_0} \frac{\partial^2 p_1}{\partial t^2} - \nabla \cdot \left(\frac{1}{\rho_0} \nabla p_1 \right) = 0 \quad (1)$$

Harmonic variations are supposed for p_1 and \mathbf{u}_1 , such that $\mathbf{u}_1 = \text{Re}(\hat{\mathbf{u}}(\mathbf{x})e^{-i\omega t})$ and $p_1 = \text{Re}(\hat{p}(\mathbf{x})e^{-i\omega t})$. In the frequency domain, the wave equation becomes the Helmholtz equation:

$$\nabla \cdot c^2 \nabla \hat{p} + \omega^2 \hat{p} = 0 \quad (2)$$

which is valid over the interior of the flow domain Ω .

Boundary conditions are formally written as a Robin condition:

$$cZ \nabla \hat{p} \cdot \mathbf{n} - i\omega \hat{p} = 0 \quad (3)$$

where \mathbf{n} is the unit vector normal to the domain boundary $\delta\Omega$, and Z is the complex impedance on $\delta\Omega$.

B. Description of the Helmholtz Solver Used in this Study

The Helmholtz solver used in this study solves the Helmholtz equation [Eq. (2)] and is fully described in [36]. A finite element method allows us to turn Eq. (2) into the following matricial form:

$$AP + \omega BP + \omega^2 P = 0 \quad (4)$$

where A is a matrix of size $N \times N$ corresponding to the operator $\nabla \cdot c^2 \nabla \hat{p}$, B is the matrix corresponding to the boundary conditions, and \mathbf{P} is the column vector of size N giving the nodal values of pressure. The size N of the problem is the number of nodes in the finite element grid that discretizes the geometrical domain. The eigenvalue problem of Eq. (4) is solved numerically using a parallel implementation of the Arnoldi method available in the P-ARPACK library [37]. The dependence in ω of Z implies that matrix B is a linear function of the frequency. The nonlinearity of the problem is handled by an iterative method; each subiteration considers that matrix B is constant and resolves a linear or quadratic eigenvalue problem [36]. The eigenelements of Eq. (4) thus provide the field of harmonic acoustic pressure and the angular frequency.

C. Acoustic Models for Multiperforated Plates

Let us consider an array of circular apertures of radius a , with an aperture spacing d , through which a mean flow parallel to the apertures axis flows at speed U . Under the excitation of an acoustic wave, Howe's model allows representation of the response of this flow under the following hypotheses [23]:

- 1) The acoustic excitation is at a low frequency, so that the wavelength is much larger than the orifices radius.
- 2) The flow has a high Reynolds number, the viscosity is then only dominant at the rims of the aperture leading to the shedding of vortices.
- 3) The Mach number of the mean flow is low, so that the flow is incompressible in the vicinity of the aperture.
- 4) The plate is infinitely thin.

5) The aperture spacing is high compared to the aperture radius, so that the interaction between the apertures is negligible and single-aperture results can be applied to the MP.

The reasoning is then done for an isolated hole. The backplane shown in Fig. 1 is located at a distance much higher than the aperture radius, thus being considered infinitely far away, making possible the application of Howe's model [23].

The Rayleigh conductivity K_a [38] of the aperture, relating the harmonic volumic flux \hat{Q} to the acoustic pressure jump across the plate, is defined by

$$K_a = \frac{i\omega \rho_0 \hat{Q}}{\hat{p}^+ - \hat{p}^-} \quad (5)$$

where ρ_0 is the mean density in the vicinity of the aperture, ω is the pulsation of the acoustic perturbation, and \hat{p}^- and \hat{p}^+ are the harmonic pressures upstream and downstream of the aperture, respectively. We have

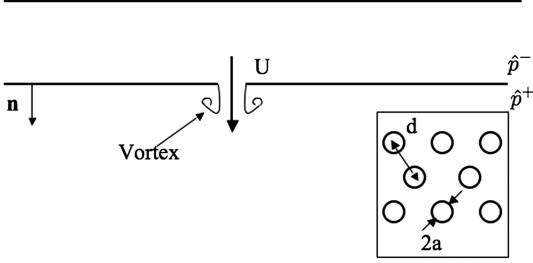


Fig. 1 Array of circular apertures, of diameter $2a$, and aperture spacing d , with a bias flow of speed U .

$$\hat{Q} = d^2 \hat{u}^\pm \quad (6)$$

where \hat{u}^\pm is the acoustic velocity on the plate, equal on both sides. Hence,

$$K_a = \frac{i\omega\rho_0 d^2 \hat{u}^\pm}{\hat{p}^+ - \hat{p}^-} \quad (7)$$

Howe [23] expresses the Rayleigh conductivity for a circular aperture in an infinitely thin plate as

$$K_a = 2a(\Gamma_a - i\Delta_a) \quad (8)$$

where

$$\Gamma_a - i\Delta_a = 1 + \frac{(\pi/2)I_1(S_t)e^{-S_t} - iK_1(S_t)\sin h(S_t)}{S_t[(\pi/2)I_1(S_t)e^{-S_t} + iK_1(S_t)\cos h(S_t)]} \quad (9)$$

with $S_t = \omega_r a/U$ and $i^2 = -1$.

The expression of the Rayleigh conductivity has been modified by Jing and Sun [33] to take into account the thickness h of the plate:

$$K_a = 2a \left(\frac{1}{\Gamma_a - i\Delta_a} + \frac{2h}{\pi a} \right)^{-1} \quad (10)$$

Luong et al. [39] proposed a simplified expression of the Rayleigh conductivity:

$$K_a = 2a \left(\frac{S_t}{S_t + (2i/\sigma_j)} \right) \quad (11)$$

where σ_j is the contraction ratio of the radius of the jet, due to the vena contracta effect [17]. The value of the Rayleigh conductivity derived by Luong et al. [39] is close to the one of Howe [23] in the case of a contraction ratio of $\sigma_j = 0.75$.

Another analytic approach was used by Melling [21] to derive the Rayleigh conductivity. This approach is based on the linearization of the Navier-Stokes equations inside each orifice. The dissipation of sound is supposed in this model to be due to viscous shear layers instead of the vortex shedding as assumed by Howe [23]. Considering $l_v = \sqrt{2\nu/\omega}$ as the thickness of the viscous shear layer, the following expression for the Rayleigh conductivity is derived:

$$K_a = \frac{\pi a^2}{h[1 + (l_v/a)(1 + i)]} \quad (12)$$

Assuming that the distance between the apertures is large enough ($d \gg a$), the interactions between the holes can be neglected. Then, the preceding expressions for K_a can be used to describe the acoustic effect of a MP submitted to an acoustic perturbation for which the wavelength is large compared to d . Besides, all the models recalled previously can be seen as Eq. (7) complemented by an expression for the Rayleigh conductivity of the form

$$K_a = K_a(a, h, \sigma_j, S_t) \quad (13)$$

where the first two parameters describe the aperture, and the last two describe the mean and unsteady components of the bias flow. In the following, we will simply refer to the Rayleigh conductivity as K_a , so

that the numerical implementation remains valid for any of the above acoustic models for MP.

D. Numerical Implementation

Using the momentum equation and Eq. (7), the following relation is obtained between the acoustic pressure jump across the plate and the acoustic pressure gradient:

$$\nabla \hat{p} \cdot \mathbf{n} = \frac{K_a}{d^2} [\hat{p}^+ - \hat{p}^-] \quad (14)$$

The numerical implementation of a MP model is first discussed in the isothermal case where the mean density can be considered the same on both sides of the plate. Figure 2 illustrates the implementation of any acoustic model of the form of Eq. (7) in a Helmholtz solver. The basic idea is to build a boundary condition imposed on each side of the homogeneous MP in such a way to retrieve Eq. (7) when computing the eigenfrequencies. Note that, since the actual MP has a finite thickness, the two sides of the plate are not located at the same place. Of course, no acoustic pressure is defined within the homogenized MP and the region between the two sides is not meshed. Let us assume that each side of the MP is oriented such that the local normal vector is pointing toward the flow domain (see \mathbf{n}^+ and \mathbf{n}^- in Fig. 2). The momentum equation under the zero Mach number assumption reads

$$i\omega\rho_0 \hat{\mathbf{u}} \cdot \mathbf{n}^\pm = \nabla \hat{p} \cdot \mathbf{n}^\pm \quad (15)$$

and the continuity of the acoustic velocity leads to

$$\nabla \hat{p} \cdot \mathbf{n}^+ = -\nabla \hat{p} \cdot \mathbf{n}^- \quad (16)$$

Equation (7) then leads to

$$\left. \begin{array}{l} i\omega\rho_0 \hat{\mathbf{u}} \cdot \mathbf{n}^+ = \frac{K_a}{d^2} [\hat{p}^+ - \hat{p}^-] \quad \text{on downstream side} \\ i\omega\rho_0 \hat{\mathbf{u}} \cdot \mathbf{n}^- = \frac{K_a}{d^2} [\hat{p}^- - \hat{p}^+] \quad \text{on upstream side} \end{array} \right\} \quad (17)$$

Finally, the acoustic model of the MP leads to the following condition valid on each side of the plate:

$$\nabla \hat{p} \cdot \mathbf{n} = -\frac{K_a}{d^2} [\hat{p}] \quad (18)$$

where $[\hat{p}]$ is the acoustic pressure jump defined as the value of \hat{p} on the opposite side minus the value of the current node.

