1 Derivation of $q/\bar{Q}$

1.1 Nomenclature

$\bar{Q}$: total power output [J/s]
T : period [s]
s_L : laminar flame speed [m/s]
t : time [s]
$a(t)$: surface area of flame at time t [m^2]
dt : infinitesimal change in time [s]
V : volume of premixed gas per unit depth consumed in a period (time T) [m^3]
E : energy output per unit volume of premixed gas [J/m^3]

1.2 Derivation

Energy output over a period $= \bar{Q}T$.
Volume of premixed gases consumed in time dt : $dV = s_L a(t)dt$
Volume of premixed gases consumed over a period $V = \int_0^T s_L a(t)dt$
Energy per unit volume of unburnt gas $E = \bar{Q}T/V$
Energy released in time dt : $q = s_L a(t)dtE/T$

$$\frac{\text{Energy released in time } dt}{\bar{Q}} = \frac{q}{\bar{Q}} = \frac{s_L a(t)dtE}{\bar{Q}T} \tag{1}$$

$$\frac{q}{\bar{Q}} = \frac{s_L a(t)dt}{V} \tag{2}$$

$$\frac{q}{\bar{Q}} = \frac{s_L a(t)dt}{\int_0^T s_L a(t)dt} \tag{3}$$

$$\frac{q}{\bar{Q}} = \frac{a(t)dt}{\int_0^T a(t)dt} \tag{4}$$

2 Calculation of $\bar{Q}$

2.1 Data available in Rankin papers

- Air flow rate $\dot{m}_{air}$ of 0.35 kg/s
- Fuel-air equivalence ratio ϕ of 1.0 (unstable case)
- Fuel: propane (C3H8, with heat of combustion 50430 kJ/kg[1])

[1] https://www.engineeringtoolbox.com/propane-d_1423.html

2.2 Calculation

Air by mass is 23.1% oxygen so $m_{O2} = 0.231 m_{air}$. For our combustion data, the fuel-air equivalence ratio ϕ is 1.0, which is defined as:

$$\phi = \frac{\text{fuel-to-oxidiser ratio}}{(\text{fuel-to-oxidiser ratio})_{stoich}} = \frac{m_{fuel}/m_{ox}}{(m_{fuel}/m_{ox})_{stoich}} = \frac{n_{fuel}/n_{ox}}{(n_{fuel}/n_{ox})_{stoich}} \tag{5}$$

where $m_{.}$ and $n_{.}$ are the molecular mass and the moles in the reaction equation:

$$C_3H_8 + 5O_2 = 3CO_2 + 4H_20. \tag{6}$$

The stoichiometric molar fuel to oxidiser ratio, $FOR_{stoich} = 0.2$ (as combustion of propane requires 5 O2 for every C3H8). The (non-stoichiometric) molar fuel to oxidiser ratio FOR is therefore $FOR = \phi FOR_{stoich} = 1.0 * 0.2 = 0.2$. Now, as

$$\frac{m_{fuel}}{m_{ox}} = \frac{n_{fuel}}{n_{ox}} \times \frac{\text{mass of } C3H8}{\text{mass of } O2} = FOR \times \frac{\text{mass of } C3H8}{\text{mass of } O2} \tag{7}$$

then $m_{fuel}/m_{ox} = 0.2 * 44.1/32 = 0.276$. Finally:

$$m_{fuel} = 0.276 m_{ox} = 0.276 * 0.231 * m_{air}. \tag{8}$$

This means that, for our combustion data which has an air mass flow rate of 0.35 kg/s the mass flow rate of fuel is $0.276 * 0.231 * 0.35 = 0.0223$ kg/s. For a energy density of 50430 kJ/kg, this suggests a power output $\bar{Q} = 50430 * 0.0223 = 1125$ kW = 1.125 MW.