

# 1 Derivation of $q/\bar{Q}$

## 1.1 Nomenclature

$\bar{Q}$ : total power output [J/s]

$T$ : period [s]

$s_L$ : laminar flame speed [m/s]

$t$ : time [s]

$a(t)$ : surface area of flame at time  $t$  [m<sup>2</sup>]

$dt$ : infinitesimal change in time [s]

$V$ : volume of premixed gas per unit depth consumed in a period (time  $T$ ) [m<sup>3</sup>]

$E$ : energy output per unit volume of premixed gas [J/m<sup>3</sup>]

## 1.2 Derivation

Energy output over a period =  $\bar{Q}T$ .

Volume of premixed gases consumed in time  $dt$ :  $dV = s_L a(t)dt$

Volume of premixed gases consumed over a period  $V = \int_0^T s_L a(t)dt$

Energy per unit volume of unburnt gas  $E = \bar{Q}T/V$

Energy released in time  $dt$ :  $q = s_L a(t)dtE/T$

$$\frac{\text{Energy released in time } dt}{\bar{Q}} = \frac{q}{\bar{Q}} = \frac{s_L a(t)dtE}{\bar{Q}T} \quad (1)$$

$$\frac{q}{\bar{Q}} = \frac{s_L a(t)dt}{V} \quad (2)$$

$$\frac{q}{\bar{Q}} = \frac{s_L a(t)dt}{\int_0^T s_L a(t)dt} \quad (3)$$

$$\frac{q}{\bar{Q}} = \frac{a(t)dt}{\int_0^T a(t)dt} \quad (4)$$

# 2 Calculation of $\bar{Q}$

## 2.1 Data available in Rankin papers

- Air flow rate  $\dot{m}_{air}$  of 0.35 kg/s
- Fuel-air equivalence ratio  $\phi$  of 1.0 (unstable case)
- Fuel: propane (C<sub>3</sub>H<sub>8</sub>, with heat of combustion 50430 kJ/kg<sup>1</sup>)

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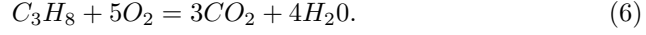
<sup>1</sup>[https://www.engineeringtoolbox.com/propane-d\\_1423.html](https://www.engineeringtoolbox.com/propane-d_1423.html)

## 2.2 Calculation

Air by mass is 23.1% oxygen so  $m_{O_2} = 0.231m_{air}$ . For our combustion data, the fuel-air equivalence ratio  $\phi$  is 1.0, which is defined as:

$$\phi = \frac{\text{fuel-to-oxidiser ratio}}{(\text{fuel-to-oxidiser ratio})_{stoich}} = \frac{m_{fuel}/m_{ox}}{(m_{fuel}/m_{ox})_{stoich}} = \frac{n_{fuel}/n_{ox}}{(n_{fuel}/n_{ox})_{stoich}} \quad (5)$$

where  $m$ , and  $n$ , are the molecular mass and the moles in the reaction equation:



The stoichiometric molar fuel to oxidiser ratio,  $FOR_{stoich} = 0.2$  (as combustion of propane requires 5 O<sub>2</sub> for every C<sub>3</sub>H<sub>8</sub>). The (non-stoichiometric) molar fuel to oxidiser ratio  $FOR$  is therefore  $FOR = \phi FOR_{stoich} = 1.0 * 0.2 = 0.2$ . Now, as

$$\frac{m_{fuel}}{m_{ox}} = \frac{n_{fuel}}{n_{ox}} \times \frac{\text{mass of } C_3H_8}{\text{mass of } O_2} = FOR \times \frac{\text{mass of } C_3H_8}{\text{mass of } O_2} \quad (7)$$

then  $m_{fuel}/m_{ox} = 0.2 * 44.1/32 = 0.276$ . Finally:

$$m_{fuel} = 0.276m_{ox} = 0.276 * 0.231 * m_{air}. \quad (8)$$

This means that, for our combustion data which has an air mass flow rate of 0.35 kg/s the mass flow rate of fuel is  $0.276 * 0.231 * 0.35 = 0.0223$  kg/s. For a energy density of 50430 kJ/kg, this suggests a power output  $\bar{Q} = 50430 * 0.0223 = 1125$  kW = 1.125 MW.