In the case of a combustion chamber, the upstream flow coming from the casing of the chamber is cold (of volumic mass ρ^-), and the downstream flow located in the combustion chamber is hot (with a volumic mass ρ^+). The implementation of an acoustic model of the form of Eq. (7) is illustrated in this case in Fig. 3. The equality of acoustic speeds through the plate then reads

$$\frac{1}{\rho^+} \nabla \hat{p} \cdot \mathbf{n}^+ = \frac{-1}{\rho^-} \nabla \hat{p} \cdot \mathbf{n}^- \quad (19)$$

with the illustration of Fig. 3.

Considering that the density to be used in Eq. (7) is the density of the upstream gas, the condition to implement on the upstream side of the plate remains the same as for the isothermal case:

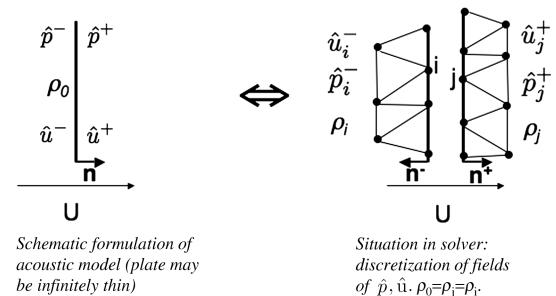


Fig. 2 Numerical implementation and model equivalence in the isothermal case.

$$\nabla \hat{p} \cdot \mathbf{n}^- = -\frac{K_a}{d^2} [\hat{p}^+ - \hat{p}^-] \quad (20)$$

To comply with the continuity of the acoustic velocity, it is then necessary to impose

$$\nabla \hat{p} \cdot \mathbf{n}^+ = -\frac{\rho^+ K_a}{\rho^- d^2} [\hat{p}^- - \hat{p}^+] \quad (21)$$

on the downstream side of the plate. The generic condition to implement for any thermal condition on a node j is then

$$\nabla \hat{p} \cdot \mathbf{n}_j = -\frac{\rho_j K_a}{\rho^- d^2} [\hat{p}]^j \quad (22)$$

where ρ_j is the mean density at node j , \mathbf{n}_j is the inward normal vector defined at node j , and $[\hat{p}]^j$ is the pressure jump at node j . Note that, in most of the cases, an interpolation procedure is necessary to get the corresponding pressure on both sides and build the appropriate pressure jump.

Equation (22) can then be used as a Neumann boundary condition in the Helmholtz solver. The normal pressure gradient on each multiperforated boundary is expressed as a function of a , d , U , and ω , so that the numerical treatment is similar to what is done for the finite impedance boundaries. Formally, this amounts to impose

$$Z^j = \frac{i\omega \hat{p}_j}{c_0 \nabla \hat{p} \cdot \mathbf{n}_j} = -\frac{i\omega \rho^- d^2 \hat{p}_j}{c_0 \rho_j K_a [\hat{p}]^j} \quad (23)$$

on both sides of the MP. The corresponding terms are then included into matrix B of Eq. (4).

III. Analytic Validation of the Coding

First, an academic configuration for which an analytic solution can be derived is presented. Some of the material presented in this section was already given in [40] and is recalled here for completeness.

The geometry depicted in Fig. 4 is computed both numerically and analytically. It consists of two coaxial cylinders, the inner one being perforated. The outer boundary of radius r_2 consists of a hard wall and will be noted as $\delta\Omega_W$. The perforated plate is located at $r = r_1$. The boundary denoted by the MP will be noted as $\delta\Omega_{MP}$, on which jump conditions apply.

A uniform mean flow of constant speed of sound c_0 follows the direction showed by the arrow in Fig. 4. In the following equations, r_1^- and r_1^+ will then denote the upstream and the downstream parts of the MP. Although the cylinder is three-dimensional (3-D), the third dimension is considered long in regard to the others so that only radial and azimuthal modes will be considered.

A. Determination of the Analytical Solution

For this simple case, Eq. (2) takes the following form:

$$\begin{cases} \Delta \hat{p} + k^2 \hat{p} = 0 & \text{on } \Omega \\ \nabla \hat{p} \cdot \mathbf{n} = 0 & \text{on } \delta\Omega_W \end{cases} \quad (24)$$

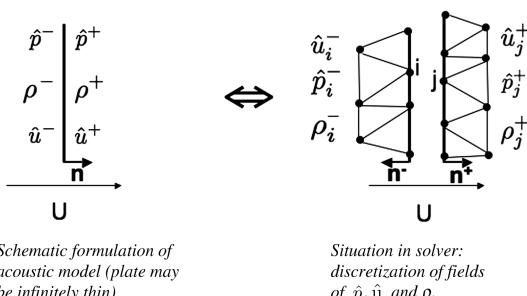


Fig. 3 Numerical implementation and model equivalence in the anisothermal case.

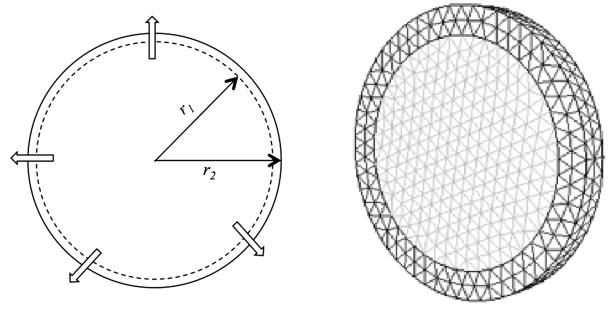


Fig. 4 Academic configuration: cylinder of radius r_2 , with a perforated plate at r_1 , and view of the mesh.

Jump conditions that apply across $\delta\Omega_{MP}$ will be detailed further in Eqs. (32) and (33).

Considering $k^2 = k_r^2 + k_z^2$ ($k = \omega/c_0$), Eq. (24) can be cast in the polar coordinates (r, θ, z) :

$$\frac{1}{R} \frac{d^2}{dr^2} R + \frac{1}{rR} \frac{d}{dr} R + \frac{1}{r^2 \Theta} \frac{d^2}{d\theta^2} \Theta + \frac{1}{Z} \frac{d^2}{dz^2} Z + (k_r^2 + k_z^2) = 0 \quad (25)$$

where $\hat{p} = R(r)\Theta(\theta)Z(z)$ [2,41].

Since we only consider the radial and azimuthal modes, the term in d^2/dz^2 can be neglected so that

$$\frac{1}{R} \frac{d^2}{dr^2} R + \frac{1}{rR} \frac{d}{dr} R + \frac{1}{r^2} \left(\frac{1}{\Theta} \frac{d^2}{d\theta^2} \Theta + n_\theta^2 \right) + k_r^2 - \frac{n_\theta^2}{r^2} = 0 \quad (26)$$

Under these conditions, the radial part of Eq. (25) can be reduced to a Bessel equation:

$$\left(\frac{d^2}{dr^2} R + \frac{1}{r} \frac{d}{dr} R \right) + R \times \left(k_r^2 - \frac{n_\theta^2}{r^2} \right) = 0 \quad (27)$$

for which the general solution is of the form

$$R(r) = AJ_{n_\theta}(k_r r) + BN_{n_\theta}(k_r r) \quad (28)$$

where J_{n_θ} and N_{n_θ} are Bessel functions of order n_θ . In the domain $r \leq r_1^-$, N_{n_θ} being singular at $r = 0$ is put aside and the pressure can then be written:

$$R(r) = AJ_{n_\theta}(k_r r) \quad \text{for } r \leq r_1^- \quad (29)$$

In the domain $r_1^+ \leq r \leq r_2$, solutions may be written:

$$R(r) = CJ_{n_\theta}(k_r r) + DN_{n_\theta}(k_r r) \quad (30)$$

A wall condition is imposed on the outer cylinder; thus, the normal acoustic speed is null on $\delta\Omega$. The condition $\hat{u} \cdot \mathbf{n} = 0$ at $r = r_2$ then leads to

$$CJ'_{n_\theta}(k_r r_2) + DN'_{n_\theta}(k_r r_2) = 0 \quad (31)$$

According to Eq. (7), jump conditions can also be written across $\delta\Omega_{MP}$:

$$\hat{p}(r = r_1^+) - \hat{p}(r = r_1^-) = \frac{i\omega \rho d^2}{K_a} \hat{u}(r = r_1^-) \quad (32)$$

$$\hat{p}(r = r_1^-) - \hat{p}(r = r_1^+) = -\frac{i\omega \rho d^2}{K_a} \hat{u}(r = r_1^+) \quad (33)$$

where \hat{u} denotes the radial acoustic velocity. We then obtain the system

$$M\mathbf{X} = 0$$

where M is the matrix obtained by using Eqs. (31–33) and is given by

Table 1 Comparison of eigenfrequencies between numerical and analytical results

	Numerical results			Analytical results
	$\text{Re}(f)$, Hz	$\text{Im}(f)$ ($\mathcal{A}\%$)	$\text{Re}(f)$, Hz	$\text{Im}(f)$ ($\mathcal{A}\%$)
$n_\theta = 1$	382.5	-18.8 s^{-1} (26.7%)	382.6	-18.9 s^{-1} (27.8%)
$n_\theta = 0$	534.1	-97.5 s^{-1} (68.2%)	533.2	-97.5 s^{-1} (68.3%)
$n_\theta = 2$	610.5	-21.4 s^{-1} (19.8%)	611	-21.6 s^{-1} (19.9%)

$$\begin{bmatrix} 0 & J'_{n_\theta}(k_r r_2) & N'_{n_\theta}(k_r r_2) \\ \frac{d^2}{K_a} k_r J'_{n_\theta}(k_r r_1^-) + J_{n_\theta}(k_r r_1^-) & -J_{n_\theta}(k_r r_1^+) & -N_{n_\theta}(k_r r_1^+) \\ J_{n_\theta}(k_r r_1^-) & -J_{n_\theta}(k_r r_1^+) + \frac{d^2 k_r}{K_a} J'_{n_\theta}(k_r r_1^+) & -N_{n_\theta}(k_r r_1^+) + \frac{d^2 k_r}{K_a} N'_{n_\theta}(k_r r_2) \end{bmatrix}$$

and $X' = [A \quad C \quad D]$.

Solving

$$\det(M) = 0 \quad (34)$$

releases the eigenvalues of the configuration.

B. Results

Let us consider the following parameters for the MP: $U = 5 \text{ m} \cdot \text{s}^{-1}$; $a = 3 \text{ mm}$; and $d = 35 \text{ mm}$, $r_1 = 0.2 \text{ m}$, and $r_2 = 0.25 \text{ m}$. The configuration calculated by the Helmholtz solver [36] contains a tetrahedric mesh of 1186 nodes, shown in Fig. 4. The sound speed is uniform and equal to $347 \text{ m} \cdot \text{s}^{-1}$.

The computed eigenfrequencies gathered in Table 1 are compared to the analytical results obtained by solving Eq. (34). The first three modes are considered; they correspond to the first radial ($n_\theta = 0$) and the two first azimuthal ($n_\theta = 1, n_\theta = 2$) modes.

The Helmholtz solver provides a complex frequency $f = \text{Re}(f) + i\text{Im}(f)$, where $\text{Re}(f)$ is the frequency of the mode, and $\text{Im}(f)$ is related to the amplification rate of the mode. With the adopted convention $p_1(\mathbf{x}, t) = \text{Re}(\hat{p}(\mathbf{x})e^{-i\omega t})$, the ratio between the value of the pressure fluctuation at $t = 0$ and $T = 1/\text{Re}(f)$ is given by Eq. (35):

$$\frac{|p_1(\mathbf{x}, t = T)|}{|p_1(\mathbf{x}, t = 0)|} = e^{i\text{Im}(\omega)T} \quad (35)$$

It is then useful to define an attenuation factor \mathcal{A} (in percent) over a period T as $\mathcal{A} = 100[1 - e^{2\pi[\text{Im}(f)/\text{Re}(f)]}]$.

As expected, the eigenfrequencies have a negative imaginary part, which means that the pressure fluctuation is damped. The numerical results are in good agreement with the theory.

1. Results on the First Azimuthal Mode

Figure 5 shows the results for the first azimuthal mode ($n_\theta = 1$, $\text{Re}(f) = 382.5 \text{ Hz}$, $\text{Im}(f) = -18.8 \text{ s}^{-1}$). Radial profiles of the real and imaginary parts of \hat{p} given by the Helmholtz solver are plotted against analytical results for comparison (bottom). A good agreement is found. The two-dimensional pressure distribution released by the Helmholtz solver as well as isolines of pressure are given (top). Note that the pressure jump across the perforated plate is visible on the real and imaginary parts of the harmonic pressure fluctuations, as well as on the discontinuity of the isolines.

This comparison was conducted for a fixed bias flow speed ($5 \text{ m} \cdot \text{s}^{-1}$), but Fig. 6 also shows good agreement between numerical and analytical results for various bias flow speeds.

2. Results on the Radial Mode

Figure 7 shows the fields of pressure of the radial mode ($n_\theta = 0$). Again, a good agreement is found. Unlike the first azimuthal mode, no discontinuity is observed on the isolines of pressure, due to the form of the pressure field.

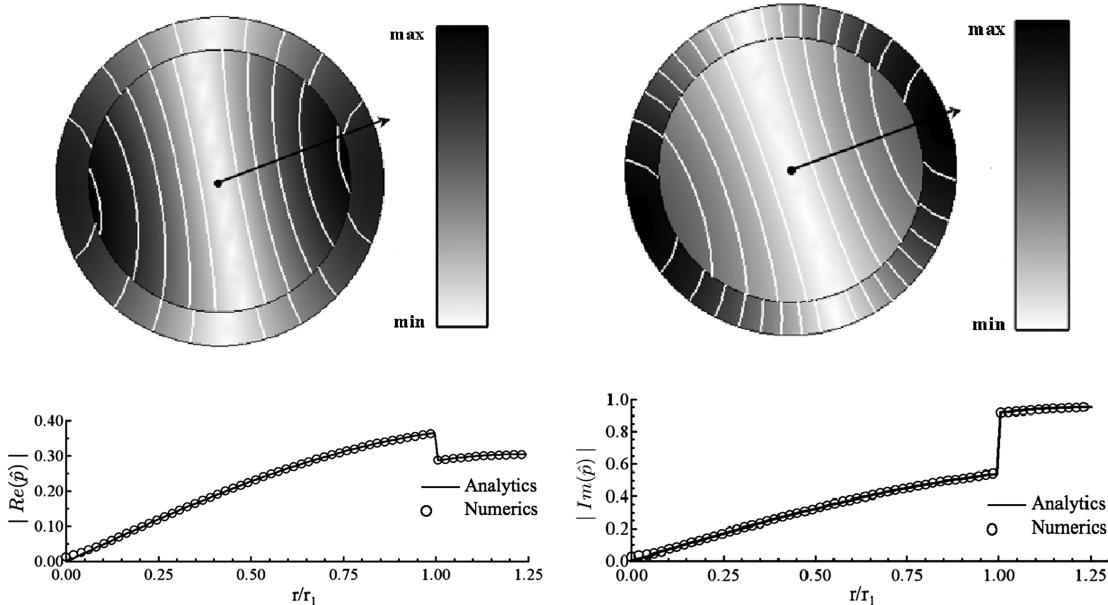


Fig. 5 First azimuthal mode: $n_\theta = 1$, $\text{Re}(f) = 382.5 \text{ Hz}$, $\text{Im}(f) = -18.8 \text{ s}^{-1}$.

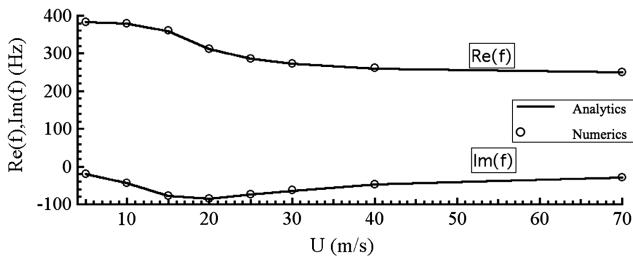


Fig. 6 Comparison between numerical and analytical results. $\text{Re}(f)$ and $\text{Im}(f)$ are shown for various speeds for the first azimuthal mode.

3. Comparison of the Evolution of Both Modes Against Bias Flow Speed

Figure 8 shows the evolution of the resonant frequencies for the radial mode and the first azimuthal mode as a function of the bias flow speed. As the bias flow speed increases, the resonant frequency of both modes decreases. But in the case of the azimuthal mode, the resonant frequencies vary in range from 380 to 260 Hz, whereas the frequency of the radial mode varies a lot, dropping from 500 to 80 Hz.

The frequency of the radial mode hence becomes close to the one of the azimuthal mode, and then is lower than it. Figure 9 displays the attenuation factor of the radial and the first azimuthal mode as a function of the bias flow speed. In the case of the radial mode, the damping raises until reaching almost 100% and keeps this very high value of damping.

On the contrary, the damping for the first azimuthal mode reaches a maximum at $U_{\text{md}} \approx 20 \text{ m} \cdot \text{s}^{-1}$ and then decreases. For this particular simple geometry, the impact of the perforated plates is thus more important on radial modes than on azimuthal modes. For the radial mode, the pressure jump is constant over all the perimeter, which makes the pressure drop important everywhere; thus, according to Eq. (14), the perforated plate has a strong impact. This will be further explained in Sec. IV.B with an energy analysis.

IV. Energy Analysis

A. General Analysis

To evaluate the dissipation induced by perforated plates, an analysis of the energy budget is done for the cylinder configuration. For a reactive case, the conservation equation for the acoustic energy $E(\mathbf{x}, t) = (1/2)\rho\mathbf{u}_1^2(\mathbf{x}, t) + (1/2\rho)p_1^2(\mathbf{x}, t)$ can be written as follows:

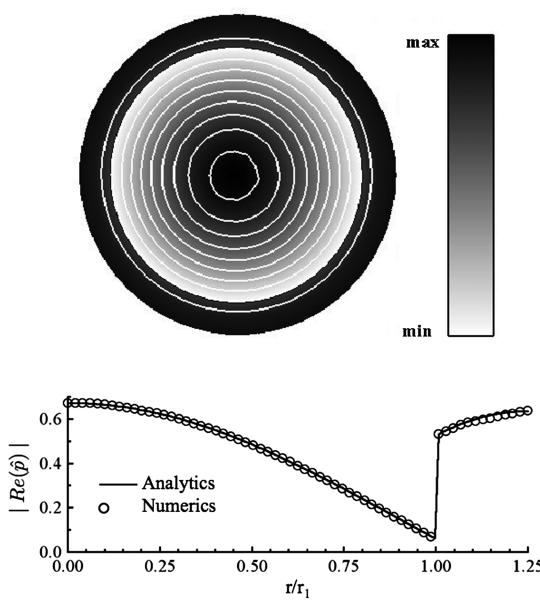


Fig. 7 Radial mode: $n_\theta = 0$, $\text{Re}(f) = 534.1 \text{ Hz}$, $\text{Im}(f) = -97.5 \text{ s}^{-1}$.

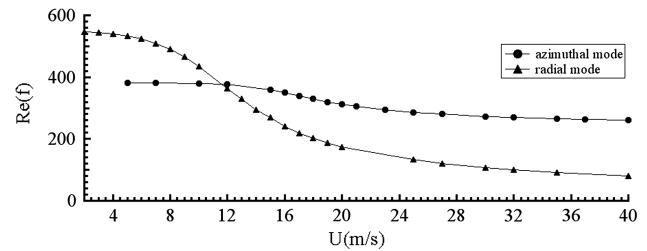


Fig. 8 Resonant frequencies as a function of bias flow speed for the radial and the first azimuthal mode. Symbols are for numerical results, while the line stands for analytical results.

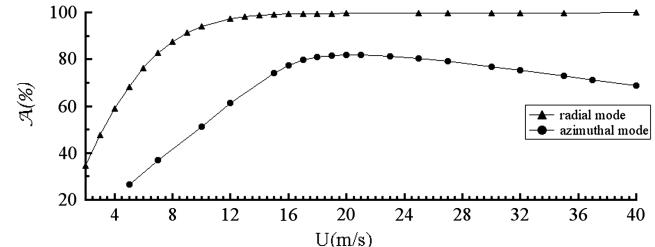


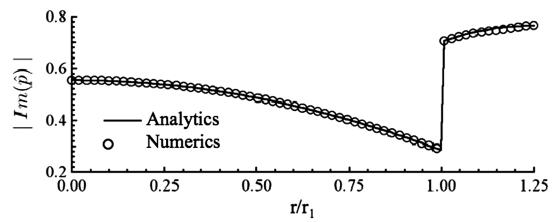
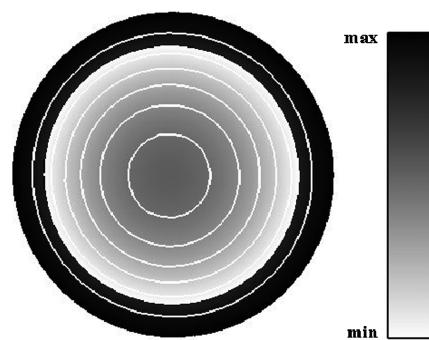
Fig. 9 Attenuation factor as a function of the bias flow speed U for the radial and the first azimuthal mode. Symbols are for numerical results, while the line stands for analytical results.

$$\frac{\partial E}{\partial t} + \nabla \cdot p_1 \mathbf{u}_1 = 0 \quad (36)$$

Integrating Eq. (36) over the flow domain and a time period T yields

$$\frac{1}{T} \int_T \frac{\partial \bar{E}}{\partial t} dt = -\frac{1}{T} \int_T \mathcal{F}_1 \quad (37)$$

where $\bar{E} = \iiint \Omega E d\Omega$ is the volume integrated acoustic energy, and Ω can be split into four parts: Ω_Z where a finite valued impedance is imposed, Ω_{MP} where a MP condition is imposed, Ω_W where an impermeable wall condition is prescribed, and Ω_P where the acoustic pressure is set to zero. The term \mathcal{F}_1 is the sum of the acoustic fluxes over the boundaries, defined by



$$\mathcal{F}_1 = \iint_{\delta\Omega} p_1 \mathbf{u}_1 \cdot \mathbf{n} dS \quad (38)$$

Combining the momentum equation $\rho i\omega \hat{\mathbf{u}} \cdot \mathbf{n} = \nabla \hat{p} \cdot \mathbf{n}$ and Eq. (22), the following relation between the acoustic velocity fluctuation and the acoustic pressure jump over a MP is derived:

$$\hat{\mathbf{u}} \cdot \mathbf{n} = \frac{-1}{\rho^- i\omega} \frac{K_a}{d^2} [\hat{p}] \quad (39)$$

As for $\delta\Omega_Z$, combining the momentum equation with Eq. (3), we obtain

$$\hat{\mathbf{u}} \cdot \mathbf{n} = \frac{\hat{p}}{\rho c Z} \quad (40)$$

Using Eqs. (39) and (40) to express the flux, the acoustic energy budget reads

$$\begin{aligned} \frac{\partial \bar{E}}{\partial t} &= - \iint_{\delta\Omega_Z} \operatorname{Re}(\hat{p}(\mathbf{x}) e^{-i\omega t}) \operatorname{Re}\left(\frac{\hat{p}(\mathbf{x})}{\rho c Z} e^{-i\omega t}\right) dS \\ &\quad - \iint_{\delta\Omega_{MP}} \operatorname{Re}(\hat{p}(\mathbf{x}) e^{-i\omega t}) \operatorname{Re}\left(\frac{-1}{\rho^- i\omega} \frac{K_a}{d^2} [\hat{p}] e^{-i\omega t}\right) dS \end{aligned}$$

The contribution of the flux over $\delta\Omega_W$ and $\delta\Omega_P$ is null because either p_1 or $\mathbf{u}_1 \cdot \mathbf{n}$ vanishes. Since there is no impedance condition, the energy loss only occurs through the perforated plates. After integrating over one period T , we have

$$\begin{aligned} \frac{\bar{E}(t=T)}{\bar{E}(t=0)} - 1 &= - \frac{1}{\bar{E}(t=0)} \int_T \iint_{\delta\Omega_{MP}} \operatorname{Re}(\hat{p}(\mathbf{x}) e^{-i\omega t}) \operatorname{Re}\left(\frac{-K_a}{d^2 \rho^- i\omega} [\hat{p}] e^{-i\omega t}\right) dS \end{aligned} \quad (41)$$

and after some algebra,

$$\begin{aligned} \int_0^T p_1 \mathbf{u}_1 \cdot \mathbf{n} dt &= \frac{1}{\rho(\omega_r^2 + \omega_i^2)} \left[\omega_i \operatorname{Im}\left(\frac{K_a}{d^2}\right) + \omega_r \operatorname{Re}\left(\frac{K_a}{d^2}\right) \right] \\ &\times [-A \operatorname{Re}(\hat{p}) \operatorname{Im}(p^+ - p^-) + B \operatorname{Re}(p^+ - p^-) \operatorname{Im}(\hat{p}) \\ &+ C (\operatorname{Re}(\hat{p}) \operatorname{Re}(p^+ - p^-) - \operatorname{Im}(p^+ - p^-) \operatorname{Im}(\hat{p})) \\ &+ \frac{1}{\rho(\omega_r^2 + \omega_i^2)} \left[\omega_i \operatorname{Re}\left(\frac{K_a}{d^2}\right) - \omega_r \operatorname{Im}\left(\frac{K_a}{d^2}\right) \right] \\ &\times [A \operatorname{Re}(\hat{p}) \operatorname{Im}(p^+ - p^-) + B \operatorname{Im}(\hat{p}) \operatorname{Im}(p^+ - p^-) \\ &+ C (\operatorname{Im}(\hat{p}) \operatorname{Re}(p^+ - p^-) + \operatorname{Re}(\hat{p}) \operatorname{Im}(p^+ - p^-))] \end{aligned} \quad (42)$$

with A , B , and C defined as follows:

$$\begin{cases} C = \int_0^T e^{2\omega_i t} \cos(\omega_r t) \sin(\omega_r t) dt = \frac{\omega_r(1-e^{2\omega_i T})}{4(\omega_r^2 + \omega_i^2)} & \text{and if } \omega_i = 0, \quad C = 0 \\ A = \int_0^T e^{2\omega_i t} \cos^2(\omega_r t) dt = \frac{e^{2\omega_i T}-1}{2\omega_i} + \frac{\omega_r}{\omega_i} C & \text{and if } \omega_i = 0, \quad A = \frac{T}{2} \\ B = \int_0^T e^{2\omega_i t} \sin^2(\omega_r t) dt = -\frac{\omega_r}{\omega_i} C = \frac{\omega_r^2(e^{2\omega_i T}-1)}{4\omega_i(\omega_r^2 + \omega_i^2)} & \text{and if } \omega_i = 0, \quad B = \frac{T}{2} \end{cases} \quad (43)$$

The contribution of a MP can be obtained by summing the fluxes on both sides of the plate. An analytical expression can be derived in the simple case where $\omega_i \ll \omega_r$ and \hat{u} do not vary along the tangential direction to the MP. Considering a perforated plate composed of two boundaries as shown in Fig. 10, an asymptotic formulation can be

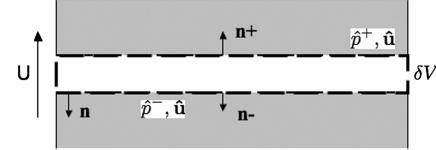


Fig. 10 Calculation of the flux difference across a perforated plate.

derived for the term \mathcal{F}_1 of Eq. (38) following the conventions defined in Fig. 10 where δV is the boundary of a control volume of small size compared to the acoustic wavelength λ .

Its expression is

$$\begin{aligned} \mathcal{F}_1 &= \oint_{\delta\Omega} \operatorname{Re}(\hat{p}^+ e^{-i\omega t}) \operatorname{Re}(\hat{\mathbf{u}}^+ \cdot \mathbf{n}^+ e^{-i\omega t}) + \operatorname{Re}(\hat{p}^- e^{-i\omega t}) \\ &\quad \times \operatorname{Re}(\hat{\mathbf{u}}^- \cdot \mathbf{n}^- e^{-i\omega t}) dS \end{aligned} \quad (44)$$

Then, making use of the fact that $\Delta \ll \lambda$, the flux density is simply

$$\begin{aligned} F_1 &= \frac{\mathcal{F}_1}{\Delta} = \operatorname{Re}(\hat{p}^+ e^{-i\omega t}) \operatorname{Re}(\hat{\mathbf{u}}^+ \cdot \mathbf{n}^+ e^{-i\omega t}) + \operatorname{Re}(\hat{p}^- e^{-i\omega t}) \\ &\quad \times \operatorname{Re}(\hat{\mathbf{u}}^- \cdot \mathbf{n}^- e^{-i\omega t}) \end{aligned} \quad (45)$$

Integrating over a period T leads to

$$\begin{aligned} \frac{1}{T} \int_T F_1 dt &= \frac{1}{T} \int_T \operatorname{Re}(\hat{p}^+ e^{-i\omega t}) \operatorname{Re}(\hat{\mathbf{u}}^+ \cdot \mathbf{n}^+ e^{-i\omega t}) \\ &\quad + \operatorname{Re}(\hat{p}^- e^{-i\omega t}) \operatorname{Re}(\hat{\mathbf{u}}^- \cdot \mathbf{n}^- e^{-i\omega t}) dt \end{aligned} \quad (46)$$

Since $\hat{u}^+ = \hat{u}^- = \hat{u}$ and \mathbf{n}^+ and \mathbf{n}^- point to opposite directions, using Eq. (39) then leads to

$$\begin{aligned} \frac{1}{T} \int_T F_1 dt &= \int_T \operatorname{Re}([\hat{p}^+ - \hat{p}^-] e^{-i\omega t}) \\ &\quad \times \operatorname{Re}\left(\frac{K_a}{\rho^- i\omega d^2} [\hat{p}^+ - \hat{p}^-] e^{-i\omega t}\right) dt \end{aligned} \quad (47)$$

The hypothesis $\omega_i \ll \omega_r$ yields

$$\frac{1}{T} \int_T F_1 dt = \frac{1}{2} \operatorname{Re}\left([\hat{p}^+ - \hat{p}^-]^* \frac{K_a}{\rho^- i\omega d^2} [\hat{p}^+ - \hat{p}^-]\right) \quad (48)$$

where $(\cdot)^*$ denotes the conjugate of a complex number. Then, it yields

$$\frac{1}{T} \int_T F_1 dt = \frac{-|\hat{p}^+ - \hat{p}^-|^2}{2\rho^- \omega_r d^2} \operatorname{Re}(i K_a) \quad (49)$$

Using Eq. (8),

$$\frac{1}{T} \int_T F_1 dt = -\frac{|\hat{p}^+ - \hat{p}^-|^2}{\rho^- \omega_r d^2} a \Delta_a \quad (50)$$

which is negative for all frequencies as soon as $\Delta_a > 0$. The net flux \mathcal{F}_1 toward the control volume (Fig. 10) is then positive (recall that the

\mathbf{n} vector points outward of the control volume of Fig. 10), inducing a loss of energy for the fluid domain surrounding the MP. Thus, any acoustic model with positive Δ_a generates some amount of damping for the acoustic modes. This result completes the result showed by Howe [42] in the case of a uniform density.

Table 2 Acoustic energy analysis on the cylinder

	$\bar{E}(t = T)\bar{E}(t = 0) / -1, \%$	$-\int_T \int_{\delta\Omega_{MP}} p_1 \mathbf{u}_1 \cdot \mathbf{n} dS, \%$	$\epsilon, \%$
First azimuthal mode $U = 5 \text{ m} \cdot \text{s}^{-1}$	-46.19	-46.37	0.4
First azimuthal mode $U = 20 \text{ m} \cdot \text{s}^{-1}$	-96.73	-96.82	0.096
Radial mode $U = 5 \text{ m} \cdot \text{s}^{-1}$	-89.8	-92	2.5
Radial mode $U = 20 \text{ m} \cdot \text{s}^{-1}$	-99.65	-99.69	0.036

B. Numerical Calculation of Dissipation Through the Perforated Plates of the Cylinder

The left-hand side (LHS) and right-hand side (RHS) of Eq. (41) were assessed numerically for the radial and the azimuthal modes for two values of the bias flow speed: $U = 5 \text{ m} \cdot \text{s}^{-1}$ and $U = 20 \text{ m} \cdot \text{s}^{-1}$. Note that the error of 0.06% caused by the Mach value of the bias flow speed of $U = 20 \text{ m} \cdot \text{s}^{-1}$ is considered negligible in front of the approximations made in this thermoacoustic study. The results are gathered in Table 2. A new damping factor A_{en} is now defined as $|\bar{E}(t = T)/\bar{E}(t = 0)| - 1|$. This damping factor refers to the damping of the acoustic energy, whereas the damping factor defined previously in Sec. III.B referred to the damping of the pressure amplitude only. In the case of the first azimuthal mode ($U = 5 \text{ m} \cdot \text{s}^{-1}$), the term A_{en} is found to be 46.19%, while

$$-\int_T \int_{\delta\Omega_{MP}} p_1 \mathbf{u}_1 \cdot \mathbf{n} dS$$

gives -46.37%. In the table, the normalized error defined by

$$\epsilon = \frac{|\bar{E}(t = T)/\bar{E}(t = 0)| - 1 + \int_T \int_{\delta\Omega_{MP}} p_1 \mathbf{u}_1 \cdot \mathbf{n} dS}{|\bar{E}(t = 0)|} \quad (51)$$

is also given; the equation of acoustic energy is well balanced for the two modes for $U = 5 \text{ m} \cdot \text{s}^{-1}$.

The same is true when considering the bias flow velocity corresponding to the maximal damping as found for the modes in Sec. III.B. In both cases, the damping increases: with a bias flow speed of $U_{md} = 20 \text{ m} \cdot \text{s}^{-1}$, the azimuthal mode is damped by 96.7% while $A_{en} = 99.65\%$ is reached for the radial mode.

Figure 11 shows $|\hat{p}^+ - \hat{p}^-|$ on each point of the MP, adimensionalized by the maximum of fluctuating pressure over the domain against the angle θ for the azimuthal mode at $U = 5 \text{ m} \cdot \text{s}^{-1}$ and $U = U_{md}$. The same value is also plotted at U_{md} for the radial mode. The profile of $|\hat{p}^+ - \hat{p}^-|$ of the radial mode is constant over all the MP, whereas for the azimuthal mode, $|\hat{p}^+ - \hat{p}^-|$ varies along the curvilinear abscissa and is globally smaller than for the radial fluctuation. According to Eq. (50), the higher $|\hat{p}^+ - \hat{p}^-|$ is, the higher the dissipation is; thus, Fig. 11 is fully consistent with the results of Table 2. Figure 11 also shows that the pressure jump for the azimuthal mode at $U = 20 \text{ m} \cdot \text{s}^{-1}$ is globally higher than for the case at $U = 5 \text{ m} \cdot \text{s}^{-1}$, which explains the increase in the damping when U is equal to U_{md} .

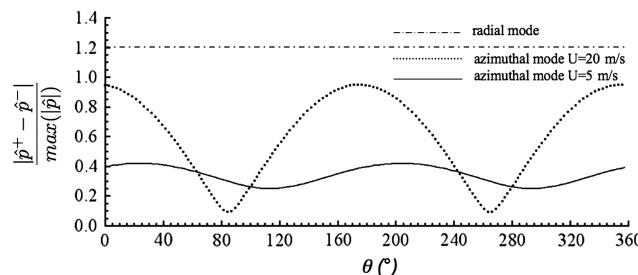


Fig. 11 $|p^+ - p^-| / \max(\hat{p})$ as a function of θ for the radial and the first azimuthal mode.

V. Acoustic Energy Analysis of an Industrial Helicopter Chamber

The purpose of this section is to illustrate the flexibility of the methodology described in the paper by computing a 3-D actual industrial chamber. Unlike the previous academic test case, the chamber contains several MPs. An analysis of the energy budget is therefore carried out in order to evaluate which perforated plates are the most responsible for the damping of the mode as well as to assess the error made on the computations of the acoustic modes. Note that some details regarding the MP geometry and bias flow are omitted for confidentiality reasons. Still, enough information is provided to illustrate the applicability of the method. At last, Howe's model [42] was used for the runs presented in the remainder of this paper as typical of the models discussed in Sec. II.C. Note that, since no detailed experimental data are available for the industrial case presented in the following, this choice does not result from a model selection based on quantitative comparisons. Using another model would lead to slightly different results but would neither change the trends presented next nor the illustration of the usefulness of such models in Helmholtz solvers. For example, the modified Howe model [42] developed by Jing and Sun [33] differs by 0.8% on the imaginary part, which does not change the behavior.

A. Description of the Chamber and First Results

The chamber studied here is a reverse-flow annular combustor and is displayed in Fig. 12. The air inlet comes out of the compressor stage through the casing. Some of this air is used to cool the chamber walls by two means: perforated plates and cooling films. Several jets let into the chamber by the orifices of these cooling devices coalesce to form a cooling coat of air on the inside walls, which protects them. The whole chamber contains 15 burners. As a first step, the computational domain is thus limited to a 24 deg sector of the chamber, corresponding to one burner. It includes the swirler, the casing, and the burner. Computations of the whole geometry with the 15 sectors will be discussed in Sec. VI. Figure 12 shows the location of the cooling devices on a sector of the chamber: the cooling films are denoted by the letter F, and the different perforated plates are denoted by MP1, MP2, and MP3. The index i stands for interior and

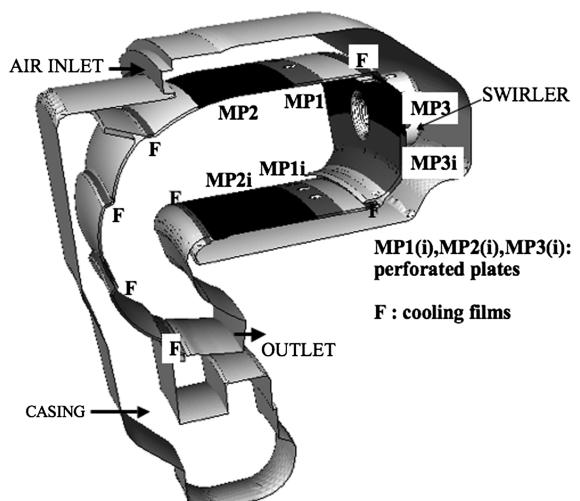


Fig. 12 Location of the MP in the chamber.

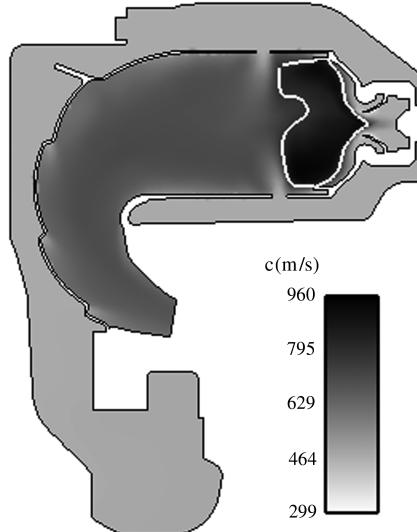


Fig. 13 Sound speed field.

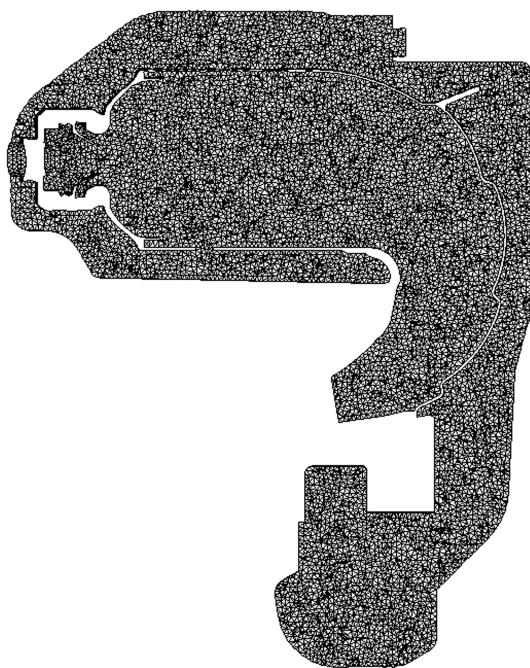


Fig. 14 Mesh view on a cut in the middle of the chamber.

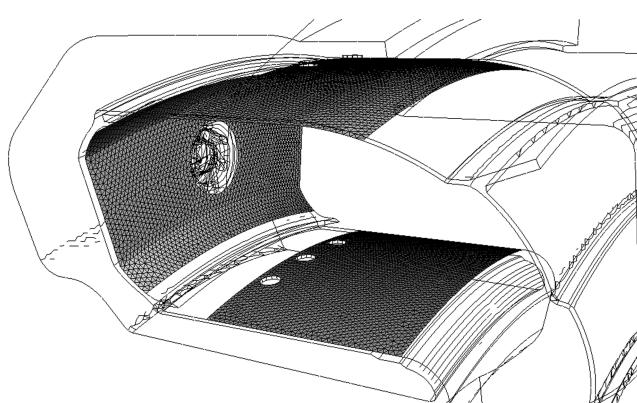
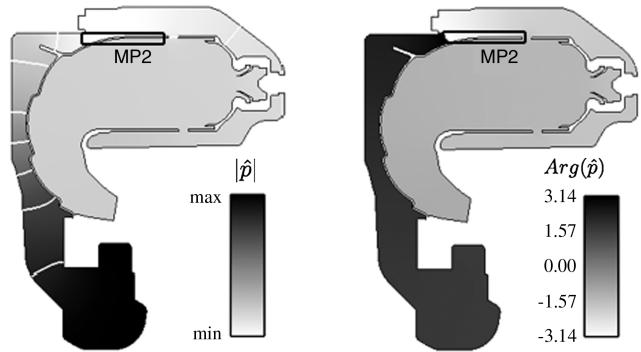


Fig. 15 Mesh on the perforated plates.

Fig. 16 Field of $|\hat{p}|$ and $\text{Arg}(\hat{p})$ for the 1L mode (the box shows the location of plate MP2).

corresponds to the plates in the inside part of the chamber. As shown in Fig. 12, the surface area of the different MP zones are different. The holes have the same diameter, but the bias flow speed is not the same for all plates and is computed using data on flow rates. The inlet of air and the outlet are considered in this computation as acoustically closed conditions, as well as all the chamber walls.

The sound speed field used by the Helmholtz solver is illustrated in Fig. 13; it is deduced from a previous LES [8] that is subsequently interpolated over the mesh used for the acoustic computation. The mesh used contains approximately 450,000 nodes and 2,500,000 tetrahedric cells. A cut of the mesh is shown in Fig. 14, while Fig. 15 shows the mesh on the perforated plates. The mesh is quite regular, with a ratio of less than 200 between the maximum and the minimum volume of the mesh cells.

Note that this chamber has already been computed in previous studies: for example, the influence of the geometry on the acoustics of this chamber was previously studied [15] and several computational fluid dynamics calculations were conducted [7,43]. The influence of the presence of perforated plates has previously been studied by the Gullaud et al. [40]. It was shown that, when using the actual parameters of bias flow through the perforated plates, a very slight influence of the plates on the first longitudinal mode is found.

The mode structure compared to a result without MPs was hardly changed, and the damping was found weak. A typical distribution of the acoustic pressure field of this result is given in Fig. 16. On the other hand, the second longitudinal mode was found more damped, due to the structure of the mode as shown in Table 3. Indeed, the pressure drop between the casing and the combustion chamber is more important for the second mode and well located to be efficient for the MP. The purpose of the next section is to illustrate how an acoustic energy analysis can be used to quantify the effect of each MP on the overall stability of the configuration.

B. Energy Analysis of the First Longitudinal Mode

An energy budget is conducted for the first longitudinal mode. The two terms of Eq. (41) can be assessed by postprocessing the results from the Helmholtz solver, and for the RHS term, each contribution of each perforated plate is computed. Results are gathered in Table 4, where all contributions are normalized by the energy of the mode integrated over the domain at $t = 0$ ($\bar{E}_1(t = 0)$) so that we have

Table 3 Frequencies of the first two longitudinal modes (1L and 2L)

Mode	1L		2L		
	f	$\text{Re}(f)$	$\text{Im}(f)$	$\text{Re}(f)$	$\text{Im}(f)$
		507.94 Hz	-2.65 Hz	1150.98 Hz	-41.95 Hz
$A_{\text{en}}, \%$		6.3		37.3	

Table 4 Acoustic energy budget on 1L mode (all values in %)

1L	$e^{2\omega_i T} - 1$	ϵ	MP1	MP1 <i>i</i>	MP2	MP2 <i>i</i>	MP3	MP3 <i>i</i>
All MPs	−6.4	0.36	0.625	$4.5 \cdot 10^{-3}$	5.36	$1.7 \cdot 10^{-2}$	0.526	0.186
			9.30	0.067	79.78	0.25	7.83	2.77
No MP2	−1.1	0.2	0.637	$4.41 \cdot 10^{-3}$	—	$1.7 \cdot 10^{-2}$	0.541	0.195
			45.68	0.32	—	1.22	38.8	13.98

Table 5 Acoustic energy analysis on the second longitudinal mode (2L)

2L	$\bar{E}(t = T/\bar{E}(t = 0 - 1))$	RHS	Error	MP1	MP1 <i>i</i>	MP2	MP2 <i>i</i>	MP3	MP3 <i>i</i>
All MP	−47.1	—	48.2	2.46	4.6 (9.54)	0.68 (1.41)	22.46 (46.6)	3.67 (7.614)	10.08 (20.9)
No MP2	−29	—	−28.2	2.97	5.28 (18.7)	0.63 (2.23)	—	4.01 (14.22)	11.3 (40.1)

$$e^{2\omega_i T} - 1 + (\text{MP1} + \text{MP1*i*} + \text{MP2} + \text{MP2*i*} + \text{MP3} + \text{MP3*i*}) = 0 \quad (52)$$

where MP_k is the value of the contribution of plate MP_k and MP_k*i* is the value of the contribution of the inside part of the plate MP_k .

The normalized error between the LHS and RHS of Eq. (52) is ϵ . The contribution of each plate to the total damping is also indicated. For example, plate MP2 contributes to approximately 80% of the total damping, while its actual damping A_{en} is of 5.4%. The damping of the first longitudinal mode (1L) mode is of 6.4% by period T . The energy balance is well closed with a normalized error of 0.36%. Table 4 shows that plate MP2 is the most efficient at damping. It is necessary to look at the pressure distribution for the 1L mode to understand this behavior. The distribution of $|\hat{p}|$ is given in Fig. 16. The pressure node is roughly located above MP2 (see the phase shift in Fig. 16), inducing the pressure drop to be maximum across plate MP2, thus inducing a significant damping from this very plate [see Eq. (50)].

Since it is responsible for almost the totality of the damping, another case is run without plate MP2 to see if it modifies the contribution of the other plates. A null acoustic speed condition is applied in this case on plate MP2. The frequency of the mode is hardly changed. The damping is now of 1%. This is almost equal to the previous damping minus the contribution of plate MP2. Since the imaginary part of the mode is now small [$\text{Im}(f) = -0.46 \text{ Hz}$], the mode can be said to be marginally stable. The contribution of the other plates is hardly changed: this can be explained by the fact that, at the actual regime, the presence of the MP does not have a strong impact on the mode structure. Figure 17 shows that replacing the plate MP2 by a wall has hardly changed the pressure field of the mode. The pressure node remains above plate MP2, preventing the other plates from improving their damping effect. The damping cannot be assured by the other plates, and the mode becomes marginally stable.

C. Energy Analysis on the Second Longitudinal Mode

Since the damping of the second longitudinal mode (2L) is more significant than the 1L mode, an energy budget is now conducted on the second longitudinal mode. It proves to be damped by about 47%. The results are gathered in Table 5. Clearly, plate MP2 is again the

most efficient at damping. The distribution of $|\hat{p}|$ and $\text{Arg}(\hat{p})$ is given in Fig. 18. The pressure node is located above MP2, inducing the pressure drop to be maximum across plate MP2, thus inducing a significant damping from this very plate [see Eq. (50)].

Since it is responsible for almost half of the damping, another case is run without plate MP2 to see if it modifies the contribution of the other plates. A null acoustic speed condition is applied in this case on plate MP2. The frequency of the mode is hardly changed. Figure 19 shows the distribution of the pressure.

As found for the first longitudinal mode, the damping drops (to 27.6% in this case), which is close to the total damping minus the contribution of plate MP2. This can be explained by the fact that, at the actual regime, the presence of the MP does not have a strong impact. The distribution of the phase changes. Once again, the contribution of the other plates is hardly changed.

A study of the influence of the bias flow speed in the holes was also conducted in [40]. A bias flow speed U_{md} inducing a maximum damping was highlighted. The energy budget will now be conducted considering this optimized speed in the next section.

D. Energy Analysis on Longitudinal Modes with Maximum Damping Speed

As shown in [40], a bias flow speed for which the effect of the MP is significantly larger can be defined. This speed is referred to as U_{md} . The same energy study is run, changing the actual speed to U_{md} . For

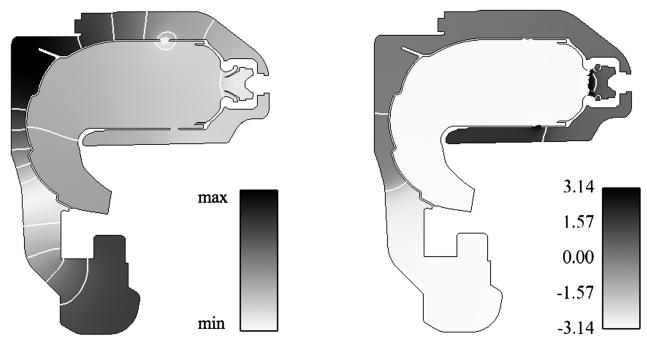


Fig. 18 $|\hat{p}|$ (left) and $\text{Arg}(\hat{p})$ (right) for 2L, all MP.

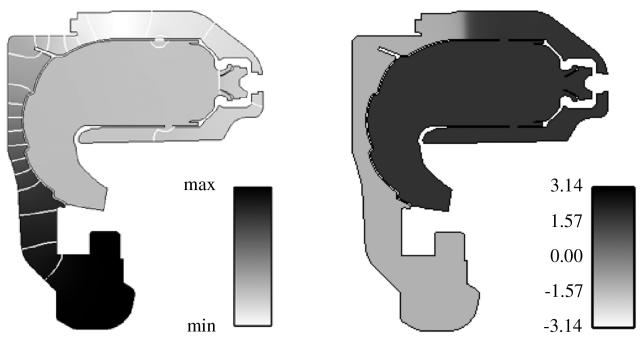


Fig. 17 Field of $|\hat{p}|$ (left) and $\text{Arg}(\hat{p})$ (right) for the 1L mode without plate MP2.

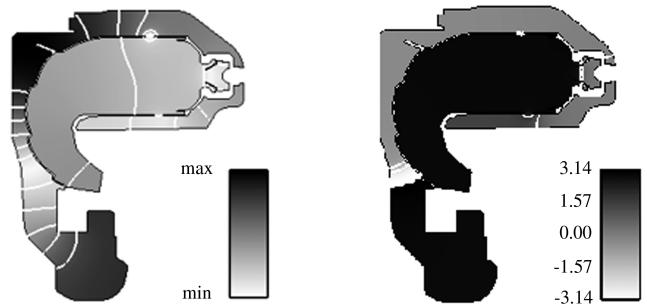


Fig. 19 $|\hat{p}|$ (left) and $\text{Arg}(\hat{p})$ (right) for 2L, no MP2.

Table 6 Acoustic energy analysis on the second longitudinal mode (2L), $U = U_{\text{md}}$

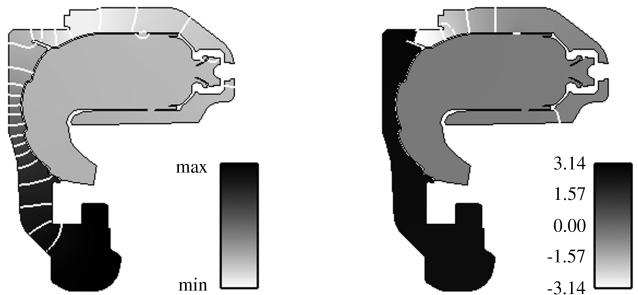
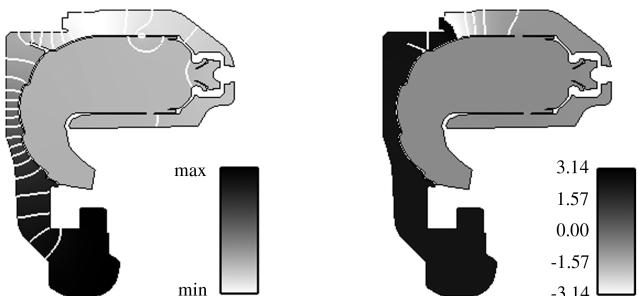
1L	$e^{2\omega_i T} - 1$	ϵ	MP1	MP1 <i>i</i>	MP2	MP2 <i>i</i>	MP3	MP3 <i>i</i>
$U = U_{\text{md}}$, all MPs	-33.4	1.54	3.3	$2.10 \cdot 10^{-2}$	28.4	0.19	2.12	0.679
				9.51	0.06	81.8	0.55	6.11 1.96
$U = U_{\text{md}}$, no MP2	-8.3	1.50	5.08	$2.6 \cdot 10^{-2}$	—	$8.6 \cdot 10^{-2}$	3.6	1
				51.9	0.266	—	0.87	36.7 10.2

Table 7 Acoustic energy analysis on the second longitudinal mode (2L), $U = U_{\text{md}}$

2L	$\tilde{E}(t = T/\tilde{E}(t = 0 - 1))$	RHS	Relative error	MP1	MP1 <i>i</i>	MP2	MP2 <i>i</i>	MP3	MP3 <i>i</i>
$U = U_{\text{md}}$, all MP	-73.1	-74.4	2	8.62 (11.6)	0.17 (0.23)	46.8 (62.9)	1.2 (1.6)	11.6 (15.6)	5.96 (8)
$U = U_{\text{md}}$, no MP2	-44.81	-46.07	2.83	15.14 (33.8)	0.33 (0.73)	—	1.1 (2.4)	19.64 (43.8)	9.84 (21.9)

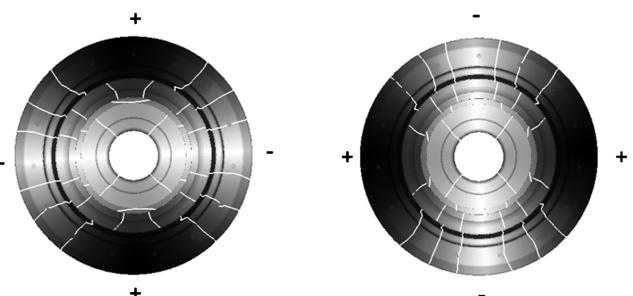
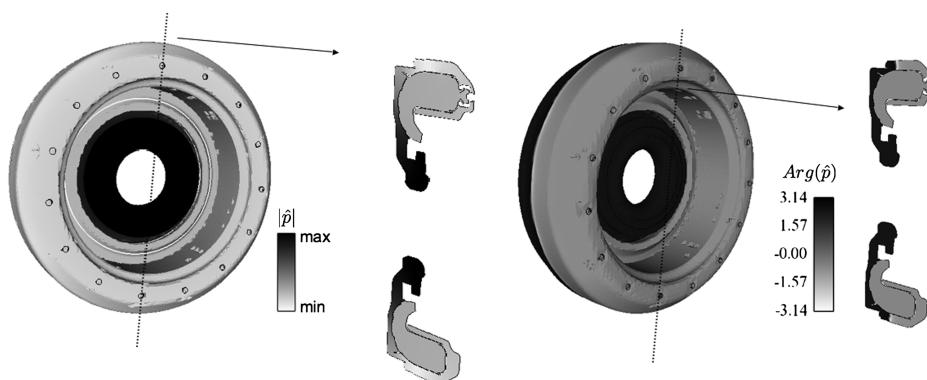
Table 8 Frequencies obtained by a 360 deg run (1L, 1A, and 2A)

Mode	1L		1A		2A	
	$\text{Re}(f)$	$\text{Im}(f)$	$\text{Re}(f)$	$\text{Im}(f)$	$\text{Re}(f)$	$\text{Im}(f)$
	509.9 Hz	-2.54 Hz	615.6 Hz	-0.92 Hz	985 Hz	-8.38 Hz
A_{en} , %	6.06		1.86		10.1	

Fig. 20 Field of $|\hat{p}|$ (left) and $\text{Arg}(\hat{p})$ (right) for the 1L mode, $U = U_{\text{md}}$.Fig. 21 Field of $|\hat{p}|$ (left) and $\text{Arg}(\hat{p})$ (right), $U = U_{\text{md}}$, without MP2.

these computations, a uniform speed equal to U_{md} has been applied on all MPs. Table 6 shows that withdrawing plate MP2 here has a strong impact on the contributions of other plates. Plate MP2 is much more efficient. Its damping contribution is up to 82% of the total damping, which has itself increased. Plates MP1, MP3, and MP3*i* are much more efficient at damping, inducing the whole damping not to be just cut down by the withdrawing of plate MP2. Thus, contrary to the previous case with $U \neq U_{\text{md}}$, the presence of MP2 modifies the mode structure so that the damping generated by the other MPs is changed. In other words, the overall damping is not the sum of all the damping obtained by considering each MP separately.

Figure 20 shows the mode structure at $U = U_{\text{md}}$ when all MPs are active, and Fig. 21 shows the case where MP2 is replaced by an impermeable wall. The pressure jump is maximum across MP2 in

Fig. 23 Field of $|\hat{p}|$ for the 1A mode.Fig. 22 Field of $|\hat{p}|$ and $\text{Arg}(\hat{p})$ (1L mode), 360 deg mesh.

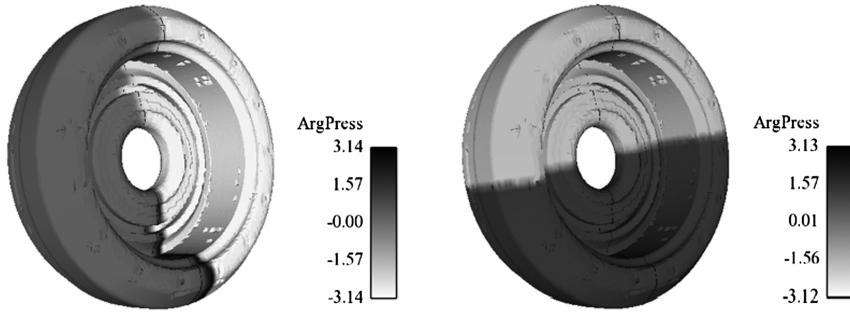
Fig. 24 Field of $\text{Arg}(\hat{p})$ for the 1A mode.Fig. 25 Field of $|\hat{p}|$ for the 2A mode. View of both sides of the chamber.

Fig. 20, but Fig. 21 shows that the mode structure has changed. The contribution of the plates are therefore all changed. Table 7 shows the same trend of results on the 2L mode. Withdrawing plate MP2 here also has a strong impact on the contributions of other plates. Plates MP1, MP3, and MP3*i* are much more efficient at damping, inducing the whole damping to not be just cut down by the withdrawing of plate MP2.

In complex configurations, the damping contributions of all MPs cannot be simply added. Since the damping effect of a MP depends on the mode structure, its contribution can be significantly changed by adding or removing another acoustic device, even if its parameters are kept the same. This has strong consequences in terms of acoustic optimization of gas turbines.

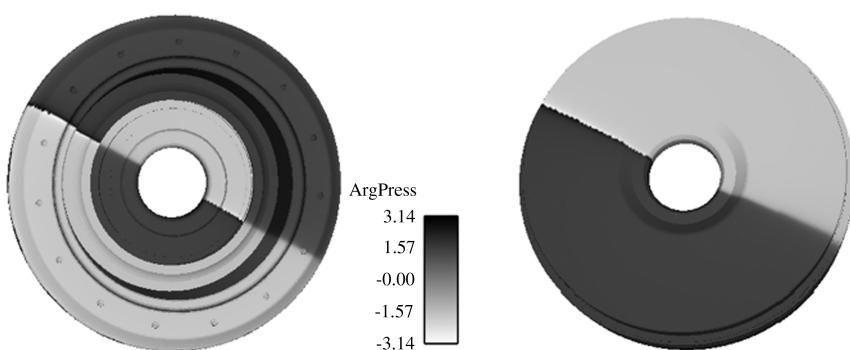
VI. Computation of the Whole Chamber: Azimuthal Modes

The frequency values of the modes computed with the 360 deg mesh are given in Table 8. The first mode of the full geometry corresponds to the first longitudinal already discussed in Sec. V.A. The value of the frequency is close to the one obtained with a one-sector computation, and Fig. 22 shows that the distributions of $|\hat{p}|$ and $\text{Arg}(\hat{p})$ do not depend of the azimuthal coordinate.

The next two modes correspond to 1A and 2A, respectively. Each of these modes are double modes in the sense that two different pressure fluctuations correspond to the same frequency of oscillation and damping rate. The modes are called orthogonal [36,44,45]. Figure 23 shows the pressure field of the two modes associated to the frequency of the 1A mode of Table 8, while Fig. 24 shows the argument field of these two modes, clearly showing their orthogonality. These two pressure fluctuations are close to marginal, indicating that none of the MP is efficient at damping them. On the contrary, the 2A mode is clearly damped by the MPs. The pressure and argument fields of the 2A mode are shown in Figs. 25 and 26, respectively. Due to the complexity of the 2A mode, two perspectives of the two sides of the annular chamber are shown for one mode. The other 2A mode corresponding to the same frequency has a shape simply rotated by 90 deg.

VII. Conclusions

A model for the acoustic behavior of perforated plates was implemented within a general-purpose Helmholtz solver. This tool allows multiperforated plates (MPs) to be taken into account when performing the acoustic analysis of industrial chambers. Besides, the budget of acoustic energy was analyzed for some of the modes computed. This allows knowing which perforated plates are the most efficient for damping any acoustic mode and gives some clues why. For the particular industrial case considered, the plate located near the inlet is found to be the most efficient. It is indeed located where the pressure drop is the most important. In cases where the perforated plates do not have too strong an influence, the damping is linear and the total damping is just and only the sum of all the MP contributions. When the plates have a stronger influence and impact on the mode structure, it is illustrated that the effect of a MP area can only be computed by accounting for all the other MPs or significant acoustic devices. This justifies the efforts in developing general-purpose Helmholtz solvers for the acoustic design of gas turbines of the future.

Fig. 26 Field of $\text{Arg}(\hat{p})$ (mode 2A).

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